Course: CSC707, Automata, Computability and Computational Theory
Group Problem: Clique $\propto$ Independent Set.
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Input: A graph G =(V,E), an integer k $\leq |V|$
Output: If there is a subset S of k vertices in G such that no pair of vertices in S is connected by an edge in G?
Decision Problem:
IS = $\{< G, k >$: Graph G has a Independent set of size k $\}$
Step 1:
We need to show that Independent Set (IS) $\in$ NP. Suppose we have been given a set of vertices V'. We can verify that V' is IS of Graph G as follows:

1. Verify $|V'|$ = k. This is O(1).
2. For each edge (u,v) $\in$ E(G), we check whether u $\in V'$ and v$\in V'$. If none of the edge satisfies this relation, then V' is IS of G, otherwise it is not. This is $O(k^2 |E|)$.

So the verification step is polynomial. Hence IS is in NP.
Step2:
Given the Clique problem is NP-Complete, we show that Clique $\propto$ IS. Given an instance of Clique problem $< G, k >$, following steps reduces it to IS problem.

1. We compute the complement $\overline{G}$ of given Graph. This take $O(n^2)$.
2. Calculate $k' = k$.

The output of reduction algorithm is an instance $< \overline{G}, k' >$ of IS problem.
Step3:
Graph G has Clique $V' \subset$ V iff $\overline{G}$ has IS V'.
$\rightarrow$
Suppose G has Clique $V' \subset$ V. If (u,v) be an edge in G and u $\in V'$ and v $\in V'$, then by definition of Clique and Complement, (u,v) $\notin \overline{E}$. Since u,v were chosen arbitrarily, the vertices that forms a clique in G forms IS in $\overline{G}$.
$\leftarrow$
Suppose G has IS $V' \subset$ V. Take any two verices u,v in G such that u $\in V'$ and v $\in V'$, then (u,v) $\notin E$ by defition of Independent Set. This implies that (u,v) $\in \overline{E}$. Since u,v were chosen arbitrarily, the vertices that forms a IS in G forms a Clique of same size in $\overline{G}$.
This proves that IS is NP-Complete problem.