Course: CSC707, Automata, Computability and Computational Theory
Homework 3: Self-reduction and Approximability.
Submission: Use Wolfware
File Format: LaTeX and PDF

Due Date: 2:00 A.M. (EST), Thursday, February 18, 2010

1. Provide any feedback/questions you may have on this homework (**optional**).
2. Using LaTeX is required.

My points: 8.25 out of 10

1. **Approximability**:

   (a) If $P \neq NP$, then for any constant $c \geq 1$, there is no polynomial-time c-approximation algorithm for a general Traveling Salesman Problem. (Hint: Show that HAM-CYCLE can be solved in polynomial time. Also, see the additional hint in the lecture slides.)

Theorem 1 *If $P \neq NP$, then for any constant $c \geq 1$, there is no polynomial-time c-approximation algorithm for a general Traveling Salesman Problem.*

-0.5 Pt: I must clearly state the theorem I want to prove.

~~**Solution:**~~ **Proof.**

(a) i. We first prove that HAM-CYCLE problem can be solved by using the c-approximation algorithm for a general Traveling Salesman Problem. Given a undirected and unweighted graph G, we can create a complete graph G' of G by adding missing edges and assign weight to each edge. For the edges in G, we assign 1 for the weight. For the edges not in G, we assign $c * |V| + 1$ for the weight. This reduction can be done in polynomial time ($O(|V| * |E|)$). (-1 Pt: Since $|E| = O(|V|^2)$, the reduction is $O(n^3)$) Using the c-approximation algorithm for TSP, we can get a TSP tour of weight W. If $W > c * |V|$, then we know this tour pick at least one edge that is not in G (since every edge in G is assigned weight of 1) and G does not have HAM-CYCLE. Otherwise, G has a HAM-CYCLE.

ii. Since the reduction from HAM-CYCLE to TSP is in polynomial time, if there exists a polynomial-time c-approximation algorithm for a general Traveling Salesman Problem, HAM-CYCLE can be solved in polynomial time. We already know that ~~HAM-CYCLE $\in NP$~~ -0.25 Pt: HAM-CYCLE is $NP-complete$ problem. If it can be solved in polynomial time, then $P = NP$, which contradicts with ~~out~~ our assumption that $P \neq NP$. Thus, there is no polynomial-time c-approximation algorithm for a general Traveling Salesman Problem.

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