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Comment:Grading is for 10 points: step 1(2 points)/ Step 2(4 points)/ Step 3(4 points)
Total Points Subtracted :6
Total Marks awarded : 4
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Prove LPATH is NP-Complete

Definition of Longest Path (LPATH):
LPATH = $\{<G, a, b, k> | G$ contains a simple path of length at least k from a to $b\}$

Step 1 (Verify): Given a LPATH of length k between points a and b on a graph G, verify that a simple path of length at least k exists from a to b in polynomial time.

-Check that the LPATH is simple
-1: Does not specify what n stands for and the use of n is ambiguous

for $i = 1 \rightarrow n$
{
  for $j = (i + 1) \rightarrow n$
  {
  make sure $vertex_i \neq vertex_j$
  }
}
This takes $O(n^2)$ time.
n stands for number of vertices in G?

-Check that all edges in path actually exist

for $i = 1 \rightarrow$ |edges in LPATH|
{
  for $j = (i + 1) \rightarrow$ |edges in G|
  {
  make sure $edge_i = edge_j$ for some $edge_j \in G$
  }
}
This takes $O(n^2)$ time.
n stands for number of edges in G?

-Count number of edges in LPATH

n stands for number of edges in LPATH?
This takes $O(n)$ time.

Thus, the entire verify step takes $O(n^2)$ time which is polynomial.
And hence LPATH $\in NP$

Step 2 (Reduction): Show that LPATH is NP-Hard. We do this by showing reduction from known NP-Complete problem to the LPATH problem.
P_{known} = UHAMPATH
P_{new} = LPATH

$UHAMPATH = \{< G, s, t > | G$ is undirected graph that contains a Hamiltonian path from s to $t\}$

Reduction:
-1:Pseudocode for reduction is not clear. Should include more details
UHAMPATH($G = (V, E), s, t$)
k = |V| - 1
Return the answer to LPATH(G, s, t, k)

The reduction step occurs in polynomial time.
-2:It is not clear how the reduction is polynomial - mention the complexity for the algorithm
Step 3 (Correctness): Graph G(V,E) has a Longest Path of size at least |V| - 1 between a and b iff G has a Hamiltonian Path between a and b.

-2: Convert any instance from P_{known} to an instance of P_{new} with the same answer i.e.yes-instance $\rightarrow$ yes-instance and no-instance $\rightarrow$ no-instance
Claim assumes yes-instance of P_{new}
-**Case 1** Assume G has Longest Path of size |V| - 1 between a and b. Show G has a Hamiltonian Path between a and b.

There exists a simple path from a to b with length |V| - 1(from given Hamiltonian Path). Therefore, there exists Comment: Not clear, it is in the assumption that we have a LPATH of size |V|-1 or the claim statement is wrong a Longest Path of at least length |V| - 1 from a to b. This Longest Path is the Hamiltonian Path.

-**Case 2** Assume G does not have a Hamiltonian Path between a and b. Show G does not have a Longest Path between a and b of size at least |V| - 1.

A longest path with length at least |V| - 1 from a to b would be a simple path which visits every vertex v, such that $v \in V$. A simple path that starts at a and ends at b, of length |V| - 1, is a Hamiltonian Path. However, no hamiltonian path from a to b exists (given). Therefore, no longest path of length |V| - 1 exists.