

**Course:** CSC707, Automata, Computability and Computational Theory

**Bonus Homework:** Bonus points assignment

**Author:** Xusheng Xiao(xxiao2@ncsu.edu), Xi Ge(xge@ncsu.edu), Da Young Lee(dlee10@ncsu.edu)

**Due Date: 2:00 AM, Monday, Feb 2nd, 2010**

- 
1. **Clique:Description of real-world problem:**  
we study the people in a party to find out groups that every two persons in a group know each other.
  2. **Vertex Cover:**
    - (a) **Description of real-world problem:**  
We study the vertex cover problem on finite connectivity random graphs by zero-temperature cavity method. The minimum vertex cover corresponds to the ground state(s) of a proposed Ising spin model.[4]
    - (b) **Identification of the corresponding graph problem:**  
 $VC = \{ \langle G, k \rangle \mid \text{a graph } G \text{ has a vertex cover of size } k \}$
  3. **Independent Set: Description of real-world problem:**  
We study the people in the party to find out group that no two people in it know each other.
  4. **Dominating Set: Description of real-world problem:**  
We study the group of people in a party that every people in the party knows at least one people in the group.
  5. **Hamiltonian Path: Description of real-world problem:**  
To tour the 10 biggest cities in the UK, starting in London (number 1) and finishing in Bristol (number 10) is an example of finding a Hamilton Path. [5]
  6. **Identification of the corresponding graph problem:**  
A Hamiltonian path (or traceable path) is a path in an undirected graph which visits each vertex exactly once.
  7. **Description of the reduction from real to graph problem:**
  8. **Hamiltonian Cycle: Description of real-world problem:**  
The 'Knight's Tour' [2] is a sequence of moves done by a knight on a chessboard. The knight is placed on an empty chessboard and, following the rules of chess, must visit each square exactly once. The Knight's Tour problem is an instance of the more general Hamiltonian path problem in graph theory. This problem of getting a closed Knight's Tour is similarly an instance of the Hamiltonian cycle problem.
  9. **Identification of the corresponding graph problem:**  
A Hamiltonian cycle (or Hamiltonian circuit) is a cycle in an undirected graph which visits each vertex exactly once and also returns to the starting vertex.
  10. **Description of the reduction from real to graph problem:**
  11. **Shortest Path:**
    - (a) **Description of real-world problem:**  
Shortest Path is used to identify where shortest paths need to be calculated, for example communications, transportation, and electronics problems or the 3D space [1].
    - (b) **Identification of the corresponding graph problem:**  
The shortest path problem is the problem of finding a path between two nodes such that the sum of the weights of its constituent edges is minimized

## 12. Longest Path:

(a) **Description of real-world problem:**

Longest path is used to identify critical path in Critical Path Method [3], a project management method. It is to address the challenge of shutting down chemical plants for maintenance and then restarting the plants once the maintenance had been completed.

(b) **Identification of the corresponding graph problem:**

The longest path problem is the problem of finding a simple path of maximum length in a given graph.

(c) **Description of the reduction from real to graph problem:**

The steps in CPM project planning includes:

- i. Specify the individual activities.
- ii. Determine the sequence of those activities.
- iii. Draw a network diagram.
- iv. Estimate the completion time for each activity.
- v. Identify the critical path (**longest path through the network**)
- vi. Update the CPM diagram as the project progresses.

The critical path is the longest-duration path through the network. The significance of the critical path is that the activities lie on it cannot be delayed without delaying the project.

## References

- [1] An approximate algorithm for solving shortest path problems for mobile robots or driver assistance. <http://www.mi.auckland.ac.nz/tech-reports/MItech-TR-37.pdf>.
- [2] New knight's tour puzzles and graphs. [http://www.archimedes-lab.org/knight\\_tour.html](http://www.archimedes-lab.org/knight_tour.html).
- [3] Cpm - critical path method, 2002.
- [4] Vertex cover problem studied by cavity method: Analytics and population dynamics, 2003. <http://www.springerlink.com/content/tlr4nqx01ae6m33f/>.
- [5] Solving a hamiltonian path problem with a bacterial computer, 2009. <http://www.jbioleng.org/content/3/1/11>.