Course: CSC707, Automata, Computability and Computational Theory
Homework 4: Finite automata (FA), DFA, NFA, regular expressions, Pumping lemma, and closure properties
Submission: Use Wolfware
File Format: LaTeX and PDF
NOTE: If you create images, make sure you submit them as well.

Due Date: 11:00 AM, Saturday, March 13, 2010

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1. Given the set of all of strings in $(0+1)^*$ such that some two zeros are separated by a string whose length is $4i$, for some some $i \geq 0$,

   (a) Give a nondeterministic finite automata accepting this set.

   (b) Provide a regular expression for L.

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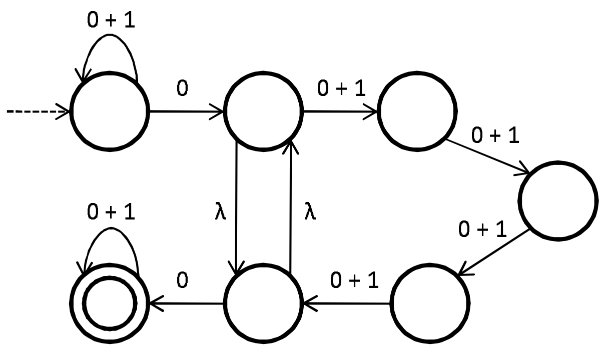


   (a)

   (b) The regular expression for L is:

$$(0+1)^*0((0+1)(0+1)(0+1)(0+1))^*0(0+1)^*$$

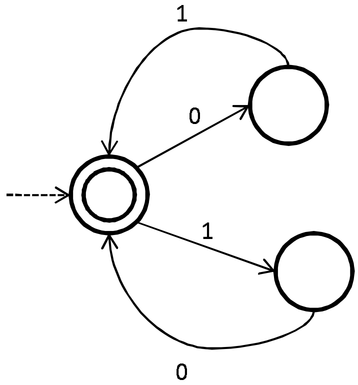
2. Given a language L of all strings over $\{0,1\}$ with an equal number of zeros and ones such that no prefix has two more zeros than ones nor has two more ones than zeros:

   (a) Construct a DFA that accepts all strings from L.

   (b) Provide a regular expression for L.

   Answer:

   (a)



(b) The regular expression for L is: $(01 + 10)^*$

3. Which of the following languages are regular? Prove your answers.

   (a) $L = \{0^j | j \bmod 3 \equiv 0\}$

   (b) $L = \{0^j 1^k | \gcd(j, k) \equiv 1\}$, where *gcd*() is the greatest common denominator.

   (c) $L = \{0^i 1^j 0^k | k \geq i + j\}$

   (a) **Theorem 1** $L = \{0^j | j \bmod 3 \equiv 0\}$ *is regular language.*

   Proof. Language L accepts strings formed by 0s whose number is the multiple of 3. We can use the regular expression $(000)^*$ to accept L. ■

   (b) **Theorem 2** $L = \{0^j 1^k | \gcd(j, k) \equiv 1\}$ *is not regular language.*

   Proof. We can use pumping lemma to prove it. Assume that L is regular. Let p be the pumping length and x is the product of the primes $\leq p + 1$. For two integer j, k, if $gcd(j, k) = 1$, they are relatively prime. Since any two consecutive integers are relatively prime, we can pick $s = 0^{x+1} 1^x$. From the pumping lemma, we can write $s = xyz$ such that $|xy| \leq p$. Thus, y must be 0^m, where $1 \leq m \leq p$. Now we consider $s' = xy^i z$ and $i = 2$. From $y = 0^m$, we can have $s' = 0^{x+1+m} 1^x$. Since $1 \leq m \leq p \Rightarrow 2 \leq m + 1 \leq p + 1$, all the prime factors of $m + 1$ are $\leq p + 1$ (if $m + 1$ is prime, then the prime factor of $m + 1$ is itself). Since x is the product of the primes $\leq p + 1$, the prime factors in $m + 1$ must be in x. Thus, there exists a common factor f between x and $m + 1$, which shows that $x + 1 + m$ has a common factor with x. Hence, $s' \notin L$. By pumping lemma, L is not regular language.

   ■

   (c) **Theorem 3** $L = \{0^i 1^j 0^k | k \geq i + j\}$ *is not regular language.*

   Proof. We can use pumping lemma to prove it. Assume that L is regular. Let p be the pumping length. We can pick $s = o^p 1^p 0^{2p}$, which is in L. From the pumping lemma, we can write $s = xyz$ such that $|xy| \leq p$. Thus, y must be 0^m, where $1 \leq m \leq p$. Now we consider $s' = xy^a z$ and $a = 2$. From $y = 0^m$, we can have $s' = 0^{p+m}1^p0^{2p}$. In s', $i = p+m, j = p$, and $k = 2p$. Since $i+j = 2p+m$ and $1 \leq m \leq p$, $2p+m \geq 2p = k$. Thus, $s' \notin L$. By pumping lemma, L is not regular language. ■

4. Assuming $L_1, L_2, ...$ are regular, which of the following languages are regular. Prove your answers.

   (a) $\bigcup_{i=1}^{n} L_i$

   (b) $\bigcup_{i=1}^{\infty} L_i$

   (c) $\bigcap_{i=1}^{n} L_i$

   (d) $\bigcap_{i=1}^{\infty} L_i$

5. Prove that the following languages are regular:

   (a) $MIN(L) = \{x \in L |$ no prefix of x is in $L\}$

   (b) $L^R = \{x|$ reverse of x is in $L\}$