Course: CSC707, Automata, Computability and Computational Theory
Reduction Homework: NP-complete problems
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Points deducted : 8

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1. Provide any feedback/questions you may have on this homework (**optional**).
2. Using LaTeX is required.

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1. Given a NP-complete problem, Vertex Cover, show that the Independent Set is NP-complete.
   Independent Set is defined as follows:
   INSTANCE: A graph $G = (V, E)$ and a positive integer $k \leq |V|$.
   QUESTION: Is there a subset S of k vertices in G such that no pair of vertices in S is connected by an edge in G?

   Solution:

   (a) (Verification):Show that Independent Set is in NP.
   Decision Problem not stated (-1)
   Given an independent set $C \subseteq V, |C| = n$ for a graph $G = (V, E)$, we can verify it using the following pseudo-code:

   $\forall u \in C$,
   $\quad \forall v \in C$,
   $\quad\quad$ check whether $< u, v > \in E$

   Checking whether $< u, v > \in E$ can be done in ~~$T(|E|)$~~ $O(|E|)$ time
   Big - O notation missing (-1)
   So the verification algorithm takes ~~$O(n^2|E|)$~~ $O(n^2|E|)$ time
   It is not specified if the Big-O notation denotes time or space or any other complexity (-1)
   The verification if $|C|$ = k , where C is IS , is not done (-1)

   (b) (Reduction):Show that Independent Set is NP-hard.
   Given a G has a VC of size k, we should construct a graph G' has Independent Set of size k'.
   Construction Process: Given VertexCover(G,k) where V_1 is the vertex cover and $k = |V_1|$, we set $G' = G$ and $k' = |V| - k$, ~~then we could return the answer to IndependentSet(G',k') where $V - V_1$ is the independent set.~~

so the problem of a VC for $< G, k >$ reduces to one of IS for $< G', k' >$
Our goal here is just reduction from a known NPC problem and not to return any answer. (-1)

This takes ~~constant~~ $O(1)$ time.
Big - O notation missing (-1)

(c) (Correctness):Show that Independent Set is NP-hard.
We need to show that G has a vertex cover of size k if and only if it has an Independent Set of size $k' = |V| - k$.

Assume $G(V, E)$ has a vertex cover C of size k. Consider two vertices $u \in V - C$ and $v \in V - C$, we can know that $e =< u, v > \notin E$ since C is a vertex cover of G. Therefore, no two vertices in $V - C$ are connected by an edge. So $V - C$ is an independent set with size $k' = |V| - k$.

Assume G has an Independent Set S of size $k' = |V| - k$.
$\forall e \in E, e =< u, v >$, S is independent set$\Rightarrow$ $u \notin S$ or $v \notin S$ or $u, v \notin S$ $\Rightarrow u \in V - S$ or $v \in V - S$ or $u, v \in V - S$
The case where $u, v \notin S$ is not considered (-1)
$\Rightarrow V - S$ covers $e =< u, v >$.
$\Rightarrow V - S$ is a VC of size k.
Inference not stated (-1)