

**Course:** CSC707, Automata, Computability and Computational Theory

**Bonus Homework:** Bonus points assignment

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**1. Clique: Description of real-world problem:**

we study the people in a party to find out groups that every two persons in a group know each other.

**2. Vertex Cover:**

**(a) Description of real-world problem:**

We study the vertex cover problem on finite connectivity random graphs by zero-temperature cavity method. The minimum vertex cover corresponds to the ground state(s) of a proposed Ising spin model.[4]

**(b) Identification of the corresponding graph problem:**

$VC = \{ \{G, k\} \mid \text{a graph } G \text{ has a vertex cover of size } k \}$

**(c) Description of the reduction from real to graph problem:**

**3. Independent Set: Description of real-world problem:**

We study the people in the party to find out group that no two people in it know each other.

**4. Dominating Set: Description of real-world problem:**

We study the group of people in a party that every people in the party knows at least one people in the group.

**5. Hamiltonian Path:**

**(a) Description of real-world problem:**

To tour the 10 biggest cities in the UK, starting in London (number 1) and finishing in Bristol (number 10) is an example of finding a Hamilton Path. [5]

**(b) Identification of the corresponding graph problem:**

A Hamiltonian path (or traceable path) is a path in an undirected graph which visits each vertex exactly once.

**(c) Description of the reduction from real to graph problem:**

We can draw a graph like this: each city represents a vertex and each road connecting each two cities represents an edge. Find a tour to visit all the cities and finish at the last city is to find out a Hamiltonian Path that visit every vertex.

**6. Hamiltonian Cycle:**

**(a) Description of real-world problem:**

The 'Knight's Tour' [2] is a sequence of moves done by a knight on a chessboard. The knight is placed on an empty chessboard and, following the rules of chess, must visit each square exactly once. The Knight's Tour problem is an instance of the more general Hamiltonian path problem in graph theory. This problem of getting a closed Knight's Tour is similarly an instance of the Hamiltonian cycle problem.

**(b) Identification of the corresponding graph problem:**

A Hamiltonian cycle (or Hamiltonian circuit) is a cycle in an undirected graph which visits each vertex exactly once and also returns to the starting vertex.

**(c) Description of the reduction from real to graph problem:**

Give a chess board, we can draw a graph from the path of a Knight's Tour: each vertex of the graph represents a square of the board and each edge represents a knight's move. The way to visit each square and have the knight finishing on a square which is just a move away from the starting square create a tour that is described as 're-entrant' or 'closed'. Such a tour is a Hamiltonian Cycle.

## 7. Shortest Path:

(a) **Description of real-world problem:**

The shortest Path is used to find where shortest paths between two places, etc. That application field is communications, transportation, electronics and 2D or 3D space. Specially, we introduce this problem to find the the shortest path between two places is of significant for mobile robots, also to identify unique paths in a given surrounding region in 3D space [1].

(b) **Identification of the corresponding graph problem:**

To find the shortest path in a graph from one nodes(or vertex) to another. The "Shortest" meaning is the least number of edges, least total weight.

(c) **Description of the reduction from real to graph problem:**

In this problem, we are used

## 8. Longest Path:

(a) **Description of real-world problem:**

Longest path is used to identify critical path in Critical Path Method [3], a project management method. It is to address the challenge of shutting down chemical plants for maintainance and then restarting the plants once the maintainance had been completed.

(b) **Identification of the corresponding graph problem:**

The longest path problem is the problem of finding a simple path of maximum length in a given graph.

(c) **Description of the reduction from real to graph problem:**

The steps in CPM project planning includes:

- i. Specify the individual activities.
- ii. Determine the sequence of those activities.
- iii. Draw a network diagram.
- iv. Estimate the completion time for each activity.
- v. Identify the critical path (**longest path through the network**)
- vi. Update the CPM diagram as the project progresses.

The critical path is the longest-duration path through the network. The significance of the critical path is that the activities lie on it cannot be delayed without delaying the project.

## References

- [1] An approximate algorithm for solving shortest path problems for mobile robots or driver assistance. <http://www.mi.auckland.ac.nz/tech-reports/MItech-TR-37.pdf>.
- [2] New knight's tour puzzles and graphs. [http://www.archimedes-lab.org/knight\\_tour.html](http://www.archimedes-lab.org/knight_tour.html).
- [3] Cpm - critical path method, 2002.
- [4] Vertex cover problem studied by cavity method: Analytics and population dynamics, 2003. <http://www.springerlink.com/content/tlr4nqx01ae6m33f/>.
- [5] Solving a hamiltonian path problem with a bacterial computer, 2009. <http://www.jbioleng.org/content/3/1/11>.