Course: CSC707, Automata, Computability and Computational Theory
Reduction Homework: NP-complete problems
Submission: Use Wolfware
File Format: LaTeX and PDF
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Due Date: 2:00 A.M. (EST), Tuesday, February 9, 2010

1. Provide any feedback/questions you may have on this homework (**optional**).
2. Using LaTeX is required.

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1. Given a NP-complete problem, Vertex Cover, show that the Independent Set is NP-complete.
   Independent Set is defined as follows:
   INSTANCE: A graph $G = (V, E)$ and a positive integer $k \leq |V|$.
   QUESTION: Is there a subset S of k vertices in G such that no pair of vertices in S is connected by an edge in G?

   Solution:

   (a) (Verification):Show that Independent Set is in NP.
   The verification algorithm guesses $C \subseteq V$ and check that whether C is an independent set of G=(V,E). If the test succeeds then the algorithm accepts, otherwise it rejects. To check whether C is an independent set of G=(V,E) only needs one travesal on G which takes $O(V + E)$, so the verification algorithm takes $O(n \cdot (V + E))$.

   (b) (Reduction):Show that Independent Set is NP-hard.
   Given a G has a VC of size k, we should construct a graph G' has Independent Set of size k'.
   Construction Process: Given VertexCover(G,k) where V_1 is the vertex cover and $k = V_1$, we set G'=G and k'=$|V| - k$, then we could return the answer to IndependentSet(G',k') where $V - V_1$ is the independent set. This takes constant time.

   (c) (Correctness):Show that Independent Set is NP-hard.
   We need to show that G has a vertex cover of size k if and only if it has an Independent Set of size $k' = |V| - k$.

   Assume G has a vertex cover C of size k. Consider two vertices $u \in V - C$ and $v \in V - C$, we find that e=(u,v)$\notin E$ since C is vertex cover. Therefore, no two vertices in V-C are connected by an edge.

So V-C is an independent set with size k'=$|V| - k$.

Assume G has an Independent Set S of size k'=$|V| - k$.
Consider an arbitrary edge e=(u,v), S is independent set$\Rightarrow$u$\notin S$ or $v \notin S$ $\Rightarrow$u$\in V - S$ or $v \in V - S \Rightarrow V - Scoverse = (u, v)$.