Course: CSC707, Automata, Computability and Computational Theory
EXAM 1: Countability, closure properties of countable sets. Complexity theory, NP-completeness, polynomial-time reducibility, self-reduction, approximability.
Submission: Home-take
FULL NAME: ____________________________

Due Date: February 25, 11:59 PM Mid-Term Exam, Spring, 2010

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1. Provide a solution to **ONE** problem in **EACH** category; a total of **FOUR** categories.
2. Solutions to the other problems in each category will **NOT** be graded.
3. This is an open-textbook (main course book), open-notes, open-homeworks, BUT **closed-internet** exam. You may **NOT** discuss the exam with any one. The exam is an **INDIVIDUAL** effort.

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1. Prove or disprove the countability of each of the following sets (**25 points**: identifying which class (5 points), proof idea (5 points), complete proof (15 points)):
   (a) The set of all regular languages over $\{0, 1\}$.
   (b) The set of all languages over the alphabet $\{0, 1\}$
   (c) The set of all infinite length strings over the three letter alphabet $\{0, 1, 2\}$
2. NP**-completeness, 25 points**: Verification Step (5 points), Reduction Step (10 points), Correctness Step (10 points): Prove that the problem is NP-complete.
   (a) BIGGER-CLIQUE $= \{\langle G_1, G_2\rangle \mid$ the largest clique of graph G_1 is larger than every clique of graph $G_2\}$
   (b) NP-PATH $= < G, s, t, k >$ — G is an undirected graph containing a simple path of length at least k from s to t. (Hint: *Use the fact that the Hamiltonian Path problem for undirected graphs is NP-complete.*)
   (c) HALF-CLIQUE $= < G >$ — G is an undirected graph having a complete subgraph with $\lfloor n/2 \rfloor$ nodes, where n is the number of nodes in G.
3. **Self-reducibility, 25 points**:

(a) Find an isomorphism between graphs G_1 and G_2 and provide time complexity, or state that none exists. (An isomorphism is a bijection $\phi : V(G_1) \to V(G_2)$ such that $(v_1, v_2) \in E(G_1)$ iff $(\phi(v_1), \phi(v_2)) \in E(G_2)$.) Assume that you have a decision algorithm $D(G, G')$ that decides whether G and G' are isomorphic in $O(f(|V(G)| + |V(G')|))$ time.

(b) Given a set of integers A, find a subset of A that sums to zero and provide time complexity, or state that none exists. Assume that you have a decision algorithm $D(S)$ that decides whether such a subset exists in $O(f(|S|))$ time.

4. **Approximability, 25 points**:

   (a) If $P \neq NP$, then Vertex Cover does not allow any absolute approximation. (There is an absolute approximation algorithm A if $A(G) \leq OPT(G) + C$ for some constant integer C and any instance, G of the vertex cover.)

   (b) The VERTEX-COVER problem and the NP-complete CLIQUE problem are complementary in the sense that an optimal vertex cover is the complement of a maximum-size clique in the complement graph. Does this relationship imply that there is a polynomial-time approximation algorithm with a constant approximation ratio for the CLIQUE problem? Justify your answer.