Course: CSC707, Automata, Computability and Computational Theory
Reduction Homework: NP-complete problems
Submission: Use Wolfware
File Format: LaTeX and PDF
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Due Date: 2:00 A.M. (EST), Tuesday, February 9, 2010

1. Provide any feedback/questions you may have on this homework (**optional**).
2. Using LaTeX is required.

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1. Given a NP-complete problem, Vertex Cover, show that the Independent Set is NP-complete.
   Independent Set is defined as follows:
   INSTANCE: A graph $G = (V, E)$ and a positive integer $k \leq |V|$.
   QUESTION: Is there a subset S of k vertices in G such that no pair of vertices in S is connected by an edge in G?

   Solution:

   (a) (Verification):Show that Independent Set is in NP.
   Given an independent set $C \subseteq V, |C| = n$ for a graph $G = (V, E)$, we can verify it using the following pseudo-code:

   $\forall u \in C$,
   $\quad \forall v \in C$,
   $\quad\quad$ check whether $< u, v >\in E$

   Checking whether $< u, v >\in E$ can be done in $T(|E|)$. So the verification algorithm takes $O(n^2|E|)$.

   (b) (Reduction):Show that Independent Set is NP-hard.
   Given a G has a VC of size k, we should construct a graph G' has Independent Set of size k'.
   Construction Process: Given VertexCover(G,k) where V_1 is the vertex cover and $k = |V_1|$, we set $G' = G$ and $k' = |V| - k$, then we could return the answer to IndependentSet(G',k') where $V - V_1$ is the independent set. This takes constant time.

   (c) (Correctness):Show that Independent Set is NP-hard.
   We need to show that G has a vertex cover of size k if and only if it has an Independent Set of size $k' = |V| - k$.

Assume $G(V, E)$ has a vertex cover C of size k. Consider two vertices $u \in V - C$ and $v \in V - C$, we can know that $e =< u, v > \notin E$ since C is a vertex cover of G. Therefore, no two vertices in $V - C$ are connected by an edge. So $V - C$ is an independent set with size $k' = |V| - k$.

Assume G has an Independent Set S of size $k' = |V| - k$.
$\forall e \in E, e =< u, v >$, S is independent set$\Rightarrow$ $u \notin S$ or $v \notin S \Rightarrow u \in V - S$ or $v \in V - S \Rightarrow V - S$ covers $e =< u, v >$.