Course: CSC707, Automata, Computability and Computational Theory
EXAM 1: Countability, closure properties of countable sets. Complexity theory, NP-completeness, polynomial-time reducibility, self-reduction, approximability.
Submission: In class
Duration: 75 minutes
FULL NAME: ______________________

Due Date: Practice Exam, Spring, 2010

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1. Provide a solution to **ONE** problem in **EACH** category; a total of **FOUR** categories.
2. Solutions to the other problems in each category will be graded on the binary basis towards your extra points: 1 (one) point for a correct solution and 0 (zero) points for the incorrect solution.
3. Specify which problem in each category should be graded as the primary (**P**) and which other ones are for extra credit (**E**). If not specified, the first provided solution will be graded as the primary one.
4. This is an open-textbook but closed-notes, closed-homeworks, closed-internet quiz.

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1. Prove or disprove the countability of each of the following sets (**25 points**: identifying which class (5 points), proof idea (5 points), complete proof (15 points)):
   (a) P/E?________ The set of all lines in the Euclidean plane that pass through at least two points whose x and y coordinates are both integers.
   (b) P/E?________ The set of all languages over the alphabet $\{0, 1\}$
   (c) P/E?________ The set of irrational numbers between 3 and 4
2. NP**-completeness, 25 points**: Verification Step (5 points), Reduction Step (10 points), Correctness Step (10 points): Prove that the problem is NP-complete.
   (a) Longest Path (Reduction from Hamiltonian Path)
   (b) MAX-CUT (See hint for Problem 7.25 in Sipser's book)
3. **Self-reducibility, 25 points**: Let G denote a finite, simple, undirected graph. Let k denote an arbitrary, positive integer.

(a) Reduce the problem of finding an independent set of size k to the problem of deciding whether such an independent set exists.

(b) Reduce the problem of finding a clique of size k in polynomial time to the problem of deciding whether such a clique exists in polynomial time.

(c) Consider the Traveling Salesman Problem, in which we seek to find a cheap tour (Hamiltonian circuit) in an edge-weighted graph. Suppose you have available a deterministic *decision* algorithm, A. Given G and k, A decides whether G ha a tour of length k or less.

- Describe how you would use A to solve the *search* version of this problem (here you will need to find a tour of length k or less if any exist).
- Describe how you would use A to solve the *optimization* version of this problem (here you need to determine the length of a shortest possible tour).

4. **Approximability, 25 points**:

(a) If $P \neq NP$, then Vertex Cover does not allow any absolute approximation. (There is an absolute approximation algorithm A if $A(G) \leq OPT(G) + C$ for some constant integer C and any instance, G of the vertex cover.)

(b) Give an approximation algorithm for a Traveling Salesman Problem with the triangle inequality. Prove its approximation performance ratio.