Preface

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From the title page:

> The author has spared himself no pains in his endeavour to present the main ideas in the simplest and most intelligible form, and on the whole, in the sequence and connection in which they actually originated. In the interest of clearness, it appeared to me inevitable that I should repeat myself frequently, without paying the slightest attention to the elegance of the presentation. I adhered scrupulously to the precept of that brilliant theoretical physicist L. Boltzmann, according to whom matters of elegance ought be left to the tailor and to the cobbler.
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> Albert Einstein, in Relativity, the Special and General Theory, (1961), p. v

Continuing:

Learning physics is hard. Part of the problem is that physics is naturally expressed in mathematical language. When we teach we use the language of mathematics in the same way that we use our natural language. We depend upon a vast amount of shared knowledge and culture, and we only sketch an idea using mathematical idioms. We are insufficiently precise to convey an idea to a person who does not share our culture. Our problem is that since we share the culture we find it difficult to notice that what we say is too imprecise to be clearly understood by a student new to the subject. A student must simultaneously learn the mathematical language and the content that is expressed in that language. This is like trying to read Les Mis´erables while struggling with French grammar.

This book is an effort to ameliorate this problem for learning the differential geometry needed as a foundation for a deep understanding of general relativity or quantum field theory. Our approach differs from the traditional one in several ways. Our coverage is unusual. We do not prove the general Stokes's Theorem— this is well covered in many other books—instead, we show how it works in two dimensions. Because our target is relativity, we put lots of emphasis on the development of the covariant derivative, and we erect a common context for understanding both the Lie derivative and the covariant derivative. Most treatments of differential geometry aimed at relativity assume that there is a metric (or pseudometric). By contrast, we develop as much material as possible independent of the assumption of a metric. This allows us to see what results depend on the metric when we introduce it. We also try to avoid the use of traditional index notation for tensors. Although one can become very adept at "index gymnastics," that leads to much mindless (though useful) manipulation without much thought to meaning. Instead, we use a semantically richer language of vector fields and differential forms.

But the single biggest difference between our treatment and others is that we integrate computer programming into our explanations. By programming a computer to interpret our formulas we soon learn whether or not a formula is correct. If a formula is not clear, it will not be interpretable. If it is wrong, we will get a wrong answer. In either case we are led to improve our program and as a result improve our understanding. We have been teaching advanced classical mechanics at MIT for many years using this strategy. We use precise functional notation and we have students program in a functional language. The students enjoy this approach and we have learned a lot ourselves. It is the experience of writing software for expressing the mathematical content and the insights that we gain from doing it that we feel is revolutionary. We want others to have a similar experience.

1 Acknowledgements

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the practice of programming is a powerful way to develop a deeper understanding of any subject. Indeed, by the act of debugging we learn about our misconceptions, and by reflecting on our bugs and their resolutions we learn ways to learn more effectively. Indeed, Turtle Geometry [2], a beautiful book about discrete differential geometry at a more elementary level, was inspired by Papert's work on education. [13]

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Gerald Jay Sussman & Jack Wisdom
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