Introduction to fracture mechanics

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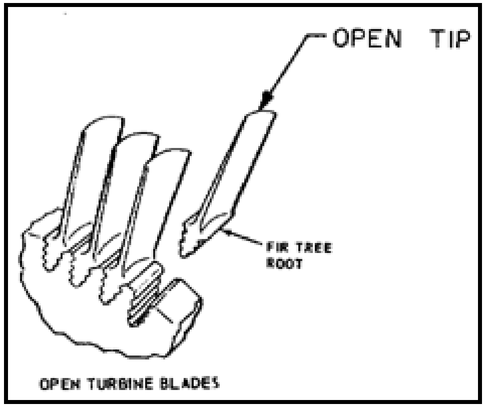
by professor Adriano

Introduction

Fracture is a failure process that involves the initiation and growth of a crack.

- the material **breaks** at a stress below its ultimate strength;
- there are **no defect-free materials** (include gas holes, shrinkage, brittle inclusions) or a crack-free structure (voids, corrosion damage);
- cracks can also initiate at regions of high stress within the material (**stress concentrators** like fastener holes, the corners of windows and doors, and the root of turbine blades);
- **inspection and maintenance costs** represent a high percentage (20% and more).



stress concentrators are inevitable in aerospace structures

Damage tolerance is the ability of structures to withstand the design load and maintain their function in the presence of cracks and other types of damage.

Modes of fracture

The **mode of fracture** depends on many factors:

- the stress level,
- type of loading (static, cyclic, strain rate),
- presence of pre-existing cracks or defects,
- material properties,
- environment and temperature.

ductile

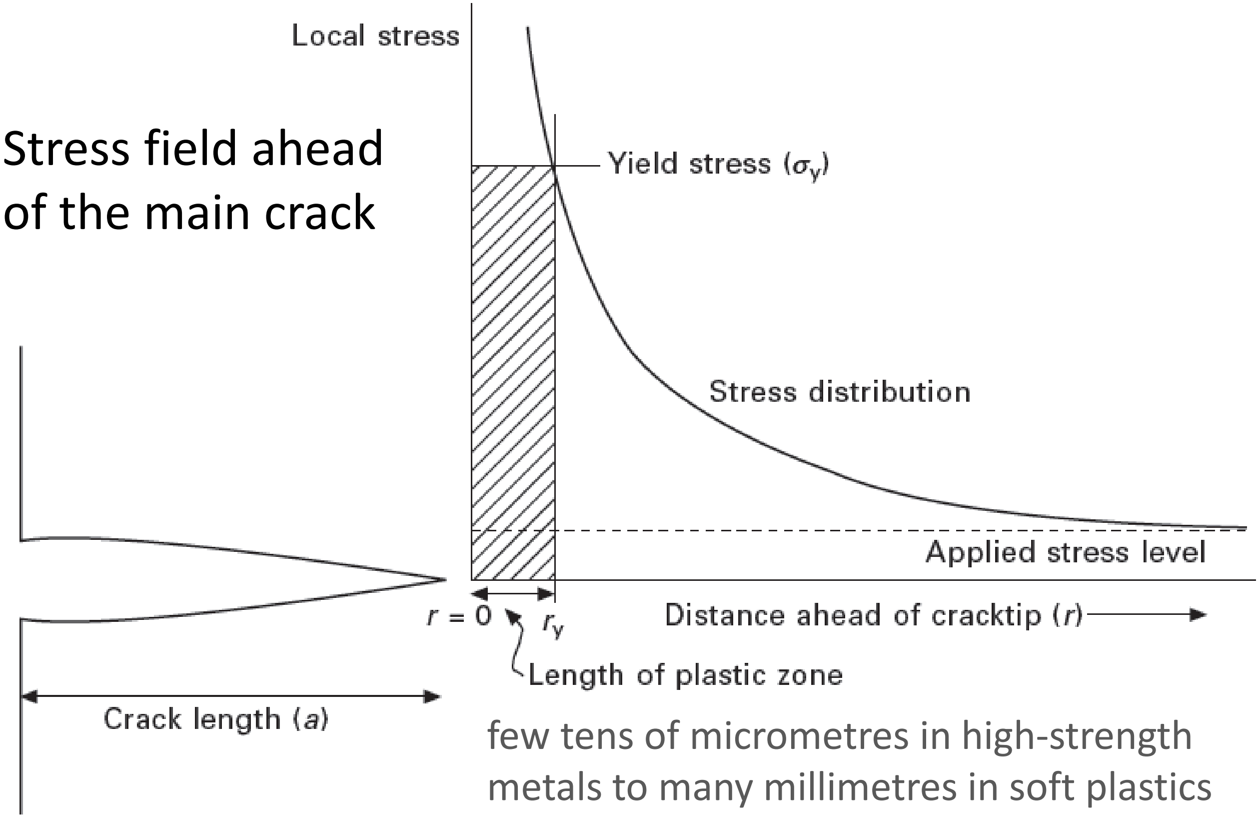
- plastic deformation
- crack growth possible under increasing applied load

brittle

- little or no plastic deformation
- leads to complete failure of the material very rapidly

Ductile fracture

Stress field ahead of the main crack



few tens of micrometres in high-strength metals to many millimetres in soft plastics

The stress to initiate a crack is lower than the stress to grow a crack

The **size of the plastic zone** depends on:

- yield strength of the material,
- the applied stress level,
- and the load conditions (e.g. tension, shear).

Brittle fracture

Stages:
(i) initiation of the crack and
(ii) rapid propagation of the crack **leading to complete fracture**.

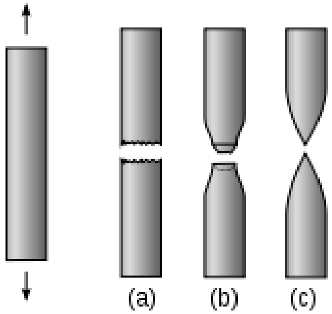
The stress needed to initiate a brittle crack is higher than the stress needed to grow the crack

The crack speed approaches the **speed of sound for the material**:
- 5 $km \cdot s^{-1}$ for Al and Ti;
- 4.5 $km \cdot s^{-1}$ for Fe-C.

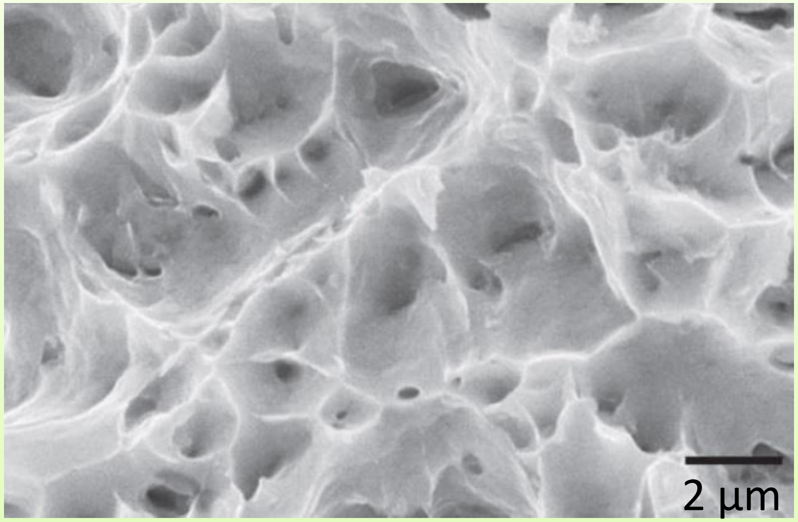
No visible signs of damage or prior warning that the material will break.

Occurs most often in **metals with high strength and low ductility**.

Ductile and brittle fracture



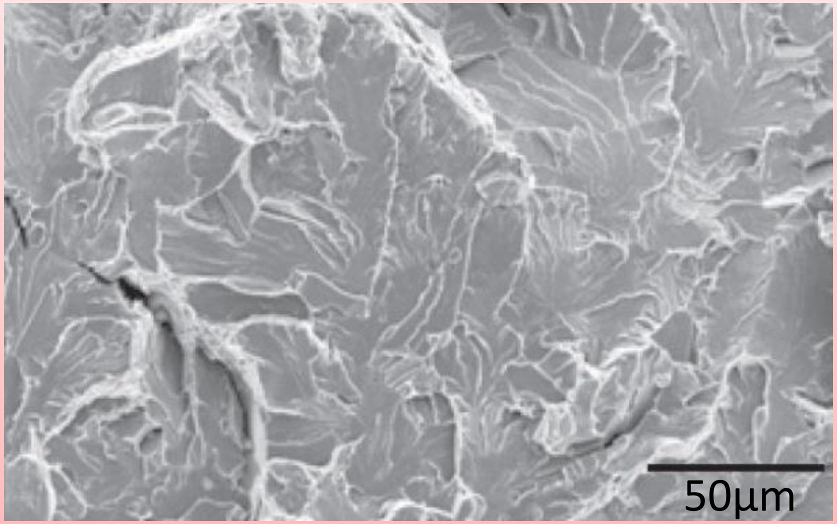
Schematic appearance of round metal bars after tensile testing.
(a) Brittle fracture
(b) Ductile fracture
(c) Completely ductile fracture



Dimpled fracture surface

Ductile fracture condition

$$\sigma_y < E/300$$



Smooth appearance of brittle fracture

Brittle fracture condition

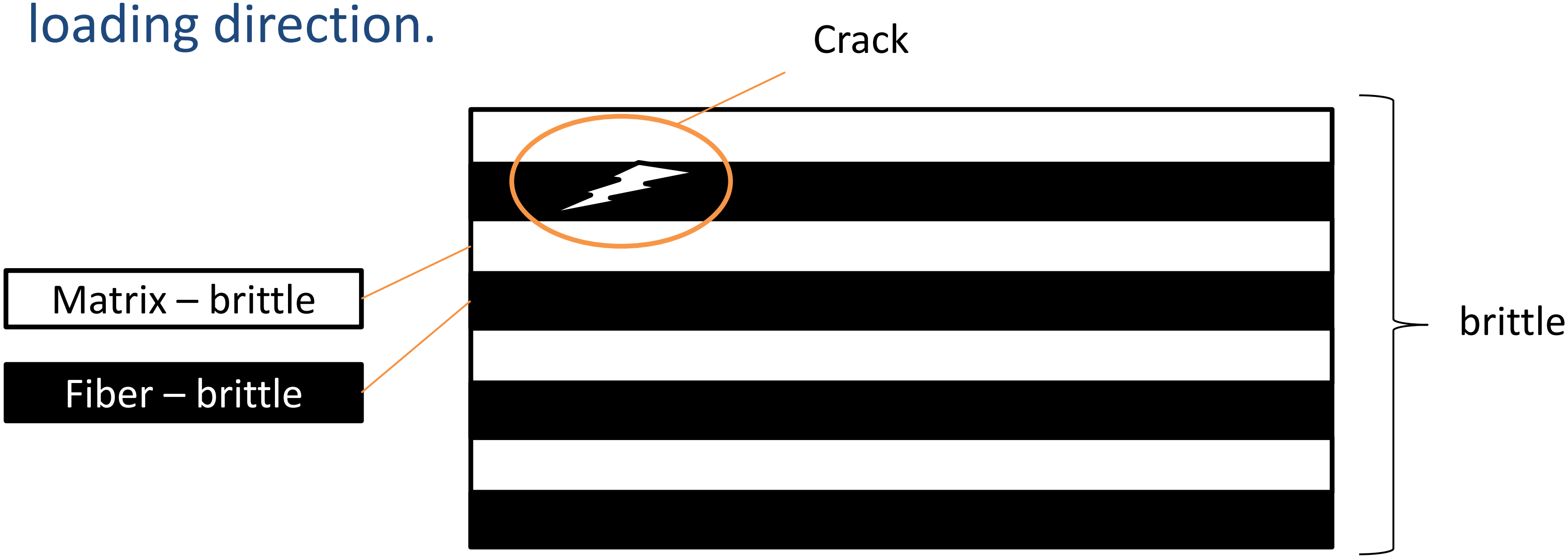
$$\sigma_y > E/150$$

Fracture of fibre–polymer composite materials

Involves a multiplicity of failure modes (**brittle**, but differs from metals)

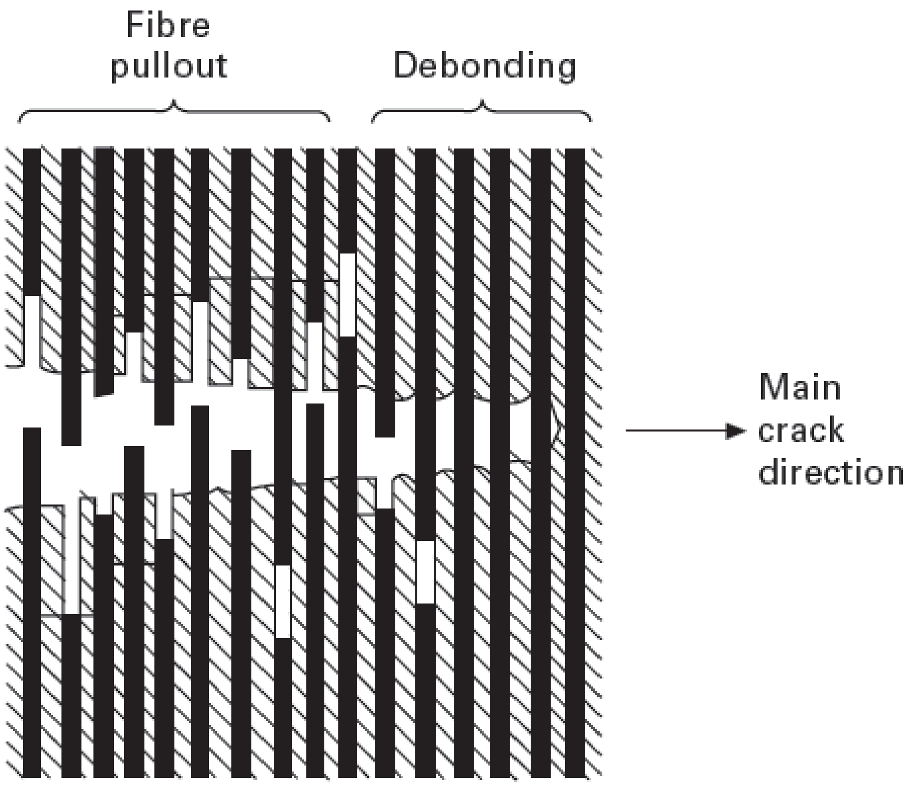
The **operative fracture modes** are dependent on the microstructure:

- the volume fraction, strength, toughness and dimensions of the fibres;
- the volume fraction, strength and ductility of the polymer matrix;
- the fibre–matrix interface;
- loading direction.



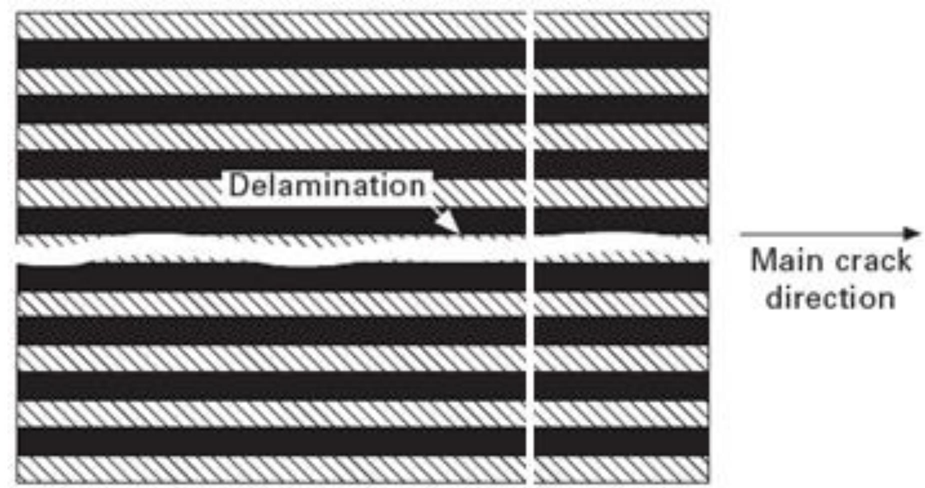
Fracture of fibre–polymer composite materials

In-plane fracture



In-plane fracture toughness of the composite is much higher than the toughness of the fibres and polymer resin on their own

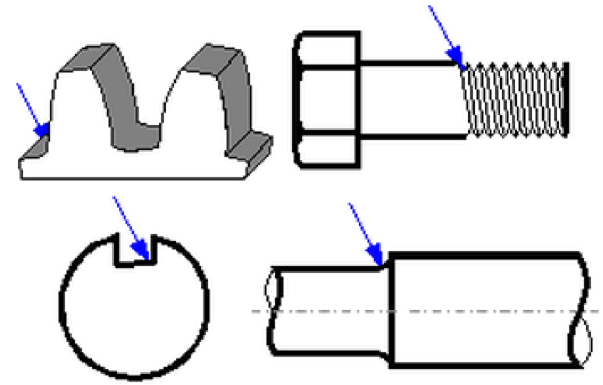
Interlaminar fracture

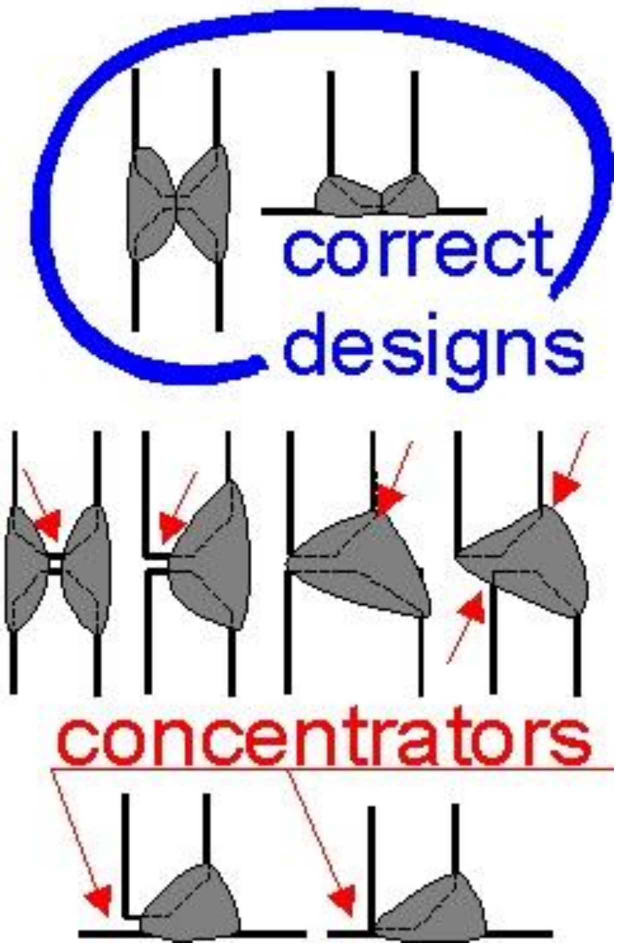


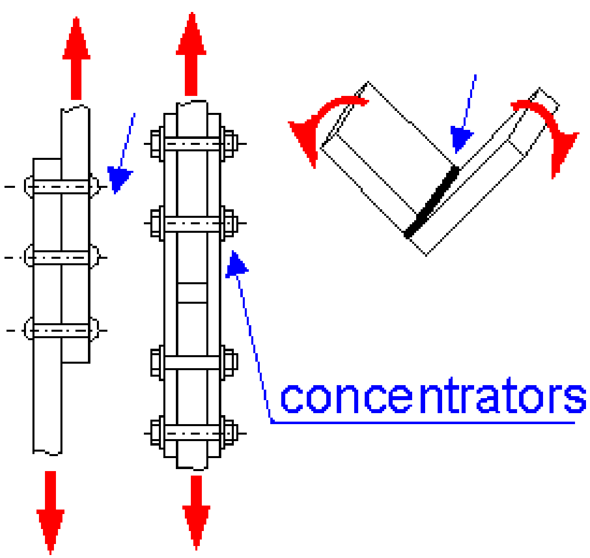
Carbon–epoxy composite:
Polymer failure: 0.1–1 kJ m^{-2}
Splitting failure: 0.1–1 kJ m^{-2}
Fibre debonding: 4–8 kJ m^{-2}
Fibre pull-out: 25–30 kJ m^{-2}
Fibre failure: 20–60 kJ m^{-2}

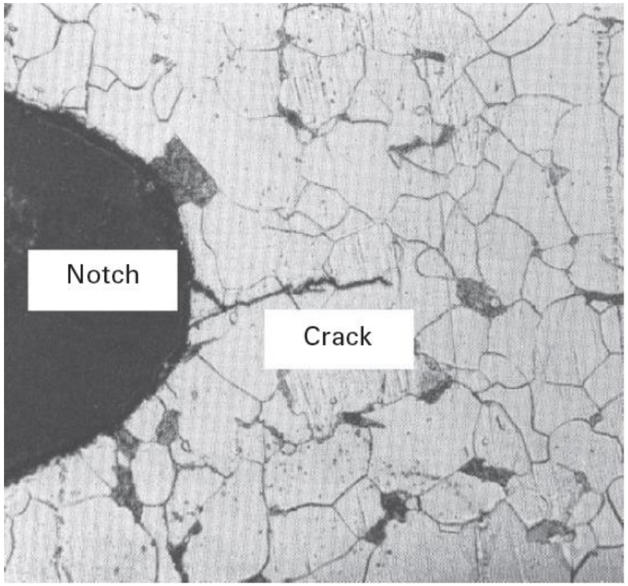
Stress concentration

Corners, holes, fillets and notches cause **cracking**

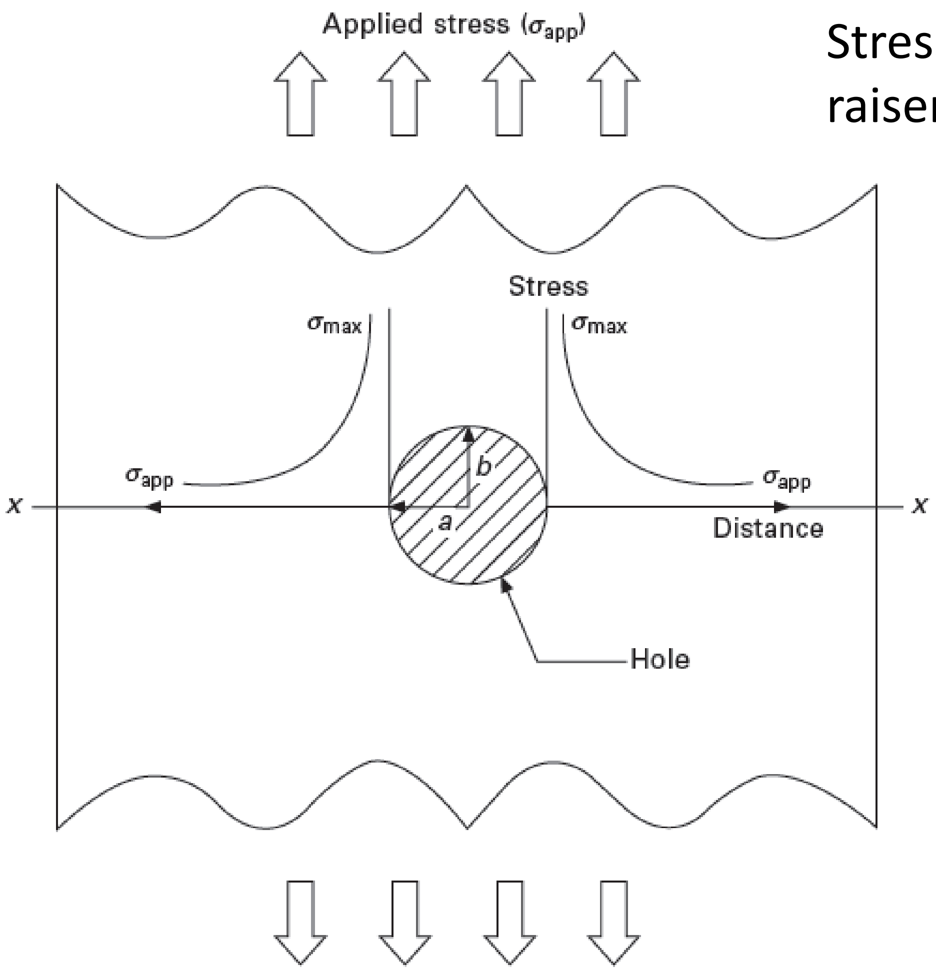








Geometric stress concentration factor



Stress distribution in a plate containing a stress raiser in the form of a circular hole

Geometric stress concentration factor

$$\boxed{K_t = \frac{\sigma_{max}}{\sigma_{app}}}$$

For elliptical hole

$$\boxed{K_t = \frac{\sigma_{max}}{\sigma_{app}} = 1 + \left(\frac{2a}{b}\right)}$$

a – half-width of the hole

b – half-height of the hole

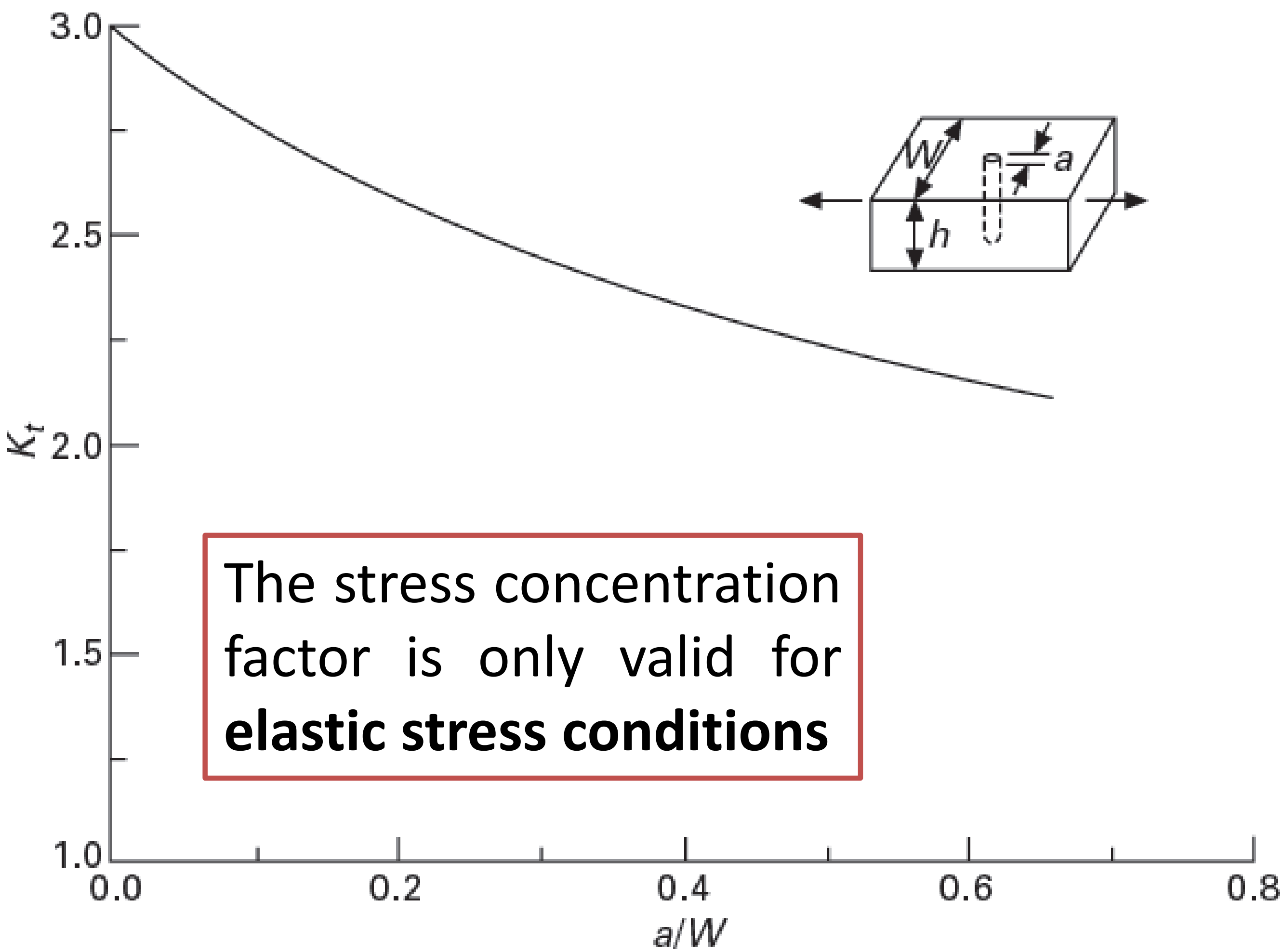
The end radius of a hole

$$\boxed{\rho = \frac{b^2}{a}}$$

$$\sigma_{max} = \sigma_{app}\,[1 + 2\sqrt{a/\rho}] \xrightarrow{a >> \rho} \boxed{\sigma_{max} \approx 2\sigma_{app}\sqrt{a/\rho}}$$

plate of infinite width

Geometric stress concentration factor



The stress concentration factor is only valid for **elastic stress conditions**

Axial loading of a flat plate with a circular hole

Geometric stress concentration factor

For anisotropic materials

$$K_t = \frac{\sigma_{max}}{\sigma_{app}} = 1 + \sqrt{2\left[\sqrt{\frac{E_x}{E_y}} - \nu_{xy}\right] + \frac{E_x}{G_{xy}}}$$

E_x – the Young's modulus in the loading direction
E_y – the Young's modulus in the transverse direction
ν_{xy} – the Poisson's ratio in the *x*–*y* plane,
G_{xy} – the in-plane shear modulus

Stress concentration factors for a **24-ply carbon-epoxy panel with a circular hole**

Lay-up		
Number of 0° plies	Number of ±45° plies	Stress concentration factor K_t
24	0	6.6
16	8	4.1
12	12	3.5
8	16	3.0
0	24	2.0

Introduction to fracture mechanics

Fracture mechanics is the mechanical analysis of materials containing one or more cracks to predict the conditions when failure is likely to occur.

- **select materials** with high resistance against cracking;
- calculate the **residual strength** of structures containing cracks;
- determine the **degree of danger** of the crack;
- determine **the cause of structural failures**;
- minimise the need for expensive **structural tests** on large aircraft components.

Methods to calculate the fracture strength of materials with cracks:

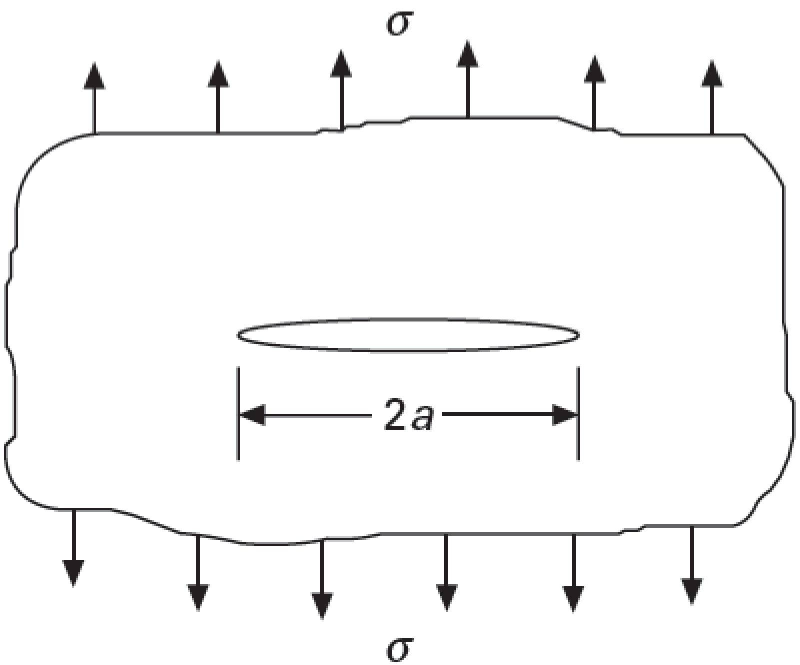
- linear **elastic** fracture mechanics (LEFM);
- **elastoplastic** fracture mechanics (EPFM).

LEFM

The fracture load is not dependent on the load-bearing area of the material, but that **cracks within the glass determine the strength.**

Fracture stress of glass: $\boxed{\sigma_{\mathrm{f}} = C/\sqrt{a}}$

a – crack length,
C – constant.



The crack of length 2*a* in an **infinitely wide plate** under tension

The crack in an **infinitely wide plate**

$$\boxed{\sigma_{\mathrm{f}} = \sqrt{\frac{2\gamma_{\mathrm{e}}E}{\pi a}}} \qquad C = \sqrt{2\gamma_{\mathrm{e}}E/\pi}$$

γ_e – the **elastic surface energy density** needed to form a new crack surface [J·m^{-2}]

EPFM

In EPFM the work of fracture for both **elastic** and **plastic** γ_p crack growth is considered:

$$\sigma_f = \sqrt{\frac{2E(\gamma_e + \gamma_p)}{\pi a}} \xrightarrow[\gamma_e = 1\text{-}20\ \mathrm{J \cdot m^{-2}}]{\gamma_p = 100\text{-}1000\ \mathrm{J \cdot m^{-2}}} \sigma_f \approx \sqrt{\frac{2E\gamma_p}{\pi a}}$$

As a value of γ_p is difficult to measure: $G_c = 2(\gamma_e + \gamma_p)$ [J·m^{-2}]

critical strain energy release rate

$$\boxed{\sigma_f = \sqrt{\frac{EG_c}{\pi a}}}$$

$$\boxed{G_c = \frac{\alpha K_c^2}{E}}$$

$a = 1$ for plane stress

$a = (1–n)^2$ for plane strain

K_c **(critical stress intensity factor [**Pa·m$^{1/2}$**]**) is the fracture toughness of a material that describes how easily a crack grows under an externally applied stress

Application of fracture mechanics

The application of fracture mechanics in materials selection depends on:

- fracture toughness K_c of the material;
- critical crack length a_c;
- operating stress σ.

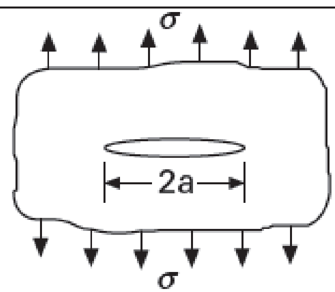
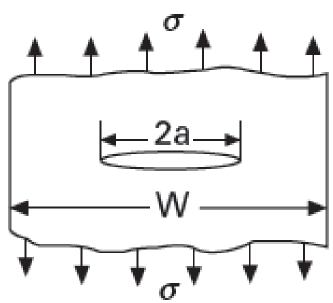
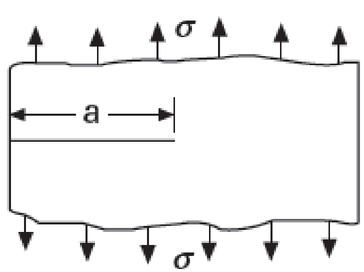
$$\boxed{K_c = \beta\sigma\sqrt{(\pi a_c)}}$$

β is a geometry factor that depends on
- the crack location and
- the shape of the component

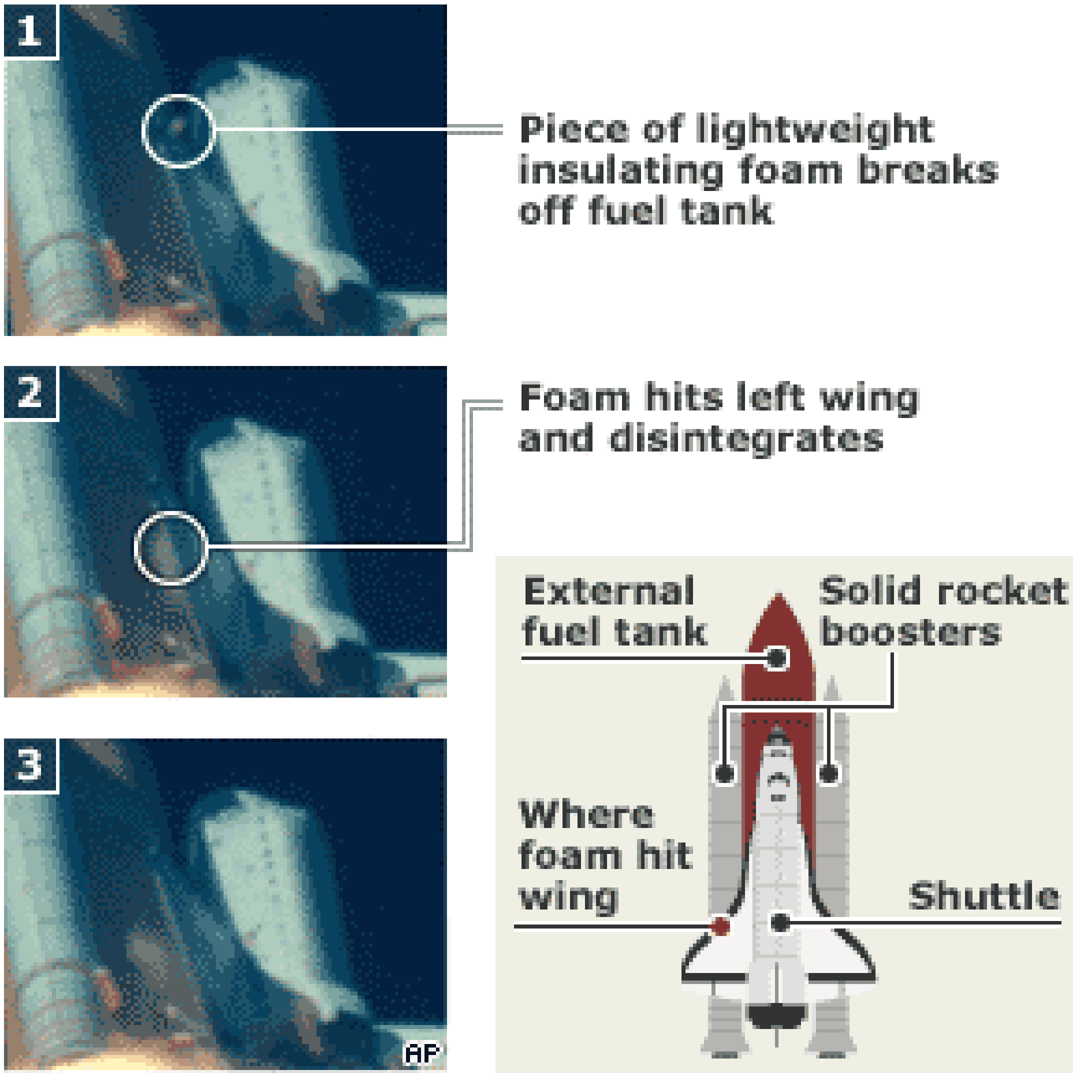
The engineer decides **what is the most important about the design**:

- certain material **properties** (e.g. E, σ_y),
- the design **stress level** (σ), or
- the **critical crack length** (a_c) that must be tolerated for safe operation of the component.

Application of fracture mechanics

Crack type		Stress intensity equation
Centre crack of length 2*a* in infinite plate		$K_I = \sigma_{app}(\pi a)^{1/2}$
Centre crack of length 2*a* in a plate of width *W*		$K_I = \sigma_{app}\left[W \tan\left(\frac{\pi a}{W}\right)\right]^{1/2}$
Edge crack of length a in semi-infinite plate		$K_I = 1.12\sigma_{app}(\pi a)^{1/2}$

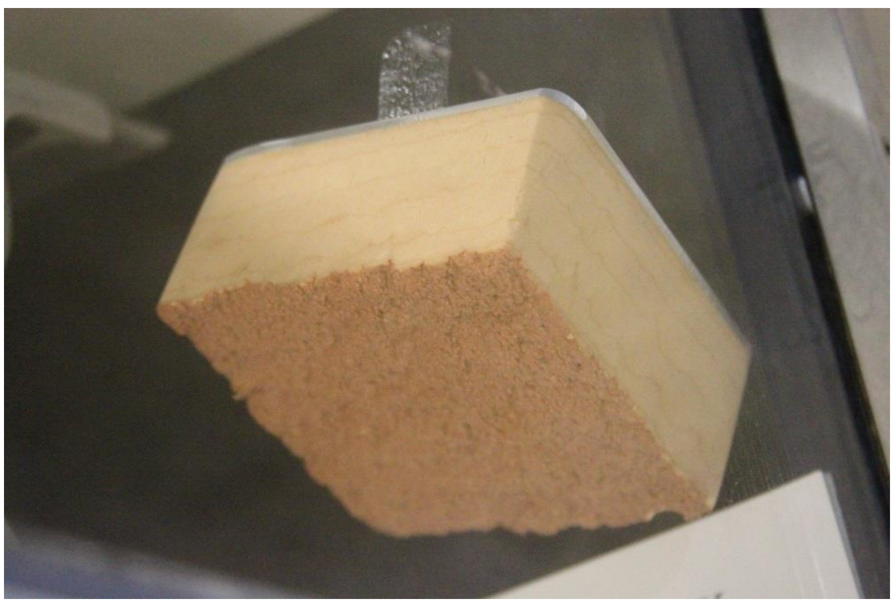
Columbia disaster



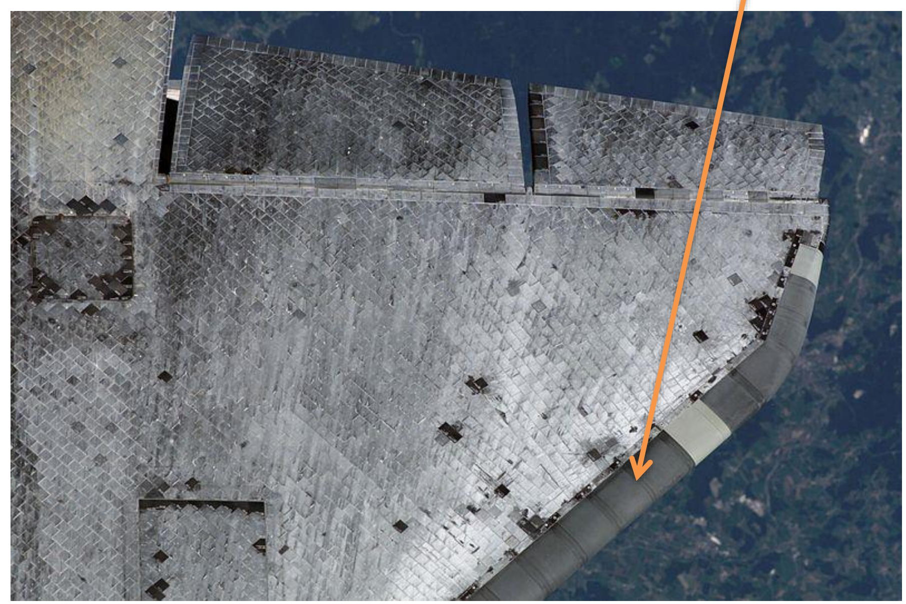
Columbia disaster

RCC was the only TPS (thermal protection system) material that also served as structural support for part of the orbiter's aerodynamic shape:

- the wing leading edges and
- the nose cap.



Space Shuttle external tank foam block

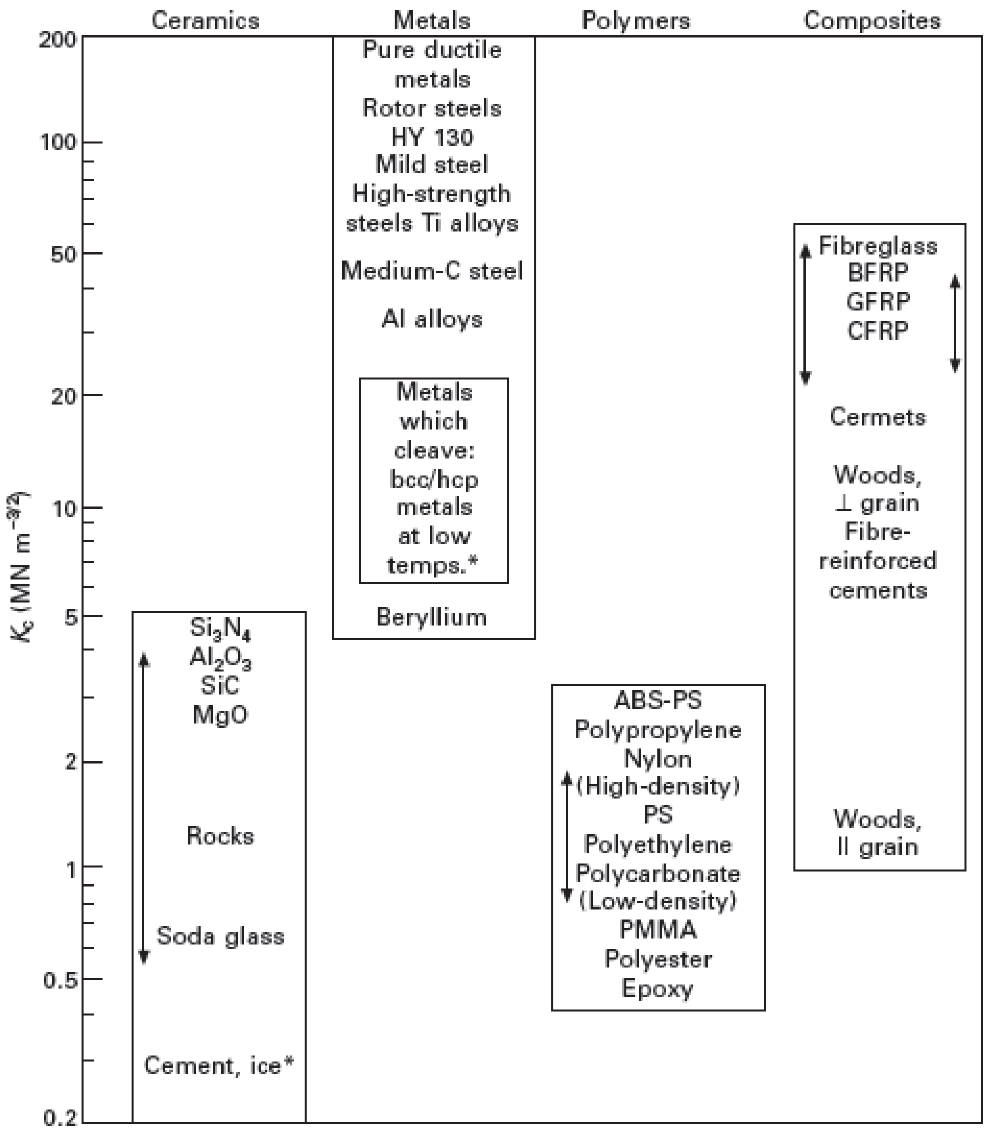


Discovery's under wing surfaces

Radome damage *F-111C*

Radome before and after encounter with pelican

Critical stress intensity factor K_c



Pure ductile metals

K_c = 100-200 MPa·m$^{1/2}$

High strength metal alloys

K_c = 20-120 MPa·m$^{1/2}$

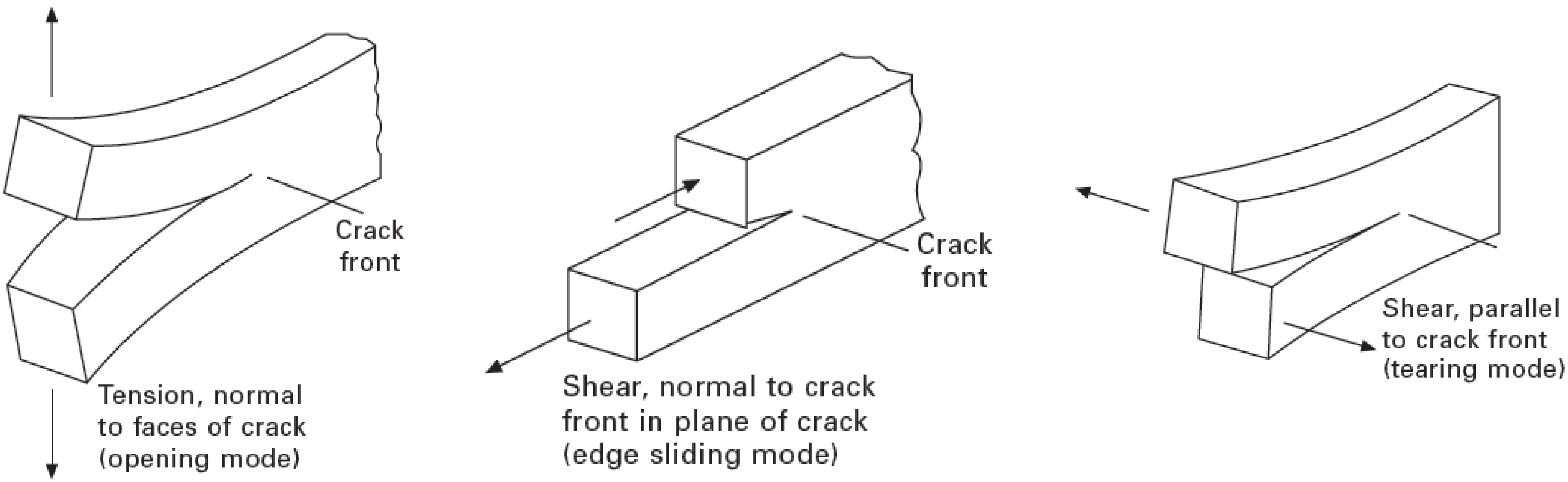
Polymers

K_c = 0.3 - 5 MPa·m$^{1/2}$

Ceramics

K_c = < 5 MPa·m$^{1/2}$

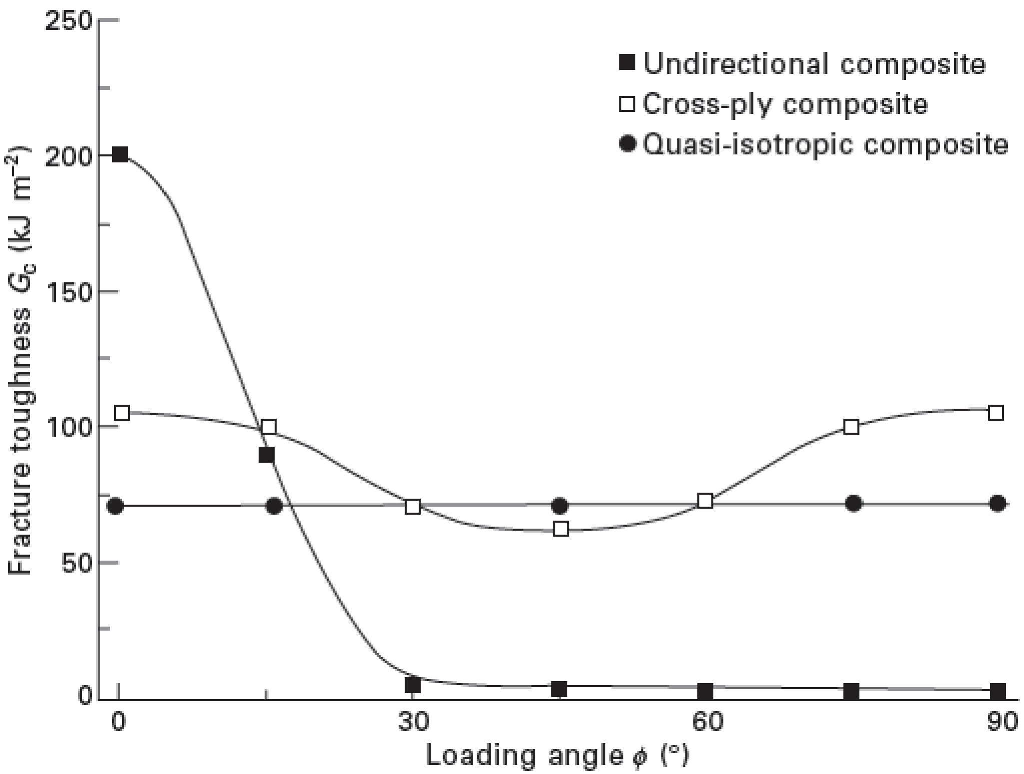
Modes of fracture toughness



Mode I   Mode II   Mode III

Alloy	K_{Ic}	K_{IIc}	K_{IIIc}	$MPa\cdot m^{1/2}$
7000	27	21	24	

Fracture toughness properties of anisotropic materials

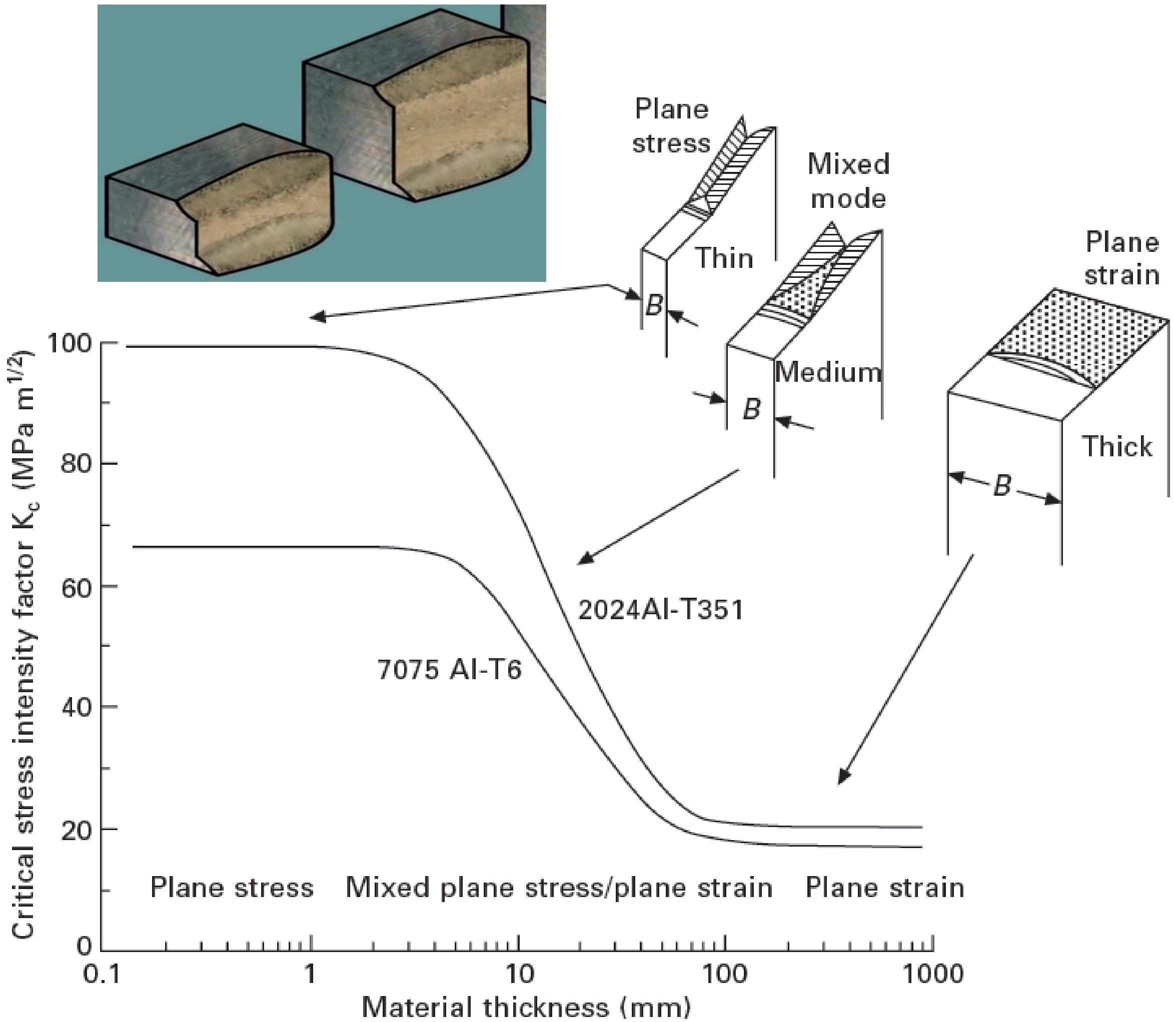


Effect of loading angle on the fracture toughness of **glass fibre-epoxy composites**

Aluminium alloy	Longitudinal (L) K_{Ic} (MPa m$^{1/2}$)	Long transverse (LT) K_{Ic} (MPa m$^{1/2}$)	Short Transverse (ST) K_{Ic} (MPa m$^{1/2}$)
2014 Al-T651	23	24	20
2024 Al-T351	32	34	24
7075 Al-T7451	31	36	27
7075-T6	23	32	21
7178-T651	22	26	16

Anisotropic fracture toughness values for **Al-alloys** in different grain directions

Fracture toughness vs thickness



The **plane strain** condition:

$$B \geq 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

$$r_p/t < 0.02$$

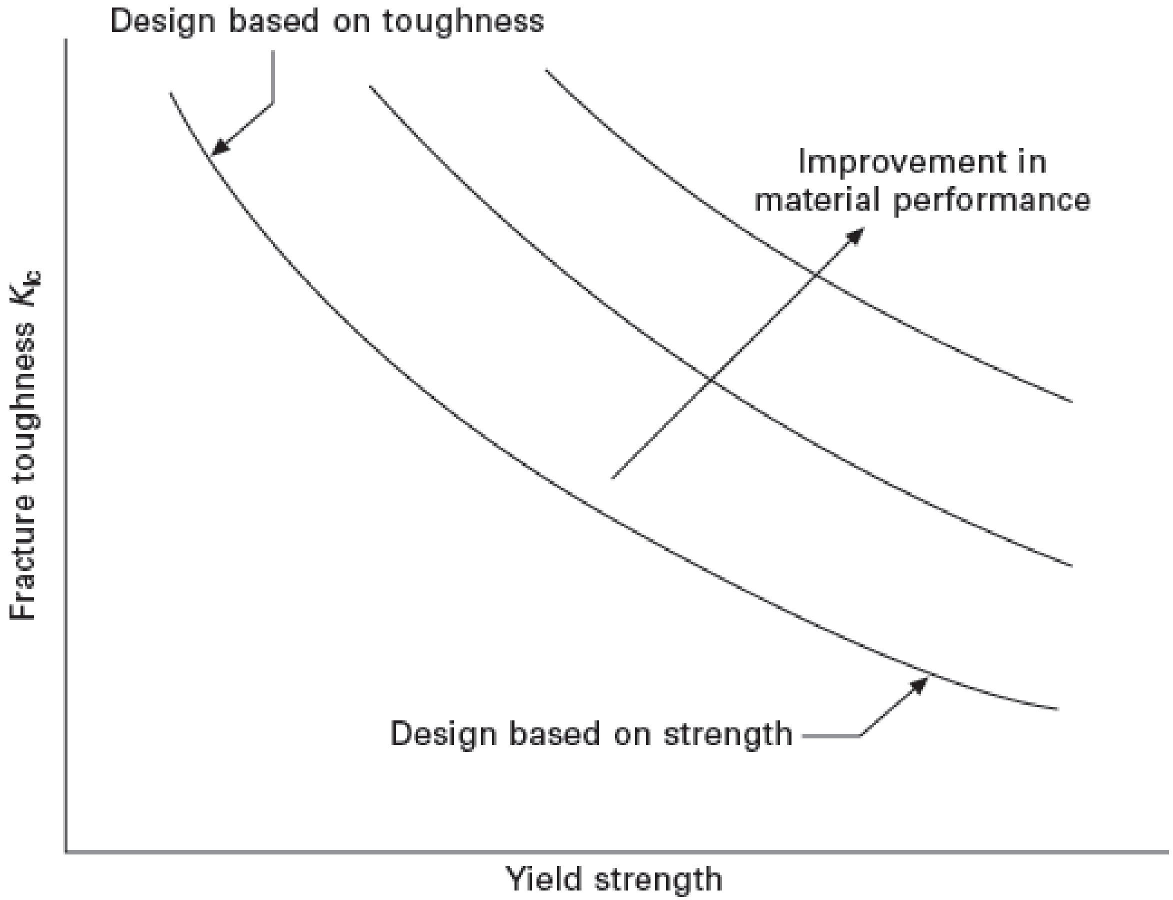
The **plane stress** condition:

$$r_p/t > 0.5$$

r_p – the radius of the crack tip plastic zone

Effect of thickness *B* on the critical stress intensity factor K_c of two aircraft-grade aluminium alloys

Fracture toughness of high-strength metals



Generalized relationship between fracture toughness and yield strength of ductile materials

The radius of the plastic zone

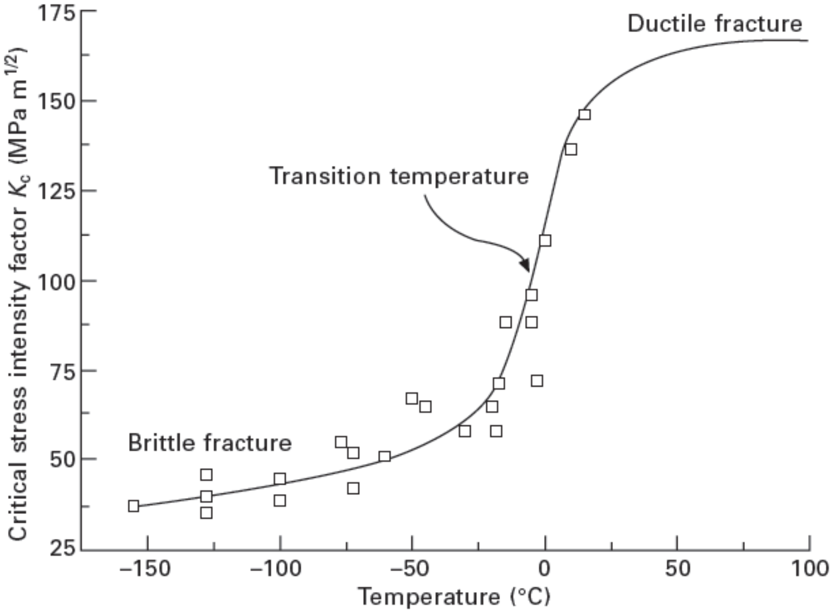
- the plane stress condition

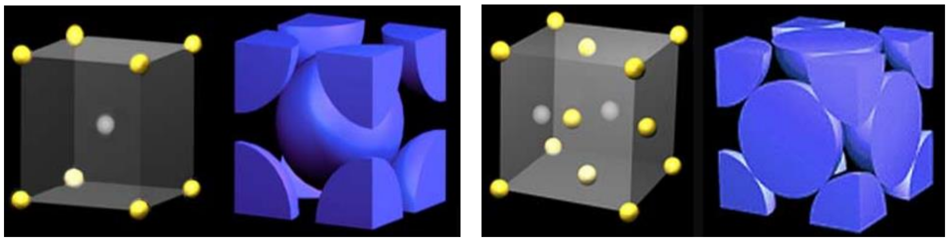
$$r_y = \frac{1}{2\pi}\left(\frac{K_c}{\sigma_y}\right)^2$$

- the plane strain condition

$$r_y = \frac{1}{6\pi}\left(\frac{K_c}{\sigma_y}\right)^2$$

Ductile/brittle transition effect

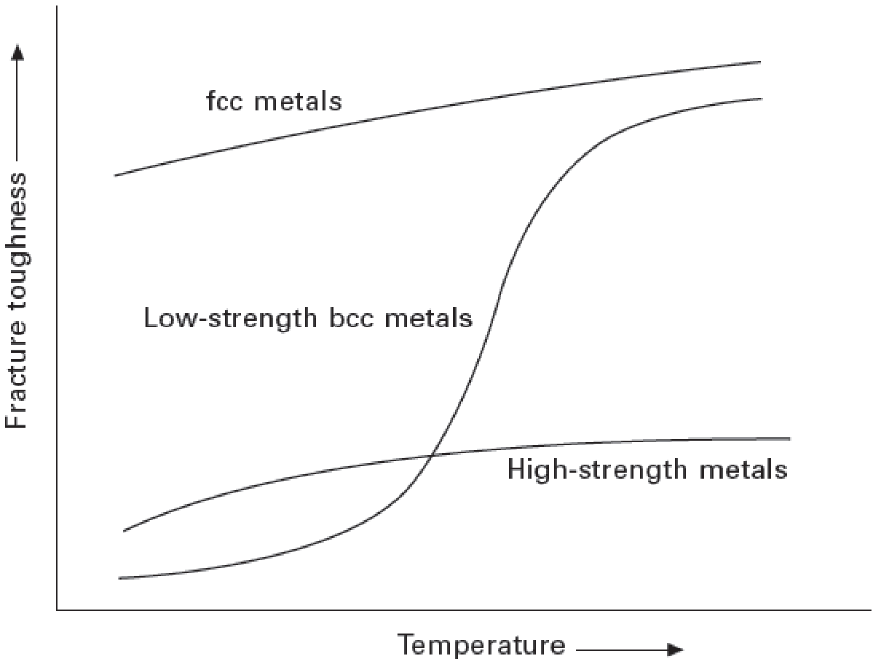




Body Centered Cubic (BCC):
Li, Cr, W, α-Fe

Face Centered Cubic (FCC):
Al, Co, Pt

↑**Ductile/brittle transition curve** for medium-strength steel



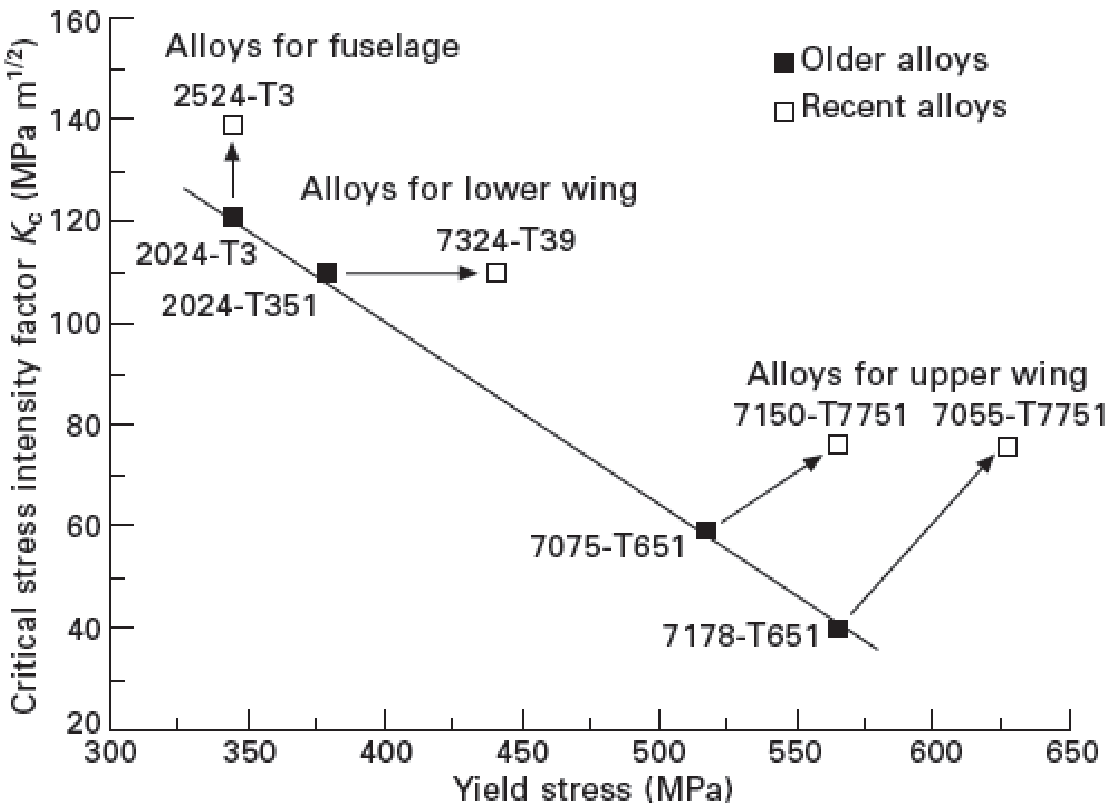
→**General trends** of the ductile/brittle transition effect for different groups of metals

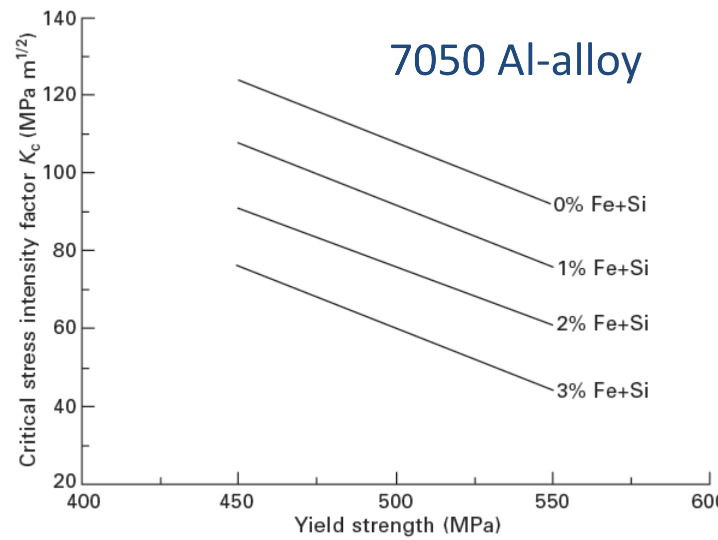
Toughening of metals

Ways to increase the fracture toughness

- alloying (diminishing of impurities),
- processing,
- heat treatment.

without significant loss in strength, fatigue resistance and other important mechanical properties





Improvements to fracture toughness and strength of aluminium alloys by alloy control and heat-treatment

Obrigado!