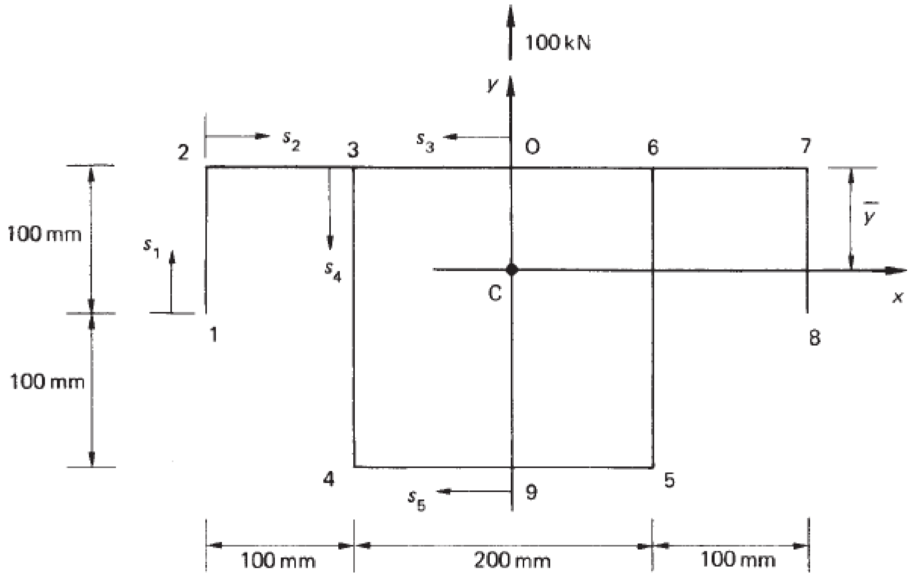
Problems of practical classes of the chapter
14 COMBINED SECTIONS

Problem 14.1

Determine the shear flow distribution in the beam section shown in Fig. below, when it is subjected to a shear load in its vertical plane of symmetry. The thickness of the walls of the section is 2mm throughout.



Solution

The centroid of area C lies on the axis of symmetry at some distance $\bar{y}$ from the upper surface of the beam section. Taking moments of area about this upper surface

$$(4 \times 100 \times 2 + 4 \times 200 \times 2)\bar{y} = 2 \times 100 \times 2 \times 50 + 2 \times 200 \times 2 \times 100 + 200 \times 2 \times 200$$

which gives $\bar{y}$=75 mm.

The second moment of area of the section about Cx is given by

$$I_{xx} = 2\left(\frac{2 \times 100^3}{12} + 2 \times 100 \times 25^2\right) + 400 \times 2 \times 75^2 + 200 \times 2 \times 125^2 + 2\left(\frac{2 \times 200^3}{12} + 2 \times 200 \times 25^2\right)$$

$$I_{xx} = 14.5 \times 10^6 \text{ mm}^4$$

The section is symmetrical about C_y so that $I_{xy} = 0$ and since $S_x = 0$ the shear flow distribution in the closed section 3456 can be simplified from Eq.

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s tx \, ds - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s ty \, ds + q_{s,0}$$

to

$$q_s = -\frac{S_y}{I_{xx}} \int_0^s ty \, ds + q_{s,0}$$

.

Also the shear load is applied through the shear centre of the complete section, i.e. along the axis of symmetry, so that in the open portions 123 and 678 the shear flow distribution is

$$q_s = -\frac{S_y}{I_{xx}} \int_0^s ty \, ds$$

since $q_{s,0} = 0$ if a new coordinate s has its origin at the free end (point 1) of the open portion 123.

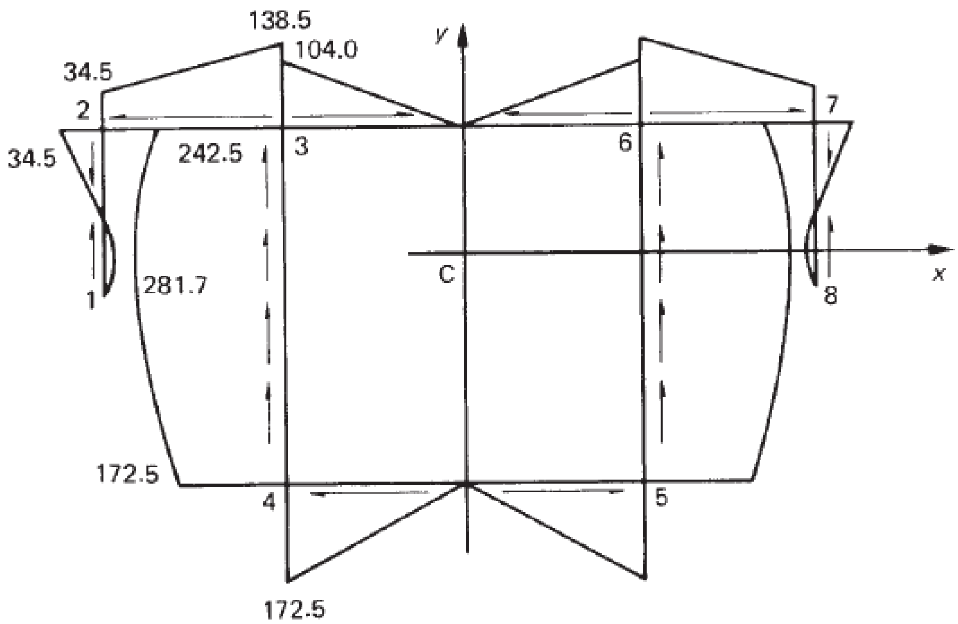
We note that the shear flow is zero at the points 1 and 8 and therefore the analysis may conveniently, though not necessarily, begin at either of these points. Thus, referring to Fig. shown above

$$q_{12} = -\frac{100 \times 10^3}{14.5 \times 10^6} \int_0^{s_1} 2(-25 + s_1) \, ds_1$$

$$q_{12} = -69.0 \times 10^{-4}(-50 s_1 + s_1^2)$$

whence $q_2 = -34.5$ N/mm.

Examination of the previous Eq. shows that q_{12} is initially positive and changes sign when $s_1 = 50$ mm. Further, q_{12} has a turning value ($dq12/ds1 = 0$) at $s_1 = 25$ mm of 4.3 N/mm.



In the wall 23

$$q_{23} = -69.0 \times 10^{-4} \int_0^{s_2} 2 \times 75 \, \mathrm{d}s_2 - 34.5$$

$$q_{23} = -1.04 s_2 - 34.5$$
.

Hence q_{23} varies linearly from a value of −34.5 N/mm at 2 to −138.5 N/mm at 3 in the wall 23.

The analysis of the open part of the beam section is now complete since the shear flow distribution in the walls 67 and 78 follows from symmetry. To determine the shear flow distribution in the closed part of the section we must use the method described in previous problem (vertical load through the shear center of rectangular section) in which the line of action of the shear load is known. Thus we 'cut' the closed part of the section at some convenient point, obtain the q_b or 'open section' shear flows for the complete section and then take moments as in Eqs

$$0 = \oint p q_b \, \mathrm{d}s + 2A q_{s,0}$$
.

However, in this case, we may use the symmetry of the section and loading to deduce that the final value of shear flow must be zero at the mid-points of the walls 36 and 45, i.e. $q_s = q_{s,0} = 0$ at these points. Hence

$$q_{03} = -69.0 \times 10^{-4} \int_0^{s_3} 2 \times 75 \, \mathrm{d}s_3$$

$$q_{03} = -1.04 s_3$$

and $q_3 = -104$ N/mm in the wall 03. It follows that for equilibrium of shear flows at 3, q_3, in the wall 34, must be equal to −138.5 −104=−242.5 N/mm. Hence

$$q_{34} = -69.0 \times 10^{-4} \int_0^{s_4} 2(75 - s_4) \, \mathrm{d}s_4 - 242.5$$

$$q_{34} = -1.04 s_4 + 69.0 \times 10^{-4} s_4^2 - 242.5$$

Examination of the previous Eq. shows that q_{34} has a maximum value of −281.7 N/mm at s_4 =75 mm; also q_4=−172.5 N/mm. Finally, the distribution of shear flow in the wall 94 is given by

$$q_{94} = -69.0 \times 10^{-4} \int_0^{s_5} 2(-125) \, \mathrm{d}s_5$$

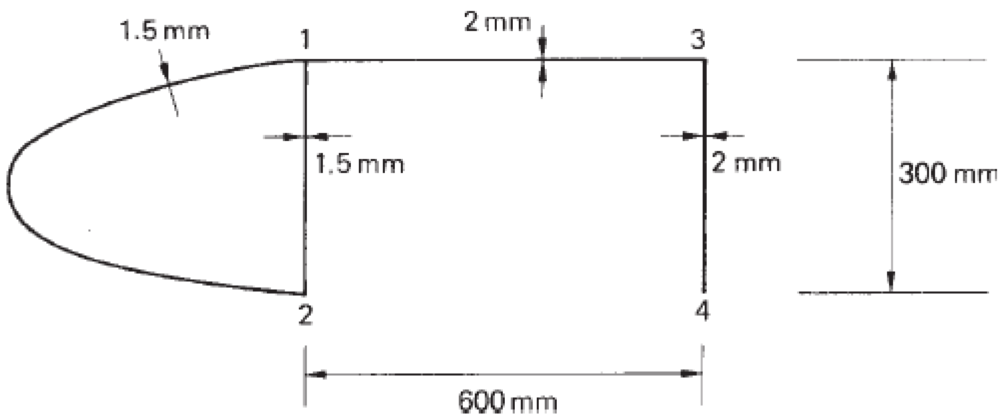
$$q_{94} = 1.73 s_5$$
.

The complete distribution is shown in Fig. above.

Problem 14.2

Find the angle of twist per unit length in the wing whose cross-section is shown in Fig. below when it is subjected to a torque of 10 kN m. Find also the maximum shear stress in the section. G=25 000 N/mm^2.

Wall 12 (outer) = 900 mm. Nose cell area = 20 000mm^2.



It may be assumed, in a simplified approach, that the torsional rigidity GJ of the complete section is the sum of the torsional rigidities of the open and closed portions.

Equation

$$\frac{d\theta}{dz} = \frac{1}{2A} \oint \frac{q_s}{Gt} ds,$$

relating the rate of twist to the variable shear flow q_s developed in a shear loaded closed section beam, is also valid for the case $q_s = q$=constant. Hence

$$\frac{d\theta}{dz} = \frac{q}{2A} \oint \frac{ds}{Gt}.$$

Since.

$$T = 2Aq$$

we can rewrite equation for rate of twist as following

$$\frac{d\theta}{dz} = \frac{T}{4A^2} \oint \frac{ds}{Gt}.$$

From previous Eq. for the closed portion the torsional rigidity is

$$(GJ)_{cl} = \frac{4A^2G}{\oint ds/t} = \frac{4 \times 20\,000^2 \times 25\,000}{(900 + 300)/1.5}$$

which gives

$$(GJ)_{cl} = 5000 \times 10^7 \text{ N mm}^2.$$

The torsional rigidity of the open portion is found using Eq.

$$J = \sum \frac{st^3}{3},$$

Thus

$$(GJ)_{op} = G \sum \frac{st^3}{3} = \frac{25\,000 \times 900 \times 2^3}{3}$$

$$(GJ)_{op} = 6 \times 10^7 \text{ N mm}^2.$$

The torsional rigidity of the complete section is then

$$GJ = 5000 \times 10^7 + 6 \times 10^7 = 5006 \times 10^7 \text{ N mm}^2$$

In all unrestrained torsion problems the torque is related to the rate of twist by the expression

$$T = GJ \frac{\mathrm{d}\theta}{\mathrm{d}z}.$$

The angle of twist per unit length is therefore given by

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{T}{GJ} = \frac{10 \times 10^6}{5006 \times 10^7} = 0.0002 \text{ rad/mm}$$

Substituting for T in Eq.

$$T = 2Aq$$

from Eq.

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{T}{4A^2} \oint \frac{\mathrm{d}s}{Gt},$$

we obtain the shear flow in the closed section. Thus

$$q_{cl} = \frac{(GJ)_{cl}}{2A} \frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{5000 \times 10^7}{2 \times 20\,000} \times 0.0002$$

$$q_{cl} = 250 \text{ N/mm}$$

The maximum shear stress in the closed section is then 250/1.5=166.7 N/mm^2.

In the open portion of the section the maximum shear stress is obtained directly from Eq.

$$\tau_{zs,max} = \pm Gt \frac{\mathrm{d}\theta}{\mathrm{d}z}$$

and is

$$\tau_{max,op} = 25\,000 \times 2 \times 0.0002 = 10 \text{ N/mm}^2.$$

It can be seen from the above that in terms of strength and stiffness the closed portion of the wing section dominates.