Trabalho Individual 4
do Tema Torsão

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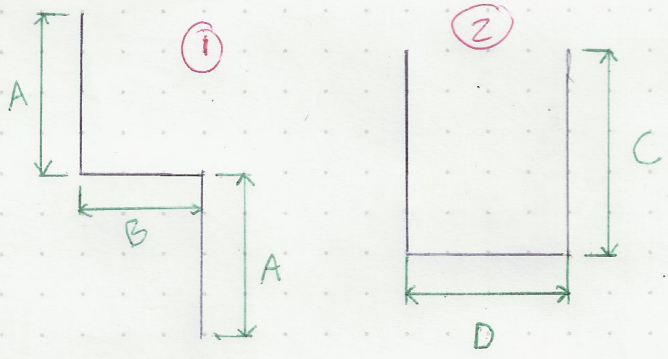

$A = 40\ mm$
$B = 38\ mm$
$C = 34\ mm$
$D = 34\ mm$

$E = 70\ GPa$
$G = 25\ GPa$
$\nu = 0.32$
$T = 1\ kN \cdot m$
$t = 1\ mm$

① $J = 2\frac{At^3}{3} + \frac{Bt^3}{3} = (2A+B)\frac{t^3}{3}$

$$T = GJ\frac{d\theta}{dz} \Rightarrow \frac{d\theta}{dz} = \frac{T}{GJ} = \frac{3T}{G(2A+B)t^3} =$$

$$= \frac{3 \cdot 1}{25(2 \cdot 40 + 38)1^3} \cdot 10^6\ m^{-1} = \boxed{1.017\ mm^{-1} = \frac{d\theta}{dz}}$$

twist rate

$$\tau_{zy,max} = \pm Gt\frac{d\theta}{dz} = \pm\frac{Tt}{J} = \pm\frac{3T}{(2A+B)t^2} = \pm\frac{3 \cdot 1}{(2 \cdot 40 + 38)1^2} \cdot 10^{12}\ Pa$$

$$\boxed{\tau_{zy,max} \approx \pm 25{,}42\ GPa}$$

maximum shear stress

② $J = 2\frac{Ct^3}{3} + \frac{Dt^3}{3} = (2C+D)\frac{t^3}{3}$

$$\frac{d\theta}{dz} = \frac{T}{GJ} = \frac{3 \cdot 1}{25 \cdot (2 \cdot 34 + 34) \cdot 1^3} \cdot 10^6\ m^{-1} \approx \boxed{1.176\ mm^{-1}}$$

twist rate

$$\tau_{zy,max} = \pm\frac{Tt}{J} = \frac{3 \cdot 1}{(2 \cdot 34 + 34)1^2} \cdot 10^{12}\ Pa \approx \boxed{29{,}41\ GPa}$$

maximum shear stress

rigidity $\propto J \propto \left(\frac{d\theta}{dz}\right)^{-1}$ $\Rightarrow$ Section ① is more rigid

Questão Teórica:

$$GJ = -\frac{4G}{\nabla^2\phi}\iint_A \phi \, dx\,dy \quad , \quad \phi = -G\frac{d\theta}{dz}\left[x^2 - \left(\frac{t}{2}\right)^2\right]$$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \underline{-G\frac{d\theta}{dz}\cdot 2}$$

$$\iint_A \phi\,dx\,dy = \int_{-s/2}^{s/2}\int_{-t/2}^{t/2}\left(-G\frac{d\theta}{dz}\left[x^2-\left(\frac{t}{2}\right)^2\right]\right)dx\,dy =$$

$$= -G\frac{d\theta}{dz}\int_{-s/2}^{s/2}\left(\frac{x^3}{3} - \left(\frac{t}{2}\right)^2 x\right)\Big|_{-t/2}^{t/2} dy = -G\frac{d\theta}{dz}\int_{-s/2}^{s/2}\left(\frac{t^3}{12} - \frac{t^3}{4}\right)dy =$$

$$= G\frac{d\theta}{dz}\left(\frac{1}{6}t^3 y\right)\Big|_{-s/2}^{s/2} = \underline{-G\frac{d\theta}{dz}\cdot\frac{1}{6}t^3 s}$$

$$GJ = -\frac{4G}{-G\frac{d\theta}{dz}\cdot 2}\left(G\frac{d\theta}{dz}\cdot\frac{1}{6}t^3 s\right) = \frac{1}{3}t^3 s \qquad \text{Q.E.D.}$$

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