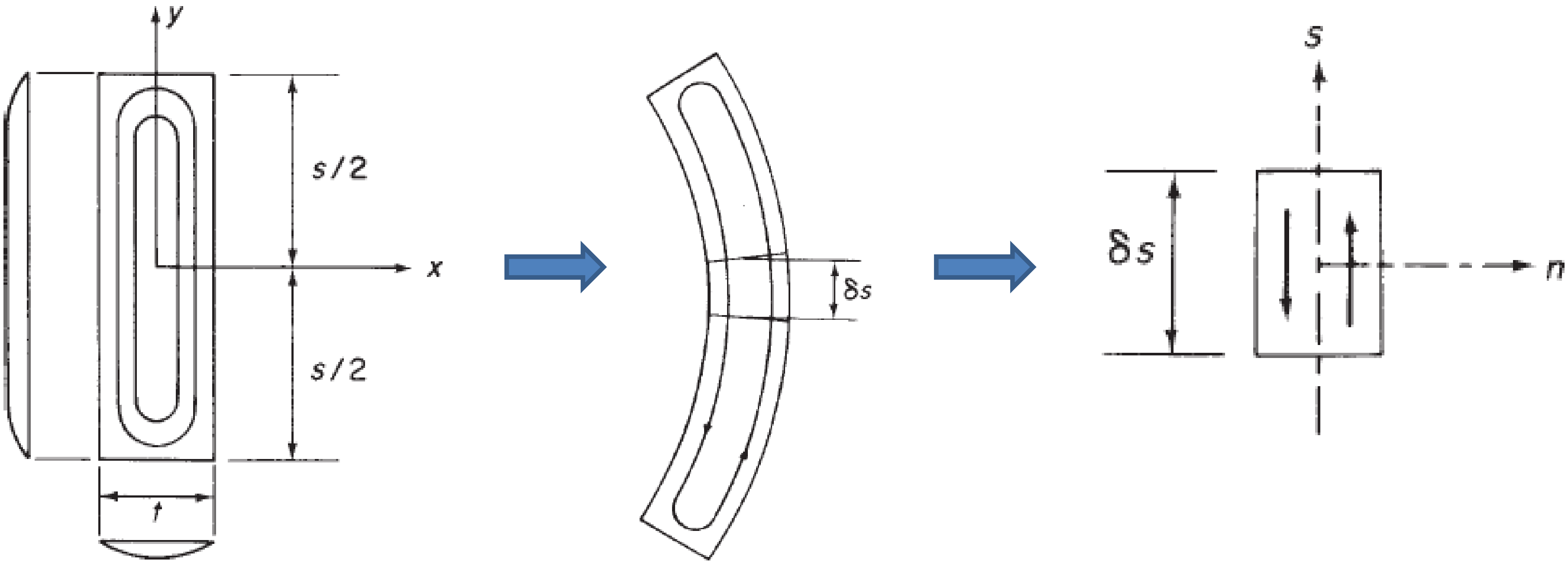
Torsion of beams

2013

Torsion of open section beams

Results of an approximate solution of thin plate under torque is used.



If ***s >> t*** then the contours of the membrane, i.e. lines of shear stress, are still approximately parallel to the inner and outer boundaries.

Equations for thin plate can be applied with reduced accuracy:

$$\tau_{zy} = 2Gx\frac{\mathrm{d}\theta}{\mathrm{d}z} \qquad \tau_{zx} = 0$$

$$\tau_{zy,\max} = \pm Gt\frac{\mathrm{d}\theta}{\mathrm{d}z}$$

$$J = \frac{st^3}{3}$$

Torsion of open section beams

The given equations can be applied in the following form:

$$\boxed{\tau_{zy} = 2Gx\frac{d\theta}{dz} \quad \tau_{zx} = 0} \Rightarrow \boxed{\tau_{zs} = 2Gn\frac{d\theta}{dz}, \quad \tau_{zn} = 0}$$

$$\boxed{\tau_{zy,\max} = \pm Gt\frac{d\theta}{dz}} \Rightarrow \boxed{\tau_{zs,\max} = \pm Gt\frac{d\theta}{dz}}$$

$$\boxed{J = \frac{st^3}{3}} \Rightarrow \boxed{J = \sum \frac{st^3}{3}} \quad \boxed{J = \frac{1}{3}\int_{\text{sect}} t^3\, ds}$$

For variable wall thickness

The rate of twist: $\boxed{T = GJ\frac{d\theta}{dz}}$

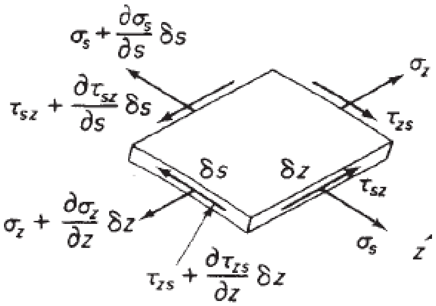
$$\tau_{zy} = 2Gx\frac{d\theta}{dz} \xrightarrow{T = GJ\frac{d\theta}{dz}} \boxed{\tau_{zs} = \frac{2n}{J}T,}$$

$$\tau_{zs,\max} = \pm Gt\frac{d\theta}{dz} \xrightarrow{T = GJ\frac{d\theta}{dz}} \boxed{\tau_{zs,\max} = \pm\frac{tT}{J}}$$

i.e. the shear stresses vary across the thickness of the beam wall

Torsion of closed section beams

A closed section beam subjected to a pure torque ***T*** does not develop a direct stress system in the absence of an axial constraint.



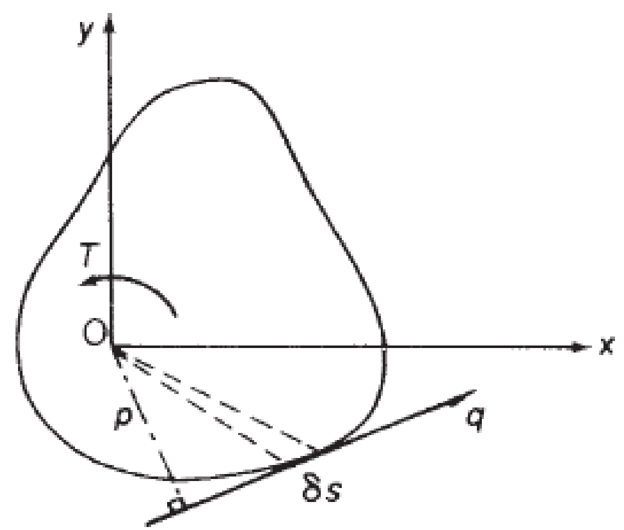
$$\frac{\partial q}{\partial s} + t\frac{\partial \sigma_z}{\partial z} = 0 \qquad \frac{\partial q}{\partial z} + t\frac{\partial \sigma_s}{\partial s} = 0$$

$$\partial q / \partial z = 0 \qquad \partial q / \partial s = 0$$

q = const

The application of a pure torque to a closed section beam results in the development of a constant shear flow in the beam wall.

As ***t = f(s)***, then the shear stress ***τ*** may vary around the cross-section.

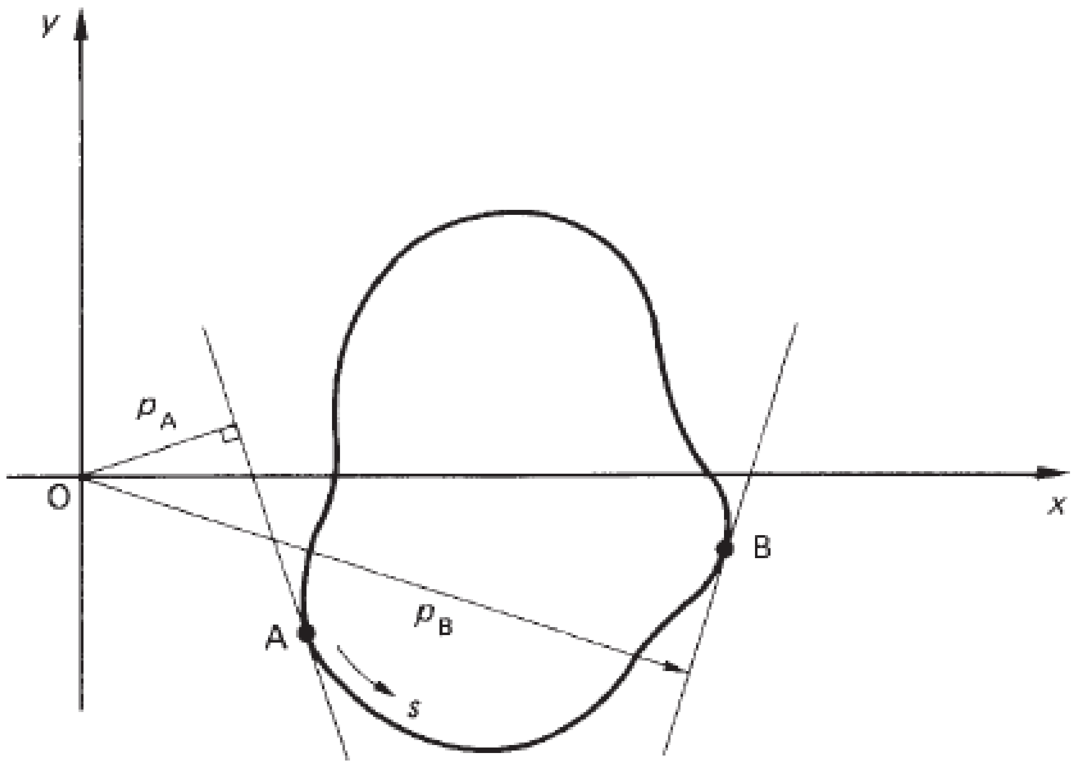


$$T = \oint pq\,\mathrm{d}s \xrightarrow{\oint p\mathrm{d}s = 2A} \boxed{T = 2Aq}$$

Bredt–Batho formula

Torsion of closed section beams

- ***p*** is positive, when the movement of its foot along the tangent at any point in the positive direction of ***s*** leads to an anticlockwise rotation of ***p*** about the origin of axes.
- the positive direction of ***s*** is in the positive direction of ***q*** which is anticlockwise (corresponding to a positive torque).



Obrigado!