Trabalho Individual 2

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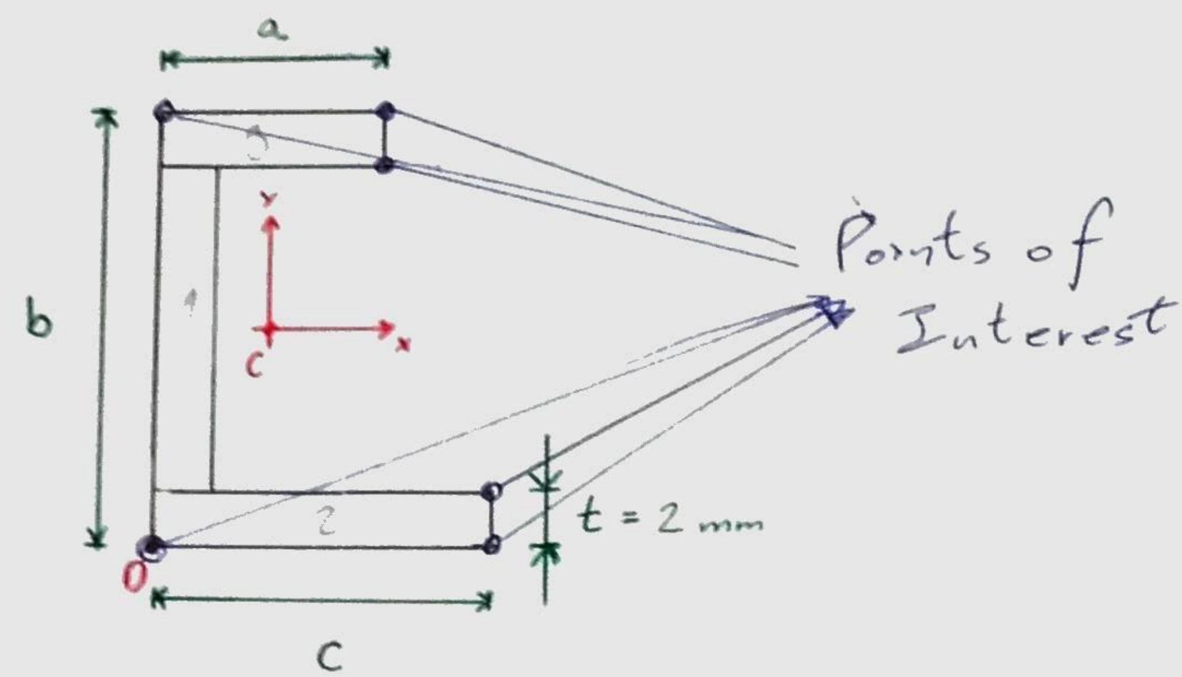

First we will find the geometric properties of the section. To find the centroid, we will express its coordinates from the arbitrary point O as $(\bar{x}, \bar{y})$. Then, knowing that the first moment of area over the centroid is null, we can find $\bar{x}$ and $\bar{y}$:

$$\bar{x}\left(at + (b-2t)t + ct\right) = at\frac{a}{2} + (b-2t)t\frac{t}{2} + ct\frac{c}{2}$$

$$\bar{x} = \frac{a^2 + bt - 2t^2 + c^2}{2(a+b+c-2t)} \approx 18.07 \text{ mm}$$

$$\bar{y}\left(at + (b-2t)t + ct\right) = at\left(b - \frac{t}{2}\right) + (b-2t)t\frac{b}{2} + ct\frac{t}{2}$$

$$\bar{y} = \frac{2ab - at + b^2 - 2tb + ct}{2(a+b+c-2t)} \approx 37.427 \text{ mm}$$

With the centroid known, we can calculate the second moments of area relative to our axes

$$I_{xx} = \frac{at^3}{12} + at\left(b - \frac{t}{2} - \bar{y}\right)^2 + (b-2t)^3\frac{t}{12} + (b-2t)t\left(\frac{b}{2} - \bar{y}\right)^2 +$$

$$+ \frac{ct^3}{12} + ct\left(\frac{t}{2} - \bar{y}\right)^2 \approx 6.3 \cdot 10^5 \text{ mm}^4$$

$$I_{xy} = at\left(\frac{a}{2} - \bar{x}\right)\left(b - \frac{t}{2} - \bar{y}\right) + (b-2t)t\left(b/2 - \bar{y}\right)\left(\frac{t}{2} - \bar{x}\right) + $$
$$+ ct\left(\frac{c}{2} - \bar{x}\right)\left(\frac{t}{2} - \bar{y}\right) \approx 2.6 \cdot 10^5 \text{ mm}^4$$

$$I_{yy} = \frac{a^3 t}{12} + at\left(\frac{a}{2} - \bar{x}\right)^2 + \frac{(b-2t)t^3}{12} +$$
$$+ (b-2t)t\left(\frac{t}{2} - \bar{x}\right)^2 + \frac{c^3 t}{12} + ct\left(\frac{c}{2} - \bar{x}\right)^2 \approx 3.7 \cdot 10^5 \text{ mm}^4$$

With all geometric properties found, we must find the moments about x and y axes. From the Forces applied at the end of the beam, we find:

$M_x = -Y_1 \cdot z$

$M_y = X_1 \cdot z$

, where z is oriented from the end of the beam, toward the wall

As we are interested in the section with maximum direct stress, we will take maximum M_x and M_y. That is, $(M_x, M_y)\Big|_{z=2000 \text{ mm}}$

Further, we know that

$$\begin{bmatrix} \sin\alpha \\ \cos\alpha \end{bmatrix} = \frac{\rho}{E} \begin{bmatrix} I_{xy} & I_{xx} \\ I_{yy} & I_{xy} \end{bmatrix}^{-1} \begin{bmatrix} M_x \\ M_y \end{bmatrix}$$

So it follows that $\tan\alpha = \dfrac{I_{xy} M_x - I_{xx} M_y}{I_{xy} M_y - I_{yy} M_x} \approx -0.264$

Taking the arctan of that result, we get the approximate value $\alpha \approx -14.77° + k \cdot 180°$

Finally, we want to find $\sigma_{z_{max}}$, which will be located at the extremities of the section. We can, therefore, test the values of each extremity. We also know that

$$\sigma_z = \frac{E}{\rho}(x \sin\alpha + y\cos\alpha)$$

But here we have to determine the correct half rotation of α. Inspecting $\sin\alpha$ and $\cos\alpha$ we see that $90° < \alpha < 180°$ and, therefore, we choose $\underline{\alpha = 165.23°}$

The interest points are, expressed as their difference to O, the following:

$x - x_0$	$y - y_0$
0	0
0	b
a	b
a	b-t
c	0
c	t

$(x_0, y_0) = -(\bar{x}, \bar{y})$

~~As $\sin\alpha > 0$ and $\cos\alpha < 0$, we can~~

Testing every point, we find that the maximum stress is

$$\sigma_{z_{máx}} = \boxed{\sigma_z\Big|_{(x,y)=(0,b)-(\bar{x},\bar{y})} \approx -42.41\ \frac{E}{\rho}\cdot mm}$$

The maximum tension stress is

$$\sigma_{z_{máx}} = \boxed{\sigma_z\Big|_{(x,y)=(c,0)-(\bar{x},\bar{y})} \approx 31.83\ \frac{E}{\rho}\cdot mm}$$