Exercício Individual 5.2 — Turma A

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April 5, 2022

Table 1: Option Variables

Variable	Value	Unit
Option	V	
a	440	mm
b	345	mm
t	0.635	mm
P	50	Pa

1 Analytical Solution

The deflections in a simply supported thin plate under uniform load can be calculated using Eq. (1), where $D = \frac{Et^3}{12(1-\nu^2)}$ is the flexural rigidity of the plate. To find D, we will take the material properties of ANSYS standard material *Structural Steel*. The stresses may be calculated according to Eq. (3), where the moments M_x and M_y are calculated using the Eq. (2). The maximum deflection and tension will occur at the center of the plate, and we can obtain numerical values for the sum by taking the first few terms. Using a python function, the sums were taken for $m, n = 1, 3, \ldots, 19$.

Equation 1: Deflections in a thin plate under uniform pressure.

$$w = \frac{1}{\pi^4 D} \sum_m \sum_n \frac{a_{mn}}{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$a_{mn} = \frac{16q}{\pi^2 mn}$$

$$w = \frac{16q}{\pi^6 D} \sum_m \sum_n \frac{\sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}}{mn\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2}$$

$$w = 0.938\,58\,\mathrm{mm}$$

Equation 2: Moments in a thin plate under uniform pressure.

$$M_x = \frac{16q}{\pi^4 D} \sum_m \sum_n \frac{\left(\frac{m}{a}\right)^2 + \nu\left(\frac{n}{b}\right)^2}{mn\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$M_x = 0.299\,34\,\mathrm{N\,m}$$
$$M_y = 0.403\,32\,\mathrm{N\,m}$$

Equation 3: Stresses in a thin plate under bending moments.

$$\sigma_{x\max} = \frac{Ez}{1-\nu^2}\frac{M_x}{D}$$
$$\sigma_{y\max} = \frac{Ez}{1-\nu^2}\frac{M_y}{D}$$

$$\sigma_{x\max} = 4.4541\,\mathrm{MPa}$$
$$\sigma_{y\max} = 6.0015\,\mathrm{MPa}$$

2 Numerical Solution

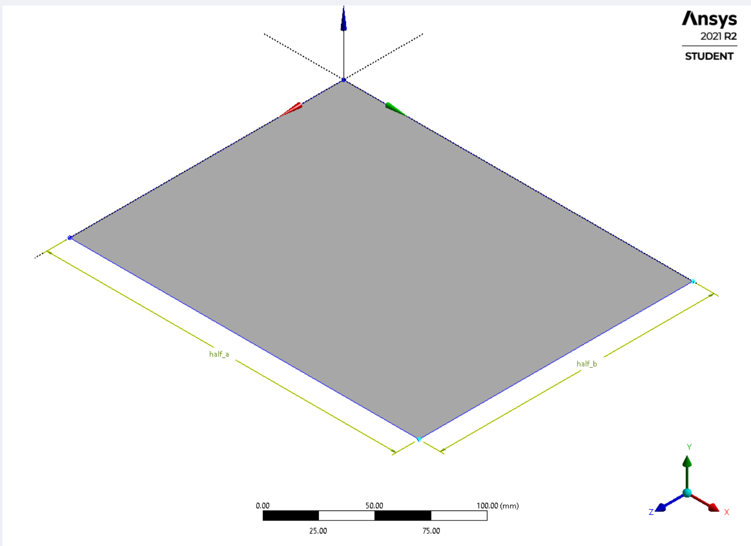
Another approach for solving the problem is to use software tools to calculate the stress distribution numerically. Here, we can use ANSYS software to solve this problem. The process can be divided into geometry modeling, mesh creation, physical setup and results exploration.

2.1 Geometry Modeling

The first step to solve this problem lies in the creation of a model of our geometries. To do that, we can use the software Design Modeler. For this thin plate, we can use a surface geometry with an assigned thickness.

To study how we could further reduce computational costs in a more complex, but still symmetric analysis, we can leverage the symmetries in the problem. Thus we will simulate only one quarter of the plate. This quarter plate geometry is shown in Fig. 1

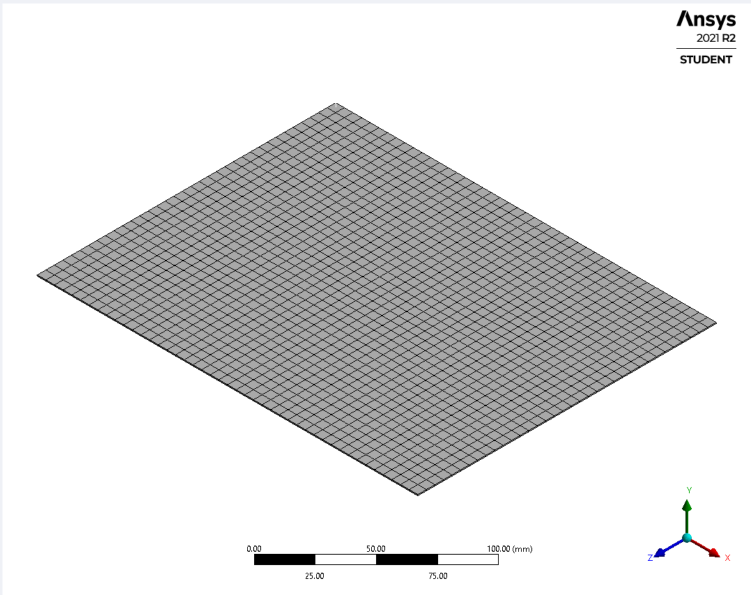
Figure 1: Plate geometry.



2.2 Mesh Creation

The *Element Size* was set to 5.0000 mm. The mesh is shown in Fig. 2

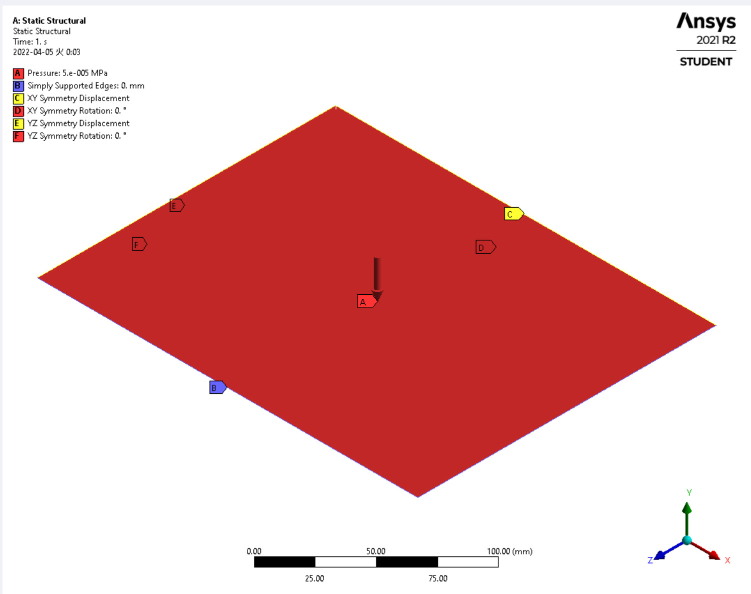
Figure 2: Mesh.



2.3 Physical Setup

The setup for our problem includes the symmetry conditions and the uniform load. The symmetry condition is satisfied by a *Displacement* and a *fixed rotation* reative to the symmetry planes. The pressure load is applied on the top face. This setup is shown in Fig. 3. We also have to apply a material, and in this case we will use the software's default material, called *Structural Steel*. The relevant properties of this material for this study are its Young's modulus, $E = 2.0000 \times 10^{11}$ Pa, and Poisson's Ratio, $\nu = 0.300\,00$, the same used in Section 1.

Figure 3: Setup conditions.



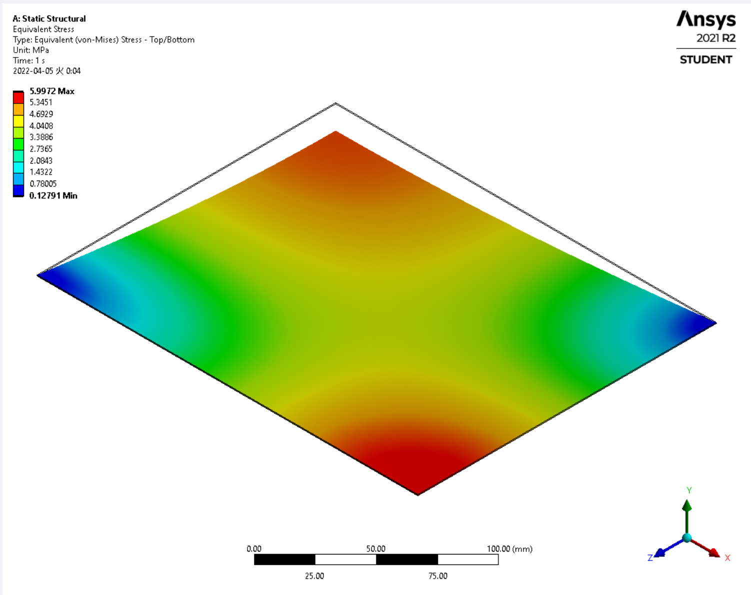
2.4 Results

Finally, the software can solve our model, and we can calculate the desired values. We are interested in the maximum deflection and maximum normal stress. A more general result, showing (scaled) deformation and Von-Mises stress is also shown in Fig. 4. The maximum deflection and maximum normal stress calculated by the software are shown in Table 2.

Table 2: Numerical Results.

Variable	Value	Unit
$w_{\max}$	0.938 69	mm
$\sigma_{\max}$	6.0030	MPa

Figure 4: Von-mises Stress distribution in the plate.



3 Method comparison

To compare the two methods, we can calculate the relative error for each value. Table 3 shows the compiled results and relative errors. All errors were smaller than 0.100 00 %.

Table 3: Error between the numerical and analytical solutions

Variable	Numerical	Analytical	Unit	Relative Error %
$\sigma_{\max}$	6.0030	6.0015	MPa	0.025 501
$w_{\max}$	0.938 69	0.938 58	mm	0.011 713