MECÂNICA DE ESTRUTURAS AEROESPACIAIS

Problems of practical classes of the chapter
BENDING OF THIN PLATES

Problem 7.1

A plate 10 mm thick is subjected to bending moments M_x equal to 10 Nm/mm and M_y equal to 5 Nm/mm. Calculate the maximum direct stresses in the plate.

Ans. $\sigma_{x,max} = \pm 600$ N/mm^2, $\sigma_{y,max} = \pm 300$ N/mm^2.

Solution

Substituting for
$((1/\rho_x) + (\nu/\rho_y))$ and $((1/\rho_y) + (\nu/\rho_x))$ from Eqs

$$M_x = D\left(\frac{1}{\rho_x} + \frac{\nu}{\rho_y}\right) \qquad M_y = D\left(\frac{1}{\rho_y} + \frac{\nu}{\rho_x}\right)$$

respectively in Eqs

$$\left.\begin{aligned} \sigma_x &= \frac{Ez}{1-\nu^2}\left(\frac{1}{\rho_x} + \frac{\nu}{\rho_y}\right) \\ \sigma_y &= \frac{Ez}{1-\nu^2}\left(\frac{1}{\rho_y} + \frac{\nu}{\rho_x}\right) \end{aligned}\right\}$$

gives

$$\sigma_x = \frac{Ez}{1-\nu^2}\frac{M_x}{D} \quad \text{and} \quad \sigma_y = \frac{Ez}{1-\nu^2}\frac{M_y}{D}.$$

Hence, since, from Eq.

$$D = \int_{-t/2}^{t/2} \frac{Ez^2}{1-\nu^2}\mathrm{d}z = \frac{Et^3}{12(1-\nu^2)}$$

Eqs for σ_x and σ_y become

$$\sigma_x = \frac{12zM_x}{t^3} \qquad \sigma_y = \frac{12zM_y}{t^3}$$

The maximum values of σ_x and σ_y will occur when $z = \pm t/2$.
Hence

$$\sigma_x(\max) = \pm\frac{6M_x}{t^2} \qquad \sigma_y(\max) = \pm\frac{6M_y}{t^2}$$

Then

$$\sigma_x(\max) = \pm\frac{6 \times 10 \times 10^3}{10^2} = \pm 600\,\mathrm{N/mm^2}$$

$$\sigma_y(\max) = \pm\frac{6 \times 5 \times 10^3}{10^2} = \pm 300\,\mathrm{N/mm^2}$$

Problem 7.2

For the plate and loading of problem **P.7.1** find the maximum twisting moment per unit length in the plate and the direction of the planes on which this occurs.

Ans. 2.5 Nm/mm at 45° to the *x* and *y* axes.

Solution

From Eq.

$$M_{\mathrm{t}} = \frac{(M_x - M_y)}{2} \sin 2\alpha + M_{xy} \cos 2\alpha$$

and since $M_{xy} = 0$

$$M_t = \frac{M_x - M_y}{2} \sin 2\alpha$$

M_t will be a maximum when $2\alpha=\pi/2$, i.e. $\alpha=\pi/4$ (45°). Thus, from Eq. (i)

$$M_t(\max) = \frac{10 - 5}{2} = 2.5\,\mathrm{Nm/mm}$$

Problem 7.3

The plate of the previous two problems is subjected to a twisting moment of 5 Nm/mm along each edge, in addition to the bending moments of M_x = 10 Nm/mm and M_y =5Nm/mm. Determine the principal moments in the plate, the planes on which they act and the corresponding principal stresses.

Ans. 13.1 Nm/mm, 1.9Nm/mm, α=–31.7◦, α=+58.3◦, ±786 N/mm^2, ±114 N/mm^2.

Solution

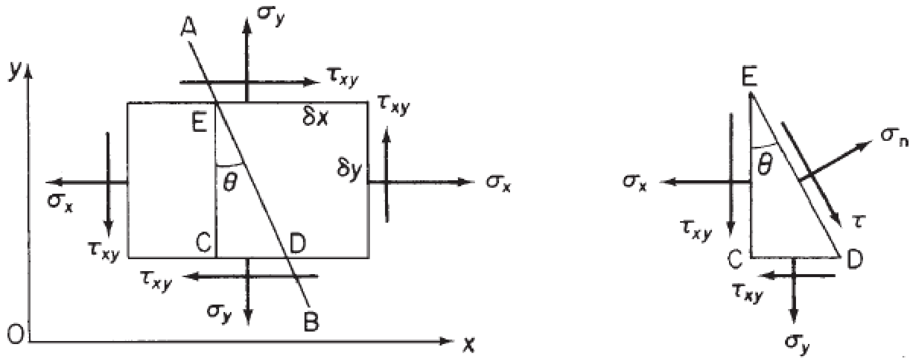
The relationship between M_n and M_x, M_y and M_{xy} in Eq.

$$M_n = M_x \cos^2\alpha + M_y \sin^2\alpha - M_{xy}\sin 2\alpha$$

and between M_t and M_x, M_y and M_{xy} in Eq.

$$M_t = \frac{(M_x - M_y)}{2}\sin 2\alpha + M_{xy}\cos 2\alpha$$

are identical in form to the stress relationships in Eqs (in accordance with the given fig.)



$$\sigma_n = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + \tau_{xy}\sin 2\theta \qquad \tau = \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

Therefore, by deduction from Eqs

$$\sigma_I = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \quad \sigma_{II} = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

we have

$$M_I = \frac{M_x + M_y}{2} + \frac{1}{2}\sqrt{(M_x - M_y)^2 + 4M_{xy}^2} \quad M_{II} = \frac{M_x + M_y}{2} - \frac{1}{2}\sqrt{(M_x - M_y)^2 + 4M_{xy}^2}$$

Further, Eq.

$$M_{\mathrm{t}} = \frac{(M_x - M_y)}{2} \sin 2\alpha + M_{xy} \cos 2\alpha$$

gives the inclination of the planes on which the principal moments occur, i.e. when $M_t = 0$. Thus

$$\tan 2\alpha = -\frac{2M_{xy}}{M_x - M_y}$$

Substituting the values $M_x = 10$ Nm/mm, $M_y = 5$ Nm/mm and $M_{xy} = 5$ Nm/mm in all derived Eqs gives

$$M_{\mathrm{I}} = 13.1\,\mathrm{Nm/mm}$$

$$M_{\mathrm{II}} = 1.9\,\mathrm{Nm/mm}$$

$$\alpha = -31.7^\circ \quad \text{or} \quad 58.3^\circ$$

The corresponding principal stresses are obtained directly from Eqs

$$\sigma_x(\max) = \pm\frac{6M_x}{t^2} \qquad \sigma_y(\max) = \pm\frac{6M_y}{t^2}$$

Hence

$$\sigma_{\mathrm{I}} = \pm\frac{6 \times 13.1 \times 10^3}{10^2} = \pm 786\,\mathrm{N/mm^2}$$

$$\sigma_{\mathrm{II}} = \pm\frac{6 \times 1.9 \times 10^3}{10^2} = \pm 114\,\mathrm{N/mm^2}$$