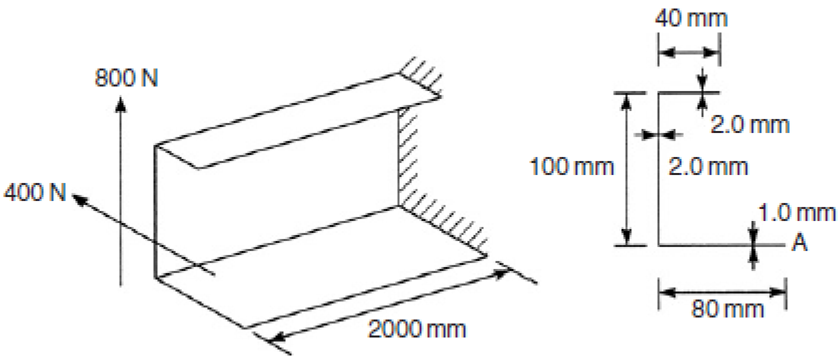
MECÂNICA DE ESTRUTURAS AEROESPACIAIS
Problems of practical classes of the chapter
Bending of the beams

Exercise 1 (Problem 2)

A thin-walled, cantilever beam of unsymmetrical cross-section supports shear loads at its free end as shown in Fig. below. Calculate the value of direct stress at the extremity of the lower flange (point A) at a section half-way along the beam if the position of the shear loads is such that no twisting of the beam occurs.

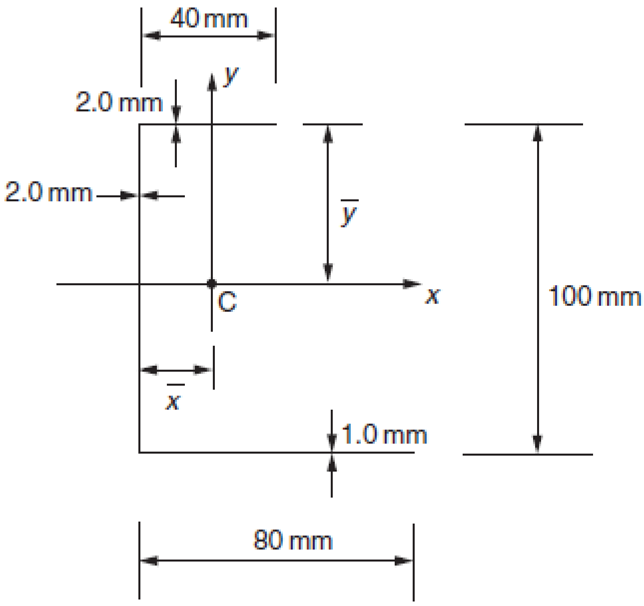


Solution

The bending moments half-way along the beam are

$$M_x = -800 \times 1000 = -800\,000 \text{ N mm}$$
$$M_y = 400 \times 1000 = 400\,000 \text{ N mm}$$

By inspection the centroid of area is midway between the flanges.
{y'·40·20 – (50 – y') ·100·2 – (100 – y') ·80·1 = 0, i.e. y' = 50 mm}



Its distance x' from the vertical web is given by

$$(40 \times 2 + 100 \times 2 + 80 \times 1)\bar{x} = 40 \times 2 \times 20 + 80 \times 1 \times 40$$

i.e. x = 13.33 mm

The second moments of area of the cross-section are calculated using the approximations for thin-walled sections. Then

$$I_{xx} = 40 \times 2 \times 50^2 + 80 \times 1 \times 50^2 + \frac{2 \times 100^3}{12} = 5.67 \times 10^5 \text{ mm}^4$$

$$I_{yy} = 100 \times 2 \times 13.33^2 + \frac{2 \times 40^3}{12} + 2 \times 40 \times 6.67^2 + \frac{1 \times 80^3}{12} + 1 \times 80 \times 26.67^2 = 1.49 \times 10^5 \text{ mm}^4$$

$$I_{xy} = 40 \times 2(6.67)(50) + 80 \times 1(26.67)(-50) = -0.8 \times 10^5 \text{ mm}^4$$

The denominator in Eq.

$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) y$$

is

$$(5.67 \times 1.49 - 0.8^2) \times 10^{10} = 7.81 \times 10^{10}.$$

Then direct stress

$$\sigma = \left(\frac{400\,000 \times 5.67 \times 10^5 - 800\,000 \times 0.8 \times 10^5}{7.81 \times 10^{10}}\right) x + \left(\frac{-800\,000 \times 1.49 \times 10^5 + 400\,000 \times 0.8 \times 10^5}{7.81 \times 10^{10}}\right) y$$

i.e.

$$\sigma = 2.08x - 1.12y$$

and at the point A where x = 66.67 mm, y= − 50mm

$$\sigma(\text{A}) = 194.7 \text{ N/mm}^2 \text{ (tension)}.$$