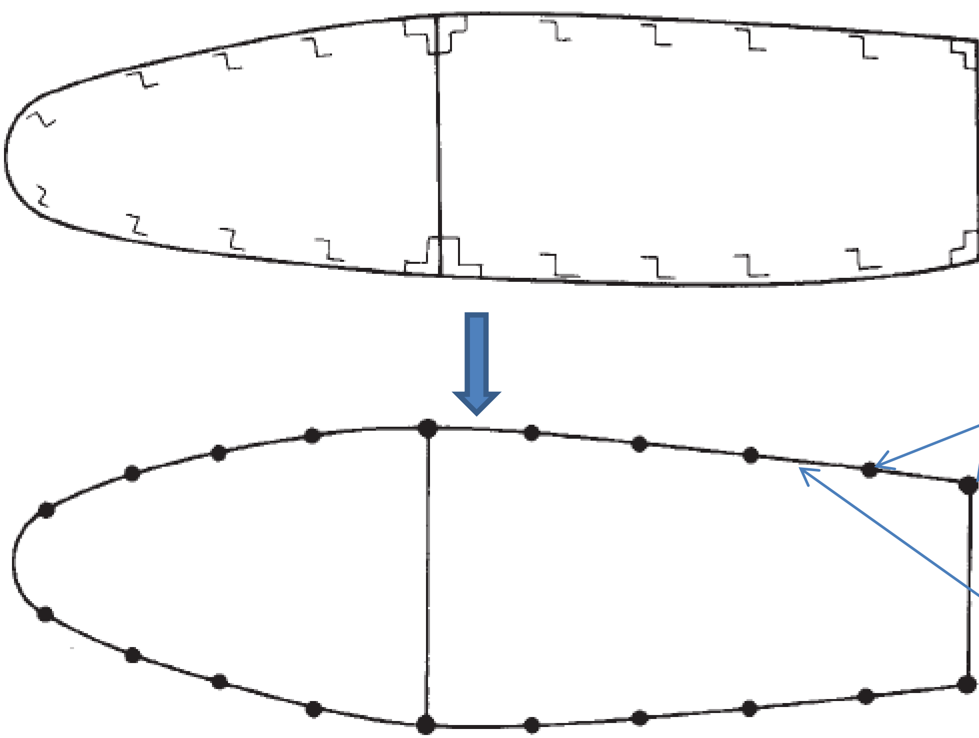
Structural idealization

2020

Principle

stringers have small cross-section compared with the complete section → variation of stress over the cross-section of a stringer is small

the difference between the distances of the stringer centroids and the adjacent skin from the wing section axis is small → the σ is constant over the stringer cross-sections



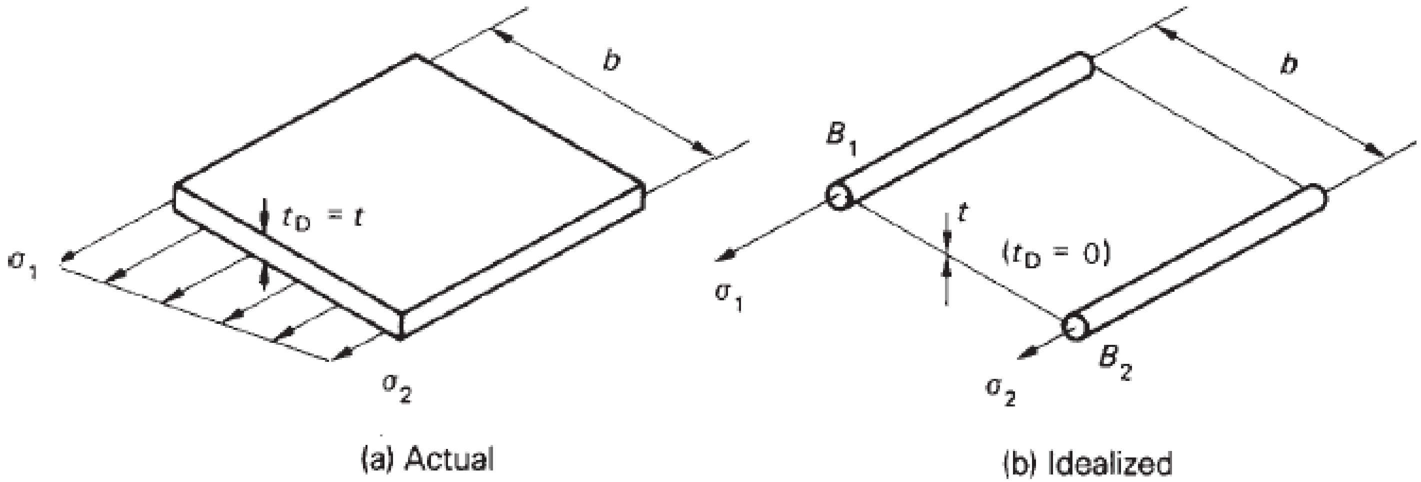
Boom:

- *σ = const;*
- *located in mid-line of skin;*
- *carries only direct stresses.*

***Skin** is effective only in shear (although can carry some of the direct stresses)*

Idealization of a panel

Idealize the panel into a combination of direct stress carrying booms and shear stress only carrying skin:



Taking moments about the right-hand edge of each panel:

$$\sigma_2 t_D \frac{b^2}{2} + \frac{1}{2}(\sigma_1 - \sigma_2) t_D b \frac{2}{3} b = \sigma_1 B_1 b$$

different loading conditions require different idealizations of the same structure

$$B_1 = \frac{t_D b}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right)$$

$$B_2 = \frac{t_D b}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right)$$

$\sigma_1/\sigma_2 = -1$ (bending moment): $\boxed{B_1 = B_2 = t_D b/6}$

$\sigma_1/\sigma_2 = 1$ (axial load): $\boxed{B_1 = B_2 = t_D b/2}$

Idealization of part of a wing section

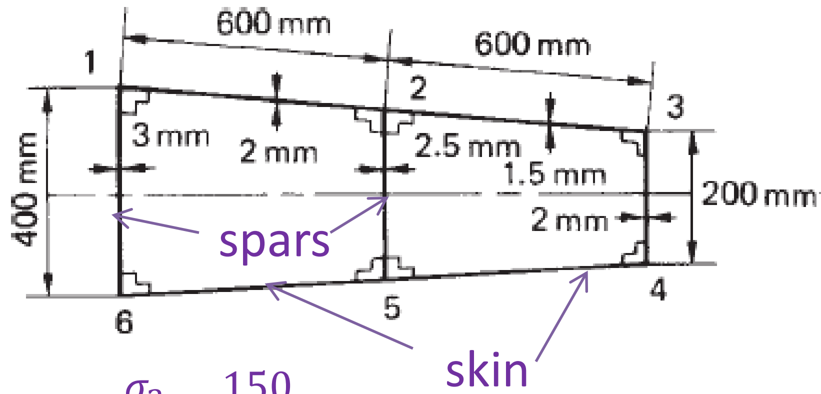
Idealize, if

- booms carry direct stresses;
- panels carry shear stresses;
- section resists bending moments in vertical plane;
- booms can be applied at the spar/skin junctions.

Solution:

Area of the booms: $\boxed{B_1 = \frac{t_D b}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right)}$

Angle sections area = 300 mm²



$$B_1 = \underbrace{300}_{\text{angle}} + \underbrace{\frac{3 \cdot 400}{6}\left(2 + \frac{\sigma_6}{\sigma_1}\right)}_{\text{spar 1-6}} + \underbrace{\frac{2 \cdot 600}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right)}_{\text{skin 1-2}}$$

$$\frac{\sigma_6}{\sigma_1} = -1 \qquad \frac{\sigma_2}{\sigma_1} = \frac{150}{200}$$

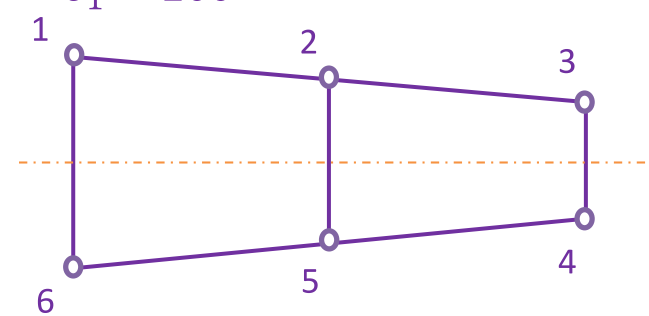
$$\boxed{B_1 = B_6 = 1050mm^2}$$

$$B_2 = 2 \cdot 300 + \frac{2 \cdot 600}{6}\left(2 + \overset{= \frac{200}{150}}{\frac{\sigma_1}{\sigma_2}}\right) + \frac{2.5 \cdot 300}{6}\left(2 + \overset{= -1}{\frac{\sigma_5}{\sigma_2}}\right) + \frac{1.5 \cdot 600}{6}\left(2 + \underset{= \frac{100}{150}}{\frac{\sigma_3}{\sigma_2}}\right)$$

$$\boxed{B_2 = B_5 = 1791.7mm^2}$$

$$B_3 = 300 + \frac{1.5 \cdot 600}{6}\left(2 + \overset{= \frac{150}{100}}{\frac{\sigma_2}{\sigma_3}}\right) + \frac{2 \cdot 200}{6}\left(2 + \overset{= -1}{\frac{\sigma_4}{\sigma_3}}\right)$$

$$\boxed{B_3 = B_4 = 891.7mm^2}$$



Idealized structure

Bending

If idealized cross-section comprises

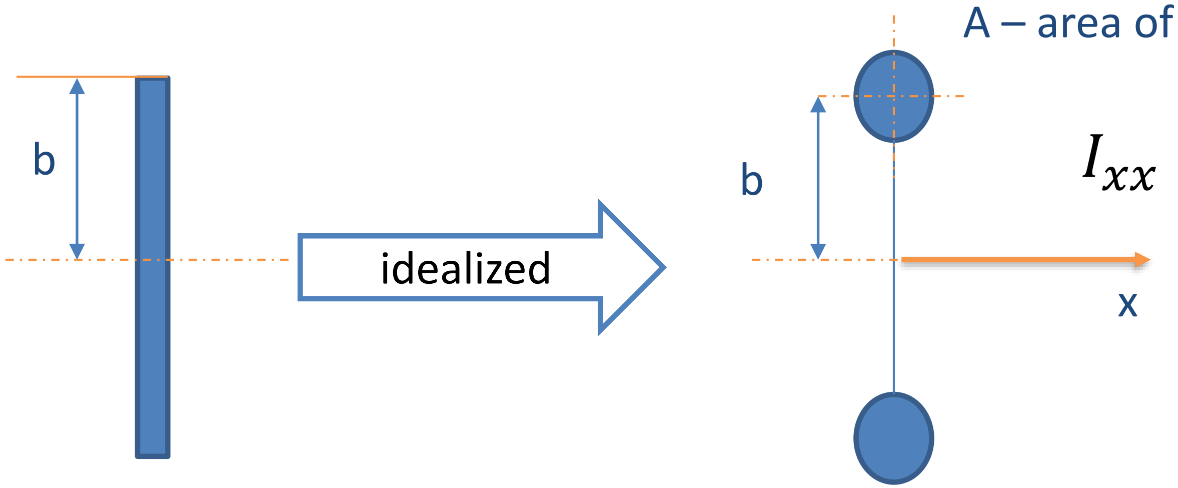
- direct stress carrying booms,
- shear stress only carrying skin ($t_D = 0$);

then

- neutral axis,
- sectional properties (I_{xx}, I_{yy}, I_{xy})

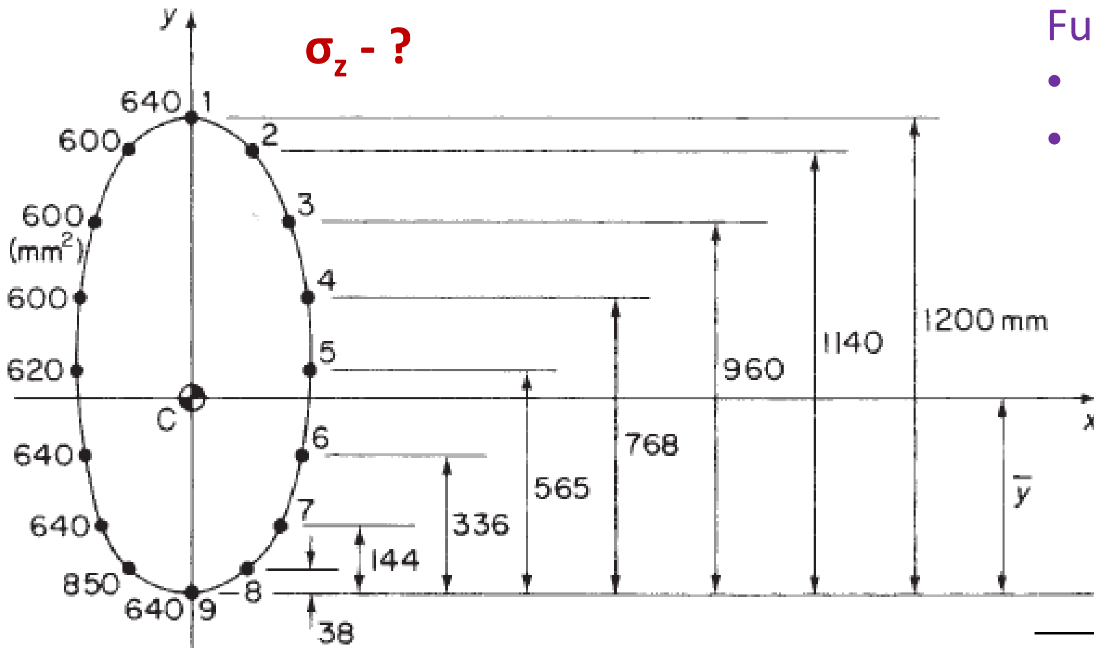
are defined only by area of booms.

For instance



$$I_{xx} = 2Ab^2$$

Bending



σ_z - ?

Fuselage section:

- subjected to M_B = 100 kN·m;
- completely idealized into a combination of
  - direct stress carrying booms,
  - shear stress carrying panels.

$$\boxed{\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)y} \Rightarrow$$

$$\sigma_z = \frac{M_x}{I_{xx}} y$$

Centroid from boom 9:

$$(6 \times 640 + 6 \times 600 + 2 \times 620 + 2 \times 850)\bar{y}$$
$$= 640 \times 1200 + 2 \times 600 \times 1140 + 2 \times 600 \times 960 + 2 \times 600 \times 768$$
$$+ 2 \times 620 \times 565 + 2 \times 640 \times 336 + 2 \times 640 \times 144 + 2 \times 850 \times 38$$

$$\Rightarrow \quad \bar{y} = 540 \text{ mm}$$

$$I_{xx.4} = By^2 = 600 \cdot (768 - 540)^2 = 31 \cdot 10^6 mm^4$$

$$\sigma_{z.4} = 100 \cdot 10^6 N \cdot mm \cdot (768 - 540) mm / 1854 \cdot 10^6 mm^4$$

$$\sigma_{z.4} = 12.3 \; N/mm^2$$

① Boom	② y (mm)	③ B (mm²)	④ $\Delta I_{xx} = By^2$ (mm⁴)	⑤ σ_z (N/mm²)
1	+660	640	278×10^6	35.6
2	+600	600	216×10^6	32.3
3	+420	600	106×10^6	22.6
4	+228	600	31×10^6	12.3
5	+25	620	0.4×10^6	1.3
6	−204	640	27×10^6	−11.0
7	−396	640	100×10^6	−21.4
8	−502	850	214×10^6	−27.0
9	−540	640	187×10^6	−29.0

Sum of terms in column 4: $I_{xx} = 1854 \times 10^6 \text{ mm}^4$

Shear of open section beams

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s t_D x \, ds - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s t_D y \, ds$$

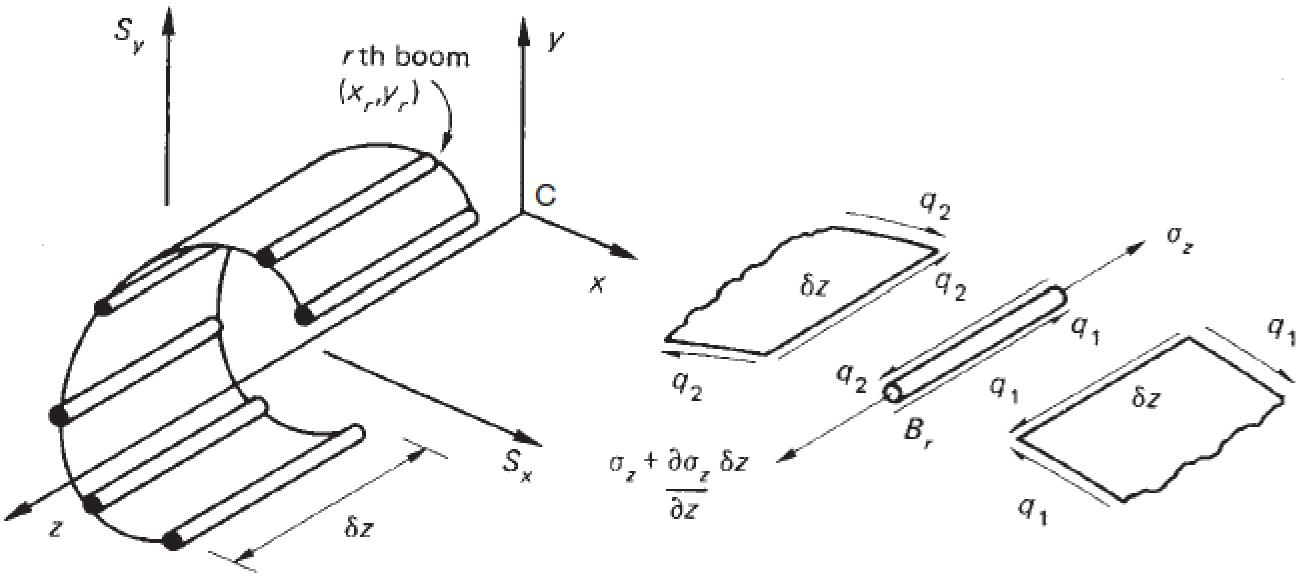
skin effective in carrying direct stress

$t_D = t$

skin carries only shear stresses

$t_D = 0$

no provision for the effects of booms



$\sum F_z = 0$ $\quad \left(\sigma_z + \frac{\partial \sigma_z}{\partial z}\delta z\right) B_r - \sigma_z B_r + q_2 \delta z - q_1 \delta z = 0 \quad \Rightarrow \quad q_2 - q_1 = -\frac{\partial \sigma_z}{\partial z} B_r$

Shear of open section beams

$$q_2 - q_1 = -\frac{\partial \sigma_z}{\partial z} B_r$$

$$\frac{\partial \sigma_z}{\partial z} = \frac{[(\partial M_y/\partial z)I_{xx} - (\partial M_x/\partial z)I_{xy}]}{I_{xx}I_{yy} - I_{xy}^2}x + \frac{[(\partial M_x/\partial z)I_{yy} - (\partial M_y/\partial z)I_{xy}]}{I_{xx}I_{yy} - I_{xy}^2}y$$

the same procedure of derivation as in lecture for shear

$$q_2 - q_1 = -\left[\frac{(\partial M_y/\partial z)I_{xx} - (\partial M_x/\partial z)I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right]B_r x_r - \left[\frac{(\partial M_x/\partial z)I_{yy} - (\partial M_y/\partial z)I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right]B_r y_r$$

$$\partial M_y/\partial z = S_x$$

Coordinates of a boom under consideration

$$\boxed{q_2 - q_1 = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)B_r x_r - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)B_r y_r}$$

the change in shear flow induced <u>by a boom</u>

At any distance ***s*** around the profile of the section, ***n*** booms have been passed, the shear flow at the point is given by

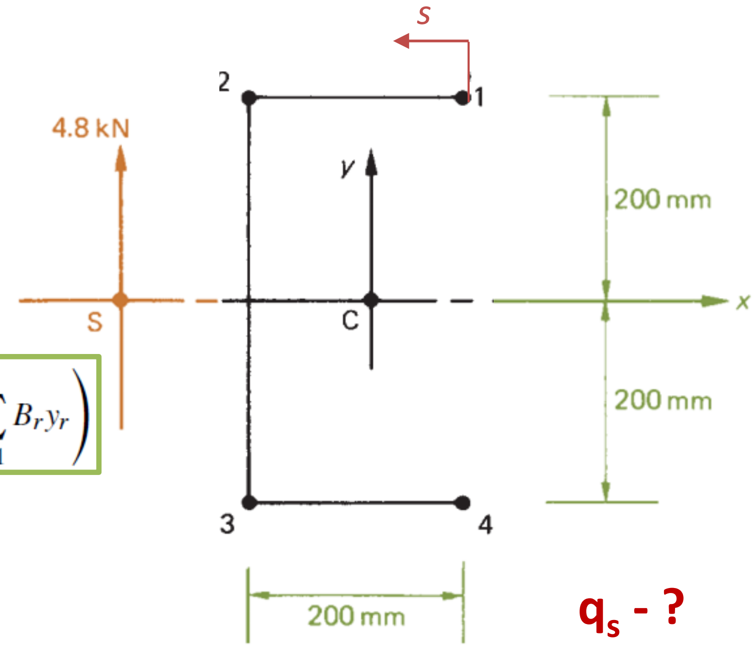
$$\boxed{q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D x \, ds + \sum_{r=1}^{n} B_r x_r\right) - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D y \, ds + \sum_{r=1}^{n} B_r y_r\right)}$$

The terms are taken into consideration only in case when **skin effective in carrying direct stress**

Shear of open section beams

Channel section with area of each boom 300 mm^2:

- shear load acts through SC;
- assume that
  - booms carry all the direct stresses,
  - walls are effective only in resisting shear stresses.



q_s - ?

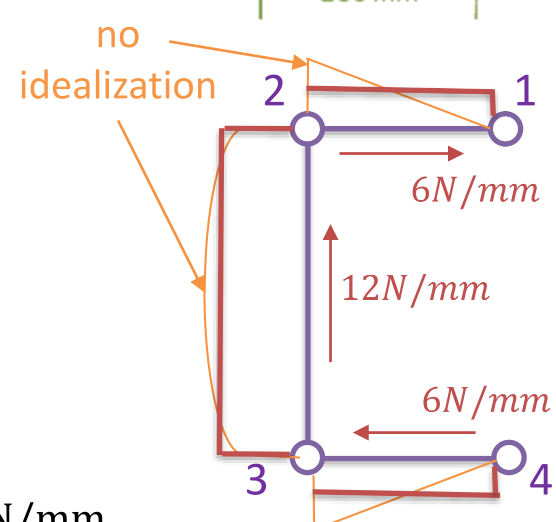
Solution:

$$\boxed{q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D x \, ds + \sum_{r=1}^{n} B_r x_r\right) - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D y \, ds + \sum_{r=1}^{n} B_r y_r\right)}$$

$$I_{xy} = 0 \qquad S_x = 0 \qquad t_D = 0$$

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r \qquad I_{xx} = 4 \cdot 300 \cdot 200^2 = 4.8 \cdot 10^7 mm^4$$

$$q_s = -\frac{4.8 \cdot 10^3 N}{4.8 \cdot 10^7 mm^4} \sum_{r=1}^{n} B_r y_r = \boxed{-0.1 \cdot 10^{-3} \sum_{r=1}^{n} B_r y_r}$$



Before the boom 1 (outside the section): $q_s = 0$

After the boom 1: $\Delta q_1 = -0.1 \cdot 10^{-3} \cdot 300 \cdot 200 = -6N/mm$

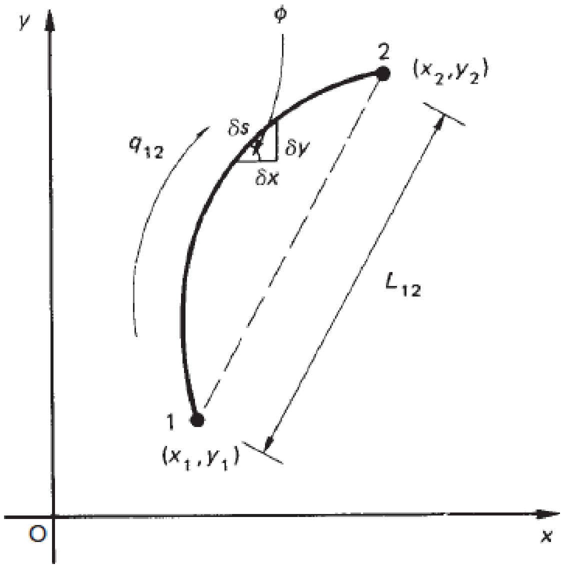
Thus $q_{12} = -6N/mm$

$$q_{23} = -6 - 0.1 \cdot 10^{-3} \cdot 300 \cdot 200 = -12\text{N/mm}$$

$$q_{34} = -12 - 0.1 \cdot 10^{-3} \cdot 300 \cdot (-200) = -6\text{N/mm}$$

Outside the boom 4 $\Delta q_4 = -6 - 0.1 \cdot 10^{-3} \cdot 300 \cdot (-200) = 0\text{N/mm}$

Resultant shear load of curved web



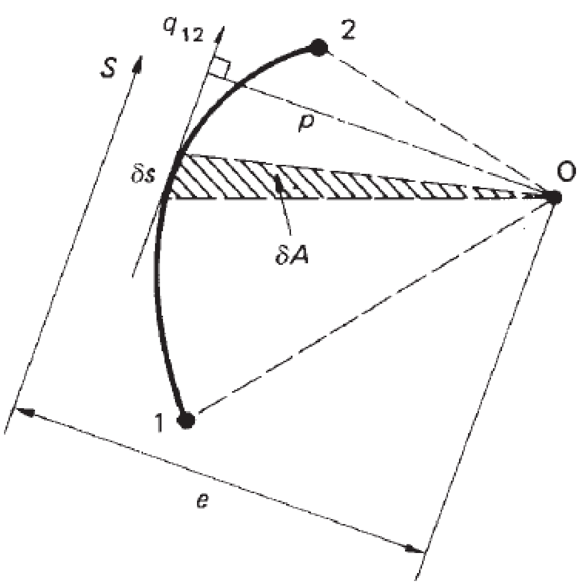
$$S_x = \int_1^2 q_{12} \cos\phi \, \mathrm{d}s \xrightarrow{q_{12} = \text{const}} S_x = q_{12} \int_1^2 \cos\phi \, \mathrm{d}s$$

$$S_x = q_{12} \int_1^2 \mathrm{d}x = q_{12}(x_2 - x_1)$$

$$S_y = q_{12}(y_2 - y_1)$$

$$S = \sqrt{S_x^2 + S_y^2} = q_{12}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\boxed{S = q_{12} L_{12}}$$



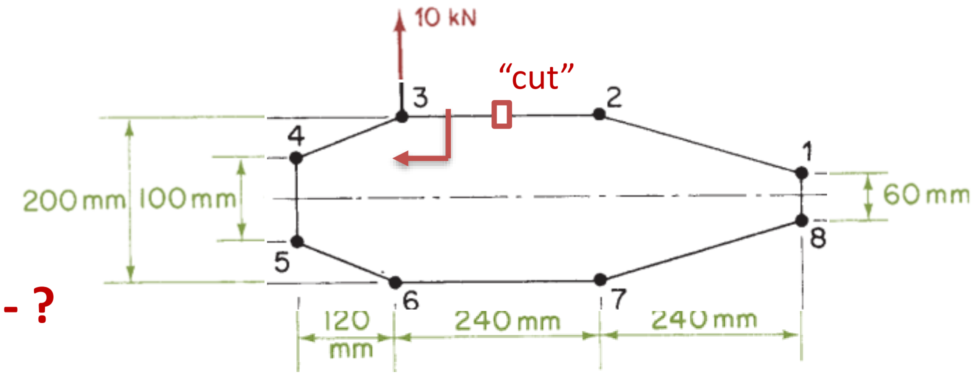
$$M_q = \int_1^2 q_{12} p \, \mathrm{d}s = q_{12} \int_1^2 2 \, \mathrm{d}A \Rightarrow M_q = 2Aq_{12}$$

$$Se = 2Aq_{12} \Rightarrow e = \frac{2A}{S} q_{12} \Rightarrow \boxed{e = \frac{2A}{L_{12}}}$$

Shear of closed section beams

Idealization

- booms carry all the direct stresses,
- walls are effective only in resisting shear stresses.



Booms area: $B_1 = B_8 = 200\ mm^2$, $B_2 = B_7 = 250\ mm^2$, $B_3 = B_6 = 400\ mm^2$, $B_4 = B_5 = 100\ mm^2$

q_s - ?

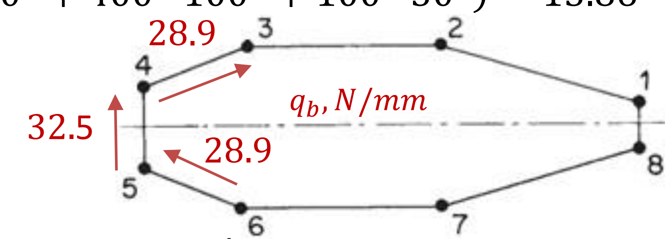
Solution:

$$\boxed{q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D x\, ds + \sum_{r=1}^{n} B_r x_r\right) - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D y\, ds + \sum_{r=1}^{n} B_r y_r\right)}$$

$$I_{xy} = 0 \quad S_x = 0 \quad t_D = 0$$

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r + q_{s,0} \qquad I_{xx} = 2(200 \cdot 30^2 + 250 \cdot 100^2 + 400 \cdot 100^2 + 100 \cdot 50^2) = 13.86 \cdot 10^6 mm^4$$

$$\boxed{q_s = -7.22 \cdot 10^{-4} \sum_{r=1}^{n} B_r y_r + q_{s,0}}$$



Web 2-3: $q_{b,23} = 0 N/mm$

Web 3-4: $y_3 = 100 \quad q_{b,34} = -7.22 \cdot 10^{-4}(400 \cdot 100) + 0 = -28.9 N/mm$

Web 4-5: $y_4 = 50 \quad q_{b,34} = -7.22 \cdot 10^{-4}(100 \cdot 50) - 28.9 = -32.5 N/mm$

Web 5-6: $q_{b,56} = q_{b,34} = -28.9 N/mm$

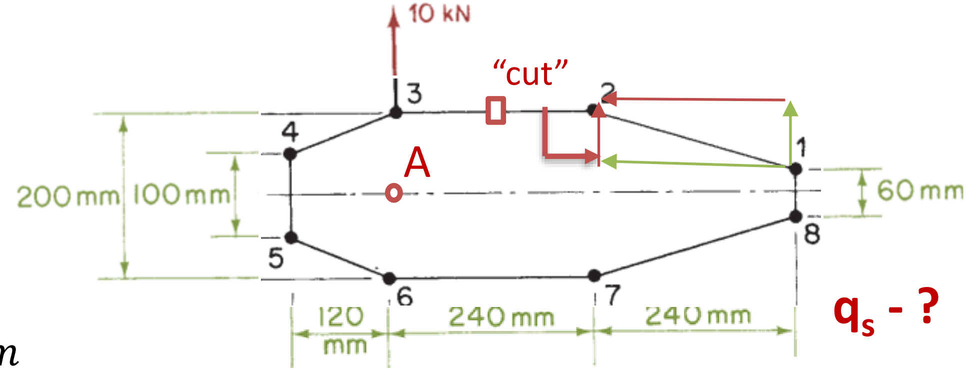
Web 6-7: $q_{b,67} = q_{b,23} = 0 N/mm$

due to symmetry

Shear of closed section beams

Booms area: $B_1 = B_8 = 200\ mm^2$, $B_2 = B_7 = 250\ mm^2$, $B_3 = B_6 = 400\ mm^2$, $B_4 = B_5 = 100\ mm^2$

$$\boxed{q_s = -7.22 \cdot 10^{-4} \sum_{r=1}^{n} B_r y_r + q_{s,0}}$$



q_s - ?

Web 2-1: $y_3 = 100$

$$q_{b,21} = -7.22 \cdot 10^{-4}(250 \cdot 100) + 0 = -18.1 N/mm$$

Web 1-8: $y_1 = 30$

$$q_{b,18} = -7.22 \cdot 10^{-4}(200 \cdot 30) - 18.1 = -22.4 N/mm$$

Web 8-7: $q_{b,87} = q_{b,21} = -18.1 N/mm$ (symmetry)

To determine $q_{s,0}$: $\boxed{0 = \oint p q_b \, ds + 2A q_{s,0}} \Rightarrow q_{s,0} = -\dfrac{\oint p q_b ds}{2A}$

Taking moment about p. A (clockwise is negative):

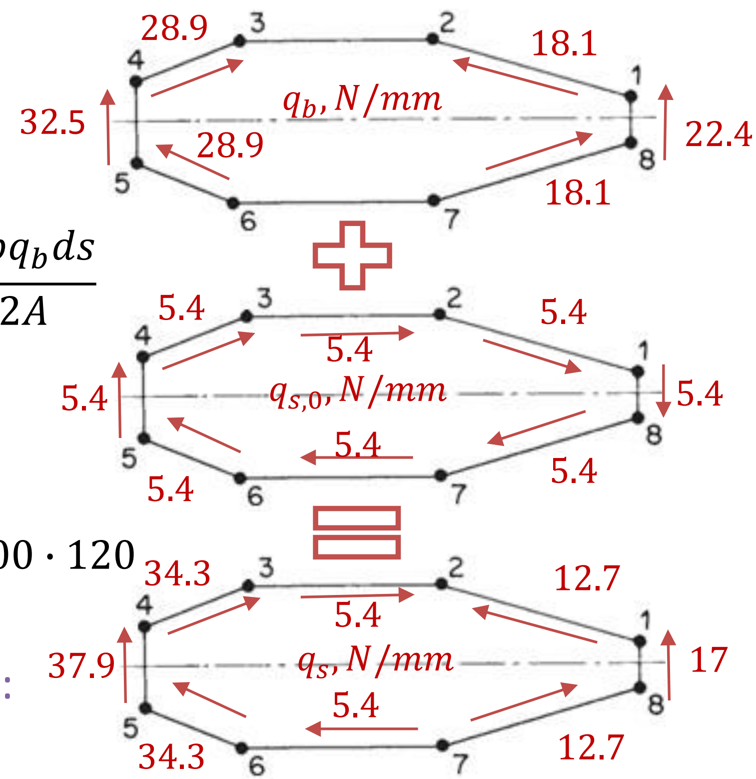
$$\oint p q_b ds = q_{b,81} \cdot \overset{ds}{60} \cdot \overset{p}{480} + 2q_{b,12} \cdot \begin{Bmatrix} 240 \cdot 100 + 70 \cdot 240 \\ 240 \cdot 30 + 70 \cdot 480 \end{Bmatrix}$$

$$+ q_{b,23} \cdot 240 \cdot 100 - 2q_{b,34} \cdot \begin{Bmatrix} 120 \cdot 50 + 50 \cdot 120 \\ 120 \cdot 100 + 50 \cdot 0 \end{Bmatrix} - q_{b,45} \cdot 100 \cdot 120$$

$$A = 97200 mm^2$$

$$q_{s,0} = -5.4 N/mm$$

Distribution of shear flow q_s:



Obrigado!