MECÂNICA DE ESTRUTURAS AEROESPACIAIS
Problems of practical classes of the chapter
Thin Shells Under Internal Pressure

Problem 1

A long steel tube, 7.5 cm internal diameter and 0.15 cm thick, has closed ends, and is subjected to internal fluid pressure of 3MPa. If E = 200 GPa and ν = 0.3, estimate the percentage increase in internal volume of the tube.

Solution

The hoop stress is

$$\boldsymbol{\sigma_1 = \frac{pr}{t} = \frac{3000000 \cdot 0.0383}{0.0015} = 76.6\ MPa}$$

The longitudinal stress is

$$\boldsymbol{\sigma_2 = \frac{pr}{2t} = 38.3\ MPa}$$

The hoop strain is

$$\boldsymbol{\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = 0.00033}$$

The longitudinal strain is

$$\boldsymbol{\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) = 0.000077}$$

The volumetric strain is

$$\boldsymbol{2\varepsilon_1 + \varepsilon_2 = 0.000737 = 0.074\%}$$

Problem 2

An air vessel, which is made of steel (E = 200GPa, is ν = 0.3) 2 m long; it has an external diameter of 45 cm and is 1 cm thick. Find the increase of external diameter and the increase of length when charged to an internal air pressure of 1 MPa.

Solution

Radius of the vessel is 0.225 m and hoop stress is

$$\boldsymbol{\sigma_1 = \frac{pr}{t} = \frac{1000000 \cdot 0.225}{0.010} = 22.5\ MPa}$$

The longitudinal stress is

$$\boldsymbol{\sigma_2 = \frac{pr}{2t} = 11.25\ MPa}$$

The hoop strain is

$$\boldsymbol{\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = 0.000096}$$

The longitudinal strain is

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) = 0.000023$$

The increase in internal diameter is
0.450·0.000096 = 0.000043 m = 0.043 mm
The increase in length is
2·0.000023 = 0.000045 m = 0.045 mm

Problem 3

A thin cylindrical shell is subjected to internal fluid pressure, the ends being closed by:

(a) two watertight pistons attached to a common piston rod;
(b) flanged ends.

Find the increase in internal diameter in each case, given that the internal diameter is 200 cm, thickness is 0.5 cm, Poisson's ratio is 0.3, Young's modulus is 200 GPa, and the internal pressure is 3.5 MPa.

Pistons	Flanged ends
The hoop stress is $\sigma_1 = \frac{pr}{t} = \frac{3.5 \cdot 10^6 \cdot 0.1}{0.005} = 70\ MPa$	The hoop stress is $\sigma_1 = \frac{pr}{t} = \frac{3.5 \cdot 10^6 \cdot 0.1}{0.005} = 70\ MPa$
The longitudinal stress is zero	The longitudinal stress is $\sigma_2 = \frac{pr}{2t} = 35\ MPa$
The hoop strain is $\varepsilon_1 = \frac{\sigma_1}{E} = \frac{70 \cdot 10^6}{200 \cdot 10^9} = 0.35 \cdot 10^{-3}$	The hoop strain is $\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = 0.298 \cdot 10^{-3}$
The increase in internal diameter is $0.2 \cdot 0.35 \cdot 10^{-3} = 0.07$ mm	The increase in internal diameter is $0.2 \cdot 0.298 \cdot 10^{-3} = 0.0596$ mm