MECÂNICA DE ESTRUTURAS AEROESPACIAIS

Examples of theoretical classes of the lecture
TWO DIMENSIONAL PROBLEMS IN ELASTICITY

PROBLEMS TO STRESS FUNCTION

Example 3.1
Consider the stress function

$$\phi = Ax^2 + Bxy + Cy^2$$

where A, B and C are constants. Biharmonic equation is identically satisfied since each term becomes zero on substituting for φ. The stresses follow from

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} = 2C$$

$$\sigma_y = \frac{\partial^2 \phi}{\partial x^2} = 2A$$

$$\tau_{xy} = -\frac{\partial^2 \phi}{\partial x\, \partial y} = -B$$

To produce these stresses at any point in a rectangular sheet we require loading conditions providing the boundary stresses shown in Fig. 2.1.

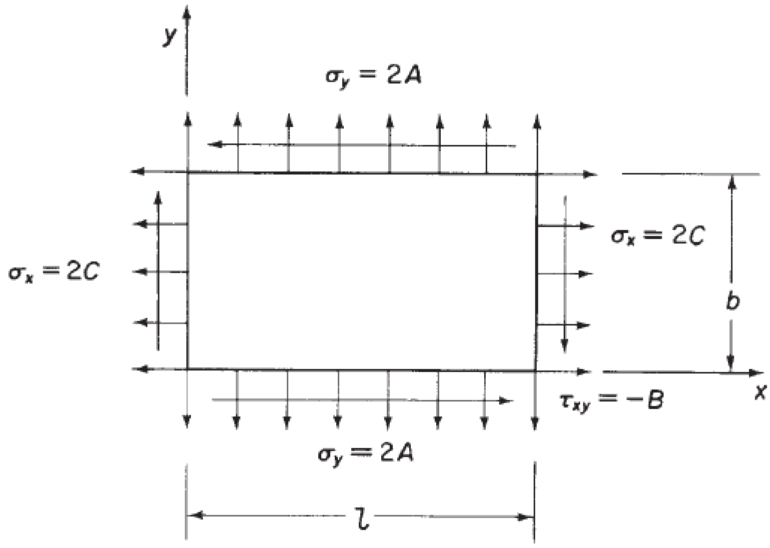

Fig. 2.1 Required loading conditions on rectangular sheet in Example 2.1.

Example 3.2
A more complex polynomial for the stress function is

$$\phi = \frac{Ax^3}{6} + \frac{Bx^2y}{2} + \frac{Cxy^2}{2} + \frac{Dy^3}{6}$$

As before

$$\frac{\partial^4\phi}{\partial x^4} = \frac{\partial^4\phi}{\partial x^2\partial y^2} = \frac{\partial^4\phi}{\partial y^4} = 0$$

so that the compatibility equation is identically satisfied.

The stresses are given by

$$\sigma_x = \frac{\partial^2\phi}{\partial y^2} = Cx + Dy$$

$$\sigma_y = \frac{\partial^2\phi}{\partial x^2} = Ax + By$$

$$\tau_{xy} = -\frac{\partial^2\phi}{\partial x\,\partial y} = -Bx - Cy$$

We may choose any number of values of the coefficients *A*, *B*, *C* and *D* to produce a variety of loading conditions on a rectangular plate.

For example, if we assume
A = B = C = 0
then
σ_x = Dy,
σ_y = 0 and
τ_{xy} = 0,
so that for axes referred to an origin at the mid-point of a vertical side of the plate we obtain the state of pure bending shown in Fig. 2.2(a).

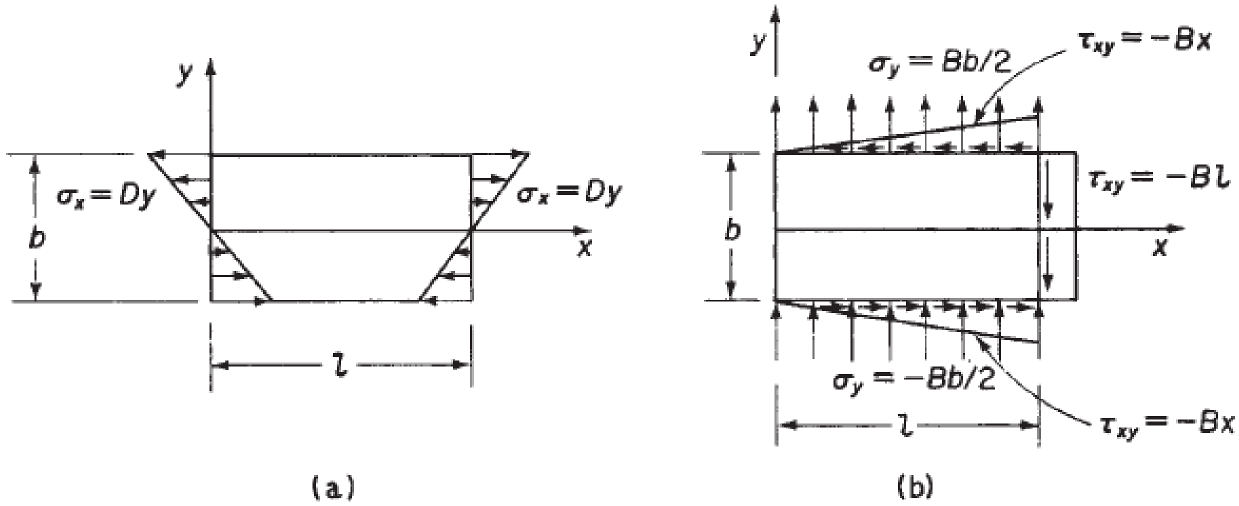

Fig. 2.2 (a) Required loading conditions on rectangular sheet for *A=B=C=0*; (b) as in (a) but *A=C=D=0*.

Alternatively, Fig. 2.2(b) shows the loading conditions corresponding to *A=C=D=0* in which $\sigma_x = 0$, $\sigma_y = By$ and $\tau_{xy} = -Bx$.