

# Thin shells (under internal pressure)

2013

# Thin cylindrical shell

Consider a cylindrical shell under internal pressure  $p$ , which:

- gives rise to a circumferential stress in the wall of the cylinder, and
- producing a longitudinal stress in the walls, if the ends of cylinder are closed.

Consider a unit length of the cylinder:

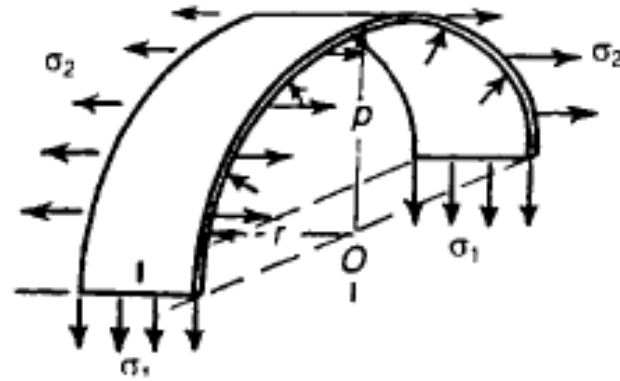
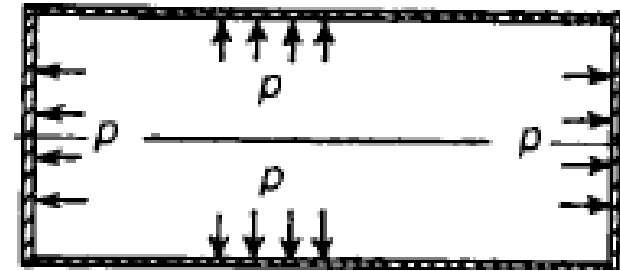
- thickness  $t$  is small compared with radius  $r$ ;
- $\sigma_1$  acts circumferentially,  $\sigma_2$  - longitudinally.

**Equilibrium circumferential:**

$$p \times (2r \times 1) = \sigma_1 \times 2(t \times 1)$$

Then circumferential (or *hoop*) stress:

$$\sigma_1 = \frac{pr}{t}$$



**Equilibrium longitudinal:**

$$p \times \pi r^2 = \sigma_2 \times 2 \pi r t$$

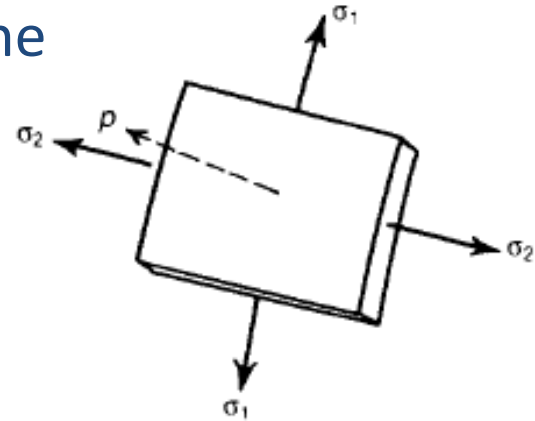
Then longitudinal stress:

$$\sigma_2 = \frac{pr}{2t}$$

# Thin cylindrical shell

The stresses acting on an element of the wall of the cylinder consists:

- circumferential stress  $\sigma_1$ ;
- longitudinal stress  $\sigma_2$ ;
- radial stress (on the internal face)  $p$ .



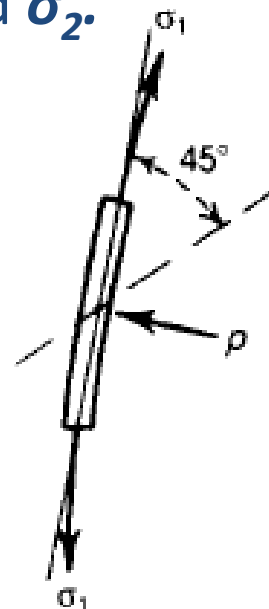
As  $r/t$  much greater than 1, then  $p$  is small compared with  $\sigma_1$  and  $\sigma_2$ , and a system is two-dimensional with principal stresses  $\sigma_1$  and  $\sigma_2$ .

The maximum shearing stress in the plane of principal stresses and in the plane of  $\sigma_2$  and  $p$  correspondingly:

$$\tau_{\max} = \frac{1}{2} (\sigma_1 - \sigma_2) = \frac{pr}{4t} \quad \tau_{\max} = \frac{1}{2} (\sigma_2) = \frac{pr}{4t}$$

The maximum shearing stress in the plane of  $\sigma_1$  and  $p$ :

$$\tau_{\max} = \frac{1}{2} (\sigma_1) = \frac{pr}{2t} \quad \text{- the greatest shearing stress at } 45^\circ$$



# Thin cylindrical shell

The corresponding strains are:

$$\epsilon_1 = \frac{1}{E} (\sigma_1 - \nu \sigma_2) = \frac{pr}{Et} \left( 1 - \frac{1}{2} \nu \right)$$

$$\epsilon_2 = \frac{1}{E} (\sigma_2 - \nu \sigma_1) = \frac{pr}{Et} \left( \frac{1}{2} - \nu \right)$$

The circumference increases by  $2\pi r\epsilon_1$ , i.e. mean radius increases by  $r\epsilon_1$ .

The increase of length is  $l\epsilon_2$ .

The change in internal volume of a unit length:

$$\delta V = \pi (r + r\epsilon_1)^2 (1 + \epsilon_2) - \pi r^2$$

The volumetric strain:  $\frac{\delta V}{\pi r^2} = (1 + \epsilon_1)^2 (1 + \epsilon_2) - 1 \doteq 2\epsilon_1 + \epsilon_2$

or 
$$\frac{\delta V}{\pi r^2} = 2\epsilon_1 + \epsilon_2 = \frac{pr}{Et} \left[ 2 \left( 1 - \frac{1}{2} \nu \right) + \left( \frac{1}{2} - \nu \right) \right] = \frac{pr}{Et} \left( \frac{5}{2} - 2\nu \right)$$

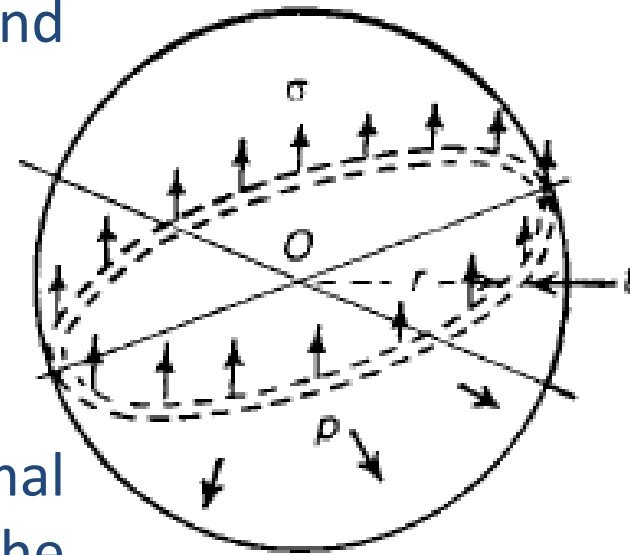
# Thin spherical shell

Consider a thin spherical shell with thickness  $t$  and radius  $r$ , which is subjected to internal pressure  $p$ .

By symmetry  $\sigma$  is the same at all points, then

$$p \times \pi r^2 = \sigma \times 2\pi r t \quad \text{then} \quad \boxed{\sigma = \frac{pr}{2t}}$$

As  $p$  is small compared with  $\sigma$ , the maximal shearing stress occurs on planes at  $45^\circ$  to the tangent plane at any point.



The circumference in any diametral plane is strained an amount

$$\epsilon = \frac{1}{E} (\sigma - \nu\sigma) = (1 - \nu) \frac{\sigma}{E}$$

The volumetric strain of the enclosed volume is

$$3\epsilon = 3(1 - \nu) \frac{\sigma}{E} = 3(1 - \nu) \frac{pr}{2Et}$$

# Cylindrical shell with hemispherical ends

Hemispherical ends have advantage of reducing the bending stresses in the cylinder when the ends are flat.

Consider a cylinder with thicknesses  $t_1$  e  $t_2$ , which proportioned so the radial expansion is the same for both cylinder and hemisphere (to eliminate bending stresses at the junction).

The circumferential strain in the cylinder:

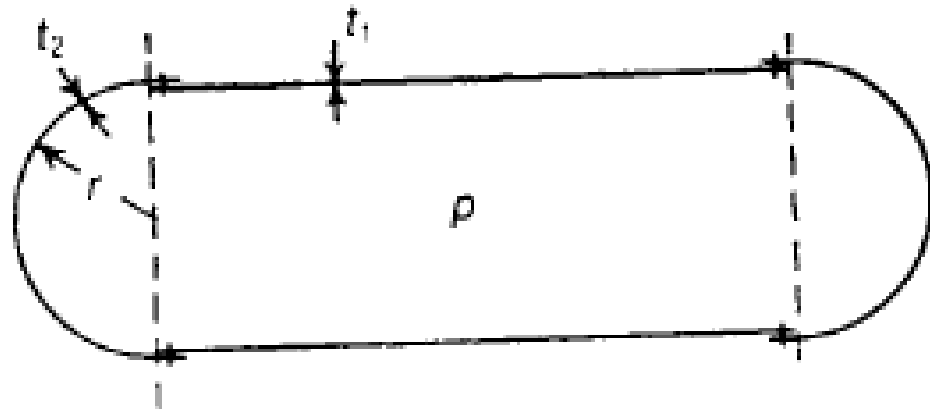
$$\frac{pr}{Et_1} \left( 1 - \frac{1}{2} \nu \right)$$

The circumferential strain in the hemisphere:

$$(1 - \nu) \frac{pr}{2Et_2}$$

If these strains are equal, than  $\frac{pr}{Et_1} \left( 1 - \frac{1}{2} \nu \right) = \frac{pr}{2Et_2} (1 - \nu)$  then

$$\frac{t_1}{t_2} = \frac{2 - \nu}{1 - \nu} \quad \text{for } \nu = 0.3, \text{ then } (t_1/t_2) = 1.7/0.7 = 2.4$$



Obrigado!