Exercício Individual 6

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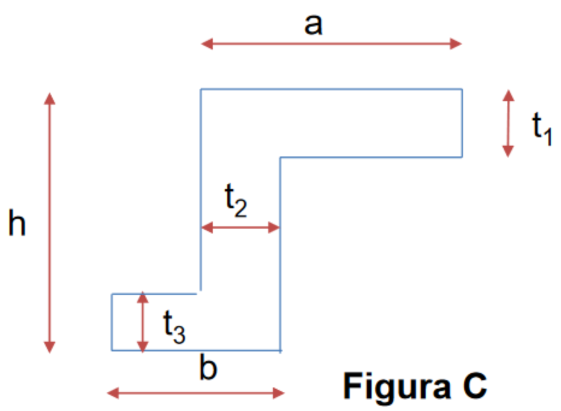
The problem values for option V were tabulated in Table 1, with the column cross-section shown in Fig. 1. We want to determine the critical buckling load P_{CR} and the buckling direction on a column with fixed/free ends, and compare the results for numerical and analytical methods.

Table 1: Values for Option V

Symbol	Value	Unit	Description
L	450	mm	Length of column
a	10	mm	Top flange length
b	10	mm	Bottom flange length
h	18	mm	Web length
t_1	4	mm	Top flange thickness
t_2	3	mm	Web thickness
t_3	5	mm	Bottom flange thickness
E	200	GPa	Modulus of Elasticity
ν	18	-	Poisson's coefficient

Figure 1: Column cross-section for option V



1 Analytical solution

The critical load P_{CR} can be calculated from Eq. (1), where I is the moment of inertia for the buckling direction and l_e is the equivalent length of the column. For our fixed/free ends, $l_e = 2L$.

$$P_{CR} = \frac{\pi^2 EI}{l_e^2} \tag{1}$$

Since the column will buckle in the direction of least resistance, and the modulus of elasticity E is constant (isotropic material), we know buckling will happen around the minimum moment of inertia. The minimum moment of inertia can be calculated from Eq. (3), while the direction of buckling can be calculated from Eq. (2). I_{xx} is the second moment of inertia through the center of gravity around the horizontal axis, while I_{yy} is the second moment of inertia through the center of gravity around the vertical axis of the cross-section.

$$\theta = \frac{1}{2}\arctan\left(\frac{2I_{xy}}{I_{yy} - I_{xx}}\right) \tag{2}$$

$$I = \frac{I_{xx} + I_{yy}}{2} - \sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + I_{xy}^2} \tag{3}$$

We can use Eqs. (4) to (6) to find the moments of inertia used in Eqs. (1) to (3) for our particular cross-section, where $c = h - t_1 - t_2$.

$$\begin{aligned} I_{xx} = {} & \frac{at_1^3}{12} + at_1\left(h - \frac{t_1}{2} - y_{cg}\right)^2 \\ & + \frac{bt_3^3}{12} + bt_3\left(\frac{t_3}{2} - y_{cg}\right)^2 \\ & + \frac{t_2c^3}{12} + t_2c\left(t_3 + \frac{c}{2} - y_{cg}\right)^2 \end{aligned} \tag{4}$$

$$\begin{aligned} I_{yy} = {} & \frac{a^3t_1}{12} + at_1\left(b - t_2 + \frac{a}{2} - x_{cg}\right)^2 \\ & + \frac{b^3t_3}{12} + bt_3\left(\frac{b}{2} - x_{cg}\right)^2 \\ & + \frac{t_2^3c}{12} + t_2c\left(b - \frac{t_2}{2} - x_{cg}\right)^2 \end{aligned} \tag{5}$$

$$\begin{aligned} I_{xy} = {} & at_1\left(b - t_2 + \frac{a}{2} - x_{cg}\right)\left(h - \frac{t_1}{2} - y_{cg}\right) \\ & + bt_3\left(\frac{b}{2} - x_{cg}\right)\left(\frac{t_3}{2} - y_{cg}\right) \\ & + t_2c\left(b - \frac{t_2}{2} - x_{cg}\right)\left(t_3 + \frac{c}{2} - y_{cg}\right) \end{aligned} \tag{6}$$

The coordinates of the center of gravity, (x_{cg}, y_{cg}) can be calculated from Eqs. (7) and (8), measured from the bottom left corner of the cross-section.

$$x_{cg} = \frac{at_1\left(b - t_2 + \frac{a}{2}\right) + bt_3\left(\frac{b}{2}\right) + t_2c\left(b - \frac{t_2}{2}\right)}{at_1 + bt_3 + t_2c} \tag{7}$$

$$y_{cg} = \frac{at_1\left(h - \frac{t_1}{2}\right) + bt_3\left(\frac{t_3}{2}\right) + t_2c\left(t_3 + \frac{c}{2}\right)}{at_1 + bt_3 + t_2c} \tag{8}$$

Substituting the values from Table 1 in Eqs. (7) and (8), we find $x_{cg} \approx 8.201\,\mathrm{mm}$ and $y_{cg} \approx 8.731\,\mathrm{mm}$. Substituting back into Eqs. (4) to (6), we find $I_{xx} \approx 4410\,\mathrm{mm}^4$, $I_{yy} \approx 1862.3\,\mathrm{mm}^4$, and $I_{xy} \approx 2108\,\mathrm{mm}^4$. Finally, substituting back into Eqs. (1) to (3), we find the minimum moment of inertia $I \approx 673.2\,\mathrm{mm}^4$, and the sought values $P_{CR} \approx 1641\,\mathrm{N}$ and $\theta \approx -29.46°$

2 Numerical solution

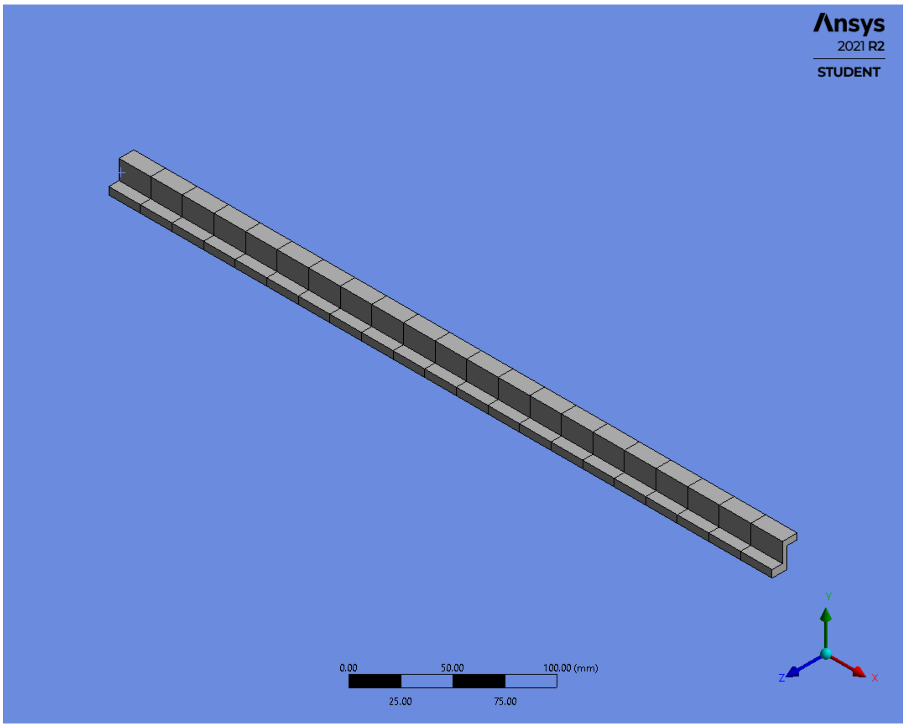
The buckling load can also be calculated numerically using finite element methods software *Ansys*. To solve the problem, we have to input the geometry (finite element mesh), the load case (boundary conditions and applied loads), and interpret the results. We have to use the *Static Structural* analysis option for a preload calculation, coupled with an *Eigenvalue Buckling* analysis for the actual buckling study.

2.1 Geometry

We can use the sofware *DesignModeler* inside *Ansys* to model our geometry and automatically mesh it. The column can be modeled by a *Line Body* of length L, with a *Z Section* cross-section of appropriate dimensions. By default, the *Z Section* is vertically mirrored relative to Fig. 1, so we turn *Reverse Orientation* option on when applying the cross-section to the *Line Body*.

Inside the *Static Structural* model, we set our mesh *Element Order* to *Quadratic* and otherwise use the software's default values, obtaining the mesh shown in Fig. 2. In the 1-D case of the *Line Body*, the mesh does not need to be refined to generate accurate results.

Figure 2: Finite Element Mesh generated by the software



2.2 Load case

In the *Static Structural* analysis, we have to input the load case for the preload. One end of the geometry is set as a fixed support, while the other receives an axial compression load. This analysis calculates the tension distribution across the body, and the *Eigenvalue Buckling* calculates the buckling load as a multiple of the preload. Therefore, we set the unitary load of 1 N to simplify calculations. The load case can be seen in Fig. 3.

We can then add an *Eigenvalue Buckling* analysis to the model, and set the pre-stress environment to the *Static Structural* analysis set up before. We can set *Max Modes to Find* to 1, as we are only interested in the first buckling mode.

2.3 Results

With the Geometry and the Load Case correctly set up, we can solve the model. We are interested in the magnitude of the critical buckling load P_{CR} and the direction of buckling θ. We can add the desired results to find the total deformation and the directional deformations perpendicular to the column axis. In any of the results, the *Load Multiplier* represents P_{CR}. We can use the values of directional deformation and basic trigonometry to find θ. It is important to note that buckling can happen to either side of the buckling axis.

The total deformation results can be seen in Fig. 4. The distribution of directional deformation is identical to the total deformation and, therefore, not relevant to show here; we are interested only in the maximum value in each direction. The buckling direction can be calculated as $\theta = \text{atan}(u_v/u_h)$, where u_v and u_h are the deformations in the vertical and horizontal directions, respectively, relative to Fig. 1. The relevant values are shown in Table 2

Figure 3: Loads and Boundary conditions applied to the geometry

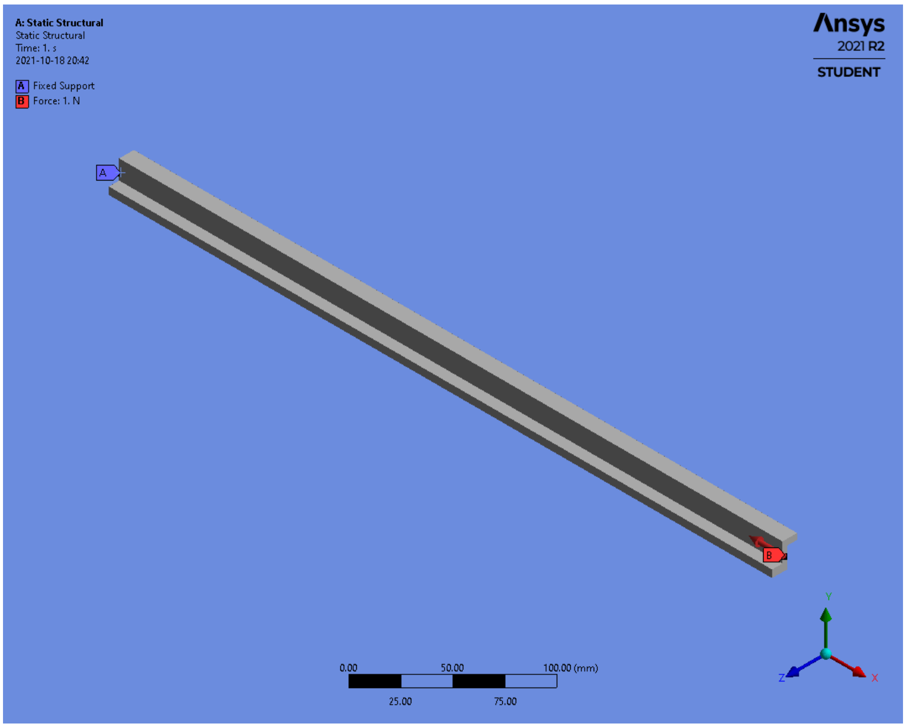
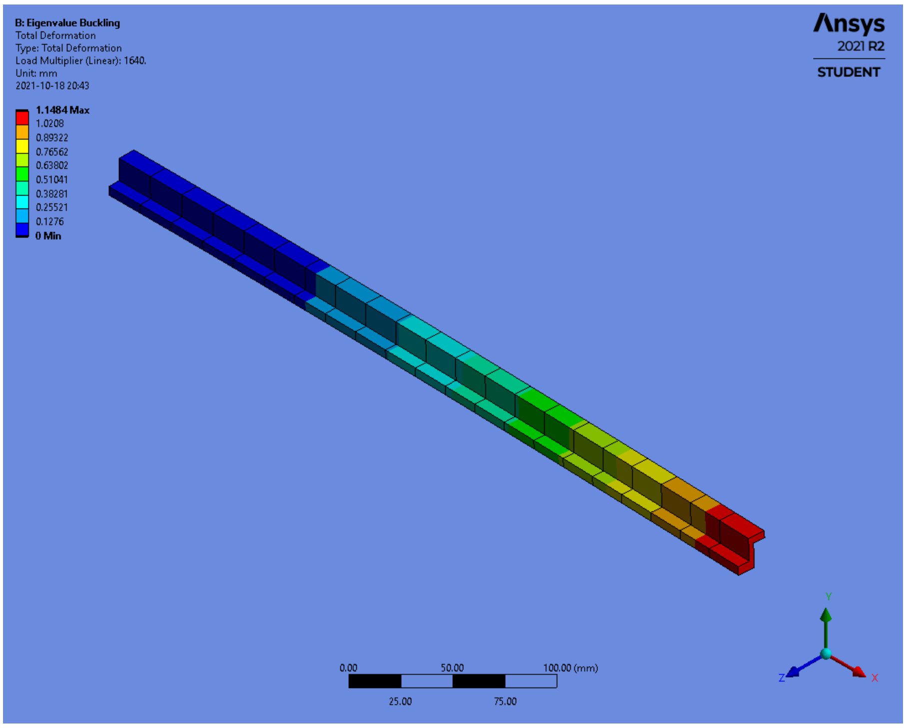


Table 2: Numerical results

P_{CR}	$1640\,\mathrm{N}$
u_v	$0.5642\,\mathrm{mm}$
u_h	$-1.000\,\mathrm{mm}$
θ	$-29.43°$

Figure 4: Total deformation across the column under critical buckling load.



3 Comparison of methods

Finally, we can compare the results found in Sections 1 and 2. We can also calculate the numerical error relative to the analytical approach. The comparison is shown in Table 3. The errors are extremely small as to be negligible.

Table 3: Comparison of analytical and numerical results

	P_{CR}	θ
Analytical Method	1641 N	−29.46°
Numerical Method	1640 N	−29.43°
Relative Error	−0.0609 %	−0.1018 %