Thin plates

2020

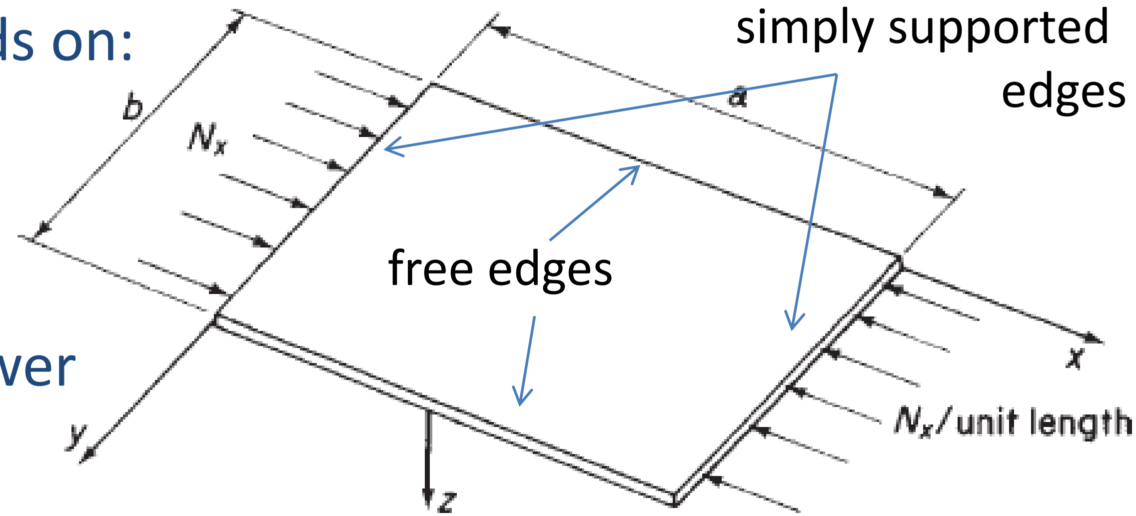
Buckling of thin plates

Buckling of thin plates

Buckling of thin plate depends on:

- its dimensions,
- the loading and
- the method of support.

Buckling loads are much lower than failure loads.

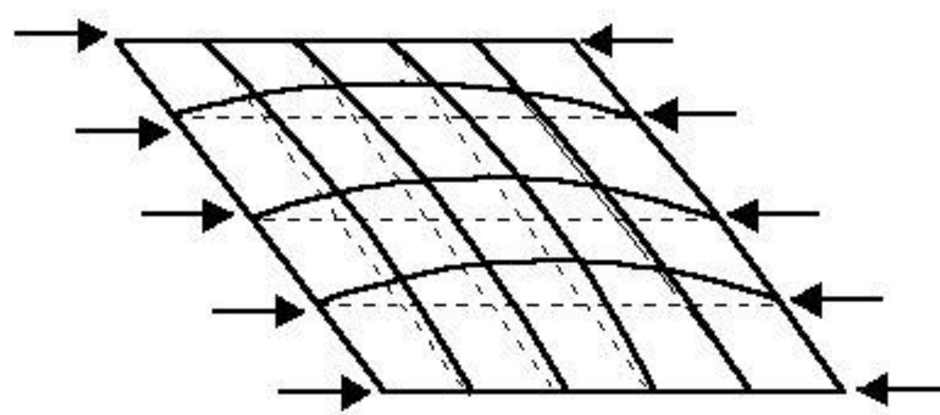


The critical load for a given plate is that predicted by the Euler theory

For thin plate with supported edges we'll use energy method with assumptions:

- only bending deflections;
- deflections are small comparing with thickness of the plate.

4 simply supported edges



Buckling of thin plates

For the given thin plate (supported in 4 edges) the deflected shape is

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

TPE is

$$\begin{aligned} U + V \quad = \quad & \frac{1}{2} \int_0^a \int_0^b \left[D \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 \right. \right. \\ & \left. \left. -2(1-\nu) \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \, \partial y} \right)^2 \right] \right\} - N_x \left(\frac{\partial w}{\partial x} \right)^2 \right] \mathrm{d}x \, \mathrm{d}y \end{aligned}$$

After integration and substitution for ***w***

$$U + V = \frac{\pi^4 abD}{8} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) - \frac{\pi^2 b}{8a} N_x \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m^2 A_{mn}^2$$

Buckling of thin plates

Assigning stationary value in the neutral equilibrium of its buckled state

$$\frac{\partial(U+V)}{\partial A_{mn}} = \frac{\pi^4 abD}{4} A_{mn} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2 - \frac{\pi^2 b}{4a} N_{x,\mathrm{CR}} m^2 A_{mn} = 0$$

For a non-trivial solution

$$\boxed{N_{x,\mathrm{CR}} = \pi^2 a^2 D \frac{1}{m^2} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2}$$

Each term in the infinite series for displacement corresponds, as in the case of a column, to a different value of critical load.

The lowest value of critical load evolves from:

- some critical combination of integers ***m*** and ***n***, i.e. the number of half-waves in the ***x*** and ***y*** directions, and
- the plate dimensions.

Clearly ***n = 1*** gives a minimum value so that no matter what the values of ***m***, ***a*** and ***b*** the plate buckles into a half sine wave in the ***y*** direction.

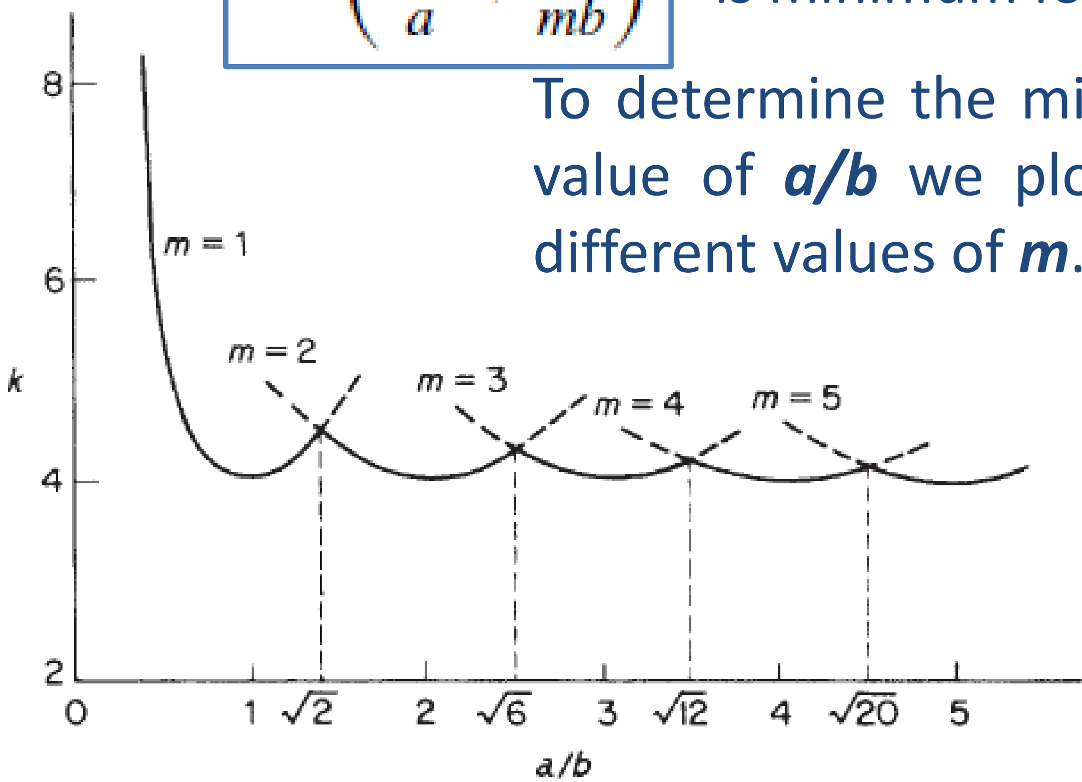
Buckling of thin plates

The last equation at ***n* = 1**

$$N_{x,\mathrm{CR}} = \pi^2 a^2 D \frac{1}{m^2}\left(\frac{m^2}{a^2} + \frac{1}{b^2}\right)^2 \quad \text{or} \quad \boxed{N_{x,\mathrm{CR}} = \frac{k\pi^2 D}{b^2}}$$

where $\boxed{k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2}$ is minimum for a given value of ***a/b***

To determine the minimum value of ***k*** for a given value of ***a/b*** we plot ***k*** as a function of ***a/b*** for different values of ***m***.



According to figure:

- ***m*** varies with the ratio ***a/b***
- ***k*** and the buckling load are a minimum when ***k* = *4*** at values of ***a/b* = *1, 2, 3,*** . . . .

Buckling of thin plates

The transition from one buckling mode to the next may be found by equating values of ***k*** for the ***m*** and ***m+1*** curves:

$$\frac{mb}{a}+\frac{a}{mb}=\frac{(m+1)b}{a}+\frac{a}{(m+1)b} \longrightarrow \frac{a}{b}=\sqrt{m(m+1)}$$

when ***m* = 1**, then ***a/b* = √2 = 1.414**

when ***m* = 2**, then ***a/b* = √6 = 2.45**

For a given value of ***a/b*** the critical stress $\sigma_{CR} = N_{x,CR}/t$

$$N_{x,\mathrm{CR}}=\frac{k\pi^2 D}{b^2} \qquad D=\int_{-t/2}^{t/2}\frac{Ez^2}{1-\nu^2}\mathrm{d}z=\frac{Et^3}{12(1-\nu^2)}$$

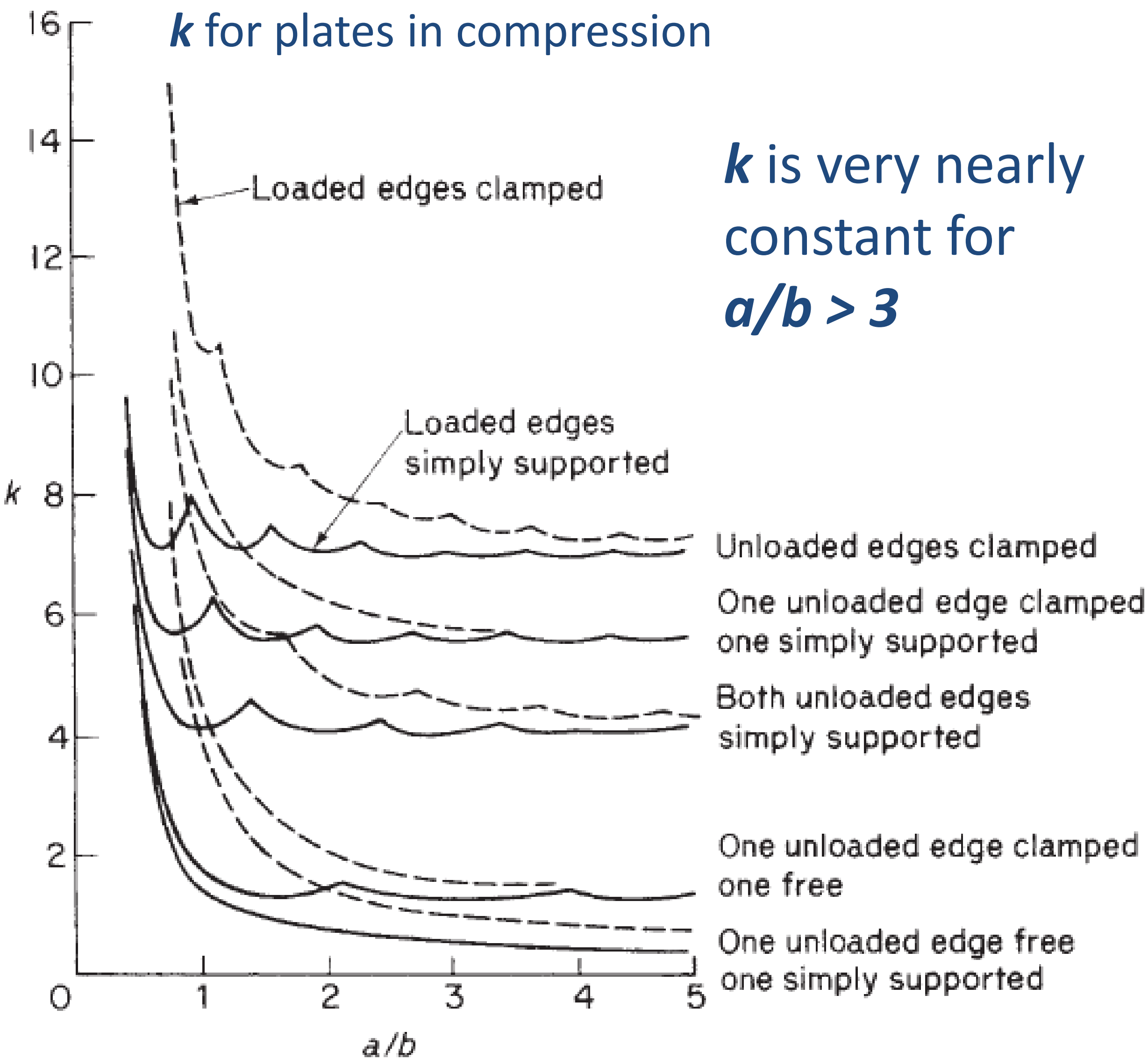
$$\sigma_{CR} = N_{x,CR}/t \longrightarrow \boxed{\sigma_{\mathrm{CR}}=\frac{k\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2}$$

The value of ***k***

- remains a function of ***a/b***;
- depends upon the type of loading;
- depends upon edge support.

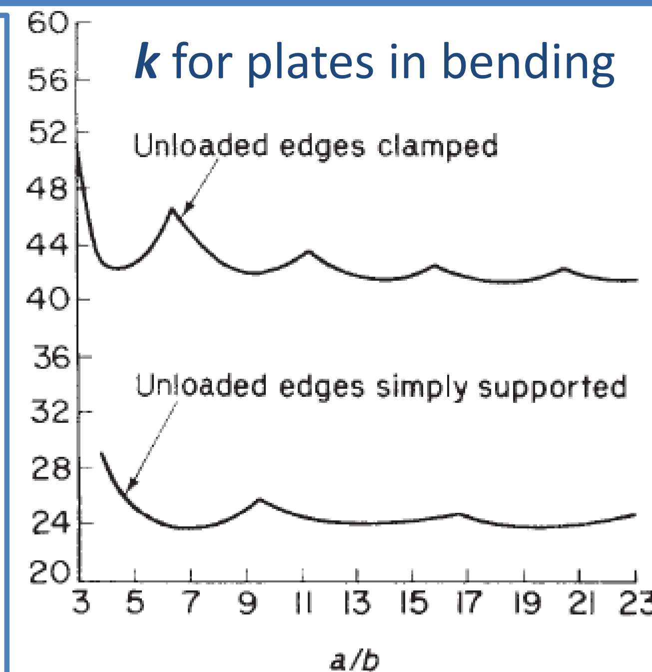
Buckling of thin plates

k for plates in compression

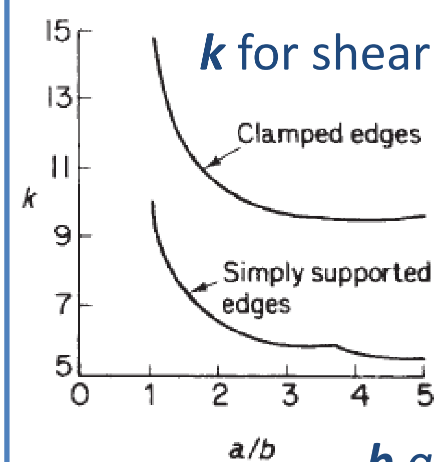


k is very nearly constant for ***a/b > 3***

k for plates in bending



k for shear buckling



b** always < **a

Inelastic buckling of plates

For small values of ***b/t*** the $\boldsymbol{\sigma_{CR}}$ may exceed the elastic limit.

$$\sigma_{\mathrm{CR}} = \frac{\eta k \pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$ where

$\boldsymbol{\eta}$ is a plasticity correction factor (in the linearly elastic region $\boldsymbol{\eta = 1}$);
E and ***ν*** are elastic values of Young's modulus and Poisson's ratio.

$$\eta = \frac{1-\nu_{\mathrm{e}}^2}{1-\nu_{\mathrm{p}}^2}\frac{E_{\mathrm{s}}}{E}\left[\frac{1}{2}+\frac{1}{2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_{\mathrm{t}}}{E_{\mathrm{s}}}\right)^{\frac{1}{2}}\right]$$ where

- $\boldsymbol{E_t}$ and $\boldsymbol{E_s}$ are the tangent modulus and secant modulus (stress/strain) of the plate in the inelastic region and
- $\boldsymbol{\nu_e}$ and $\boldsymbol{\nu_p}$ are Poisson's ratio in the elastic and inelastic ranges.

Local instability

Conditions for different types of buckling:

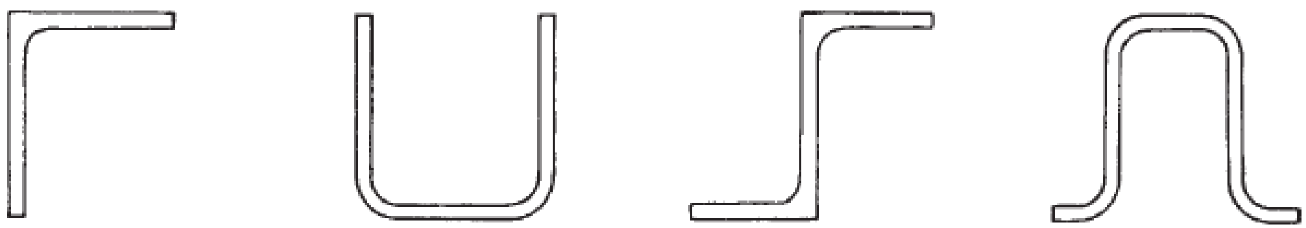
$l_e/r < 20$ secondary (local) instability (*thin walled columns*);

$l_e/r > 80$ primary instability;

$l_e/r = 20 - 80$ a combination of both primary and secondary modes.

Thin-walled columns are encountered in aircraft structures in the shape of longitudinal stiffeners with cross-sections in form:

- angle,
- channel,
- Z-section,
- top hat’ section.



The plate elements fall into two distinct categories:

- **flanges** which have a free unloaded edge and
- **webs** which are supported by the adjacent plate elements on both unloaded edges.

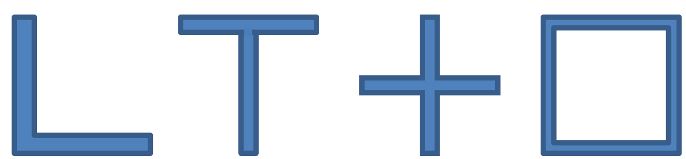
Local instability

In local instability:

- the flanges and webs buckle like plates with a change in cross-section;
- the wavelength of the buckle is of the order of the widths of the plate elements;
- the corresponding critical stress is generally independent of the length ***a*** of the column (when ***a* ≥ 3*b***, where ***b*** – the width of the largest plate element in the column cross-section).

Mechanics of buckling of plate elements:

- buckling occurs in the weakest plate element (usually a flange);
- the rotational restraint provided by adjacent elements to each other disappears;
- as a result the elements behave as though they are simply supported along their common edges.
- common cross-section of the case:



Local instability

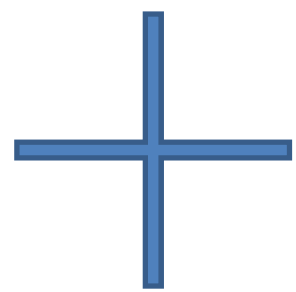
For local instability may use the equation for inelastic buckling:

$$\sigma_{CR} = \frac{\eta k \pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

For example:

k for a cruciform section column is obtained from appropriate figure for a plate:

- simply supported on three sides;
- one edge free;
- ratio ***a/b>3.***



Hence ***k = 0.43*** and if the section buckles elastically then ***η = 1*** and

$$\sigma_{CR} = 0.388E \left(\frac{t}{b}\right)^2 \quad (\nu = 0.3)$$

There is no universal method for solving local instability problems, most of data is experimental.

Primary instability of stiffened panels

$$\sigma_{CR} = \frac{k\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

Low values of critical stress will be at large values of ***b/t*** or low ***k***.

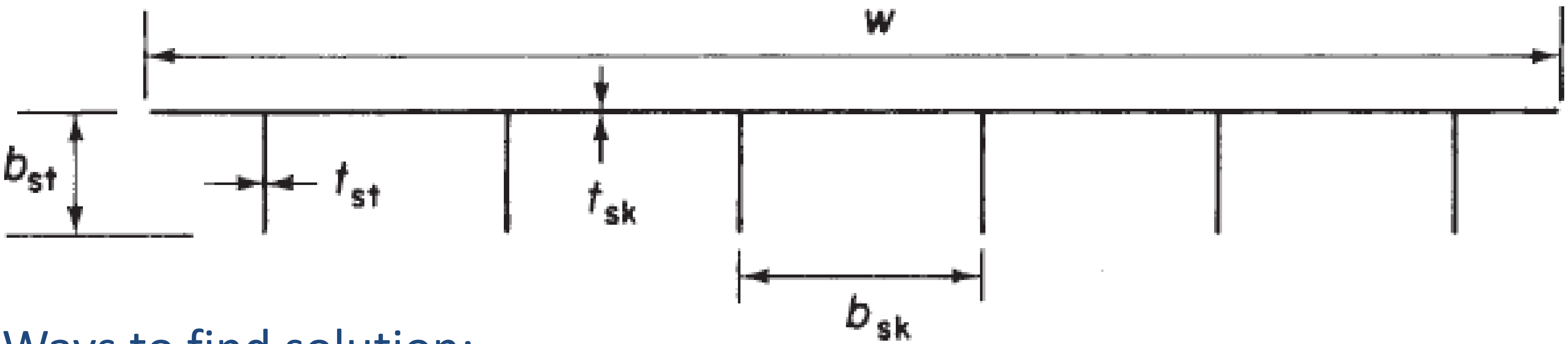
Methods to increase $\boldsymbol{\sigma_{CR}}$:

- to introduce stiffeners along the length of the plate thereby dividing a wide sheet into a number of smaller and more stable plates (longitudinal stiffeners carry part of the compressive load);
- to divide the plate into a series of wide short columns by stiffeners attached across its width (all the load is supported by the plate);
- to form a grid-stiffened structure (more common case) by combining both methods of stiffening.

A more efficient structure is obtained by adjusting the stiffener sections so that buckling occurs in both stiffeners and skin at about the same stress. In this case the complete panel must be considered as a unit.

Primary instability of stiffened panels

We present a relatively simple approach suggested by **Gerard**.



Ways to find solution:

- it is possible for the panel to behave as an Euler column (with a cross-section shown above): $$\sigma_{\mathrm{CR,E}} = \frac{\pi^2 E}{(l_e/r)^2}$$
- individual plate elements comprising the panel cross-section may buckle as long plates $$\sigma_{\mathrm{CR}} = \frac{\eta k \pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

where the values of ***k***, ***t*** and ***b*** depend upon the particular portion of the panel being investigated.

Primary instability of stiffened panels

For example,

1) the portion of skin between stiffeners may buckle as a plate simply supported on all four sides.

Thus, for ***a/b>3, k = 4, η = 1*** *(elastic range)*

$$\sigma_{CR} = \frac{4\pi^2 E}{12(1-\nu^2)}\left(\frac{t_{sk}}{b_{sk}}\right)^2$$

2) the stiffeners may buckle as long plates simply supported on three sides with one edge free

$$\sigma_{CR} = \frac{0.43\pi^2 E}{12(1-\nu^2)}\left(\frac{t_{st}}{b_{st}}\right)^2$$

Compressive load is applied to the panel over its complete cross-section

$\sigma_A = \dfrac{N_x}{\bar{t}}$ where $\bar{t} = \dfrac{A_{st}}{b_{sk}} + t_{sk}$ - equivalent skin thickness

A_{st} – the stiffener area.

Obrigado!