Examples of theoretical classes of the lecture
TWO DIMENSIONAL PROBLEMS IN ELASTICITY

Example 6.1

P.2.1 A metal plate has rectangular axes Ox, Oy marked on its surface. The point O and the direction of Ox are fixed in space and the plate is subjected to the following uniform stresses:

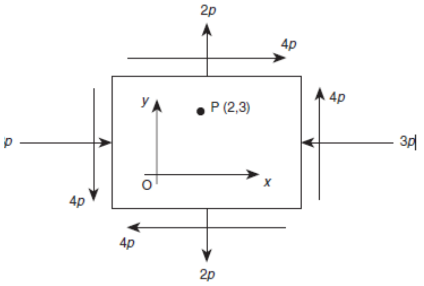
compressive, 3p, parallel to Ox,
tensile, 2p, parallel to Oy,
shearing, 4p, in planes parallel to Ox and Oy
in a sense tending to decrease the angle xOy.

Determine the direction in which a certain point on the plate will be displaced; the coordinates of the point are (2, 3) before straining. Poisson's ratio is 0.25.

Ans. 19.73 ◦ to Ox.

Solution

The stress system applied to the plate is shown in Fig. S.2.1. The origin, O, of the axes may be chosen at any point in the plate; let **P** be the point whose coordinates are (2, 3).



This is the case of plane stress (σz =0)
From 2 first Eqs

$$\left.\begin{aligned}
\varepsilon_x &= \frac{1}{E}(\sigma_x - \nu\sigma_y) \\
\varepsilon_y &= \frac{1}{E}(\sigma_y - \nu\sigma_x) \\
\varepsilon_z &= \frac{-\nu}{E}(\sigma_x - \sigma_y) \\
\gamma_{xy} &= \frac{\tau_{xy}}{G}
\end{aligned}\right\}$$

we have

$$\varepsilon_x = -\frac{3p}{E} - \nu\frac{2p}{E} = -\frac{3.5p}{E}$$
$$\varepsilon_y = \frac{2p}{E} + \nu\frac{3p}{E} = \frac{2.75p}{E}$$

Hence, from Eqs

$$\left.\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} \\ \varepsilon_y &= \frac{\partial v}{\partial y} \\ \varepsilon_z &= \frac{\partial w}{\partial z} \end{aligned}\right\}$$

$$\frac{\partial u}{\partial x} = -\frac{3.5p}{E} \quad \text{so that} \quad u = -\frac{3.5p}{E}x + f_1(y)$$ where $f1(y)$ is a function of y.

Also

$$\frac{\partial v}{\partial y} = \frac{2.75p}{E} \quad \text{so that} \quad v = -\frac{2.75p}{E}y + f_2(x)$$

in which $f2(x)$ is a function of x.

From the last of Eqs of the first set (in the solution) and Eq. for the γ_{xy}

$$\left.\begin{aligned} \gamma_{xz} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \\ \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\ \gamma_{yz} &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \end{aligned}\right\}$$

we have

$$\gamma_{xy} = \frac{4p}{G} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \frac{\partial f_2(x)}{\partial x} + \frac{\partial f_1(y)}{\partial y}$$

Suppose

$$\frac{\partial f_1(y)}{\partial y} = A$$

then

$$f_1(y) = Ay + B$$

in which A and B are constants.

Similarly, suppose1

$$\frac{\partial f_2(x)}{\partial x} = C$$

then

$$f_2(x) = Cx + D$$

in which *C* and *D* are constants

Substituting for *f*1(*y*) and *f*2(*x*) in Eqs ∂*u*/∂*x* and ∂*v*/∂*y* gives

$$u = -\frac{3.5p}{E}x + Ay + B$$
$$v = \frac{2.75p}{E}y + Cx + D$$

Since the origin of the axes is fixed in space it follows that when
x=*y*=0, *u*=*v*=0.
Hence, from the last Eqs, *B*=*D*=0.

Further, the direction of O*x* is fixed in space so that, when
y=0, ∂*v*/∂*x* =0.
Therefore, from last Eq. *C* =0.

Thus, from Eqs

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

and for u, when *x* =0

$$\frac{\partial u}{\partial y} = \frac{4p}{G} = A$$

From the last Eqs for *u* and *v* we have

$$u = -\frac{3.5p}{E}x + \frac{4p}{G}y$$

and

$$v = \frac{2.75p}{E}y$$

From Eq.
G=*E*/2(1+*v*)=*E*/2.5
and the last Eq for u

$$u = \frac{p}{E}(-3.5x + 10y)$$

At the point (2,3)

$$u = \frac{23p}{E}$$

and

$$v = \frac{8.25p}{E}$$

The point **P** therefore moves at an angle α to the x axis given by

$$\alpha = \tan^{-1} \frac{8.25}{23} = 19.73°$$