Lista 2

1) $\left[\dot{\vec{V}}_{NED}\right]_b = \dot{\vec{V}}_b + \vec{\omega}_b \times \vec{V}_b$

$$= \begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{bmatrix} + \begin{bmatrix} 0 & -R & Q \\ R & 0 & -P \\ -Q & P & 0 \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

$$= \begin{bmatrix} \dot{U} + QW - RV \\ \dot{V} + RU - PW \\ \dot{W} + PV - QU \end{bmatrix}$$

2) $\dot{\vec{V}}_b = \frac{1}{m}\vec{F}_b - \vec{\omega}_b \times \vec{V}_b$

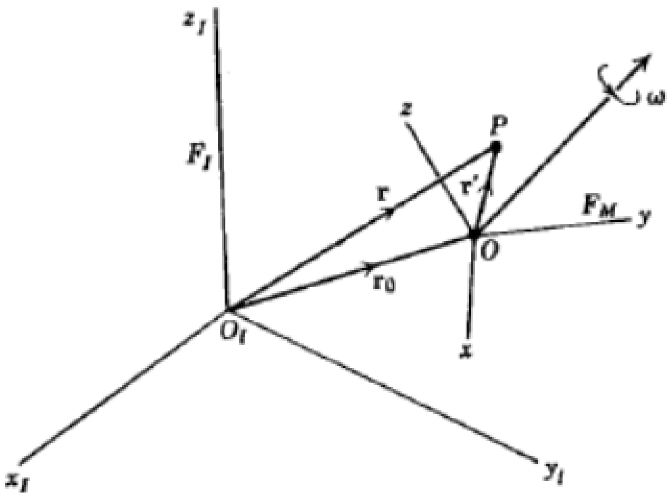
$$= \begin{bmatrix} \frac{1}{m}F_{x_b} + RV - QW \\ \frac{1}{m}F_{b_y} + PW - RU \\ \frac{1}{m}F_{b_z} + QU - PV \end{bmatrix}$$

4) $$\vec{a}'_b = \left[\dot{\vec{V}}_{NED}\right]_b - \vec{g}_b$$

$$\vec{a}'_b = \dot{\vec{V}}_b + \vec{\omega}_b \times \vec{V}_b - D_b^{NED}\,\vec{g}_{NED} = \frac{1}{m}\left(\vec{F}_{b,A} + \vec{F}_{b,T}\right)$$

5) $$\dot{\vec{V}}_b = \vec{a}'_b + D_b^{NED}\,\vec{g}_{NED} - \vec{\omega}_b \times \vec{V}_b$$

6)



$$[\dot{\vec{r}}'_{NED}]_b = \dot{\vec{r}}'_b + \vec{\omega}_b \times \vec{r}'_b$$

Como por hipótese, $\dot{\vec{r}}'_b = \vec{0}$

$$[\dot{\vec{r}}'_{NED}]_b = \vec{\omega}_b \times \vec{r}'_b$$

Derivando novamente

$$[\ddot{\vec{r}}'_{NED}]_b = \dot{\vec{\omega}}_b \times \vec{r}'_b + \vec{\omega}_b \times \dot{\vec{r}}'_b + \vec{\omega}_b \times (\vec{\omega}_b \times \vec{r}'_b)$$

$$[\ddot{\vec{r}}'_{NED}]_b = \dot{\vec{\omega}}_b \times \vec{r}'_b + \vec{\omega}_b \times (\vec{\omega}_b \times \vec{r}'_b)$$

i) Como:

$$\vec{a}'_b = \left[\dot{\vec{V}}_{NED}\right]_b - \vec{g}_b + \left[\ddot{\vec{r}}'_{NED}\right]_b$$

$$\vec{a}'_b = \dot{\vec{V}}_b + \vec{\omega}_b \times \vec{V}_b - D_b^{NED}\,\vec{g}_{NED} + \dot{\vec{\omega}}_b \times \vec{r}'_b + \vec{\omega}_b \times \left(\vec{\omega}_b \times \vec{r}'_b\right)$$

ii) $$\dot{\vec{V}}_b = \vec{a}'_b - \vec{\omega}_b \times \vec{V}_b + D_b^{NED}\,\vec{g}_{NED} - \dot{\vec{\omega}}_b \times \vec{r}'_b - \vec{\omega}_b \times \left(\vec{\omega}_b \times \vec{r}'_b\right)$$

7) i) $\vec{a}'_b = -D_b^{NED}\, \vec{a}_{NED} = -\vec{g}_b$

$$\vec{a}'_b = g \begin{bmatrix} \operatorname{sen}\theta \\ -\operatorname{sen}\phi \cos\theta \\ -\cos\phi \cos\theta \end{bmatrix}$$

iii) $\theta = \arcsin\left(\dfrac{a'_{bx}}{g}\right)$

$$\phi = \arcsin\left(-\frac{a'_{by}}{|\vec{a}'_b| \cos\theta}\right)$$

8) i) $\vec{T}_b = \left(\dot{\vec{H}}_{NED}\right)_b$

$$\vec{T}_b = \dot{\vec{H}}_b + \vec{w}_b \times \vec{H}_b$$

$$\vec{T}_b = J_b \dot{\vec{w}}_b + \vec{w}_b \times J_b \vec{w}_b$$

ii) $J_b \dot{\vec{w}}_b = \vec{T}_b - \vec{w}_b \times J_b \vec{w}_b$

$$J_b^{-1} J_b \dot{\vec{w}}_b = J_b^{-1}\left(\vec{T}_b - \vec{w}_b \times J_b \vec{w}_b\right)$$

$$\dot{\vec{w}}_b = J_b^{-1}\left(\vec{T}_b - \vec{w}_b \times J_b \vec{w}\right)$$