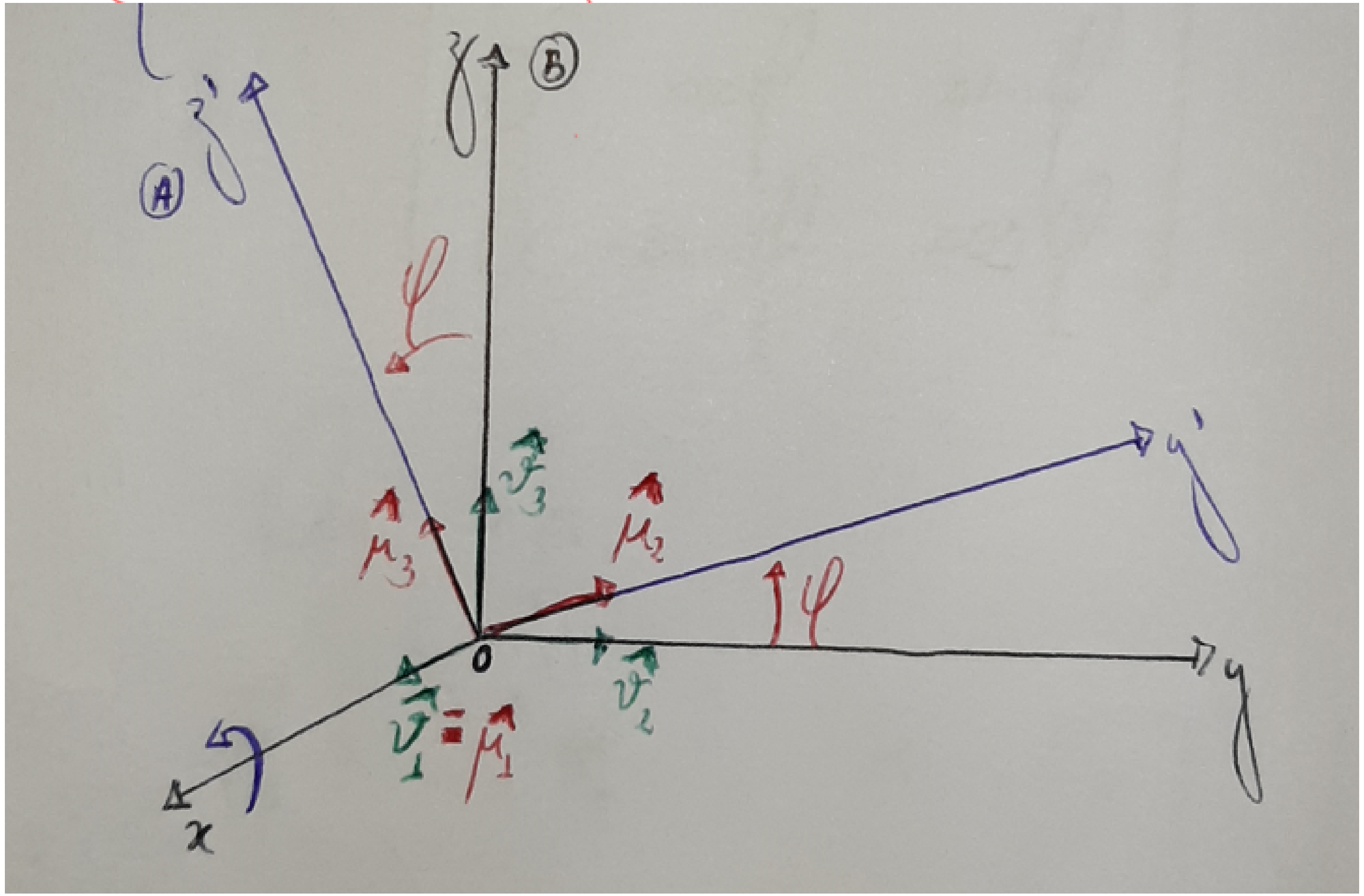
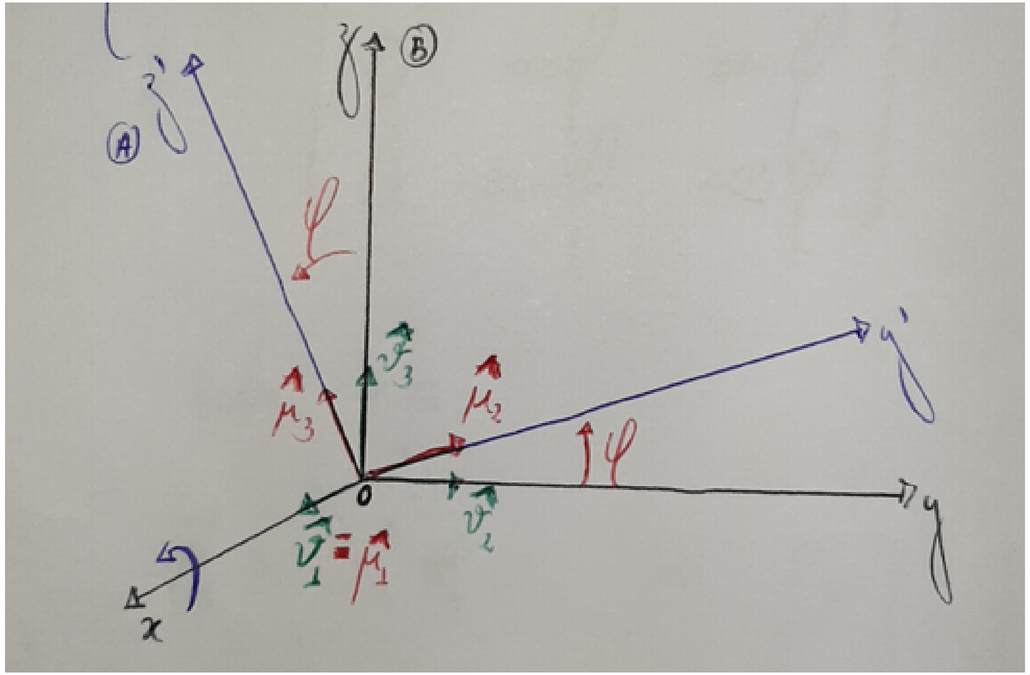
Exercícios - Lista 1

1)





Temos que

$$\vec{v}_B = D_B^A \, \vec{v}_A$$

$$D_B^A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$\hat{\vartheta}_1$, $\hat{\vartheta}_2$ e $\hat{\vartheta}_3$ representados na base A

$$\begin{cases} \hat{\vartheta}_1 = \hat{\mu}_1 \\ \hat{\vartheta}_2 = \cos\varphi\,\hat{\mu}_2 - \operatorname{sen}\varphi\,\hat{\mu}_3 \\ \hat{\vartheta}_3 = \operatorname{sen}\varphi\,\hat{\mu}_2 + \cos\varphi\,\hat{\mu}_3 \end{cases} \Rightarrow \begin{cases} \hat{\vartheta}_1 = a_{11}\hat{\mu}_1 + a_{12}\hat{\mu}_2 + a_{13}\hat{\mu}_3 \\ \hat{\vartheta}_2 = a_{21}\hat{\mu}_1 + a_{22}\hat{\mu}_2 + a_{23}\hat{\mu}_3 \\ \hat{\vartheta}_3 = a_{31}\hat{\mu}_1 + a_{32}\hat{\mu}_2 + a_{33}\hat{\mu}_3 \end{cases}$$

Por comparação

$$D_B^A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\operatorname{sen}\varphi \\ 0 & \operatorname{sen}\varphi & \cos\varphi \end{bmatrix} = R_x(-\varphi)$$

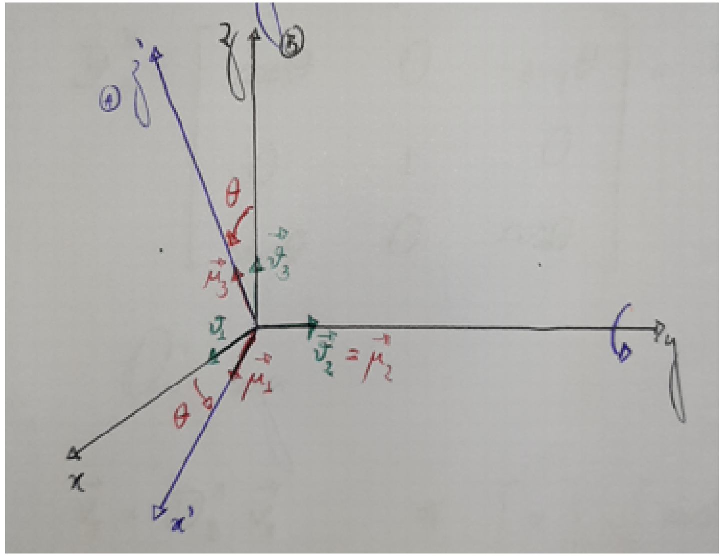
$$D_A^B = \left(D_B^A\right)^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \operatorname{sen}\varphi \\ 0 & -\operatorname{sen}\varphi & \cos\varphi \end{bmatrix} = R_x(\varphi)$$

Ou seja

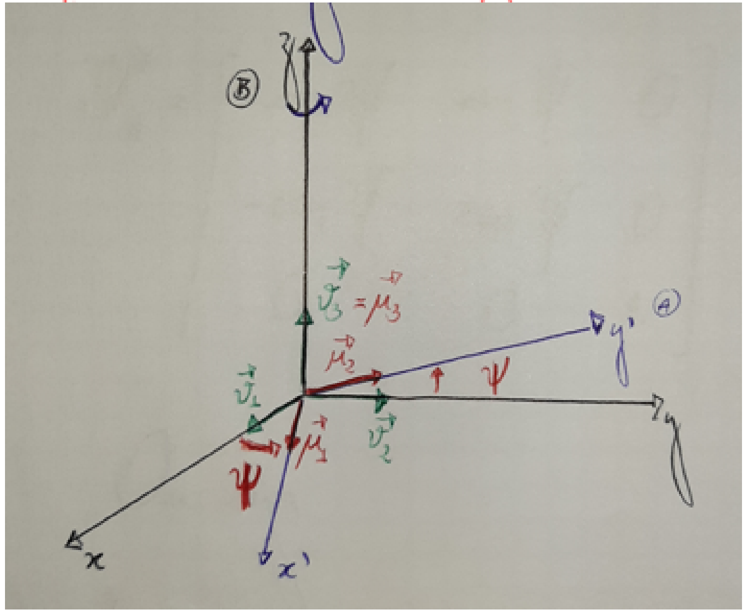
$$\vec{v}_B = D_B^A \vec{v}_A \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\operatorname{sen}\varphi \\ 0 & \operatorname{sen}\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$\vec{V}_A = D_A^B \, \vec{V}_B \Rightarrow \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \operatorname{sen}\varphi \\ 0 & -\operatorname{sen}\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

No eixo Oy



No eixo Oz



2) $D_b^{NED} = D(\psi, \theta, \varphi) = D_x(\varphi)\, D_y(\theta)\, D_z(\psi)$

$$D_b^{NED} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \operatorname{sen}\varphi \\ 0 & -\operatorname{sen}\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\operatorname{sen}\theta \\ 0 & 1 & 0 \\ \operatorname{sen}\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \operatorname{sen}\psi & 0 \\ -\operatorname{sen}\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D_b^{NED} = \begin{bmatrix} \cos\theta\cos\psi & c\theta s\psi & -s\theta \\ -\cos\varphi\operatorname{sen}\psi + \operatorname{sen}\varphi\operatorname{sen}\theta\cos\psi & c\varphi c\psi + s\varphi s\theta s\psi & s\varphi c\theta \\ \operatorname{sen}\varphi\operatorname{sen}\psi + \cos\varphi\operatorname{sen}\theta\cos\psi & -s\varphi c\psi + c\varphi s\theta s\psi & c\varphi c\theta \end{bmatrix}$$

Ou seja

$$\vec{v}_b = D_b^{NED}\, \vec{v}_{NED}$$

3) $D_{NED}^{b} = \left(D_{b}^{NED}\right)^{-1} = \left(D_{b}^{NED}\right)^{T}$

Logo:

$\vec{A}_{NED} = D_{NED}^{b} \vec{A}_{b}$

4) $D_{b}^{NED} = D(\psi, \theta, 0)$

Para $\phi = 0$

$$D_{b}^{NED}(\psi, \theta, 0) = \begin{bmatrix} \cos\theta \cos\psi & \cos\theta \operatorname{sen}\psi & -\operatorname{sen}\theta \\ -\operatorname{sen}\psi & \cos\psi & 0 \\ \operatorname{sen}\theta \cos\psi & \operatorname{sen}\theta \operatorname{sen}\psi & \cos\theta \end{bmatrix}$$

Temos que

$$\left(D_b^{NED}(\psi,\theta,0)\right)^{-1} = \frac{1}{\det\left[D_b^{NED}(\psi,\theta,0)\right]}\left[\operatorname{cof} D_b^{NED}(\psi,\theta,0)\right]^T$$

$$= \left(D_b^{NED}(\psi,\theta,0)\right)^T = D_{NED}^{b}(\psi,\theta,0)$$

7) $D_b^w = D_y(\alpha)\, D_z(-\beta)$

$$D_b^w = \begin{bmatrix} \cos\alpha & 0 & -\operatorname{sen}\alpha \\ 0 & 1 & 0 \\ \operatorname{sen}\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos(-\beta) & \operatorname{sen}(-\beta) & 0 \\ -\operatorname{sen}(-\beta) & \cos(-\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D_b^w = \begin{bmatrix} \cos\alpha\cos\beta & -\cos\alpha\operatorname{sen}\beta & -\operatorname{sen}\alpha \\ \operatorname{sen}\beta & \cos\beta & 0 \\ \operatorname{sen}\alpha\cos\beta & -\operatorname{sen}\alpha\operatorname{sen}\beta & \cos\alpha \end{bmatrix}$$

8)

i) $\vec{V}_b = D_b^w \vec{V}_w = D_b^w \begin{bmatrix} V_T \\ 0 \\ 0 \end{bmatrix}$

ii) $\vec{V}_{NED} = D_{NED}^b \vec{V}_b = D_{NED}^b D_b^w \vec{V}_w$

9) $$D_{NED}^{ECEF} = \begin{bmatrix} \cos(-\varphi) & 0 & -\operatorname{sen}(-\varphi) \\ 0 & 1 & 0 \\ \operatorname{sen}(-\varphi) & 0 & \cos(-\varphi) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\lambda & \operatorname{sen}\lambda \\ 0 & -\operatorname{sen}\lambda & \cos\lambda \end{bmatrix} \begin{bmatrix} \cos\left(-\frac{\pi}{2}\right) & 0 & -\operatorname{sen}\left(-\frac{\pi}{2}\right) \\ 0 & 1 & 0 \\ \operatorname{sen}\left(-\frac{\pi}{2}\right) & 0 & \cos\left(-\frac{\pi}{2}\right) \end{bmatrix}$$

$$D_{NED}^{ECEF} = \begin{bmatrix} -\operatorname{sen}\varphi\cos\lambda & -\operatorname{sen}\varphi\operatorname{sen}\lambda & \cos\varphi \\ -\operatorname{sen}\lambda & \cos\lambda & 0 \\ -\cos\varphi\cos\lambda & -\cos\varphi\operatorname{sen}\lambda & -\operatorname{sen}\varphi \end{bmatrix}$$

11) $D_{ENU}^{ECEF} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\pi/2 & s\pi/2 \\ 0 & -s\pi/2 & c\pi/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(-\varphi) & s(-\varphi) \\ 0 & -s(-\varphi) & c(-\varphi) \end{bmatrix} \begin{bmatrix} c\,\pi/2 & s\,\pi/2 & 0 \\ -s\,\pi/2 & c\,\pi/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\lambda & s\lambda & 0 \\ -s\lambda & c\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$D_{ENU}^{ECEF} = \begin{bmatrix} -\operatorname{sen}\lambda & \cos\lambda & 0 \\ -\operatorname{sen}\varphi\cos\lambda & -\operatorname{sen}\varphi\operatorname{sen}\lambda & \cos\varphi \\ \cos\varphi\cos\lambda & \cos\varphi\operatorname{sen}\lambda & \operatorname{sen}\varphi \end{bmatrix}$$

É ainda

$$D_{ECEF}^{END} = \left(D_{ENU}^{ECEF} \right)^T$$

12) Encontrando $D_{t_1}^{ENU}$ e $D_{t_2}^{ENU}$

$$D_{t_1}^{ENU} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \operatorname{sen}\beta \\ 0 & -\operatorname{sen}\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & -\operatorname{sen}\alpha \\ 0 & 1 & 0 \\ \operatorname{sen}\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$D_{t_1}^{ENU} = \begin{bmatrix} \cos\alpha & 0 & -\operatorname{sen}\alpha \\ \operatorname{sen}\beta \operatorname{sen}\alpha & \cos\beta & \operatorname{sen}\beta\cos\alpha \\ \cos\beta \operatorname{sen}\alpha & -\operatorname{sen}\beta & \cos\beta\cos\alpha \end{bmatrix}$$

$$D_{ENU}^{t_1} = \left(D_{t_1}^{ENU}\right)^T$$

$$D_{t_2}^{ENU} = \begin{bmatrix} \cos\delta & 0 & -\operatorname{sen}\delta \\ 0 & 1 & 0 \\ \operatorname{sen}\delta & 0 & \cos\delta \end{bmatrix} \begin{bmatrix} \cos\gamma & \operatorname{sen}\gamma & 0 \\ -\operatorname{sen}\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D_{ENU}^{t_2} = \left(D_{t_2}^{ENU} \right)^T$$