Prova Substitutiva

Luiz Georg 15/0041390

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1.1

$$V_w = V_T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$D_b^w = \begin{bmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) 0 & \cos(\alpha) & \end{bmatrix} \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D_b^w = \begin{bmatrix} \cos(\alpha)\cos(\beta) & -\sin(\beta)\cos(\alpha) & -\sin(\alpha) \\ \sin(\beta) & \cos(\beta) & 0 \\ \sin(\alpha)\cos(\beta) & -\sin(\alpha)\sin(\beta) & \cos(\alpha) \end{bmatrix}$$

$$V_b = D_b^w V_w$$

$$V_b = V_T \begin{bmatrix} \cos(\alpha)\cos(\beta) \\ \sin(\beta) \\ \sin(\alpha)\cos(\beta) \end{bmatrix}$$

$$D_{NED}^b = R_x(-\phi) R_y(-\theta) R_z(-\phi)$$

$$D_{NED}^b = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix}$$

$$D_{NED}^b = \begin{bmatrix} \cos(\psi)\cos(\theta) & \sin(\phi)\sin(\theta)\cos(\psi) - \sin(\psi)\cos(\phi) & \sin(\phi)\sin(\psi) + \sin(\theta)\cos(\phi)\cos(\psi) \\ \sin(\psi)\cos(\theta) & \sin(\phi)\sin(\psi)\sin(\theta) + \cos(\phi)\cos(\psi) & -\sin(\phi)\cos(\psi) + \sin(\psi)\sin(\theta)\cos(\phi) \\ -\sin(\theta) & \sin(\phi)\cos(\theta) & \cos(\phi)\cos(\theta) \end{bmatrix}$$

$$V_{NED} = D_{NED}^b V_b$$

$$V_{NED} = V_T \begin{bmatrix} (\sin(\phi)\sin(\psi) + \sin(\theta)\cos(\phi)\cos(\psi))\sin(\alpha)\cos(\beta) + (\sin(\phi)\sin(\theta)\cos(\psi) - \sin(\psi)\cos(\phi))\sin(\beta) + \cos(\alpha)\cos(\beta)\cos(\psi)\cos(\theta) \\ -(\sin(\phi)\cos(\psi) - \sin(\psi)\sin(\theta)\cos(\phi))\sin(\alpha)\cos(\beta) + (\sin(\phi)\sin(\psi)\sin(\theta) + \cos(\phi)\cos(\psi))\sin(\beta) + \sin(\psi)\cos(\alpha)\cos(\beta)\cos(\theta) \\ \sin(\alpha)\cos(\beta)\cos(\phi)\cos(\theta) + \sin(\beta)\sin(\phi)\cos(\theta) - \sin(\theta)\cos(\alpha)\cos(\beta) \end{bmatrix}$$

2) $\vec{\omega} = \omega \hat{z}$; $\vec{r} = r \hat{\rho}$

i) $\vec{V} = 0$; $\dot{\vec{\omega}} = 0$

$$\ddot{r} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) = \underline{-\omega^2 r \hat{\rho}}$$ (aceleração centrípeta)

ii) $\vec{V} = 0$;

$$\ddot{r} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \dot{\vec{\omega}} \times \vec{r} = -\omega^2 r \hat{\rho} + \underbrace{\dot{\omega} r \hat{\theta}}_{\substack{\text{(aceleração tangencial} \\ \text{devido a } \dot{\omega})}}$$

iii) $\vec{V} = -v \hat{\rho}$

$$\ddot{r} = \dot{\vec{V}} + 2 \vec{\omega} \times \vec{V} + \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$= \underbrace{-\dot{v} \hat{\rho}}_{\substack{\text{(aceleração} \\ \text{direta)}}} \underbrace{- 2 \omega v \hat{\theta}}_{\substack{\text{(aceleração de} \\ \text{Coriolis)}}} + \dot{\omega} r \hat{\theta} - \omega^2 r \hat{\rho}$$

iv) $\vec{V} = v \hat{\theta}$

$$\ddot{r} = \underbrace{\dot{v} \hat{\theta}}_{\substack{\text{(aceleração} \\ \text{direta)}}} \underbrace{- 2 \omega v \hat{\rho}}_{\substack{\text{(aceleração} \\ \text{Coriolis)}}} + \dot{\omega} r \hat{\theta} - \omega^2 r \hat{\rho}$$

3) i) $s^2 + \left(m_q + \frac{L\alpha}{V_e} + \frac{g}{V_e E'}\right)s + \left(m_\alpha + m_q\left(\frac{L\alpha}{V_e} + \frac{g}{V_e E'}\right)\right) = 0$

ii) $\xi = \frac{m_q + \frac{L\alpha}{V_e} + \frac{g}{V_e E'}}{2\omega_0}$ $\qquad \omega_0 = \sqrt{m_\alpha + m_q\left(\frac{L\alpha}{V_e} + \frac{g}{V_e E'}\right)}$

$\omega = \omega_0\sqrt{1-\xi^2}$

iii) $\xi \propto \sqrt{\lambda}$

iv) $\begin{bmatrix} q \\ \bar{\alpha} \end{bmatrix} = e^{-\omega_0 \xi t}\left(\begin{bmatrix} A_q \\ A_\alpha \end{bmatrix} \operatorname{sen}(\omega t) + \begin{bmatrix} B_q \\ B_\alpha \end{bmatrix} \cos(\omega t)\right)$

4) $\beta < 0$

$C_{y\beta} < 0$; $C_{y\delta_r} > 0$ $\left(-\frac{\pi}{2} < \phi_1 < \frac{\pi}{2}\right)$

$C_{l\beta} < 0$; $C_{l\delta_a} < 0$

$C_{n\beta} > 0$; $C_{n\delta_r} < 0$

$$\delta_r = -\frac{\overset{\oplus}{C_{n\beta}}}{\underset{\ominus}{C_{n\delta_r}}}\overset{\ominus}{\beta} < 0 \Rightarrow$$ <u>leme à direita</u>

$$\delta_a = -\frac{\overset{\ominus}{C_{l\beta}}}{\underset{\ominus}{C_{l\delta_a}}}\overset{\ominus}{\beta} > 0 \Rightarrow$$ <u>aileron esquerdo para cima</u>

$$\sin\phi_1 = -\underbrace{\left(\frac{\rho_e S V_e^2}{mg}\right)}_{\oplus}\left(\overset{\ominus}{C_{y\beta}}\overset{\ominus}{\beta} + \overset{\oplus}{C_{y\delta_r}}\overset{\ominus}{\delta_r}\right)$$

$$\left(C_{y\beta}\beta + C_{y\delta_r}\delta_r\right) = \frac{\beta}{C_{n\delta_r}}\left(C_{y\beta}C_{n\delta_r} - C_{n\beta}C_{y\delta_r}\right) = \frac{\overset{\ominus}{\beta}}{\underset{\ominus}{C_{n\delta_r}}}\left(\frac{\overset{?}{(l_F - a)}}{\underset{\oplus}{l}}\overset{\ominus}{C_{y\beta}}\overset{\oplus}{C_{y\delta_r}}\right)$$

$l_F < 0$; $|l_F| >> |a| \Rightarrow (l_F - a) < 0 \Rightarrow (C_{y\beta}\beta + C_{y\delta_r}\delta_r) > 0$

$= \sin\phi_1 < 0 \Rightarrow$ asa direita acima do plano horizontal

5) $$\begin{cases} \Omega V_e \cos\phi - Y_\beta \cancel{\beta}^{0} - g \,\mathrm{sen}(\phi) = 0 \\ l_\beta \cancel{\beta}^{0} + l_r \Omega + l_{\delta_a} \delta_a = 0 \\ n_\beta \cancel{\beta}^{0} + n_r \Omega + n_{\delta_r} \delta_r = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \tan(\phi) = \dfrac{\overset{\oplus}{\Omega} \overset{\oplus}{V_e}}{g_{\oplus}} > 0 & \text{asa direita abaixo do plano horizontal} \\ \delta_a = -\dfrac{l_r^{\oplus}}{l_{\delta_a \ominus}} \Omega^{\oplus} > 0 & \text{aileron esquerdo para cima} \\ \delta_r = -\dfrac{n_r^{\ominus}}{n_{\delta_r \ominus}} \Omega^{\oplus} < 0 & \text{leme à direita} \end{cases}$$

$\Omega = 0.5°/s = \frac{\pi}{360}$ rad/s
$V_e = 200$ m/s
$g = 9,7711$ m/s
$\rho = 0,4663$ kg/m³
$S = 260$ m²
$Z = \rho S V_e^2 / 2$
$l = 6,61$ m
$l_r = Z l^2 C_{l_r} / I_x$ $\qquad C_{l_r} = 2,9$
$l_{\delta a} = Z l C_{l_{\delta a}} / I_x$ $\qquad C_{l_{\delta a}} = -0,33$

$n_r = Z l^2 C_{n_r} / I_z$ $\qquad C_{n_r} = -7,5$
$n_{\delta r} = Z l C_{n_{\delta r}} / I_z$ $\qquad C_{n_{\delta r}} = -1,0$

$$\Rightarrow \begin{cases} \phi = \arctan\left(\dfrac{\Omega V_e}{g}\right) \approx 0.1768 \text{ rad} \approx 10.13° \\ \delta_a = -l C_{l_r} / C_{l_{\delta a}} \cdot \Omega \approx 29,04° \\ \delta_r = -l C_{n_r} / C_{n_{\delta r}} \cdot \Omega \approx -24,79° \end{cases}$$