Movimento de Curto Período

i) Resposta a Perturbação externa $\delta_p = 0$

$$q(t) = e^{-w_0 \xi t}\left(A_q \operatorname{sen} w_0\sqrt{1-\xi^2}\,t + B_q \cos w_0\sqrt{1-\xi^2}\,t\right) \qquad \text{I}$$

$$\alpha(t) = e^{-w_0 \xi t}\left(A_\alpha \operatorname{sen} w_0\sqrt{1-\xi^2}\,t + B_\alpha \cos w_0\sqrt{1-\xi^2}\,t\right) \qquad \text{II}$$

Derivando as equações I e II em relação ao tempo

$$\dot{q}(t) = -w_0 \xi e^{-w_0 \xi t}\left(A_q \operatorname{sen} w_0\sqrt{1-\xi^2}\,t + B_q \cos w_0\sqrt{1-\xi^2}\,t\right)$$
$$+ e^{-w_0 \xi t}\left(A_q w_0\sqrt{1-\xi^2} \cos w_0\sqrt{1-\xi^2}\,t - B_q w_0\sqrt{1-\xi^2} \operatorname{sen} w_0\sqrt{1-\xi^2}\,t\right) \quad \text{III}$$

$$\dot{\alpha}(t) = -w_0 \xi e^{-w_0 \xi t}\left(A_\alpha \operatorname{sen} w_0\sqrt{1-\xi^2}\,t + B_\alpha \cos w_0\sqrt{1-\xi^2}\,t\right)$$
$$+ e^{-w_0 \xi t}\left(A_\alpha w_0\sqrt{1-\xi^2} \cos w_0\sqrt{1-\xi^2}\,t - B_\alpha w_0\sqrt{1-\xi^2} \operatorname{sen} w_0\sqrt{1-\xi^2}\,t\right) \quad \text{IV}$$

Dada as condições iniciais $\alpha(0) = \alpha_0$ e $q(0) = q_0$, usamos

$$\begin{bmatrix} \dot{q}_0 \\ \dot{\alpha}_0 \end{bmatrix} = A \begin{bmatrix} q_0 \\ \alpha_0 \end{bmatrix} \quad \text{V}$$

$$\begin{bmatrix} \dot{q}_0 \\ \dot{\alpha}_0 \end{bmatrix} = \begin{bmatrix} -M_q & -M_\alpha \\ 1 & -\left(\dfrac{L_\alpha}{V_e} + \dfrac{\cancel{g}}{V_e (E)}\right) \end{bmatrix} \begin{bmatrix} q_0 \\ \alpha_0 \end{bmatrix} \qquad (VI)$$

Dessa forma, encontramos as constantes em (I), (II), (III) e (IV)

$$q(0) = B_q = q_0$$

$$\alpha(0) = B_\alpha = \alpha_0$$

$$\dot{q}(0) = -w_0 \xi B_q + A_q w_0 \sqrt{1-\xi^2} = \dot{q}_0 \qquad (VII)$$

$$\dot{\alpha}(0) = -w_0 \xi B_\alpha + A_\alpha w_0 \sqrt{1-\xi^2} = \dot{\alpha}_0$$

Ou seja

$B_q = q_0$

$B_\alpha = \alpha_0$

$$A_q = \frac{\dot{q}_0 + \omega_0 \xi q_0}{\omega_0 \sqrt{1-\xi^2}}$$

$$A_\alpha = \frac{\dot{\alpha}_0 + \omega_0 \xi \alpha_0}{\omega_0 \sqrt{1-\xi^2}}$$

VIII