Exercícios 8

5) Equações do movimento para um voo horizontal estabilizado com $C_{l_{\delta r}} = C_{n_{\delta a}} = C_{y_{\delta a}} = 0$

$$\begin{cases} \frac{1}{2}\rho_e S V_e^2 \left(C_{y_\beta}\beta + C_{y_{\delta r}}\delta r\right) + mg \operatorname{sen}\phi_1 = 0 \\ C_{l_\beta}\beta + C_{l_{\delta a}}\delta a = 0 \\ C_{n_\beta}\beta + C_{n_{\delta r}}\delta r = 0 \end{cases}$$

Analisando

$$\Rightarrow \frac{\delta_r}{\beta_e} = -\frac{C_{n_\beta}}{C_{n_{\delta r}}} \Rightarrow \frac{\delta_r}{\beta_e} > 0 \Rightarrow \begin{cases} \text{Se } \beta_e < 0 & \Rightarrow \delta_r < 0 \\ \text{Se } \beta_e > 0 & \Rightarrow \delta_r > 0 \end{cases}$$

$$\Rightarrow \frac{\delta_a}{\beta_e} = -\frac{C_{l_\beta}}{C_{l_{\delta a}}} \Rightarrow \frac{\delta_a}{\beta_e} < 0 \Rightarrow \begin{cases} \text{Se } \beta_e < 0 & \Rightarrow \delta_a > 0 \\ \text{Se } \beta_e > 0 & \Rightarrow \delta_a < 0 \end{cases}$$

$$\Rightarrow \frac{\operatorname{sen}\varphi_1}{\beta_e} = -\frac{1}{2mg\,C_{n_{\delta r}}}\rho_e S V_e^2 \left(C_{n_{\delta r}} C_{y_\beta} - C_{y_{\delta r}} C_{n_\beta}\right) > 0$$

$$\varphi_1 = \operatorname{arc\,sen}\left(-\frac{\beta_e}{2mg\,C_{n_{\delta r}}}\rho_e S V_e^2 \left(C_{n_{\delta r}} C_{y_\beta} - C_{y_{\delta r}} C_{n_\beta}\right)\right)$$

$$\Rightarrow \begin{cases} \text{Si} \quad \beta_e < 0 & \Rightarrow \quad \varphi_1 < 0 \\ \text{Si} \quad \beta_e > 0 & \Rightarrow \quad \varphi_1 > 0 \end{cases}$$

Logo para $\beta_e < 0$

$\delta_r < 0 \Rightarrow$ pedal direito

$\delta_a > 0 \Rightarrow$ manche à esquerda

$\phi_1 < 0 \Rightarrow$ asa esquerda abaixada

6) Sem hipóteses simplificadoras

$$\frac{1}{2}\rho_e S V_e^2 \left(C_{y_\beta}\beta + C_{y_{\delta a}}\delta a + C_{y_{\delta r}}\delta r \right) + mg\,\mathrm{sen}\,\phi_t = 0$$

$$C_{l_\beta}\beta + C_{l_{\delta a}}\delta a + C_{l_{\delta r}}\delta r = 0$$

$$C_{n_\beta}\beta + C_{n_{\delta a}}\delta a + C_{n_{\delta r}}\delta r = 0$$

Escalonamento

$$\begin{bmatrix} mg & \frac{1}{2}\rho_e S V_e^2 C_{y_{\delta a}} & \frac{1}{2}\rho_e S V_e^2 C_{y_{\delta r}} \\ 0 & C_{l_{\delta a}} & C_{l_{\delta r}} \\ 0 & C_{n_{\delta a}} & C_{n_{\delta r}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\rho_e S V_e^2 C_{y_\beta}\beta \\ -C_{l_\beta}\beta \\ -C_{n_\beta}\beta \end{bmatrix}$$

Fazer: $2L \cdot \left(\frac{1}{C_{l_{\delta a}}}\right)$

$3L \cdot \left(\frac{1}{C_{n_{\delta a}}}\right)$

$1L \cdot \left(\frac{1}{mg}\right)$

$$\begin{bmatrix} 1 & \dfrac{\rho_e S V_e^2 C_{y_{\delta a}}}{2mg} & \dfrac{\rho_e S V_e^2 C_{y_{\delta r}}}{2mg} \\ 0 & 1 & \dfrac{C_{l_{\delta r}}}{C_{l_{\delta a}}} \\ 0 & 1 & \dfrac{C_{n_{\delta r}}}{C_{n_{\delta a}}} \end{bmatrix} = \begin{bmatrix} -\dfrac{\rho_e S V_e^2 C_{y_\beta}\beta}{2mg} \\ -\dfrac{C_{l_\beta}\beta}{C_{l_{\delta a}}} \\ -\dfrac{C_{n_\beta}\beta}{C_{n_{\delta a}}} \end{bmatrix}$$

Fazer: $3L \leftarrow 2L$

$$\begin{bmatrix} 1 & \frac{\rho_e S V_e^2 C_{y_{\delta a}}}{2mg} & \frac{\rho_e S V_e^2 C_{y_{\delta r}}}{2mg} \\ 0 & 1 & \frac{C_{l_{\delta r}}}{C_{l_{\delta a}}} \\ 0 & 0 & \frac{C_{n_{\delta r}}}{C_{n_{\delta a}}} - \frac{C_{l_{\delta r}}}{C_{l_{\delta a}}} \end{bmatrix} = \begin{bmatrix} -\frac{\rho_e S V_e^2 C_{y_\beta} \beta}{2mg} \\ -\frac{C_{l_\beta} \beta}{C_{l_{\delta a}}} \\ -\frac{C_{n_\beta} \beta}{C_{n_{\delta a}}} + \frac{C_{l_\beta} \beta}{C_{l_{\delta a}}} \end{bmatrix}$$

Fazer: $3L \cdot \left(\frac{1}{\frac{C_{n_{\delta r}}}{C_{n_{\delta a}}} - \frac{C_{l_{\delta r}}}{C_{l_{\delta a}}}} \right)$

$$\begin{bmatrix} 1 & \dfrac{\rho_e S V_e^2 C_{y_{\delta a}}}{2mg} & \dfrac{\rho_e S V_e^2 C_{y_{\delta r}}}{2mg} \\ 0 & 1 & \dfrac{C_{l_{\delta r}}}{C_{l_{\delta a}}} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\dfrac{\rho_e S V_e^2 C_{y_\beta} \beta}{2mg} \\ -\dfrac{C_{l_\beta} \beta}{C_{l_{\delta a}}} \\ \left(\dfrac{C_{l_\beta} C_{n_{\delta a}} - C_{n_\beta} C_{l_{\delta a}}}{C_{n_{\delta r}} C_{l_{\delta a}} - C_{l_{\delta r}} C_{n_{\delta a}}} \right) \beta \end{bmatrix}$$

Fazer: $3L\left(-\frac{C_{l_{\delta r}}}{C_{l_{\delta a}}}\right)+2L$

$$3L\left(-\frac{\rho_e S V_e^2 C_{y_{\delta r}}}{2mg}\right)+1L$$

$$2L\left(-\frac{\rho_e S V_e^2 C_{y_{\delta a}}}{2mg}\right)+1L$$

Logo:

$$\delta_r = \beta\left(\frac{C_{l_\beta} C_{n_{\delta a}} - C_{n_\beta} C_{l_{\delta a}}}{C_{n_{\delta r}} C_{l_{\delta a}} - C_{n_{\delta a}} C_{l_{\delta r}}}\right)$$

$$\delta_a = \beta\left(\frac{C_{n_\beta} C_{l_{\delta r}} - C_{l_\beta} C_{n_{\delta r}}}{C_{n_{\delta r}} C_{l_{\delta a}} - C_{n_{\delta a}} C_{l_{\delta r}}}\right)$$

$$\operatorname{sen}\phi_1 = -\frac{\rho_e S V_e^2 \beta}{2mg}\left(C_{y_\beta} + \frac{\left(C_{l_\beta} C_{n_{\delta a}} - C_{n_\beta} C_{l_{\delta a}}\right) C_{y_{\delta r}} + \left(C_{n_\beta} C_{l_{\delta r}} - C_{l_\beta} C_{n_{\delta r}}\right) C_{y_{\delta a}}}{C_{n_{\delta r}} C_{l_{\delta a}} - C_{n_{\delta a}} C_{l_{\delta r}}}\right)$$

h) Voo horizontal (não simétrico) com $C_{l_{\delta r}} = C_{n_{\delta a}} = C_{y_{\delta a}} = 0$

$$\frac{1}{2}\rho_e S V_e^2 \left(C_{y_\beta}\beta + C_{y_{\delta r}}\delta r\right) + mg\,\text{sen}\,\phi_1 = 0$$

$$C_{l_\beta}\beta + C_{l_{\delta a}} = 0$$

$$C_{n_\beta}\beta + C_{n_{\delta r}}\delta r + C_{n_F} = 0$$

i) $\beta = 0$

$$\frac{1}{2}\rho_e S V_e^2 C_{y_{\delta r}} \delta r + mg \sin\phi_1 = 0$$

$$C_{l_{\delta a}} \delta a = 0$$

$$C_{n_{\delta r}} \delta r + C_{n_F} = 0$$

temos

$$\Rightarrow \delta a = 0$$

$$\Rightarrow \frac{\delta r}{C_{n_F}} = -\frac{1}{C_{n_{\delta r}}} \Rightarrow \frac{\delta r}{C_{n_F}} > 0 \Rightarrow \begin{cases} \text{Se } C_{n_F} < 0 & \Rightarrow \delta r < 0 \\ \text{Se } C_{n_F} > 0 & \Rightarrow \delta r > 0 \end{cases}$$

$$\Rightarrow \operatorname{sen}\phi_{\perp} = -\frac{1}{2}\frac{\rho_0 S V_e^2}{mg} C_{y_{\delta r}}\,\delta r = \frac{1}{2}\frac{\rho_0 S V_e^2}{mg} C_{y_{\delta r}}\frac{C_{n_F}}{C_{n_{\delta r}}}$$

$$\frac{\operatorname{sen}\phi_{\perp}}{C_{n_F}} < 0 \Rightarrow \begin{cases} \text{Se } C_{n_F} < 0 \Rightarrow \phi_{\perp} > 0 \\ \text{Se } C_{n_F} > 0 \Rightarrow \phi_{\perp} < 0 \end{cases}$$

Logo para $C_{n_F} < 0$

$\delta a = 0 \Rightarrow$ aileron neutro

$\delta r < 0 \Rightarrow$ pedal da esquerda

$\phi_1 > 0 \Rightarrow$ asa direita baixa

12) Para um voo curvilíneo horizontal sem os efeitos parasitas

$$l_{\delta r} = n_{\delta a} = Y_{\delta r} = Y_{\delta a} = 0$$

$$\begin{cases} \Omega V_e \cos\phi - Y_\beta \beta - g \operatorname{sen}\phi = 0 \\ l_\beta \beta + l_r \Omega + l_{\delta a} \delta a = 0 \\ n_\beta \beta + n_r \Omega + n_{\delta r} \delta r = 0 \end{cases}$$

i) $\phi = 0$

$$\frac{\beta}{\Omega} = \frac{V_e}{Y_\beta} < 0$$

$$\frac{\delta_r}{\Omega} = \frac{-\frac{n_\beta \beta}{\Omega} + n_r}{n_{\delta r}} < 0$$

$$\frac{\delta_a}{\Omega} = \frac{-\frac{l_\beta \beta}{\Omega} + l_r}{l_{\delta a}} > 0$$

- Se $r < 0$

  $\beta < 0 \Rightarrow$ vento da esquerda

  $\delta_r < 0 \Rightarrow$ pedal da direita

  $\delta_a > 0 \Rightarrow$ manche à esquerda

- Se $r > 0$

  $\beta > 0 \Rightarrow$ vento da direita

  $\delta_r > 0 \Rightarrow$ pedal da esquerda

  $\delta_a < 0 \Rightarrow$ manche à direita

ii) $\beta = 0$

$$\frac{\tan\phi}{r} = \frac{V_e}{g} > 0$$

$$\frac{\delta_r}{r} = -\frac{n_r}{n_{\delta r}} < 0$$

$$\frac{\delta_a}{r} = -\frac{l_r}{l_{\delta a}} > 0$$

- Se $\Omega > 0$

  $\phi > 0 \Rightarrow$ asa direita abaixada

  $\delta_r < 0 \Rightarrow$ pedal da direita

  $\delta_a > 0 \Rightarrow$ manche à esquerda

- Se $\Omega < 0$

  $\phi < 0 \Rightarrow$ asa esquerda abaixada

  $\delta_r > 0 \Rightarrow$ pedal da esquerda

  $\delta_a < 0 \Rightarrow$ manche à direita