Lista 7

(5) equação do movimento para um voo horizontal estabilizado $(C_{l_{\delta r}} = C_{n_{\delta a}} = C_{y_{\delta a}} = 0)$

$$\frac{1}{2}\rho_e S V_e^2 (C_{y\beta} + C_{y_{\delta r}}\delta r) + mg\,\operatorname{sen}\varphi_1 = 0$$

$$C_{l_\beta}\beta + C_{l_{\delta a}}\delta a = 0$$

$$C_{n_\beta}\beta + C_{n_{\delta r}}\delta r = 0$$

$$P.\ \beta_e < 0$$

$$\frac{\delta r}{\beta_e} = -\frac{C_{n\beta}}{C_{n_{\delta r}}} \Rightarrow \frac{\delta r}{\beta_e} > 0 \Rightarrow \begin{cases} \text{se } \beta_e < 0 \Rightarrow \delta_r < 0 \\ \beta_e > 0 \Rightarrow \delta r > 0 \end{cases}$$

$$\frac{\delta_a}{\beta_e} = -\frac{C_{l\beta}}{C_{l_{\delta a}}} \Rightarrow \frac{\delta a}{\beta_e} < 0 \Rightarrow \begin{cases} \text{se } \beta_e < 0 \Rightarrow \delta a > 0 \\ \beta_e > 0 \Rightarrow \delta a < 0 \end{cases}$$

$$\frac{\operatorname{sen}\varphi_1}{\beta_e} = -\frac{1}{2mg\,C_{n_{\delta r}}}\,\rho_e S V_e^2 (C_{n_{\delta e}} C_{y_\beta} - C_{n_{\delta r}} C_{n\beta}) > 0 \Rightarrow \begin{cases} \text{Se } \beta_e < 0 \Rightarrow \varphi_1 < 0 \\ \beta_e > 0 \Rightarrow \varphi_1 > 0 \end{cases}$$

(6)
$$\frac{1}{2}\rho_e S V_e^2 \left(C_{y_\beta}\beta + C_{y_{\delta a}}\delta a + C_{y_{\delta r}}\delta r \right) + mg\,\operatorname{sen}\varphi_1 = 0$$

$$C_{l_\beta}\cdot\beta + C_{l_{\delta a}}\delta a + C_{l_{\delta r}}\delta r = 0$$

$$C_{n_\beta}\beta + C_{n_{\delta a}}\delta a + C_{n_{\delta r}}\delta r = 0$$

→ Escalonamento

$$\begin{bmatrix} mg & \frac{1}{2}\rho_e S V_e^2 \cdot C_{y\delta a} & \\ 0 & C_{l_{\delta a}} & C_{l_{\delta r}} \\ 0 & C_{n_{\delta a}} & C_{n_{\delta r}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\rho_e V_e^2 S\, C_{y_\beta}\beta \\ -C_{l_\beta}\cdot\beta \\ -C_{n_\beta}\beta \end{bmatrix}$$

$$1^\circ L\cdot\left(\frac{1}{mg}\right) \Rightarrow 2^\circ L\cdot\left(\frac{1}{C_{l_{\delta a}}}\right) \Rightarrow 3^\circ L\cdot\left(\frac{1}{C_{n_{\delta a}}}\right) \Rightarrow 3^\circ L - 2^\circ L \Rightarrow 3^\circ L / \left(\frac{C_{n_{\delta r}} - C_{l_{\delta r}}}{C_{n_{\delta a}} - C_{l_{\delta a}}}\right)$$

$$\Rightarrow 3^\circ L\left(-\frac{C_{l_{\delta r}}}{C_{l_{\delta a}}}\right) + 2^\circ L \Rightarrow 3^\circ L\cdot\left(-\frac{\rho_e S V_e^2 C_{y_{\delta r}}}{2mg}\right) + 1^\circ L \Rightarrow 2^\circ L\left(-\frac{\rho S_e^2 C_{y_{\delta r}}}{2mg}\right) + 1^\circ L$$

$$\delta r = \beta\left(\frac{C_{l\beta} C_{n_{\delta a}} - C_{n\beta} C_{l_{\delta a}}}{C_{n_{\delta r}} C_{l_{\delta a}} - C_{n_{\delta a}} C_{l_{\delta r}}}\right);\quad \delta a = \beta\left(\frac{C_{n\beta} C_{l_{\delta r}} - C_{l\beta} C_{n_{\delta r}}}{C_{n_{\delta r}} \cdot C_{l_{\delta a}} - C_{n_{\delta a}} C_{l_{\delta r}}}\right)$$

$$\rightsquigarrow \operatorname{sen}\varphi_1 = -\frac{\rho_e S V_e^2 \beta}{2mg}\left(C_{y_\beta} + \frac{(C_{l_\beta} C_{m_{\delta a}} - C_{m_\beta} C_{l_{\delta a}}) C_{y_{\delta r}} + (C_{n_\beta} C_{l_{\delta r}} - C_{l_\beta} C_{n_{\delta r}}) C_{y_{\delta a}}}{C_{n_{\delta r}} C_{l_{\delta a}} - C_{m_{\delta a}} C_{l_{\delta r}}}\right)$$

7) $(C_{l_{\delta r}} = C_{m_{\delta a}} = C_{y_{\delta a}} = 0)$

$$\frac{1}{2}\rho_e S V_e^2 (C_{y_\beta}\cdot\beta + C_{y_{\delta r}}\delta r) + mg \operatorname{sen}\varphi_1 = 0$$

$$C_{l_\beta}\beta + C_{l_{\delta a}}\delta a = 0$$

$$C_{n_\beta}\cdot\beta + C_{m_{\delta r}}\delta r + C_{m_F} = 0$$

⇝ Pane no motor da esquerda $C_{m_F} < 0$

i) $\beta = 0$

$$\frac{1}{2}\rho_e S V_e^2 C_{y_{\delta r}}\delta r + mg \operatorname{sen}\varphi_1 = 0$$

$$C_{l_{\delta a}}\delta a = 0$$

$$C_{m_{\delta r}}\cdot\delta r + C_{m_F} = 0$$

⇝ Temos:

$\Rightarrow \delta a = 0$

$$\Rightarrow \frac{\delta r}{C_{m_F}} = \frac{-1}{C_{m_{\delta r}}} \Rightarrow \frac{\delta r}{C_{m_F}} > 0 \Rightarrow \begin{cases} \text{se } C_{m_F} < 0 \Rightarrow \delta r < 0 \\ C_{m_F} > 0 \Rightarrow \delta r > 0 \end{cases}$$

$$\Rightarrow \operatorname{sen}\varphi_1 = -\frac{1}{2}\frac{\rho_e S V_e^2}{mg} C_{y_{\delta r}}\cdot\delta r = \frac{1}{2}\frac{\rho_e S V_e^2}{mg} C_{y_{\delta r}}\cdot\frac{C_{m_F}}{C_{m_{\delta r}}}$$

$$\Rightarrow \frac{\operatorname{sen}\varphi_1}{C_{m_F}} < 0 \Rightarrow \begin{cases} \text{se } C_{m_F} < 0 \Rightarrow \varphi_1 > 0 \\ C_{m_F} > 0 \Rightarrow \varphi_1 < 0 \end{cases}$$

ii) $\varphi_1 = 0$

$$\frac{1}{2}\rho_e S V_e^2 (C_{y_\beta}\beta \pm C_{y_{\delta r}}\delta r) = 0$$

$$C_{l_\beta}\beta + C_{l_{\delta a}}\cdot\delta a = 0$$

$$C_{m_\beta}\cdot\beta + C_{m_{\delta r}}\cdot\delta r + C_{m_F} = 0$$

Temos:

$$\frac{\beta}{\delta r} = -\frac{C_{y_{\delta r}}}{C_{y_\beta}} \Rightarrow \frac{\beta}{\delta r} > 0 \Rightarrow \begin{cases} \text{se } \beta < 0 \Rightarrow \delta r < 0 \\ \beta > 0 \Rightarrow \delta r > 0 \end{cases}$$

$$\Rightarrow \frac{\delta_a}{\beta} = -\frac{C_{l\beta}}{C_{l\delta a}} \Rightarrow \frac{\delta_a}{\beta} < 0 \Rightarrow \begin{cases} \text{se } \beta < 0 \Rightarrow \delta_a > 0 \\ \beta > 0 \Rightarrow \delta_a < 0 \end{cases}$$

$$\Rightarrow \frac{\delta_r}{C_{nF}} = \frac{C_{y\beta}}{(C_{m\beta} C_{y\delta r} - C_{m\delta r} C_{y\beta})} > 0 \Rightarrow \begin{cases} \text{se } C_{nF} < 0 \Rightarrow \delta_r < 0 \\ C_{mF} > 0 \Rightarrow \delta_r > 0 \end{cases}$$

(8) Sem hipóteses simplificadoras

$$\frac{1}{2}\rho_0 V_e^2 S \left(C_{y\beta} \cdot \beta + C_{y\delta a} \cdot \delta_a + C_{y\delta r} \delta_r \right) + mg \operatorname{sen} \varphi_1 = 0$$

$$C_{l\beta} \cdot \beta + C_{l\delta a} \delta_a + C_{l\delta r} = 0$$

$$C_{m\beta} \cdot \beta + C_{m\delta a} \delta_a + C_{m\delta r} \cdot \delta_r + C_{mF} = 0$$

→ escalonamento

i) $\beta = 0$

$$\operatorname{sen} \varphi_1 = -\frac{1}{2} \frac{\rho_0 S V_e^2}{mg} \left(C_{y\delta a} \cdot \delta_a + C_{y\delta r} \cdot \delta_r \right)$$

$$\delta_r = \frac{C_{mF} \cdot C_{l\delta a}}{\Delta_2}$$

$$\delta_a = -C_{mF} \cdot \frac{C_{l\delta r}}{\Delta_2}$$

$$\Delta_2 = C_{m\delta a} C_{l\delta r} - C_{m\delta r} C_{l\delta a}$$

ii) $\varphi_1 = 0$

$$\beta = -\frac{C_{mF}}{K}$$

$$\delta_r = \frac{\beta \cdot C_{y\beta} \cdot C_{l\delta a} - C_{y\delta a} \cdot C_{l\beta}}{\Delta_1}$$

$$\delta_a = \beta \frac{C_{y\delta r} C_{l\beta} - C_{y\beta} \cdot C_{l\delta r}}{\Delta_1}$$

$$K = C_{m\beta} + \frac{(C_{y\beta} C_{l\delta a} - C_{y\delta a} C_{l\beta}) C_{m\delta r} + (C_{y\delta r} C_{l\beta} - C_{y\beta} C_{l\delta r}) C_{m\delta a}}{\Delta_1}$$

$$\Delta_1 = C_{m\delta a} C_{l\delta r} - C_{y\delta r} \cdot C_{l\delta a}$$

1) → Quando $\beta > 0 \Rightarrow C_{y_\beta} < 0$

→ Quando $\delta_r > 0 \Rightarrow C_{y_{\delta_r}} > 0$

→ Se o controle de rolamento é constituído por ailerons ou spoilers situados longe da fuselagem, nenhuma força lateral resultará da deflexão destes elementos e, por conseguinte, $C_{y_{\delta a}}$ é nulo em geral. No caso onde tais superfícies estiverem localizadas perto da fuselagem, suas deflexões podem provocar uma dissemetria do escoamento em torno da fuselagem que conduz a uma força lateral. Portanto, é impossível de se prever, sem ensaios, o sentido desta força lateral, e portanto, de deduzir uma regra geral para o sinal de $C_{y_{\delta a}}$

2) → Para $\delta_a > 0 \Rightarrow C_{l_{\delta a}} < 0$ (Pequenos valores)

→ $\delta_a > 0 \Rightarrow C_{n_{\delta a}} > 0$

3) → $\delta_r > 0 \Rightarrow C_{n_{\delta r}} < 0$ (Pequenos valores)

→ $\delta_r > 0 \Rightarrow C_{l_{\delta r}} > 0$

4) → $\beta > 0 \Rightarrow C_{L_\beta} < 0$ (Pequenos valores)

→ $\beta > 0 \Rightarrow C_{n_\beta} > 0$

$\varphi_1 = -1,13$
$\beta = 2^\circ,30$

9
$V_e = 60 \, m/s$
$g = 9,809 \, m/s^2$
$\rho = 1,112 \, kg/m^3$
$FE = 60\,000 \, N$
$FD = -2000 \, N$
$S = 260 \, m^2$
$l = 6,61$

i) Para $\beta = 0$

Falta o valor da constante b!

11 → Equações para um voo curvilíneo horizontal 11

$$\Omega V_e \cos\varphi - Y_\beta \beta - Y_{\delta r}\delta r - Y_{\delta a}\delta_a - g \operatorname{sen}\varphi = 0$$

$$l_\beta \beta + l_r \Omega + l_{\delta a}\delta a + l_{\delta r}\delta r = 0$$

$$n_\beta \beta + n_r \Omega + m_{\delta a}\delta a + m_{\delta r}\cdot\delta r = 0$$

→ Sem os efeitos parasitas. $(l_{\delta r} = m_{\delta a} = Y_{\delta r} = Y_{\delta a} = 0)$

$$\Omega V_e \cos\varphi - Y_\beta \beta - g \operatorname{sen}\varphi = 0$$

$$l_\beta \cdot \beta + l_r \Omega + l_{\delta a}\delta a = 0$$

$$n_\beta \cdot \beta + n_r \Omega + m_{\delta r}\cdot \delta r = 0$$

i) $\varphi = 0 \therefore \quad \dfrac{\beta}{\Omega} = \dfrac{V_e}{Y_\beta} < 0$

$$\frac{\delta r}{\Omega} = \frac{-n_\beta \cdot \frac{\beta}{\Omega} + n_r}{n_{\delta r}} < 0$$

$$\frac{\delta a}{\Omega} = \frac{-l_\beta \cdot \frac{\beta}{\Omega} + l_r}{l_{\delta a}} > 0$$

- se $\Omega > 0$ (curva para direita)
  - $\beta < 0$ ⇒ vento da esquerda
  - $\delta_r < 0$ ⇒ pedal da direita
  - $\delta_a < 0$ ⇒ manche para a direita

- se $\Omega < 0$ (curva para a esquerda)
  - $\beta > 0$ ⇒ vento da direita
  - $\delta_r > 0$ ⇒ pedal da esquerda
  - $\delta_a > 0$ ⇒ manche para à esquerda.

ii) $\beta = 0 \therefore \quad \dfrac{tg\,\varphi}{\Omega} = \dfrac{V_e}{g} > 0$

$$\frac{\delta_r}{\Omega} = \frac{-n_r}{n_{\delta r}} < 0$$

$$\frac{\delta_a}{\Omega} = \frac{-l_r}{l_{\delta a}} > 0$$

- se $\Omega > 0$
  - $\varphi > 0$ ⇒ asa direita abaixada
  - $\delta_r < 0$ ⇒ pedal da direita
  - $\delta_a > 0$ ⇒ manche à esquerda

- se $\Omega < 0$
  - $\varphi < 0$ ⇒ asa esquerda abaixada
  - $\delta_r > 0$ ⇒ pedal da esquerda
  - $\delta_a < 0$ ⇒ manche à direita

iii) $\delta_r = 0$ ∴ $\frac{\beta}{\Omega} = -\frac{\eta_r}{\eta_\beta} > 0$

$$\frac{tg\,\varphi}{\Omega} = \frac{V_e}{g}\left[1 - \frac{Y_\beta \beta}{V_e \Omega \cos\varphi}\right] > 0$$

$$\frac{\delta_a}{\Omega} = -\frac{(\eta_\beta l_r - \eta_r l_\beta)}{\eta_\beta l_{\delta a}} \quad ?$$

• se $\Omega < 0$

$\beta < 0 \Rightarrow$ vento da esquerda

$\varphi < 0 \Rightarrow$ asa esquerda abaixada, maior que no caso coordenado.

δ_r: sinal oposto de $(\eta_\beta l_r - n_r l_\beta)$

• se $\Omega > 0$

$\beta > 0 \Rightarrow$ vento da direita

$\varphi > 0 \Rightarrow$ asa direita abaixada, maior que na curva coordenada

δ_a: mesmo sinal de $(\eta_\beta l_r - n_r l_\beta)$

iv) $\delta_a = 0$ ∴ $\frac{\beta}{\Omega} = \frac{-l_r}{l_\beta} > 0$

$$\frac{tg\,\varphi}{\Omega} = \frac{V_e}{g}\left[1 - \frac{Y_\beta \beta}{V_e \Omega \cos\varphi}\right] > 0$$

$$\frac{\delta_r}{\Omega} = \frac{\eta_\beta l_r - n_r l_\beta}{l_\beta n_{\delta a}} \quad ?$$

• se $\Omega > 0$

$\beta > 0 \Rightarrow$ vento da direita

$\varphi > 0 \Rightarrow$ asa direita abaixada, maior que no caso coordenado

δ_r: mesmo sinal de $(\eta_\beta l_r - \eta_r l_\beta)$

• se $\Omega < 0$

$\beta < 0 \Rightarrow$ vento da esquerda

$\varphi < 0 \Rightarrow$ asa esquerda abaixada, maior que no caso coordenado

δ_r: sinal oposto de $(\eta_\beta l_r - n_r l_\beta)$

(12) → Dados:

$\Omega = 30°/min$

$H = 9000$ km

$V_e = 200$ m/s

$g = 9{,}7711$ m/s²

$\rho = 0{,}4663$ Kg/m³

→ Dados do Airbus (apêndice C)

$S = 260$ m²

$l = 6{,}61$ m

$m = 120\,000$ Kg

$I_x = 5{,}55 \cdot 10^6$ kg·m²

$I_z = 14{,}51 \cdot 10^6$ kg·m²

$C_{y_\beta} = -1{,}5$

$C_{l_\beta} = -1{,}3$

$C_{l_r} = 2{,}9$

$C_{l_{\delta a}} = -0{,}33$

$C_{m_\beta} = 1{,}75$

$C_{m_{\delta r}} = -1{,}0$

$C_{m_r} = -7{,}5$

• Cálculos preliminares:

$$Y_\beta = \frac{\rho S V_e^2}{2m} \cdot C_{y_\beta} = -30{,}31$$

$$l_\beta = \frac{\rho S V_e^2 l}{2 I_x} \cdot C_{l_\beta} = -3{,}75$$

$$l_r = \frac{\rho S V_e l^2}{2 I_x} \cdot C_{l_r} = 0{,}28$$

$$l_{\delta a} = \frac{\rho S V_e^2 l}{2 I_x} \cdot C_{l_{\delta a}} = -0{,}95$$

$$n_\beta = \frac{\rho S V_e^2 l}{2 I_z} \cdot C_{m_\beta} = 1{,}93$$

$$n_r = \frac{\rho S V_e l^2}{2 I_z} C_{m_r} = -0{,}27$$

$$n_{\delta r} = \frac{\rho S V_e^2 l}{2 I_z} C_{m_{\delta r}} = -1{,}10$$

• Usando as eq. obtidas na questão 11.

i) $\varphi = 0$

$$\rightarrow \beta = \frac{\Omega \cdot V_e}{Y_\beta} = \frac{0{,}5°/s \cdot 200 \text{ m/s}}{-30{,}31} = -3{,}30° \quad \text{(vento da esquerda)}$$

$$\rightarrow \delta_r = \left[\frac{\frac{-n_\beta \cdot \beta}{\Omega} + n_r}{n_{\delta r}}\right] \cdot \Omega = -5{,}67° \quad \text{(pedal da direita)}$$

$$\rightarrow \delta_a = \left[\frac{\frac{-l_\beta \cdot \beta}{\Omega} + l_r}{l_{\delta a}}\right] \Omega = 13{,}17° \quad \text{(manche p/ esquerda)}$$

ii) $\beta = 0$

$$\rightarrow \varphi = \operatorname{arctg}\left(\frac{V_e \cdot \Omega}{g}\right) = 10{,}13° \quad \text{(asa direita abaixada)}$$

$$\rightarrow \delta_r = \frac{-n_r \cdot \Omega}{n_{\delta r}} = -0{,}12° \quad \text{(pedal da direita)}$$

$$\rightarrow \delta_a = \frac{-l_r \cdot \Omega}{l_{\delta a}} = 0{,}15° \quad \text{(manche a esquerda)}$$

$^\circ \xrightarrow{\cdot \frac{\pi}{180}} rad$

iii) $\delta_r = 0$

$$\rightarrow \beta = -\frac{n_r}{n_\beta}\,\Omega = 0{,}07^\circ = 1{,}22\cdot 10^{-3}\ rad$$

$$\rightarrow \phi = \operatorname{artg}\left[\frac{V_e\cdot\Omega}{g}\left(1 - \frac{y_\beta\cdot\beta}{V_e\cdot\Omega\cos\Omega}\right)\right] = 0{,}197\,rad = 11{,}30^\circ$$

$$\rightarrow \delta_a = -\frac{(n_\beta l_r - n_r l_\beta)}{n_\beta\cdot l_{\delta a}}\,\Omega = -0{,}13^\circ$$

(iv) $\delta_a = 0$

$$\rightarrow \beta = -\frac{l_r}{l_\beta}\,\Omega = 0{,}04^\circ = 6{,}52\cdot 10^{-4}\ rad$$

$$\rightarrow \phi = \operatorname{artg}\left[\frac{V_e\cdot\Omega}{g}\left(1 - \frac{y_\beta\cdot\beta}{V_e\,\Omega\cos\phi}\right)\right] \Rightarrow \text{(calculando iterativamente) } \phi = 0{,}179\,rad = 10{,}29^\circ$$

$$\rightarrow \delta_r = \frac{n_\beta l_r - n_r l_\beta}{l_\beta\, n_{\delta r}}\,\Omega = -0{,}06^\circ$$

13 → Dados:

$\Omega = -60^\circ/min$

$H = 4000$ km

$V_e = 720{,}53$ m/s

$g = 9{,}7864\ m/s^2$

$\rho = 0{,}8191\ kg/m^3$

→ Dados do Mirage III (apêndice C)

$S = 36\ m^2$

$l = 5{,}25$ m

$m = 7400$ kg

$I_x = 0{,}9\cdot 10^4\ kg/m^2$

$I_z = 6\cdot 10^7\ kg/m^2$

$C_{y_\beta} = -0{,}6$

$C_{l_\beta} = -0{,}05$

$C_{l_r} = 0{,}06$

$C_{l_{\delta a}} = -0{,}30$

$C_{m_\beta} = 0{,}15$

$C_{m_r} = -0{,}7$

$C_{m_{\delta r}} = -0{,}085$

- Cálculos preliminares (análoga a questão 12)

$y_\beta = -620{,}63$

$l_\beta = -223{,}25$

$l_r = 1{,}95$

$l_{\delta a} = -1339{,}5$

$n_\beta = 100{,}46$

$n_r = -3{,}42$

$n_{\delta r} = -56{,}93$

i) $\varphi = 0$

- $\beta = 1,16°$ (vento direita)
- $\delta_r = 117,30°$ (pedal esquerda)
- $\delta_a = -11,08°$ (manche direita)

ii) $\beta = 0$

- $\varphi = -52,10°$
- $\delta_r = 0,06°$
- $\delta_a = -0,0015°$

iii) $\delta_r = 0$

- $\beta = -0,039°$
- $\varphi = -52,90°$
- $\delta_a = 0,0042°$

iv) $\delta_a = 0$

- $\beta = -0.0087°$ (vento esquerda)
- $\varphi = -52,39°$ (asa esq. abaixada)
- $\delta_r = 0,0496°$