Lista 7

3) Movimento de curto período

$$\dot{q} = -M_q q - M_\alpha \bar{\alpha} - M_\delta \bar{\delta}_p$$

$$\dot{\bar{\alpha}} = q - \left(\frac{L_\alpha}{V_e} + \cancel{\frac{g}{V_e}}\right)\bar{\alpha} - \frac{L_\delta}{V_e}\bar{\delta}_p$$

$$\begin{bmatrix} \dot{q} \\ \dot{\bar{\alpha}} \end{bmatrix} = \begin{bmatrix} -M_q & -M_\alpha \\ 1 & -\left(\frac{L_\alpha}{V_e} + \cancel{\frac{g}{V_e}}\right) \end{bmatrix} \begin{bmatrix} q \\ \bar{\alpha} \end{bmatrix} + \begin{bmatrix} -M_\delta \\ -\frac{L_\delta}{V_e} \end{bmatrix} \bar{\delta}_p$$

Considerando $\bar{\delta}_p = 0$

$$\begin{bmatrix} \dot{q} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} -m_q & -m_\alpha \\ 1 & -\left(\frac{L_\alpha}{V_e} + \frac{g}{V_e \cancel{E'}}\right) \end{bmatrix} \begin{bmatrix} q \\ \alpha \end{bmatrix}$$

i) $\det(A - \mathbb{I}s) = 0$

$$\begin{vmatrix} -m_q - s & -m_\alpha \\ 1 & -\left(\frac{L_\alpha}{V_e} + \frac{g}{V_e E'}\right) - s \end{vmatrix} = 0$$

$$s^2 + \left(m_q + \frac{L_\alpha}{V_e} + \frac{g}{V_e \cancel{E'}}\right)s + m_\alpha + \left(\frac{L_\alpha}{V_e} + \frac{g}{V_e \cancel{E'}}\right) m_q = 0$$

ii) $s^2 + 2\omega_0 \xi s + \omega_0^2 = 0$

Frequência natural

$$\omega_0 = \sqrt{m_\alpha + m_q \left(\frac{L_\alpha}{V_e} + \frac{g}{V_e \cancel{\xi'}} \right)}$$

Amortecimento

$$\xi = \frac{\left(m_q + \frac{L_\alpha}{V_e} + \frac{g}{V_e \cancel{\xi'}} \right)}{2\omega_0}$$

Dessa forma, temos a solução para s

$$s = \frac{-2\omega_0\xi \pm \sqrt{4\omega_0^2\xi^2 - 4\omega_0^2}}{2}$$

$$s = -\omega_0\xi \pm \omega_0\sqrt{\xi^2 - 1}$$

iii) Multiplicando os comprimentos por λ

- ω_0 é dividido por $\sqrt{\lambda}$ $\Rightarrow$ diminui com o aumento de λ
- T é multiplicado por $\sqrt{\lambda}$ $\Rightarrow$ aumenta com o aumento de λ
- ξ é multiplicado por $\sqrt{\lambda}$ $\Rightarrow$ aumenta com o aumento de λ

iv) Temos

$$s = -\omega_0 \xi \pm \omega_0 \sqrt{\xi^2 - 1}$$

Portanto

- Se $\xi > 1 \Rightarrow s$ é real $\Rightarrow s_1 \neq s_2$

não oscilatório

SFS: $\{e^{s_1 t}, e^{s_2 t}\}$

- Se $\xi = 1 \Rightarrow s$ é real $\Rightarrow s_1 = s_2$

não oscilatório

SFS: $\{e^{s_1 t}, t e^{s_2 t}\}$

- Se $\xi < 1 \Rightarrow s$ é complexo

$$s = -w_0 \xi \pm i w_0 \sqrt{1-\xi^2} = a \pm ib$$

Movimento oscilatório

SFS: $\{e^{at}\cos bt,\ e^{at}\operatorname{sen} bt\}$

5.) $$\begin{cases} \dot{q} = -M_q q - M_\alpha \bar{\alpha} - M_\delta \bar{\delta}_p \\ \dot{\bar{\alpha}} = q - \left(\frac{L_\alpha}{V_e} - \frac{g}{V_e t'} \right) \bar{\alpha} - \frac{L_\delta}{V_e} \bar{\delta}_p \end{cases}$$

Aplicando a transformada de Laplace $\mathcal{L}[f(t)] = F(s)$ com $\bar{\alpha}(0) = 0$ e $q(0) = 0$

$$\begin{cases} (s + M_q)\, q(s) + M_\alpha \bar{\alpha}(s) = -M_\delta \bar{\delta}_p(s) \\ q(s) + \left(\left(\frac{L_\alpha}{V_e} + \frac{g}{V_e t'} \right) + s \right) \bar{\alpha}(s) = -\frac{L_\delta}{V_e} \bar{\delta}_p(s) \end{cases}$$

Luego

$$\bar{\alpha}(s) = - \frac{\frac{s L_\delta}{V_e} + m_q \frac{L_\delta}{V_e} + m_q}{s^2 + \left(\frac{L_\alpha}{V_e} + \frac{g}{V_e \cancel{t'}} + m_q\right)s + m_q\left(\frac{L_\alpha}{V_e} + \frac{g}{V_e \cancel{t'}}\right) + m_\alpha} \bar{\delta}_p(s) = G_{\alpha\delta}\, \bar{\delta}_p(s)$$

$$\bar{q}(s) = - \frac{s\, m_\delta + \left(\frac{L_\alpha}{V_e} + \frac{g}{V_e \cancel{t'}}\right) m_\delta - \frac{L_\delta}{V_e} m_\alpha}{s^2 + \left(\frac{L_\alpha}{V_e} + \frac{g}{V_e \cancel{t'}} + m_q\right)s + m_q\left(\frac{L_\alpha}{V_e} + \frac{g}{V_e \cancel{t'}}\right) + m_\alpha} \bar{\delta}_p(s) = G_{q\delta}\, \bar{\delta}_p(s)$$

i) $\bar{\delta}_p(s) = \frac{1}{s}\left[1 - e^{-st_0}\right]$

$$G_{\alpha f} = \frac{Qs + R}{(s-p)^2 + q^2}$$

$$G_{qf} = \frac{Q's + R'}{(s-p)^2 + q^2}$$

Dessa forma

$$\bar{\alpha}(s) = \eta(s) + \beta(s) = \frac{1}{s}\,\frac{Qs + R}{(s-p)^2 + q^2}\left[1 - e^{-s}\right]$$

$$q(s) = \eta'(s) + \beta'(s) = \frac{1}{s}\,\frac{Q's + R'}{(s-p)^2 + q^2}\left[1 - e^{-s}\right]$$

sendo:

$$p = -w_0 \xi$$

$$q = w_0 \sqrt{1 - \xi^2}$$

Usando frações parciais

$$\frac{1}{s}\,\frac{Qs+R}{(s-p)^2+q^2} \equiv \frac{A}{s} + \frac{Cs+D}{(s-p)^2+q^2}$$

$$Qs+R = A\left((s-p)^2+q^2\right) + Cs^2 + Ds$$

$$Qs+R = A\left(s^2-2sp+p^2+q^2\right) + Cs^2 + Ds$$

$$\begin{cases} A+C = 0 \\ -2pA + D = Q \\ A(p^2+q^2) = R \end{cases}$$

$$A = \frac{R}{p^2+q^2} = -C$$

$$D = Q + \frac{2pR}{p^2+q^2}$$

Logo

$$\begin{cases} \bar{x}(s) = \dfrac{A}{s} - \dfrac{A}{s}e^{-s} + \dfrac{Cs+D}{(s-p)^2+q^2} - \dfrac{Cs+D}{(s-p)^2+q^2}e^{-s} \\ \varphi(s) = \dfrac{A'}{s} - \dfrac{A'}{s}e^{-s} + \dfrac{C's+D'}{(s-p)^2+q^2} - \dfrac{C's+D'}{(s-p)^2+q^2}e^{-s} \end{cases}$$

Aplicando $\mathcal{L}^{-1}[F(s)] = f(t)$

$$\begin{cases} x(t) = A - A\,\mathcal{U}(t-1) + e^{pt}\left(\dfrac{pC+D}{q}\,\operatorname{sen} qt + C\cos qt\right) \\ \qquad + e^{p(t-1)}\left(\dfrac{pC+D}{q}\,\operatorname{sen} q(t-1) + C\cos q(t-1)\right) \\ y(t) = A' - A'\,\mathcal{U}(t-1) + e^{pt}\left(\dfrac{pC'+D'}{q}\,\operatorname{sen} qt + C'\cos qt\right) \\ \qquad + e^{p(t-1)}\left(\dfrac{pC'+D'}{q}\,\operatorname{sen} q(t-1) + C'\cos q(t-1)\right) \end{cases}$$