# Self-oscilation in vectorial model of nonequilibrium magnetization

.....

Istituto Nazionale di Ricerca Metrologica (I.N.Ri.M.), Strada delle Cacce, 91, 10135 Torino, ITALY

### Abstract

The combined capability to inject, manipulate and detect spin polarized currents in bilayers formed by a non magnetic conductor with a large spin Hall effect (i.e. Pt) and a magnetic insulator Yttrium Iron Garnet (i.e. YIG) represents a milestone in the field of spintronics. This paper presents theoretical predictions and a possible mechanism to excite self-oscillation induced by the spin Hall torque. The spin Hall torque is caused by the nonequilibrium magnetization (i.e. spin accumulation) occurring at the interface between the magnetic conductor and a magnetic insulator layer and it may generate magnetization dynamics in the ferrimagnet [1]. The microscopic origin of this effect has been searched in the transmission of a spin current through the Pt/YIG interface [2]. However, this effect represents a conversion between a DC pure spin current into an AC magnetization dynamics at heavy-metal magnetic-insulator interface. To this aim, it is appropriate to employ the nonequilibrium thermodynamic approach originally developed by Johnson and Silsbee to describe the joined transport of heat, electric charge and magnetization current [3]. The theory here presented is a possible generalization of the scalar one recently applied in [4, 5].

Keywords: spin hall torque, magnetic moment, magnetic conductor, spin waves, nonequilibrium magnetization

2017 MSC: 00-01, 99-00

## 1. Thermodynamic Model

25

In order to extend the approach of [3, 4, 5] to vector magnetization M, the continuity equation for a ferromagnetic material with precession is therefore written as:

$$\partial_t \mathbf{M} + \nabla \mathbf{J}_{\mathbf{M}} = -\mu_0 \gamma_L \mathbf{M} \times \mathbf{H}_{\text{eff}}^* + \tau_{\mathbf{M}}^{-1} \mathbf{H}_{\text{eff}}^*$$
 (1)

Here we consider the equation where the left-hand side of the Eq.1 contains the local variation of magnetization and the divergence of the magnetization current  $J_M$  and the right-hand side includes a source/sink terms proportional to the thermodynamic effective field  $\mathbf{H}_{\text{eff}}^*$  expressing the distance from equilibrium and characterized by the damping coefficient  $\alpha$ , and a precessional term written in the Landau-Lifshitz style. In the previous equation,  $\gamma_{\rm L}$  is the gyromagnetic ratio, M its amplitude. The proportionality of the magnetic moment current is taken into account by an appropriate constitutive relation  $\mathbf{J}_{\mathbf{M}} = \mu_0 \sigma_{\mathbf{M}} \nabla \cdot \mathbf{H}_{\text{eff}}^*$ with  $\mathbf{J}_{\mathbf{M}}$  the gradient of the potential  $\nabla \cdot \mathbf{H}^*$  and the conductivity tensor  $\sigma_{\mathbf{M}}$ . In particular, Eq.1 reduces to the case considered in [4], [5] if the precessional term  $\mathbf{M} \times \mathbf{H}_{\text{eff}}^*$  is set equal to zero. To study the spin Hall torque problem, Eq.1 shall be solved for  $\mathbf{H}_{\mathrm{eff}}^*$  and thus it is necessary to add the constitutive relations between  $J_M$  and  $\nabla \cdot H_{\text{eff}}^*$ , which is the thermodynamic driving force for the magnetization currents [4, 5]. The conduction of the magnetic moment in ferromagnets can be caused by spin polarized electrons (in metals) and spin waves. In both cases the symmetry is broken by the presence of the spontaneous magnetization. Here we limit to the case in which the magnetic moment can be transferred by means of spin waves. The spin wave picture is applied both for general magnetization or to the cases in which the magnetization has only a small deviation from its equilibrium.

The effect of the spin accumulation is to reduce the value of the magnetization's module  $|\mathbf{M}|$  during this time the magnetization is directed along the z-axis then projecting Eq.1 in the direction of the magnetization (i.e. scalar multiplying the Eq.1 with the vector  $\mathbf{e}_z$ ) we have the following diffusion equation

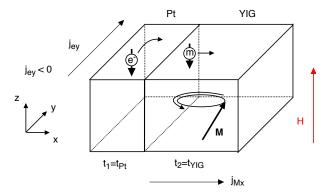


Figure 1: figure

vector representation of a magnetization and an applied external and spin orbit torque fields

for the modules of the magnetization M:

$$\tau_{\mathcal{M}} \frac{\partial |\mathbf{M}|}{\partial t} + l_{\mathcal{M}}^2 \nabla^2 \mathbf{H}_{\mathbf{z}}^* = \mathbf{H}_{\mathbf{z}}^* \tag{2}$$

In the direction perpendicular to the magnetization by cross multiplying the Eq.1 with the vector  $-\mathbf{m} \times (\mathbf{m} \times \mathbf{we})$  can rewrite Eq.1 as

$$\frac{\partial \mathbf{m}}{\partial t} + \nabla \mathbf{J}_{\mathbf{M}}^{\perp} = -\mu_0 \gamma_L \, \mathbf{m} \times \left( \mathbf{H}_{\text{eff}}^{*\perp} + \alpha \mathbf{m} \times \mathbf{H}_{\text{eff}}^{*\perp} \right)$$
(3)

where  $\mathbf{m}$  is the magnetization direction  $\mathbf{m} = \mathbf{M}/M$  and  $\alpha = (\mu_0 \gamma_L M \tau_M)^{-1}$ . In the previous equation only the component of  $\mathbf{H}_{\text{eff}}^*$  perpendicular to  $\mathbf{M}$  is active.

# 35 2. Spin Orbit Torque

30

The spin Hall torque effect is realized in Pt/YIG systems in which the Pt is the spin Hall conductor able to inject a magnetic moment current into YIG. A constant field along the z-direction is applied to YIG. As a result, a potential is generated in YIG by the absorption of the current. To solve the problem one first solve the transport of magnetic moment current along the z-direction axis.

We take the applied field along z-direction only and there no applied field in the plane. The equation for z-direction component is the simple diffusion one.

$$\nabla \mathbf{J}_{\mathbf{M}}^{\perp} = \tau^{-1} \mathbf{H}_{\mathbf{z}}^{*} \mathbf{e}_{z} = \tau_{\mathbf{M}}^{-1} \mathbf{H}_{\text{sot}}(x)$$
 (4)

Ones derived this result we can plug it into the full equation and normalizing the equation to the saturation magnetization wit  $\tau = \mu_0 \gamma_L M_s$ 

$$\frac{\partial \mathbf{m}}{\partial \tau} = -\mathbf{m} \times \{\mathbf{h}_{\mathbf{a}} + \alpha \mathbf{m} \times [\mathbf{h}_{\mathbf{a}} - \mathbf{h}_{\text{sot}}(x)]\}$$
 (5)

## 3. Self Oscillations

The spin Hall torque effects realized in Pt/YIG systems in which the Pt is the spin Hall conductor able to inject a magnetic moment current into YIG. A constant field along the z-direction is applied to YIG. As a result, a potential  $H_z^*$  is generated in YIG by the absorption of the current. To solve the problem one first solve the transport of magnetic moment current along the z- axis. We take the applied field along z only,  $H_z$ , and there no applied field in the plane. The equation for z- component is the simple diffusion one. The solution is given in [?] and shows that the divergence of the current is an extra term (the spin orbit torque term)

$$\nabla \cdot \mathbf{J_M} = \tau_{\mathbf{M}}^{-1} \mathbf{H_{sot}}(x) \tag{6}$$

where

$$\mathbf{H}_{\text{sot}}(x) = -v_{\text{eff}} \frac{\cosh((x - t_{\text{YIG}})/l_{\text{YIG}})}{v_2 \sinh(t_{\text{YIG}}/l_{\text{YIG}})} \frac{j_{MS}}{v_{\text{Pt}}} \tanh(t_{\text{Pt}}/(2l_{\text{Pt}})) \mathbf{e}_z$$
 (7)

After having derived this result we can plug it into the full equation and then project it on the plane perpendicular to the magnetization direction. With the applied field,  $\mathbf{H}_a$ , along z, the dynamic equation for the magnetization direction  $\mathbf{m}$  becomes

$$\frac{d\mathbf{m}}{dt} = -\mu_0 \gamma_L \,\mathbf{m} \times (\mathbf{H}_a + \alpha \mathbf{m} \times (\mathbf{H}_a - \mathbf{H}_{\text{sot}}(x))) \tag{8}$$

Hoever expanding in Tylor series up to the first zero order of  $t_{\rm YIG}/l_{\rm YIG}$  and  $\mathbf{H}_{\rm sot}(x) = v_1^{-1} J_{\rm MS} \tanh(t_{\rm Pt}/2l_{\rm Pt})$  the threshold for auto oscillations at  $\mathbf{H}_a = \mathbf{H}_{\rm sot}$ , i.e.

In this condition the magnetization m may be driven to an instability leading to self oscillations.

#### 65 References

- [1] M. Collet, X. De Milly, O. d. Kelly, V. V. Naletov, R. Bernard, P. Bortolotti, J. B. Youssef, V. Demidov, S. Demokritov, J. Prieto, et al., Generation of coherent spin-wave modes in yttrium iron garnet microdiscs by spin-orbit torque, Nature communications 7.
- [2] Y. Tserkovnyak, A. Brataas, G. E. Bauer, Spin pumping and magnetization dynamics in metallic multilayers, Physical Review B 66 (22) (2002) 224403.
  - [3] M. Johnson, R. Silsbee, Thermodynamic analysis of interfacial transport and of the thermomagnetoelectric system, Physical Review B 35 (10) (1987) 4959.
- [4] V. Basso, E. Ferraro, A. Magni, A. Sola, M. Kuepferling, M. Pasquale, Nonequilibrium thermodynamics of the spin seebeck and spin peltier effects, Physical Review B 93 (18) (2016) 184421.
  - [5] V. Basso, E. Ferraro, M. Piazzi, Thermodynamic transport theory of spin waves in ferromagnetic insulators, Physical Review B 94 (14) (2016) 144422.