1 a) Nech $f: V \to U$ je lineárne zobrazenie.

Množina $Ker(f) = \{\vec{v} \in V: f(\vec{v}) = \vec{0}\}$ sa volá jadro f.

Množina $Im(f) = \{\vec{y} \in U: f(\vec{v}) = \vec{y}$ pre nejaké $\vec{v} \in V\}$ sa volá obraz f.

Nech $f: V \to U$ je lineárne zobrazenie konečnorozmerných vektorových priestorov, potom $dim(V) = dim(Ker(f)) + dim(Im(f))$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & -1 & 0 \end{array}\right)$$

$Ker(f) = \{\vec{v} \in \mathbb{R}^3: f(\vec{v}) = \vec{0}\} = \{t(1,1,-2): t \in \mathbb{R}\}$ $\quad dim(Ker(f)) = 1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 1 & -1 & 0 & b \\ -1 & 1 & 0 & c \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 1 & -1 & 0 & b \\ 0 & 0 & 0 & c+b \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 0 & -2 & -1 & b-a \\ 0 & 0 & 0 & c+b \end{array}\right)$$

$c+b = 0$

$c = -b$

$Im(f) = \{f(\vec{v}): \vec{v} \in \mathbb{R}^3\} = \{(a,b,-b): a,b \in \mathbb{R}\}$ $\quad dim(Im(f)) = 2$

- ortonormálna báza $Ker(f) = \left(\frac{\vec{v}}{\sqrt{6}}\right) = \left(\frac{(1,1,-2)}{\sqrt{6}}\right)$

$\vec{v} = (1,1,-2)$

ortonormálna báza $Im(f) = \left(\frac{(1,0,0)}{\sqrt{1}}, \frac{(0,1,-1)}{\sqrt{2}}\right)$

$X = ((1,0,0),(0,1,-1))$