ŠÁLO RIEŠENIE

1) $\alpha \in \mathbb{R}$ $\quad A: \mathbb{R}^3 \to \mathbb{R}^3$ $\quad \begin{pmatrix} 1 & 2 & 1 \\ 0 & \alpha & 0 \\ 1 & 2 & 1 \end{pmatrix}$

$C = \begin{pmatrix} 1-\lambda & 2 & 1 \\ 0 & \alpha-\lambda & 0 \\ 1 & 2 & 1-\lambda \end{pmatrix} \Rightarrow \det(C) = (1-\lambda)(\alpha-\lambda)(1-\lambda) - (\alpha-\lambda) =$

$(\alpha-\lambda)((1-\lambda)^2-1) = (\alpha-\lambda)(-\lambda)(2-\lambda)$

$\lambda_1 = 0 \quad \lambda_2 = \alpha \quad \lambda_3 = 2$

$\lambda_1 = 0$

$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & \alpha & 0 & 0 \\ 1 & 2 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & \alpha & 0 & 0 \end{array}\right)$ ak $\alpha \neq 0 \Rightarrow$ dostaneme nulový vektor

$\lambda_2 = \alpha$

$\left(\begin{array}{ccc|c} 1-\alpha & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 2 & 1-\alpha & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1-\alpha & 2 & 1 & 0 \\ 1 & 2 & 1-\alpha & 0 \end{array}\right)$ ak $\alpha \neq 1$, nedostaneme nulový vektor

$\lambda_3 = 2$

$\left(\begin{array}{ccc|c} -1 & 2 & 1 & 0 \\ 0 & \alpha-2 & 0 & 0 \\ 1 & 2 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 2 & 1 & 0 \\ 0 & \alpha-2 & 0 & 0 \end{array}\right)$ ak $\alpha \neq 2$, dostaneme nulový vektor

$\alpha \in \mathbb{R} \setminus \{0, 2\}$