Geometric Calibration for X-ray Fluorescence Tomography using Multi-Support Consistency

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We propose a novel geometric calibration approach for computed tomography (CT) based on data consistency conditions that rely solely on sinogram supports, i.e., the regions where the sinogram values are non-zero. This approach enables the estimation of shifts and of the center of rotation. It is agnostic to attenuation and applies to sets of supports arising from different spatial, spectral, and other measurement domains. While the formulation is general, X-ray fluorescence CT serves as the primary application considered in this work.

Introduction

Geometric calibration is essential for accurate computed tomography (CT) reconstruction, as geometric errors induce artifacts and resolution loss. In transmission CT, analytical calibration based on data consistency conditions (DCCs) provides data-driven correction, but these conditions break down in modalities such as X-ray fluorescence CT (XFCT).

In XFCT, isotropically emitted fluorescence photons generated by X-ray excitation are recorded over translation and rotation and, after spectral fitting, yield elemental sinograms from a single measurement (Fig. 1). Both the incident beam and the emitted fluorescence are attenuated along their paths, thereby eliminating the general validity of existing analytical calibration approaches.

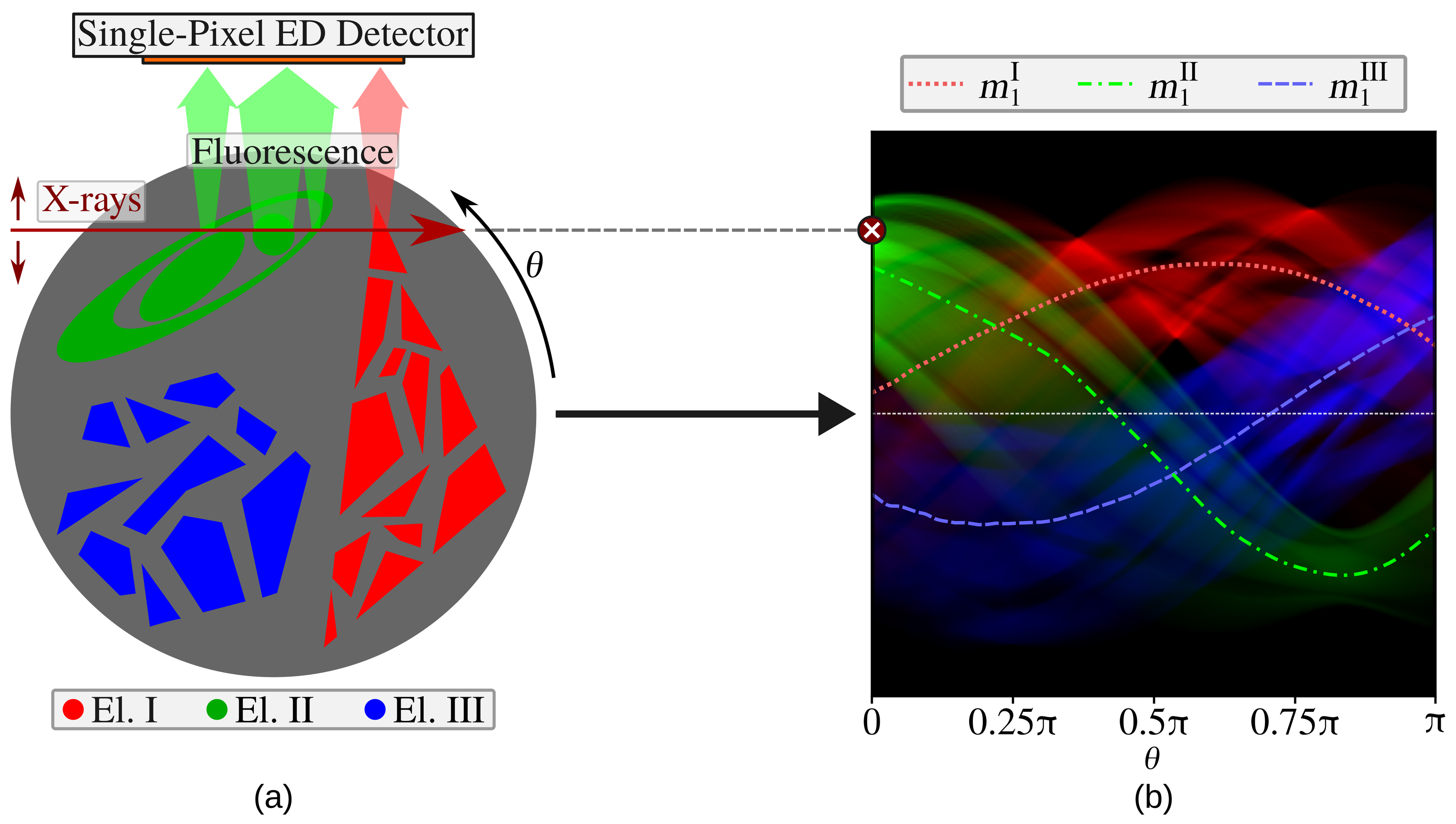

Fig. 1: XFCT: (a) Experimental setup in which an X-ray pencil beam traverses the sample at each projection angle and excites atoms in a phantom comprising distinct elements (here, three), generating fluorescence photons that are recorded by a single-pixel energy-dispersive (ED) detector positioned normal to the beam direction. (b) Combined X-ray fluorescence sinograms with first-order moments (m_1), exhibiting attenuation effects. The white cross indicates the pencil beam position in (a).

Radii of Curvature DCCs

The support of a sinogram, Y, is determined by the convex envelope of the corresponding fluorescence-emitting region in object space, X, via the support Radon mapping $\mathcal{R}_{\mathrm{sup}}: \ X \mapsto Y$. The support can be characterized by its upper and lower boundaries, $h_Y^+(\theta)$ and $h_Y^-(\theta)$, which are related to the radii of curvature of the object points delineating them [1]. Since the radii of curvature of convex envelopes do not change sign, these consequently become

$$\rho_X^{\pm}(\theta) = \mathcal{L}h_Y^{\pm}(\theta), \qquad \pm\rho_X^{\pm}(\theta) \geq 0, \qquad \mathcal{L} := \mathrm{Id} + \frac{\partial^2}{\partial \theta^2}. \tag{1}$$

Physically consistent data must satisfy this condition, thereby constituting a DCC. An example case is illustrated in Fig. 2.

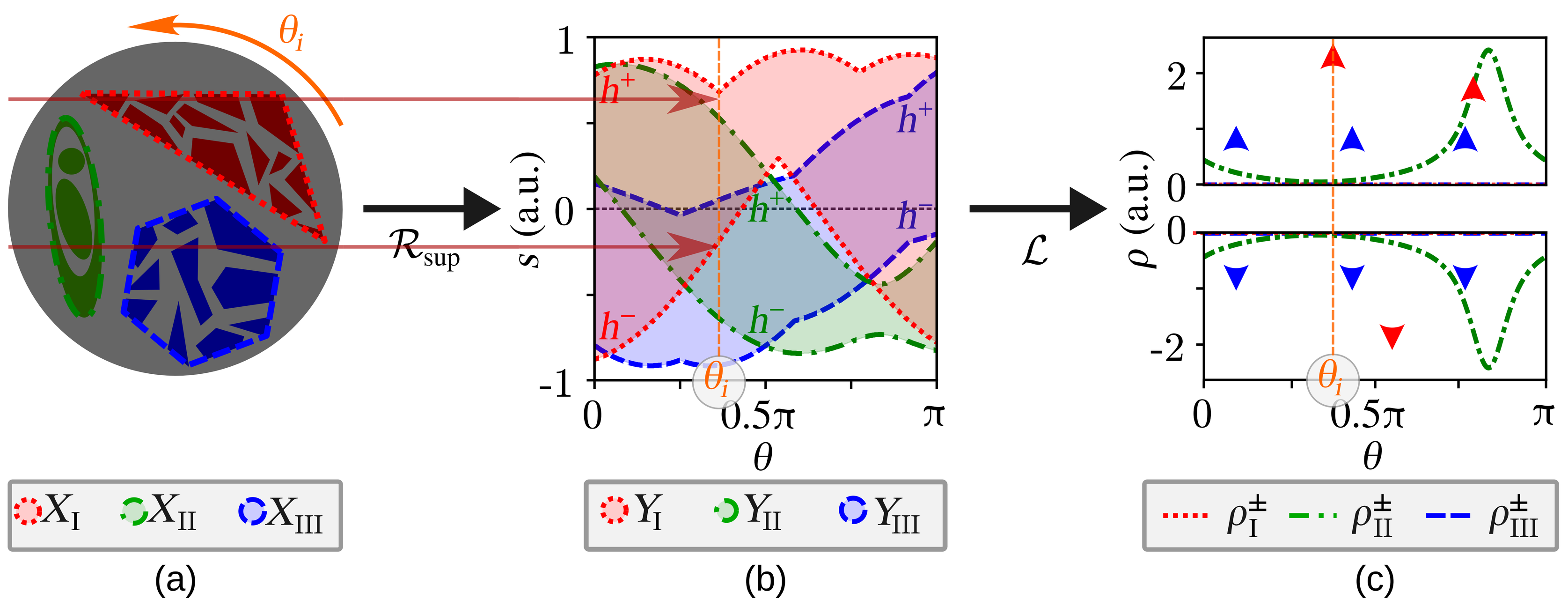

Fig. 2: Radii of curvature. (a) Phantom exhibiting convex envelopes X of three fluorescent supports, within a gray capillary. The two pencil beams correspond to irradiated extreme points of X_{I} at angle θ_i with infinity (top) and zero (bottom) radii of curvature. (b) Support sinograms. The vertical dashed line highlights Y_{I} at angle θ_i featuring a cusp and a sinusoidal segment at the boundaries. (c) Corresponding radii of curvature (ρ^+ and ρ^- in top and bottom plot, respectively), with arrows indicating Dirac delta contributions.

Perturbations and DCC-based Calibration

Perturbations can induce angle-dependent projection shifts (Fig. 3), degrading the reconstruction. This results in the perturbed boundaries $\tilde{h}_Y^{\pm}(\theta) := h_Y^{\pm}(\theta) + \Delta(\theta)$, where $\Delta(\theta)$ denotes the shift at angle θ. Such perturbations may cause violation of the DCC stated in Eq. 1. However, combining this definition with Eq. 1 yields bound on the admissible shifts,

$$\mathcal{L}\tilde{h}_Y^-(\theta) \leq \mathcal{L}\Delta(\theta) \leq \mathcal{L}\tilde{h}_Y^+(\theta) \quad \forall Y \in \mathbb{Y}. \tag{2}$$

For a single sinogram, these inequalities typically impose only weak constraints. When considered over a set $\mathbb{Y}$ of support sinograms, however, the remaining ambiguity may be substantially reduced. Enforcing shifts that restore the DCC of Eq. 1 therefore provides an estimate of $\Delta(\theta)$ within the admissible range specified by Eq. 2.

For discretely sampled data, Eq. 1 may be enforced by fitting both shifts and slope-coupled sinusoids (with period 2π) through all boundary points. These sinusoids (Fig. 3) correspond to sinograms generated by infinitesimal object elements and must consequently remain fully contained within the overall sinogram support. Deviations from this condition can be penalized via an appropriate cost function, thereby enabling data calibration.

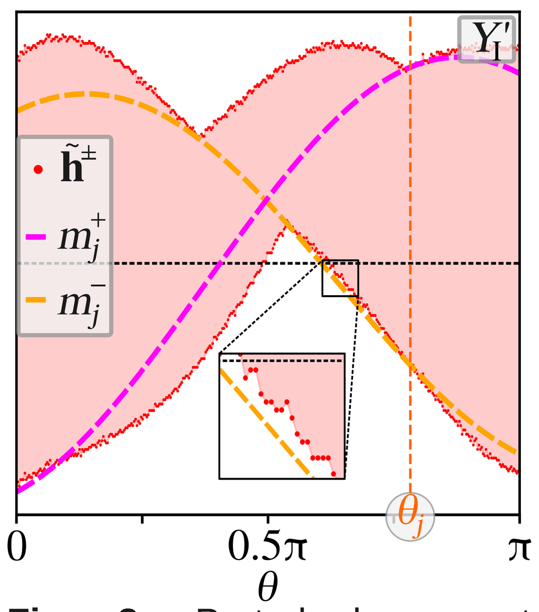

Fig. 3: Perturbed support sinogram Y_{I}' with the best-fitting sines ($m_j^{\pm}$) at an angle θ_j .

Simulation and Results

Simulations were performed for the sample shown in Fig. 1a using the `PyCorrectedEmissionCT` package [2], with 256 translational and 360 rotational steps over a half turn. Ground-truth (GT) and estimated shifts are shown in Fig. 4, and the corresponding reconstructions in Fig. 5. Note that attenuation effects were not corrected for in these reconstructions.

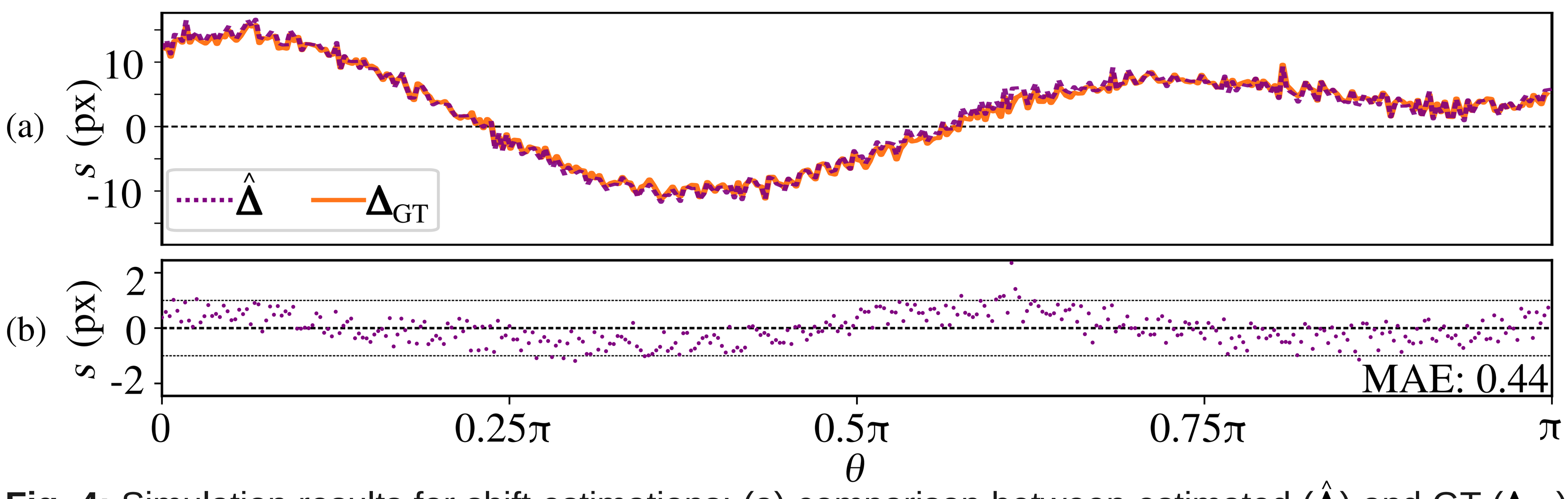

Fig. 4: Simulation results for shift estimations: (a) comparison between estimated ($\hat{\mathbf{\Delta}}$) and GT ($\mathbf{\Delta}_{\mathrm{GT}}$) shifts, (b) residuals between found and GT shifts and their mean absolute error (MAE).

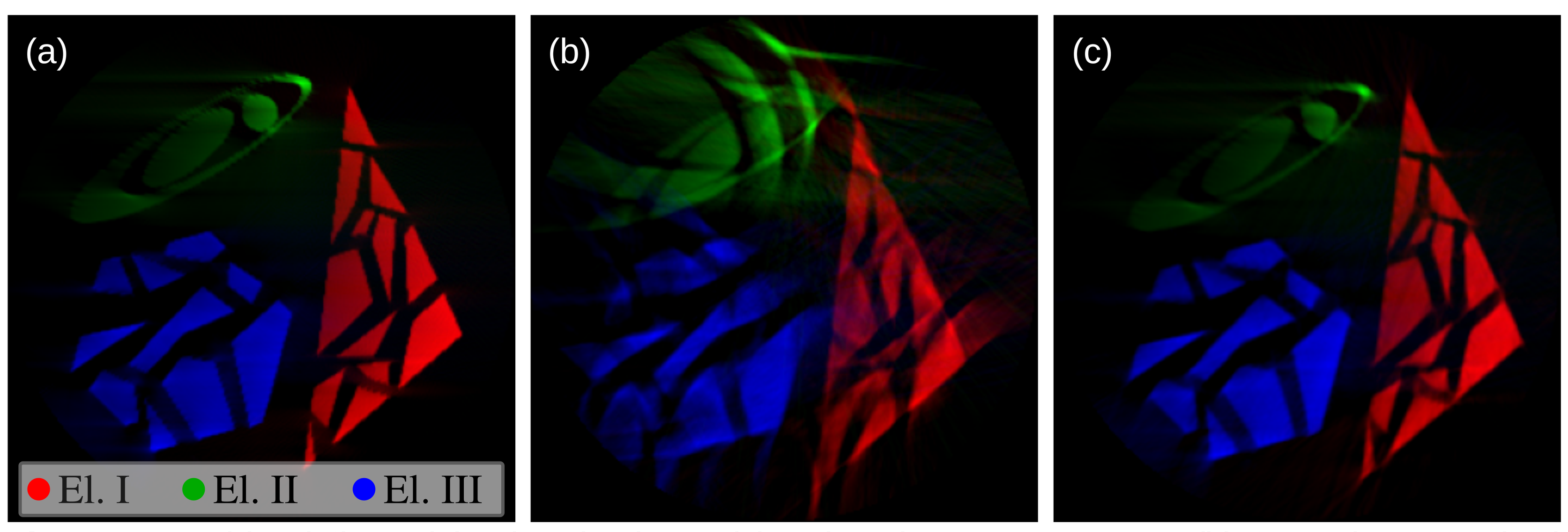

Fig. 5: Combined filtered backprojections: (a) using the GT geometry, (b) from the perturbed sinograms, and (c) from the sinograms corrected with the proposed method.

Conclusion

A novel tomographic calibration method is proposed and validated numerically. It exploits sinogram support boundaries to recover shifts relative to a set center of rotation. Provided these boundaries can be reliably extracted, the method is invariant to attenuation, applicable to half-rotation (or smaller) acquisitions, and requires no reconstruction. Simulations on shift-affected XFCT data demonstrate its effectiveness. Although applicable more broadly, this work constitutes the first analytical calibration approach that is agnostic to attenuation effects in XFCT.

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[2] N. Viganò and V. A. Solé. "Physically corrected forward operators for induced emission tomography: a simulation study," *Measurement Science and Technology*, vol. 29, no. 3, p. 034005, 2018.