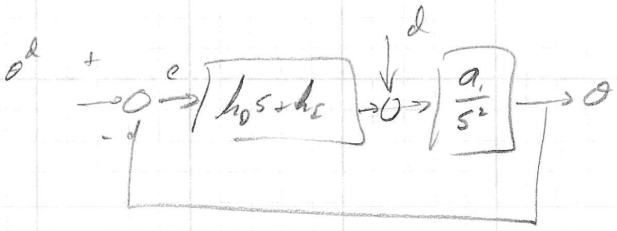
Homework E.13 - Solution

E.13

The inner loop is



where $a_1 = \frac{l}{\frac{1}{3}m_2 l^2 + m_1 y_e^2}$

The open loop TF is

$$P(s)C(s) = \left(\frac{a}{s^2}\right)(k_0 s + k_p)$$

two free integrators $\Rightarrow$ Type 2 $\therefore$

ss error to step $= 0$

ss error to ramp $= 0$

ss error to parabola $= \lim_{s\to 0} \frac{1}{s^2 P(s)C(s)} = \frac{1}{a k_p}$

The TF from d to e is

$$E(s) = \frac{P(s)}{1 + P(s)C(s)} D(s) = \frac{a_1/s^2}{1 + \left(\frac{a}{s^2}\right)(k_0 s + k_p)} D(s)$$

$$= \frac{a_1}{s^2 + a_1 k_0 s + a_1 k_p} D(s)$$

If $D(s) = \frac{1}{s^2 + 1}$ then the ss error is

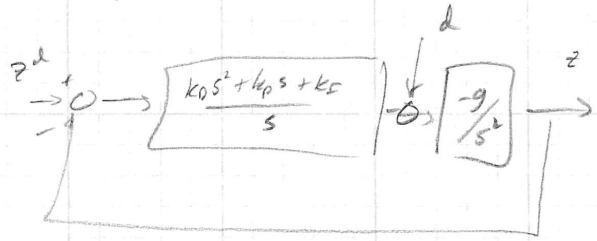
$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{a_1}{s^2 + a_1 k_0 s + a_1 k_p} \frac{1}{s^2} = \lim_{s\to 0} \frac{1}{k_p} \frac{1}{s^2}$$

which is finite when $g = 0$ $\Rightarrow$ Type 0 wrt disturbance

and the error to a step in d is $\frac{1}{k_p}$

E.13

The outer loop is



The open loop TF is

$$P(s)C(s) = \left(\frac{-g}{s^2}\right)\left(\frac{k_D s^2 + k_P s + k_I}{s}\right)$$

without integrator! Type 2

ss error to step = 0

ss error to ramp = 0

ss error to parabol $= \frac{1}{\lim_{s\to 0} s^2 P(s)C(s)} = \frac{1}{-g k_P}$

with integrator: Type 3

ss error to parabola = 0

ss error to $t^3 = \frac{1}{\lim_{s\to 0} s^3 PC} = \frac{1}{g k_I}$

The TF from d to e is

$$E(s) = \frac{P(s)}{1+PC} D = \frac{-g/s^2}{1+\left(\frac{-g}{s^2}\right)\left(\frac{k_D s^2 + k_P s + k_I}{s}\right)} D(s) = \frac{-g s}{s^3 + (-g k_D)s^2 + (-g k_P)s + (-g k_I)} D(s)$$

If $D(s) = \frac{1}{s^g}$ then ss error is

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} \frac{-g s}{s^3 + (-g k_D)s^2 + (-g k_P)s + (-g k_I)} \frac{1}{s^g} = \lim_{s\to 0} \frac{(-g)s}{(-g k_P)s + (-g k_I)} \frac{1}{s^g}$$

without integrator: ($k_I = 0$) Type 0 and ss error to a step is $\frac{1}{k_P}$

with integrator: Type 1 and ss error to a ramp is $\frac{1}{k_I}$