Homework G.8 - Solution

6.8 The altitude dynamics from $\tilde{F}$ to h is

$$H(s) = \left(\frac{\left(\frac{1}{m_c+2m_r}\right)}{s^2}\right)\tilde{F}(s)$$

Using the parameters in the problem description,

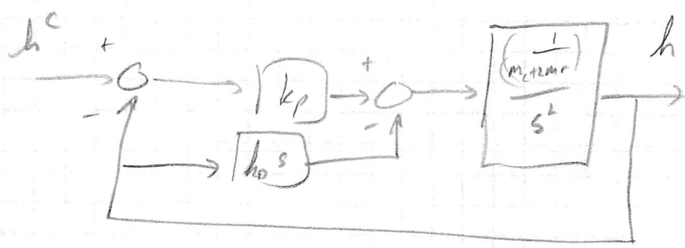
$$H(s) = \left(\frac{0.667}{s^2}\right)\tilde{F}(s)$$

The open loop char polynomial is

$$\Delta_{OL} = s^2$$

with two roots at zero.

The block diagram for PD control is



and so

$$H(s) = \frac{\left(\frac{1}{m_c+2m_r}\right)}{s^2}\left(k_p\left(H^c - H\right) - k_d s H\right)$$

$$\Rightarrow \left(s^2 + \frac{k_d}{m_c+2m_r}s + \frac{k_p}{m_c+2m_r}\right)H = \frac{k_p}{m_c+2m_r}H^c$$

6.8

$\Rightarrow$

$$H = \left(\frac{\frac{k_P}{m_c + 2m_r}}{s^2 + \frac{k_d}{m_c + 2m_r} s + \frac{k_P}{m_c + 2m_r}} \right) H_c$$

The char polynomial for the closed loop system is

$$\Delta_{cl} = s^2 + \left(\frac{k_d}{m_c + 2m_r} \right) s + \left(\frac{k_P}{m_c + 2m_r} \right)$$

Given the desired closed loop poles of -0.3, -0.2

The desired closed loop char poly is

$$\Delta_{cl}^d = (s + 0.1)(s + 0.2) = s^2 + 0.5 s + 0.06$$

Equating terms and solving for k_P and k_D gain

$$\boxed{\begin{aligned} k_P &= (m_c + 2m_r)\, 0.06 = 0.09 \\ k_D &= (m_c + 2m_r)\, 0.5 = 0.75 \end{aligned}}$$