Homework G.5 - Solution

Soln ①

6.5

Using the expression $F = f_r + f_\ell$ and $\tau = d(f_r - f_\ell)$, the equations of motion are given by

$$\begin{pmatrix} (m_c+2m_r) & 0 & 0 \\ 0 & (m_c+2m_r) & 0 \\ 0 & 0 & (J_c+2m_r d^2) \end{pmatrix} \begin{pmatrix} \ddot{z} \\ \ddot{h} \\ \ddot{\theta} \end{pmatrix} = \begin{pmatrix} -F\sin\theta - \mu\dot{z} \\ -(m_c+2m_r)g + F\cos\theta \\ \tau \end{pmatrix}$$

At equilibrium when

$$\ddot{z} = \ddot{h} = \ddot{\theta} = \dot{z} = \dot{h} = \dot{\theta} = 0$$

we have

$$-F_0 \sin\theta_0 = 0 \qquad (1)$$

$$-(m_c+2m_r)g + F_0\cos\theta_0 = 0 \qquad (2)$$

$$\tau_0 = 0 \qquad (3)$$

$\therefore$ $\boxed{\tau_0 = 0}$ from (3)

From (1) either $\theta_0 = 0$ or $F_0 = 0$. Since $F_0 = 0$ would make (2) impossible to satisfy, we conclude that $\boxed{\theta_0 = 0}$.

From (2) $\boxed{F_0 = (m_c + 2m_r)g}$

Soln (2)

6.5

To linearize, let

$$\tilde{\theta} = \theta - \theta_0 \quad \Rightarrow \quad \theta = \theta_0 + \tilde{\theta}$$

$$\tilde{F} = F - F_0 \quad \Rightarrow \quad F = F_0 + \tilde{F}$$

We also have

$$F \sin\theta \cong F_0 \sin\theta_0 + \frac{\partial}{\partial F}(F\sin\theta)\Big|_0 \tilde{F} + \frac{\partial}{\partial \theta}(F\sin\theta)\Big|_0 \tilde{\theta} + H.O.T$$

$$= F_0 \sin\theta_0 + \sin\theta_0 \tilde{F} + F_0 \cos\theta_0 \tilde{\theta}$$

$$= F_0 \tilde{\theta}$$

$$F \cos\theta \simeq F_0 \cos\theta_0 + \frac{\partial}{\partial F}(F\cos\theta)\Big|_0 \tilde{F} + \frac{\partial}{\partial \theta}(F\cos\theta)\Big|_0 \tilde{\theta}$$

$$= F_0 \cos\theta_0 + \cos\theta_0 \tilde{F} - F_0 \sin\theta_0 \tilde{\theta}$$

$$= F_0 + \tilde{F}$$

The linearized equations of motion are therefore

$$(m_c + 2m_r)\ddot{z} = -F_0 \tilde{\theta} - \mu \dot{z}$$

$$(m_c + 2m_r)\ddot{h} = -(m_c + 2m_r)g + F_0 + \tilde{F} = \tilde{F}$$

$$(J_c + 2m_r d^2)\ddot{\theta} = \tau$$

Noting that $\tilde{\theta} = \theta - \theta_0 = \theta$ we write

Soln

G.5

(3)

$$\boxed{\begin{aligned} (m_c + 2m_r)\,\ddot{z} &= -f_0\,\theta - \mu\dot{z} \\ (m_c + 2m_r)\,\ddot{h} &= \tilde{F} \\ (J_c + 2m_r d^2)\,\ddot{\theta} &= \tau \, . \end{aligned}}$$