

Homework D.7 - Solution

The linearized equation of motion is

$$\ddot{z} + \frac{b}{m}\dot{z} + \frac{k}{m}z = \frac{1}{m}F.$$

Solving for \ddot{z} gives

$$\ddot{z} = \frac{1}{m}F - \frac{b}{m}\dot{z} - \frac{k}{m}z.$$

Let

$$\begin{aligned}\mathbf{x} &= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \triangleq \begin{bmatrix} z \\ \dot{z} \end{bmatrix} \\ u &\triangleq F \\ y &\triangleq z\end{aligned}$$

then

$$\begin{aligned}\dot{x}_1 &= \dot{z} = x_2 \\ \dot{x}_2 &= \ddot{z} = \frac{1}{m}F - \frac{b}{m}\dot{z} - \frac{k}{m}z \\ &= \frac{1}{m}u - \frac{b}{m}x_2 - \frac{k}{m}x_1 \\ y &= z.\end{aligned}$$

In matrix form

$$\begin{aligned}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}}_{\triangleq \mathbf{A}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{\triangleq \mathbf{B}} u \\ y &= \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\triangleq \mathbf{C}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{\triangleq \mathbf{D}} u.\end{aligned}$$