Homework G.6 - Solution

Taking the Laplace transform and setting the initial conditions to zero gives

$$(m_c + 2m_r)\, s^2 Z(s) = -F_0\tilde{\Theta}(s) - \mu s Z(s) \tag{1}$$

$$(m_c + 2m_r)\, s^2 H(s) = \tilde{F}(s) \tag{2}$$

$$\left(J_x + 2m_r d^2\right) s^2 \Theta(s) = \tau(s). \tag{3}$$

Solving (2) for $H(s)$ gives

$$H(s) = \frac{\left(\frac{1}{m_c+2m_r}\right)}{s^2}\tilde{F}(s)$$

which are the linearized longitudinal dynamics. Solving (3) for $\Theta(s)$ gives

$$\Theta(s) = \frac{\left(\frac{1}{J_x+2m_r d^2}\right)}{s^2}\tilde{\tau}(s)$$

Solving (1) for $Z(s)$ gives

$$\begin{aligned}
&\left((m_c + 2m_r)\, s^2 + \mu s\right) Z(s) = -F_0\Theta(s) \\
&\Rightarrow Z(s) = \frac{-F_0}{(m_c + 2m_r)\, s^2 + \mu s}\Theta(s) \\
&\Rightarrow Z(s) = \frac{\left(-\frac{F_0}{m_c+2m_r}\right)}{s^2 + \left(\frac{\mu}{m_c+2m_r}\right) s}\Theta(s).
\end{aligned}$$

The block diagram of the longitudinal motion is shown in Figure 1 and the block diagram of the lateral motion is shown in Figure 2.

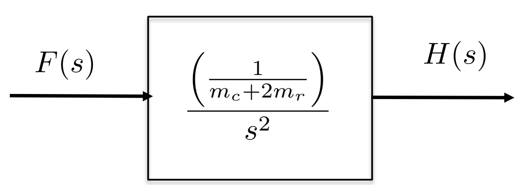

Figure 1: Block diagram showing the linearized longitudinal dynamics

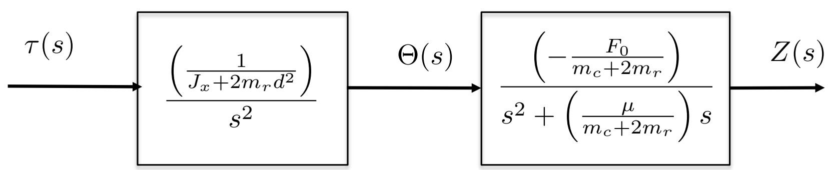

Figure 2: Block diagram showing the linearized lateral dynamics