Homework D.8 - Solution

D.8

The open loop transfer function is given by

$$\tilde{Y}(s) = \frac{1/m}{s^2 + b/m\, s + k/m} \tilde{F}(s)$$

using the parameters in the problem description given

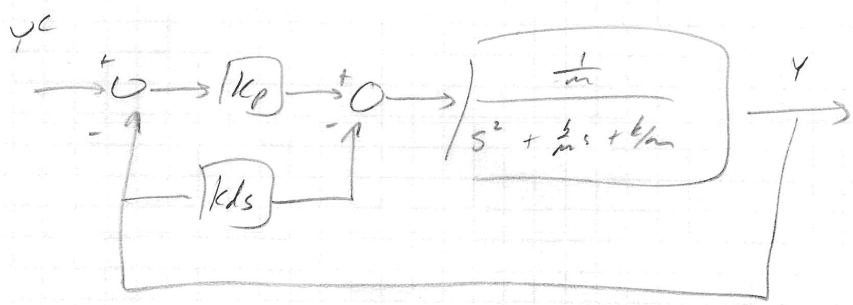
$$\tilde{Y}(s) = \frac{0.2}{s^2 + 0.1 s + 0.6} \tilde{F}(s)$$

The open loop char polynomial is

$$\Delta_{ol} = s^2 + 0.1 s + 0.6$$

with roots $-0.05 \pm j0.773.$

The block diagram for PD control is



So

$$Y(s) = \left(\frac{\frac{1}{m}}{s^2 + b/m\, s + k/m}\right)\left(k_p(Y^c - Y) - k_d s Y\right)$$

$$\Rightarrow (s^2 + \tfrac{b}{m}s + \tfrac{k}{m}) Y = \tfrac{k_p}{m}(Y^c - Y) - \tfrac{k_d}{m} s Y$$

D.8

$$\Rightarrow \left(s^2 + \left(\frac{b}{m} + \frac{k_d}{m}\right)s + \left(\frac{k}{m} + \frac{k_p}{m}\right)\right)Y = \frac{k_p}{m} Y^c$$

$$\Rightarrow Y = \left(\frac{k_p/m}{s^2 + \left(\frac{b}{m} + \frac{k_d}{m}\right)s + \left(\frac{k}{m} + \frac{k_p}{m}\right)}\right) Y^c$$

The char polynomial for the closed loop system is therefore

$$\Delta_{cl} = s^2 + \left(\frac{b}{m} + \frac{k_d}{m}\right) s + \left(\frac{k}{m} + \frac{k_p}{m}\right)$$

Given the desired closed loop poles of -0.5, -0.2 the desired closed loop char polynomial is

$$\Delta_{cl}^d = (s+0.5)(s+0.2)$$

$$= s^2 + 0.7 s + 0.1$$

Equating terms gives

$$\frac{k}{m} + \frac{k_p}{m} = 0.1$$

$$\frac{b}{m} + \frac{k_d}{m} = 0.7$$

$$\Rightarrow \boxed{\begin{aligned} k_p &= 0.1\, m - k = -2.5 \\ k_d &= 0.7\, m - b = 3 \end{aligned}}$$