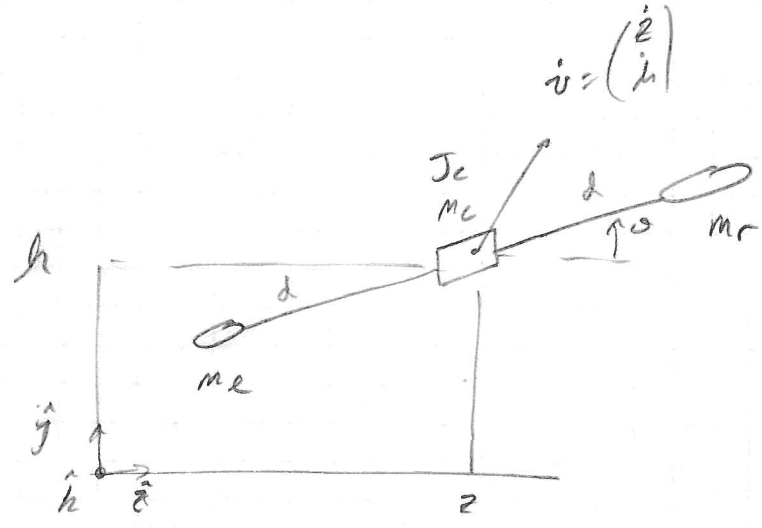
Soln G.1



There are three masses in the system, + the center pod is large enough that the inertia matters.

Therefore the kinetic energy is given by

$$K = \tfrac{1}{2} m_c v_c^T v_c + \tfrac{1}{2} \omega_c^T J \omega_c + \tfrac{1}{2} m_r v_r^T v_r + \tfrac{1}{2} m_\ell v_\ell^T v_\ell$$

The positions of each mass are given by

$$p_c = \begin{pmatrix} z \\ h \\ 0 \end{pmatrix}$$

$$p_r = \begin{pmatrix} z + d\cos\theta \\ h + d\sin\theta \\ 0 \end{pmatrix}$$

$$p_\ell = \begin{pmatrix} z - d\cos\theta \\ h - d\sin\theta \\ 0 \end{pmatrix}$$

which implies that

$$v_c = \begin{pmatrix} \dot z \\ \dot h \\ 0 \end{pmatrix}$$

$$v_r = \begin{pmatrix} \dot z - d\dot\theta \sin\theta \\ \dot h + d\dot\theta\cos\theta \\ 0 \end{pmatrix}$$

$$v_\ell = \begin{pmatrix} \dot z + d\dot\theta\sin\theta \\ \dot h - d\dot\theta\cos\theta \\ 0 \end{pmatrix}$$

Also $\omega = \begin{pmatrix} 0 \\ 0 \\ \dot\theta \end{pmatrix}$

$$\therefore K = \tfrac{1}{2} m_c \left[\dot z^2 + \dot h^2\right]$$

$$+ \tfrac{1}{2} J_c \dot\theta^2$$

$$+ \tfrac{1}{2} m_r \left[(\dot z - d\dot\theta\sin\theta)^2 + (\dot h + d\dot\theta\cos\theta)^2\right]$$

$$+ \tfrac{1}{2} m_\ell \left[(\dot z + d\dot\theta\sin\theta)^2 + (\dot h - d\dot\theta\cos\theta)^2\right]$$

$$= \tfrac{1}{2} \underline{m_c \dot z^2} + \tfrac{1}{2} m_c \dot h^2 + \tfrac{1}{2} J_c \dot\theta^2$$

$$+ \tfrac{1}{2} m_r \left[\underline{\dot z^2} - 2d\dot z\dot\theta\sin\theta + d^2\dot\theta^2\sin^2\theta + \underline{\dot h^2} + \underline{2d\dot h\dot\theta\cos\theta} + \underline{d^2\dot\theta^2\cos^2\theta}\right]$$

$m_r = m_\ell$

$$+ \tfrac{1}{2} m_\ell \left[\underline{\dot z^2} + 2d\dot z\dot\theta\sin\theta + \underline{d^2\dot\theta^2\sin^2\theta} + \underline{\dot h^2} - \underline{2d\dot h\dot\theta\cos\theta} + \underline{d^2\dot\theta^2\cos^2\theta}\right]$$

$$\therefore \boxed{K = \tfrac{1}{2}(m_c + 2m_r)\dot z^2 + \tfrac{1}{2}(m_c + 2m_r)\dot h^2 + \tfrac{1}{2}(J_c + 2m_r d^2)\dot\theta^2}$$