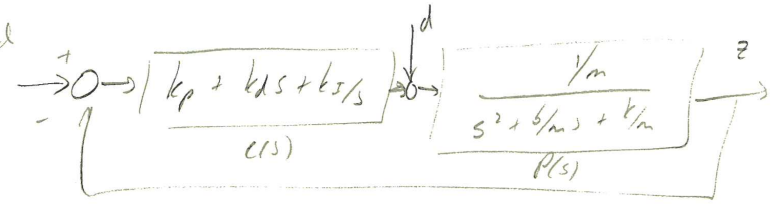
Homework D.13 - Solution

a)



The loop gain is $L(s) = \left(k_p + k_d s + k_I/s\right)\left(\frac{1/m}{s^2 + b/m\, s + k/m}\right)$

Without the integrator, the system is type 0 and

ss error to a unit step is

$$e_{ss} = \lim_{s\to 0} \frac{1}{1 + L(s)} = \frac{1}{1 + \frac{k_p/m}{k/m}} = \frac{k}{k + k_p}$$

ss error to ramp $= \infty$

With the integrator the system is type 1 and

ss error to unit step $= 0$

ss error to unit ramp $= \lim_{s\to 0} \frac{1}{sL(s)} = \frac{1}{\frac{k_I/m}{k/m}} = \frac{k}{K_I}$

ss error to parabola $= \infty$

b) The transfer function from d to z is

$$Z = P(D - CZ) \Rightarrow Z = \frac{P}{1 + PC} D(s)$$

For a unit step on d, ie $D(s) = 1/s$ the steady state value of $Z(s)$ is

$$z_{ss} = \lim_{t\to\infty} z(t) = \lim_{s\to 0} sZ(s) = \lim_{s\to 0} \frac{sP(s)}{1 + P(s)C(s)} \frac{1}{s} = \lim_{s\to 0} \frac{P(s)}{1 + P(s)C(s)}$$

$$= \lim_{s\to 0} \frac{1/m / k/m}{1 + \left(\frac{1/m}{k/m}\right)\left(k_p + k_I/s\right)} = \lim_{s\to 0} \frac{1}{k + k_p + k_I/s}$$

Without an integrator ($k_I = 0$)

$$z_{ss} = \frac{1}{k + k_p}$$

With an integrator

$$z_{ss} = 0$$