Homework E.6 - Solution

The simplified equations of motion are

$$
\begin{aligned}
m_1\ddot{z}_1 + m_1 g\tilde{\theta} &= 0 \\
\left(\frac{m_2 l^2}{3} + m_1 z_e^2\right)\ddot{\tilde{\theta}} + m_1 g\left(\tilde{z}\right) &= l\tilde{F}.
\end{aligned}
$$

(a)

Rearranging the linear equations of motion to get $\ddot{\tilde{z}}$ and $\ddot{\tilde{\theta}}$ alone gives

$$
\begin{aligned}
\ddot{\tilde{z}} &= -g\tilde{\theta} \\
\ddot{\tilde{\theta}} &= \frac{l}{\frac{m_2 l^2}{3} + m_1 z_e^2}\tilde{F} - \frac{m_1 g}{\frac{m_2 l^2}{3} + m_1 z_e^2}\tilde{z}.
\end{aligned}
$$

Taking the Laplace transform gives

$$
\begin{aligned}
s^2 Z(s) &= -g\tilde{\Theta}(s) \\
s^2\tilde{\Theta}(s) &= \frac{l}{\frac{m_2 l^2}{3} + m_1 z_e^2}\tilde{F}(s) - \frac{m_1 g}{\frac{m_2 l^2}{3} + m_1 z_e^2}\tilde{Z}(s).
\end{aligned}
$$

(b)

Solving for $Z(s)$ and $\tilde{\Theta}(s)$ gives

$$
\begin{aligned}
Z(s) &= -\frac{g}{s^2}\tilde{\Theta}(s) \\
\tilde{\Theta}(s) &= \frac{\frac{l}{\frac{m_2 l^2}{3} + m_1 z_e^2}}{s^2}F(s) - \underbrace{\frac{\frac{m_1 g}{\frac{m_2 l^2}{3} + m_1 z_e^2}}{s^2}\tilde{Z}(s)}_{=d(s)}
\end{aligned}
$$

where $d(s)$ can be thought of as a disturbance. The block diagram for this system is shown in Figure 1.

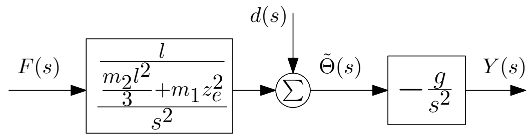

Figure 1: Block diagram