Machine Learning Assignment-1 Report

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October 4, 2014

1. 平方和的結果會把誤差大的嚴重性放大，除了要求錯誤差距少以外更希望的是錯誤的差距不能太大，寧可是比較多小錯誤，而不太容許有大錯誤的出現；第二個取絕對值得和反而就沒有這些考量，只是單純的希望平均錯誤差距小而已。

2.

$$
\begin{aligned}
l(\beta) &= \sum_{t=1}^{N}\left\{y^{(t)} log \pi\left(x^{(t)};\beta\right) + \left(1-y^{(t)}\right) log\left(1-\pi\left(x^{(t)};\beta\right)\right)\right\} \\
&= \sum_{t=1}^{N}\left\{y^{(t)} log\left(\frac{e^{\beta^T\tilde{x}^{(t)}}}{e^{\beta^T\tilde{x}^{(t)}}+1}\right) + \left(1-y^{(t)}\right) log\left(1-\frac{e^{\beta^T\tilde{x}^{(t)}}}{e^{\beta^T\tilde{x}^{(t)}}+1}\right)\right\} \\
&= \sum_{t=1}^{N}\left\{y^{(t)} log\left(e^{\beta^T\tilde{x}^{(t)}}\right) - y^{(t)} log\left(e^{\beta^T\tilde{x}^{(t)}}+1\right) + \left(y^{(t)}-1\right) log\left(e^{\beta^T\tilde{x}^{(t)}}+1\right)\right\} \\
&= \sum_{t=1}^{N}\left\{y^{(t)}\beta^T\tilde{x}^{(t)} - log\left(1+e^{\beta^T\tilde{x}^{(t)}}\right)\right\}
\end{aligned}
$$

3. (a) $\forall x, y \in C_i \cap C_j$

   Because $x, y \in C_i$, we have $(1-\theta)x + \theta y \in C_i$ for any $\theta \in [0,1]$
   Because $x, y \in C_j$, we have $(1-\theta)x + \theta y \in C_j$ for any $\theta \in [0,1]$
   Then, we know $(1-\theta)x + \theta y \in C_i \cap C_j$ for any $\theta \in [0,1]$

   It shows that intersection of two convex sets is convex. We can also show that the intersection of convex sets is convex by continue to intersect two convex sets.

   (b) According Appendix C, we know that affine function and $f(x) = -log(x)$ are both convex functions. Moreover, we know addition of convex functions is convex function.

   $\left(y^{(t)}\beta^T\tilde{x}^{(t)}\right)$ is an affine function.
   $\left(-log\left(1+e^{\beta^T\tilde{x}^{(t)}}\right)\right)$ is a $f(x) = -log(x)$.
   So the log-likelihood function for logistic regression is concave.

4. (a)

$$X = \begin{bmatrix} 1 & x^{(1)} \\ 1 & x^{(2)} \\ \vdots & \vdots \\ 1 & x^{(N)} \end{bmatrix} r = \begin{bmatrix} r^{(1)} \\ r^{(2)} \\ \vdots \\ r^{(N)} \end{bmatrix} L = \frac{1}{2} \begin{bmatrix} l^{(1)} & 0 & \cdots & 0 \\ 0 & l^{(2)} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & l^{(N)} \end{bmatrix}$$

$$\begin{aligned}
(Xw-r)^T L (Xw-r) &= (Xw-r)^T \frac{1}{2} \begin{bmatrix} l^{(1)} & 0 & \cdots & 0 \\ 0 & l^{(2)} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & l^{(N)} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} 1 & x^{(1)} \end{bmatrix} w - r^{(1)} \\ \begin{bmatrix} 1 & x^{(2)} \end{bmatrix} w - r^{(2)} \\ \vdots \\ \begin{bmatrix} 1 & x^{(N)} \end{bmatrix} w - r^{(N)} \end{bmatrix} \\
&= \frac{1}{2} \begin{bmatrix} \begin{bmatrix} 1 & x^{(1)} \end{bmatrix} w - r^{(1)} \\ \begin{bmatrix} 1 & x^{(2)} \end{bmatrix} w - r^{(2)} \\ \vdots \\ \begin{bmatrix} 1 & x^{(N)} \end{bmatrix} w - r^{(N)} \end{bmatrix} \begin{bmatrix} l^{(1)} \left(\begin{bmatrix} 1 & x^{(1)} \end{bmatrix} w - r^{(1)} \right) \\ l^{(2)} \left(\begin{bmatrix} 1 & x^{(2)} \end{bmatrix} w - r^{(2)} \right) \\ \vdots \\ l^{(N)} \left(\begin{bmatrix} 1 & x^{(N)} \end{bmatrix} w - r^{(N)} \right) \end{bmatrix} \\
&= \frac{1}{2} \sum_{i=1}^{N} l^{(i)} \left(w^T \begin{bmatrix} 1 \\ x^{(i)} \end{bmatrix} - r^{(i)} \right)^2
\end{aligned}$$

(b) let $L = L'^T L'$

$$\begin{aligned}
Xw-r)^T L (Xw-r) &= (Xw-r)^T L'^T L' (Xw-r) \\
&= (L'^T (Xw-r))^T L (L'^T (Xw-r))
\end{aligned}$$

let $X' = L'X$ and $r' = L'r$, the formula will become $(X'w-r')^T(X'w-r') = \|X'w-r'\|^2$, so

$$w = (X'^T X')^{-1} X'^T r' = (X^T L X T)^{-1} X T^L$$

(c)

$$\begin{aligned}
\prod_{i=1}^{N} p\left(r^{(i)} | x^{(i)}; w\right) &\propto \prod_{i=1}^{N} \exp \left(- \frac{\left(r^{(i)} - w^T \begin{bmatrix} 1 \\ x^{(i)} \end{bmatrix} \right)^2}{2\sigma^{(i)2}} \right) \\
&\propto \sum_{i=1}^{N} - \frac{\left(r^{(i)} - w^T \begin{bmatrix} 1 \\ x^{(i)} \end{bmatrix} \right)^2}{2\sigma^{(i)2}} \\
&= \frac{1}{2} \sum_{i=1}^{N} l^{(i)} \left(w^T \begin{bmatrix} 1 \\ x^{(i)} \end{bmatrix} - r^{(i)} \right)^2 \quad where \quad l^{(i)} = \frac{-1}{\sigma^{(i)2}}
\end{aligned}$$

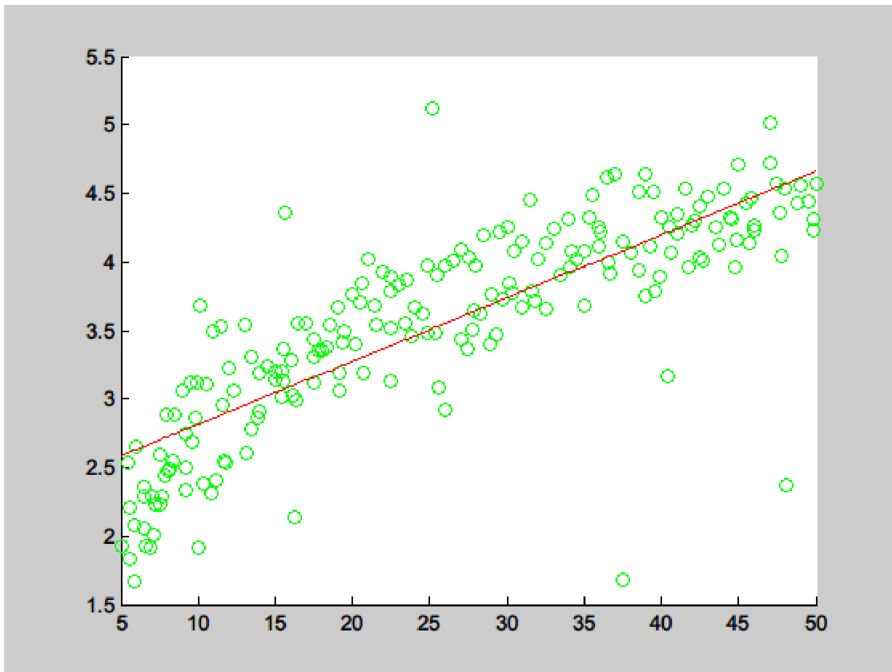
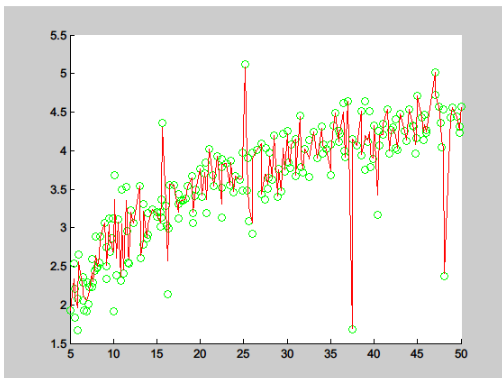


Figure 1: 4.(d)

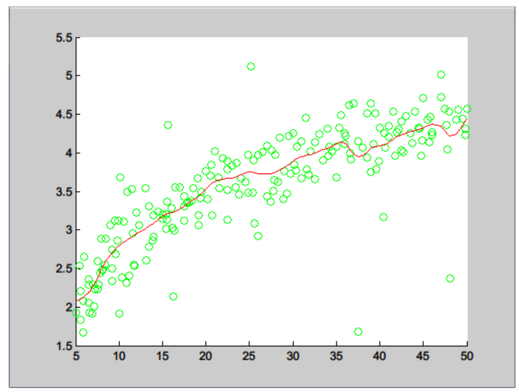
(d) see Figure 1, 在 train 時，用左除算出 w

(e) see Figure 2, 在 predict 時，對於每一個 instance $\tilde{x}$，算出 L', X', r' (4-b 提到的)，再用 X', r' 左除算出的 w 後，拿去預測 $\tilde{x}$

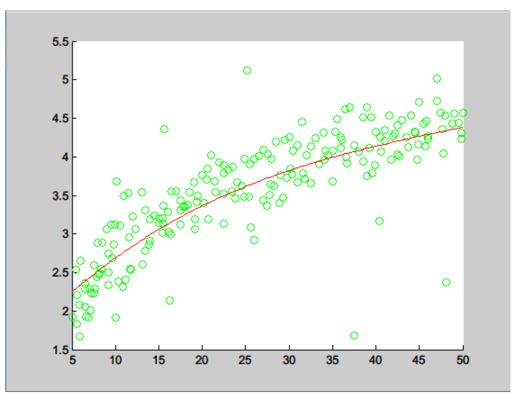
(f) 當 τ 越小的話預測的曲線看起來越複雜，比較接近 training dataset 分布的樣子；反之，則是越接近直線，結果會越接近單純的 Linear Regression 得出的結果。



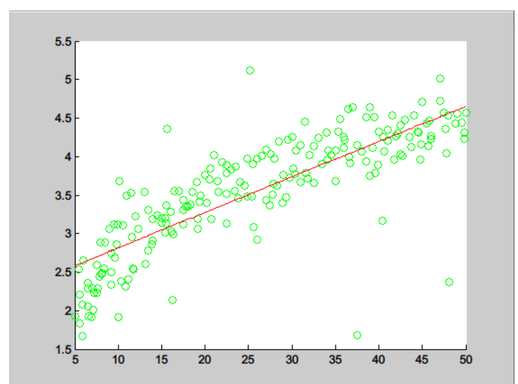
(a) $\tau = 0.1$



(b) $\tau = 1$



(c) $\tau = 10$



(d) $\tau = 100$

Figure 2: 4.(e)