convex optimization P69~P70

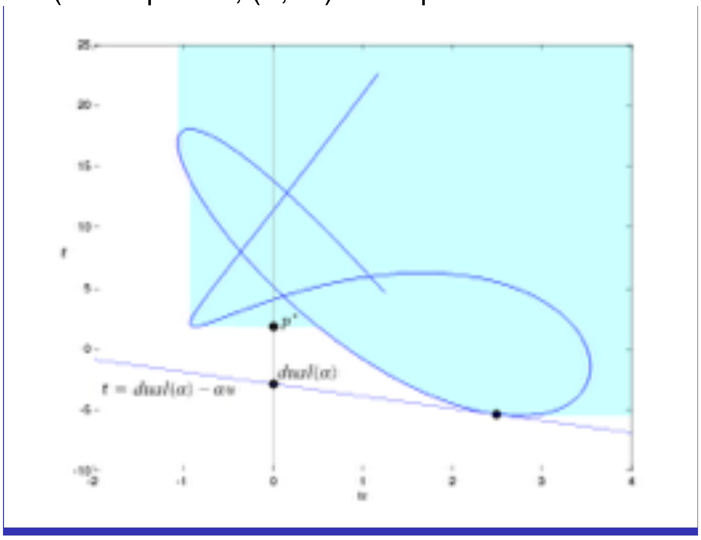
Remind that we want to solve the problem:

$$\max_{\alpha \geqslant 0} \inf_{x} f(x) + \alpha g(x)$$

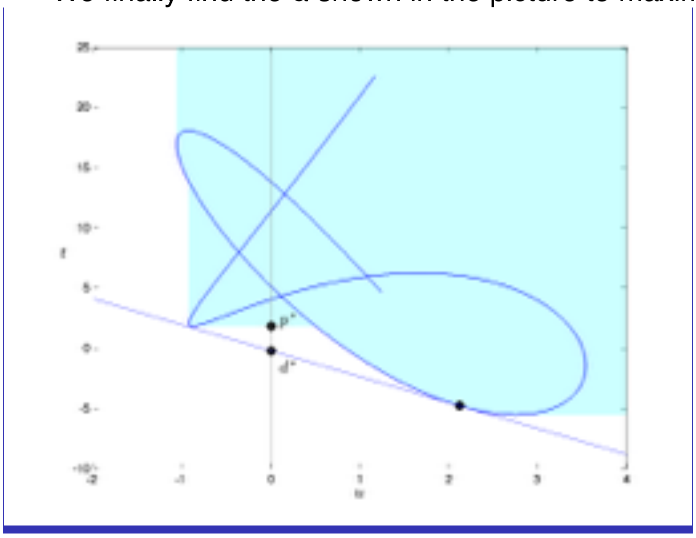
We must solve the inner problem first then solve the outer problem.

Follow the steps below and the problem solved.

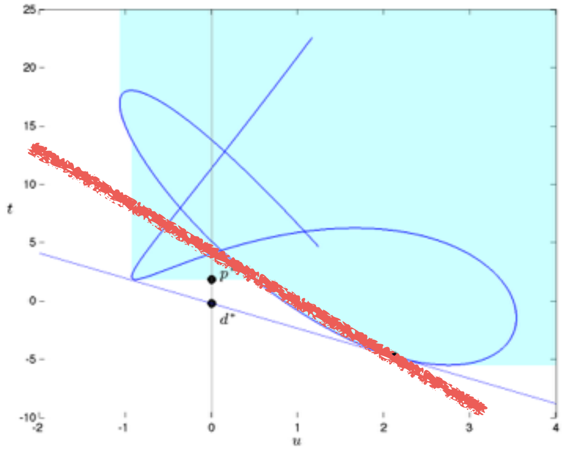
(1) Fixing α(>=0), then modify x to find f(x*) = t*, g(x*) = u* minimizing the dual(α).
In geometric interpretation, it means that we fix the slope of the line then search the t*, u*.
(in this picture, (t*, u*) is the point of contact of the line and the figure)



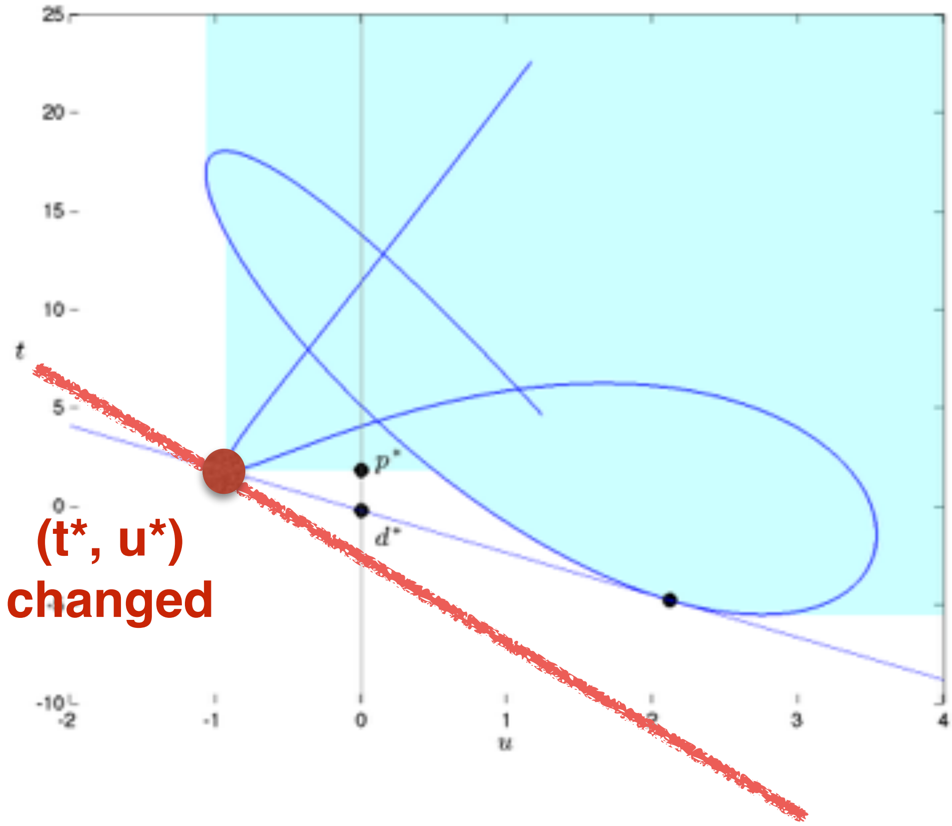
(2) Fixing t*, u* found in step (1), then modify α to maximize the hole problem.
In geometric interpretation, the value of dual(α) is the point of contact of the line and g(x) = 0.
(See point 2 in page 70, if you have questions)
We finally find the α shown in the picture to maximize the hole problem.



Notice that in step (2), the range of α must not change the value of t and u* found in step (1), so we can't find an α like the red line shows.



If we use such α, then t*, u* will be changed in the first step, and the hole problem changed. (like the picture shows)



A briefer explanation is that the α that we can choose in step (2) are bounded by the result of step (1)(the inner problem), although there is only one constrain that (α >= 0) in the beginning.