

3A1.2 FIRST ORDER DECAY (FOD) MODEL – BASIC THEORY

The basis for a first order decay reaction is that the reaction rate is proportional to the amount of reactant remaining (Barrow and Gordon, 1996), in this case the mass of degradable organic carbon decomposable under anaerobic conditions (DDOC_m). The DDOC_m reacted over a period of time dt is described by the differential equation 3A.1.1:

$$\begin{aligned} &\text{EQUATION 3A1.1} \\ &\text{DIFFERENTIAL EQUATION FOR FIRST ORDER DECAY} \\ &d(\text{DDOC}_m) = -k \cdot \text{DDOC}_m \cdot dt \end{aligned}$$

Where:

DDOC_m = mass of degradable organic carbon (DOC) in the disposal site at time t

k = decay rate constant in y^{-1}

The solution to this equation is the basic FOD equation.

$$\begin{aligned} &\text{EQUATION 3A1.2} \\ &\text{FIRST ORDER DECAY EQUATION} \\ &\text{DDOC}_m = \text{DDOC}_{m_0} \cdot e^{-kt} \end{aligned}$$

Where:

DDOC_m = mass of degradable organic carbon that will decompose under anaerobic conditions in disposal site at time t

DDOC_{m₀} = mass of DDOC in the disposal site at time 0, when the reaction starts

k = decay rate constant in y^{-1}

t = time in years.

Substituting $t=1$ into Equation 3A1.2 shows that at the end of year 1 (the year after disposal), the amount of DDOC_m remaining in the disposal site is:

$$\begin{aligned} &\text{EQUATION 3A1.3} \\ &\text{DDOC}_m \text{ REMAINING AFTER 1 YEAR OF DECAY} \\ &\text{At } t = 1, \text{ DDOC}_m = \text{DDOC}_{m_0} \cdot e^{-k} \end{aligned}$$

Sample Data

Note – at the end of year 1 the initial 100 will have decomposed to 90.5

| year | DDOCm disposed | DDOCm accumulated | DDOCm decomposed |
|------|----------------|-------------------|------------------|
| 0 | 100 | 100 | 0 |
| 1 | 100 | 190.5 | 9.5 |
| 2 | 100 | 272.4 | 18.1 |
| 3 | 100 | 346.4 | 25.9 |
| 4 | 100 | 413.5 | 33.0 |
| 5 | 100 | 474.1 | 39.3 |
| 6 | 100 | 529.0 | 45.1 |

Equation 2A1.2 Expressed in Corticon

| | |
|--|-------------------------------------|
| Actions | |
| Post Message(s) | |
| Reactant.DDOCm_t = Reactant.DDOCm_0 * (Constant.e ** (-Reactant.k * Reactant.t)) | <input checked="" type="checkbox"/> |
| Constant.e = 2.71828 | <input checked="" type="checkbox"/> |

Result from Corticon = 90.48

| Input | Output |
|--|--|
| <div><div>Constant [1]</div><div>e [2.718280]</div><div>Reactant [1]</div><div>DDOCm_0 [100.000000]</div><div>k [0.100000]</div><div>t [1]</div></div> | <div><div>Constant [1]</div><div>e [2.718280]</div><div>Reactant [1]</div><div>DDOCm_0 [100.000000]</div><div>DDOCm_t [90.483748]</div><div>k [0.100000]</div><div>t [1]</div></div> |