Lecture 08 – Loop-Carried Dependences

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Anti Dependence and Output Dependence

- False dependence로 불림
  - 실제 연산의 실행 순서와 관련되어 있는 것이 아니라 메모리 위치의 재사용에 의해 일어나는 디펜던스
- 변수 재명명(variable renaming)으로 제거할 수 있음
  - Flow dependence: S1 → S2, S3 → S4
  - Anti dependence: S2 → S3

```
S1: t = a + b
S2: sum1 = t + s1
S3: t = c + d
S4: sum2 = t + s2
```

```
S1: t1 = a + b
S2: sum1 = t1 + s1
S3: t2 = c + d
S4: sum2 = t2 + s2
```

Loop-Independent Dependences

- S1 and S2 both reference the same location on the same loop iteration, but with S1 preceding S2 during execution of the loop iteration

```
for (i=0; i<N; i++) {
    A[i] = B[i];
    F[i+1] = A[i];
}
```

Loop-carried Dependences

- S1 references a location on one iteration of a loop
- On subsequent iteration, S2 references the same location

```
sum = 0;
for (i=0; i<N; i++) {
    sum = sum + A[i];
}
```

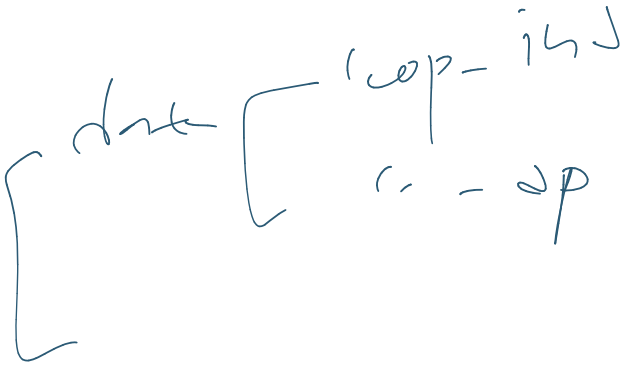
```
for (i=0; i<N; i++) {
    A[i+1] = F[i];
    F[i+1] = A[i];
}
```

```
S1 sum = sum + A[0];
S1 sum = sum + A[1];
   sum = sum + A[2];
   sum = sum + A[3];
   …
```

```
A[1] = F[0];
F[1] = A[0];
A[2] = F[1];
F[2] = A[1];
…
```

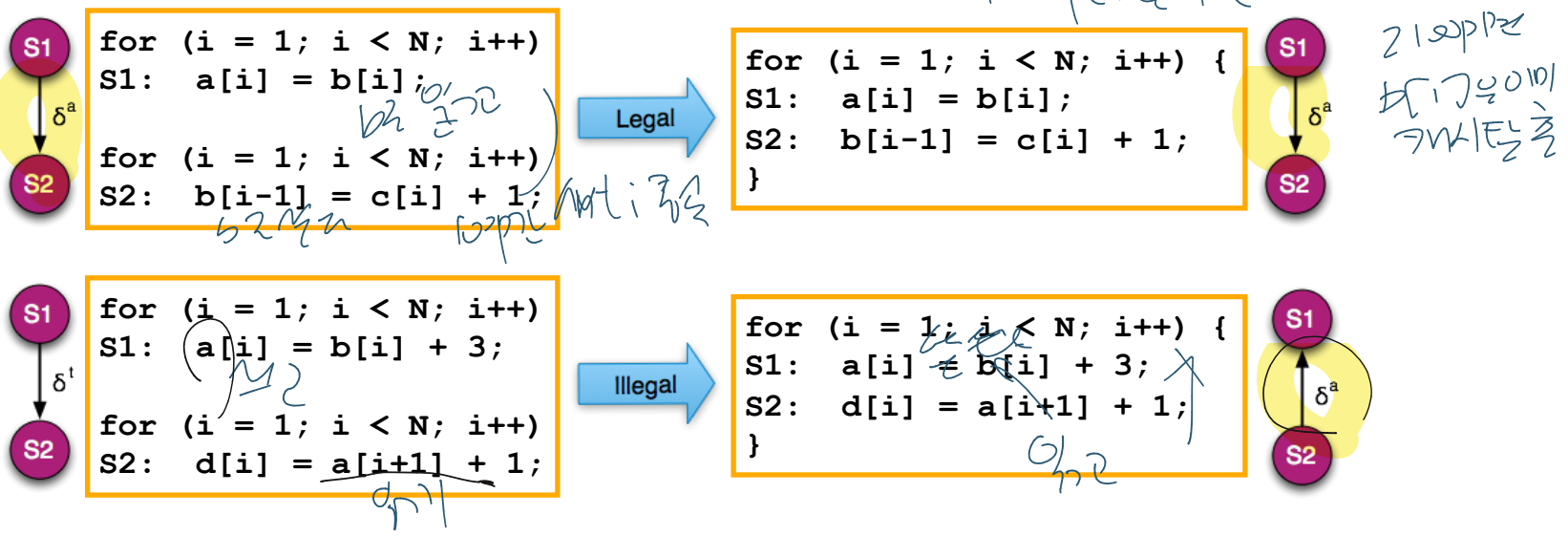
Fundamental Theorem of Dependence

- Any reordering transformation that preserves every dependence in a program preserves the meaning of that program



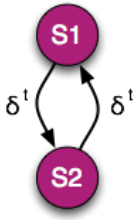
Loop Fusion

- Takes two adjacent loops that have the same iteration space traversal and combines their bodies into a single loop
- Fusion is illegal if fusing two loops causes the dependence direction to be changed



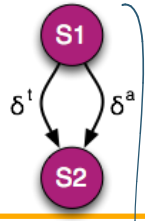
Loop Distribution (Fission)

- Takes a loop that contains multiple statements and splits it into two loops with the same iteration-space traveral
- Can be used to convert a sequential loop to multiple parallel loops
  - Converts loop-carried dependences to loop-independent dependences
- Legal when it does not result in breaking any cycles in the dependence graph of the original loop




```
for (i = 1; i < N; i++) {
S1:  a[i] = b[i] + 3;
S2:  b[i-1] = a[i+1] + 1;
}
```

Illegal

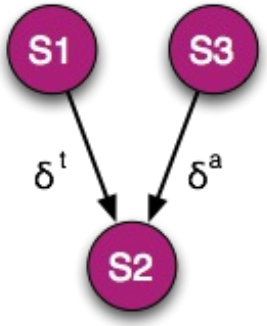



```
for (i = 1; i < N; i++)
S1:  a[i] = b[i] + 3;

for (i = 1; i < N; i++)
S2:  b[i-1] = a[i+1] + 1;
```

Loop Distribution (cont'd)

dep
δ
S2 S1




```
for (i = 1; i < N; i++) {
S1:  a[i] = b[i] + 3;
S2:  c[i] = a[i] + 1;
S3:  d[i] = c[i+1];
}
```

Legal

```
for (i = 1; i < N; i++)
S1:  a[i] = b[i] + 3;

for (i = 1; i < N; i++)
S3:  d[i] = c[i+1];

for (i = 1; i < N; i++)
S2:  c[i] = a[i] + 1;
```

Reduction

- 어떤 연산을 이용해 여러 개의 값을 모아서 하나의 값을 생성하는 과정
- 대표적인 병렬 컴퓨팅 패턴
- +, *, min, max 등의 연산에 대하여 정의됨
  - 교환 법칙, 결합 법칙이 성립하고 항등원을 가지는 연산
  - 항등원 : 연산이 정의된 집합에 대하여 임의의 원소 a 와 항등원을 연산한 결과는 항상 a

```
sum = 0;
for(i = 0; i < 8; i = i+1){
    sum = sum + x[i];
}
```

Reduction (cont'd)

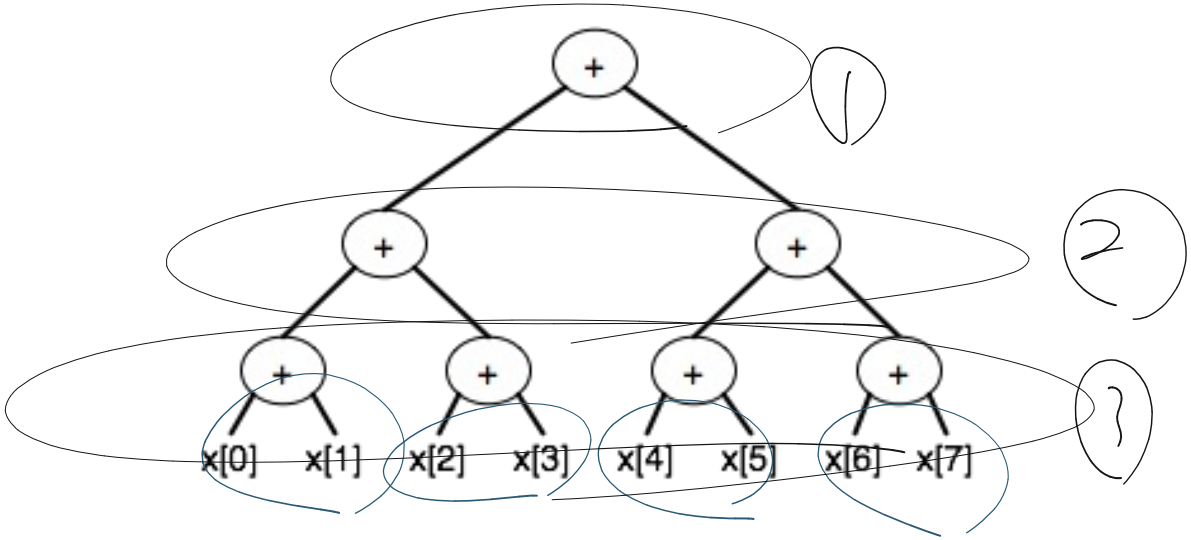
- 루프 캐리드 디펜던스 때문에 for 루프를 순차적으로 실행할 수 밖에 없음
- N 개의 데이터 아이템이 있다면 모두 N 번의 덧셈 연산이 필요함
  - N 번의 순차적인 덧셈 스텝이 필요

```
sum = 0;
for(i = 0; i < 8; i = i+1){
    sum = sum + x[i];
}
```

Parallel Reduction

- Parallel reduction을 적용하면 $\log N$ 번의 덧셈 스텝만 필요

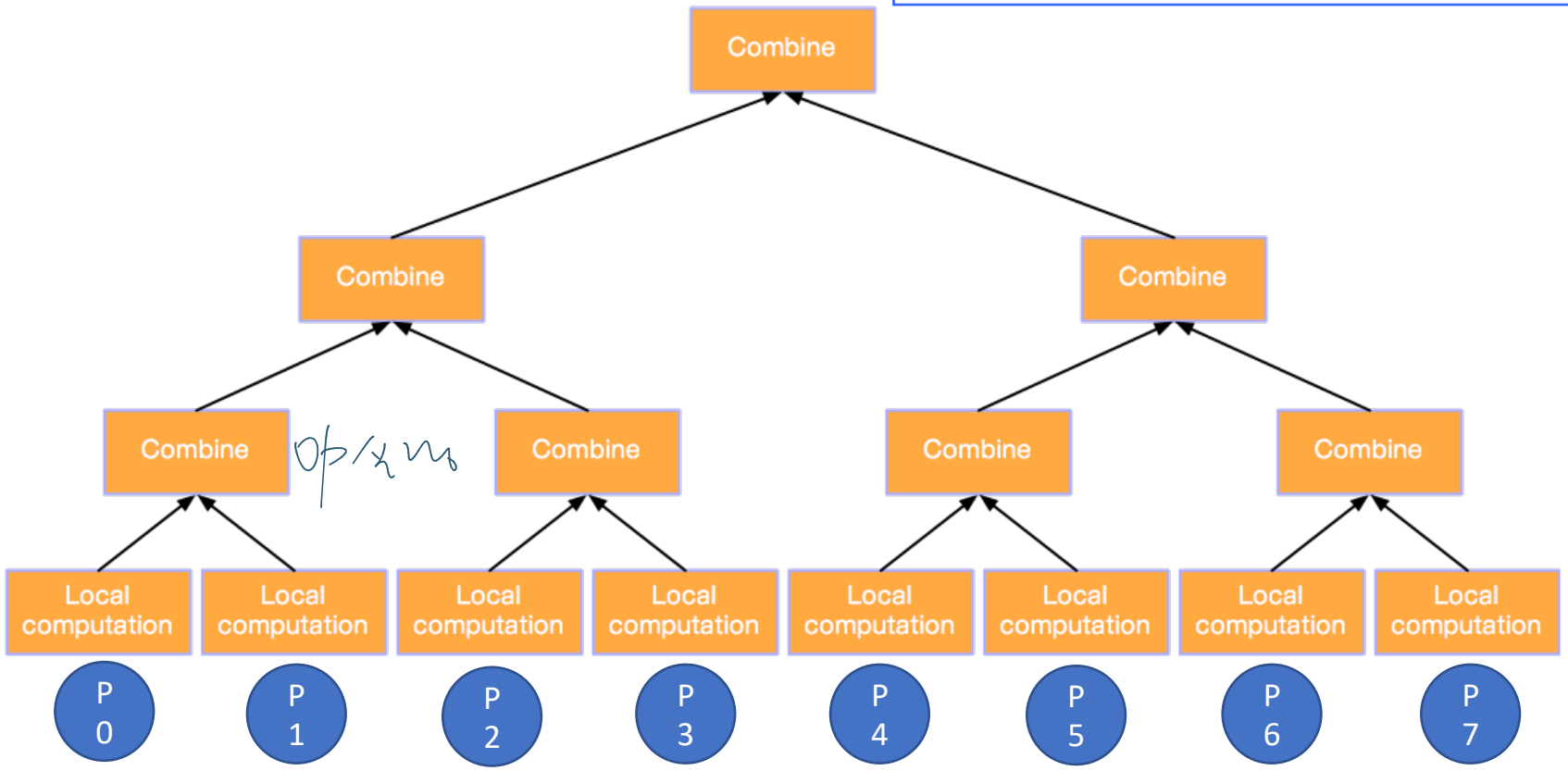
```
sum = ((x[0]+x[1])+(x[2]+x[3]))
      +((x[4]+x[5])+(x[6]+x[7]))
```



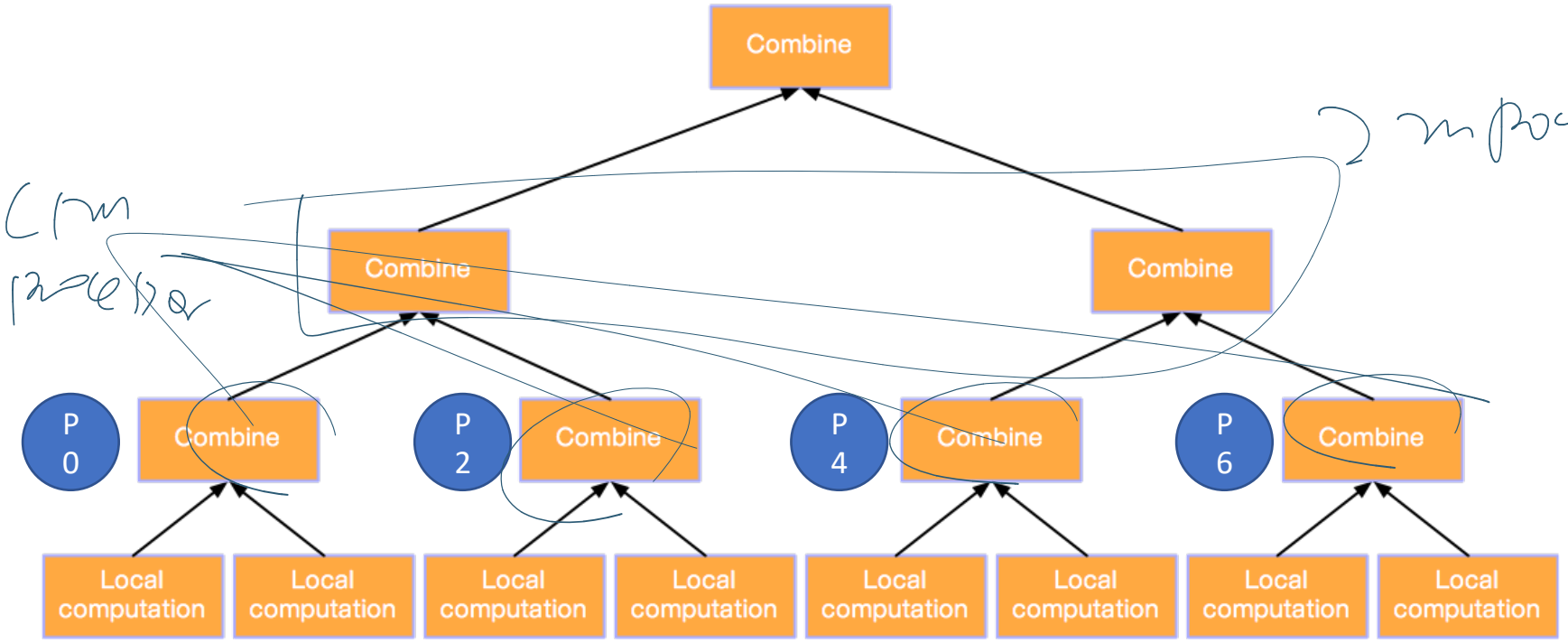
Parallel Reduction의 구조

- Tree 구조

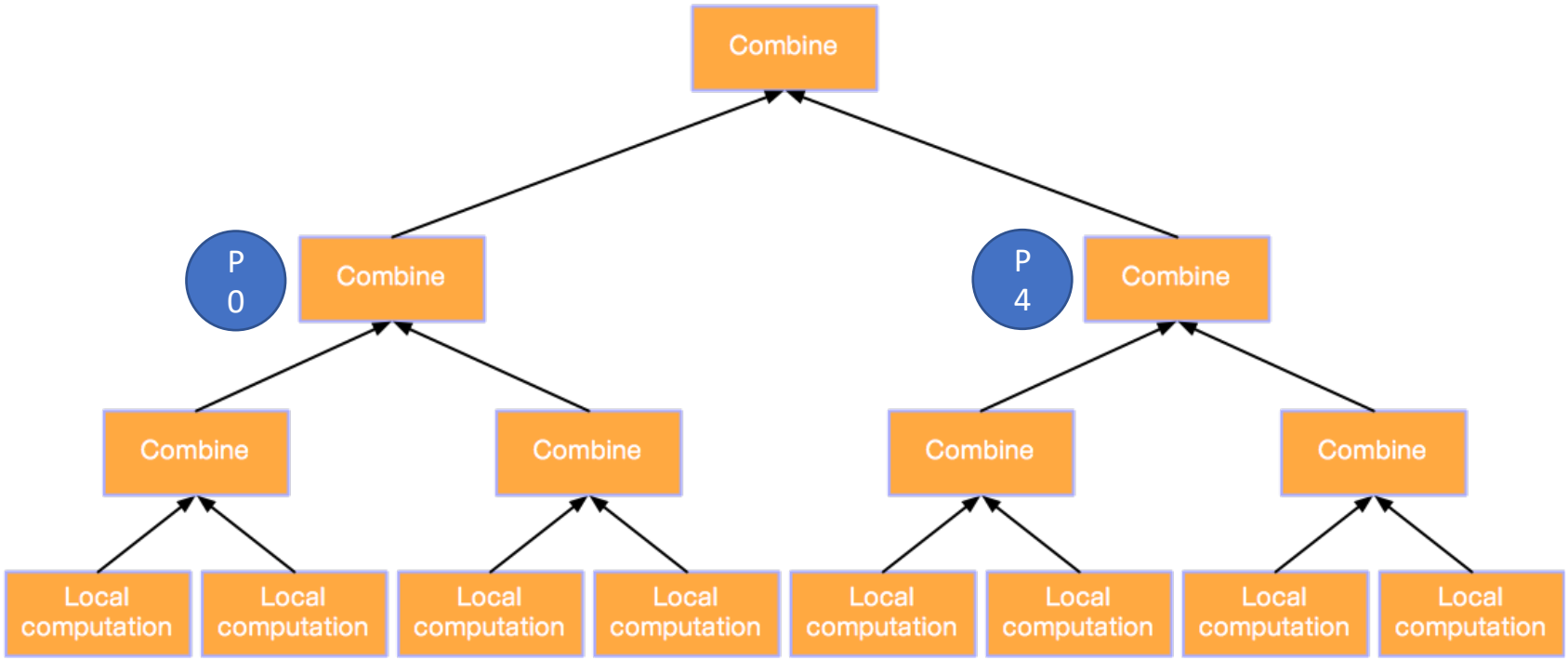
```
sum = 0;
for(i = 0; i < 8; i = i+1){
    sum = sum + f(x[i]);
}
```



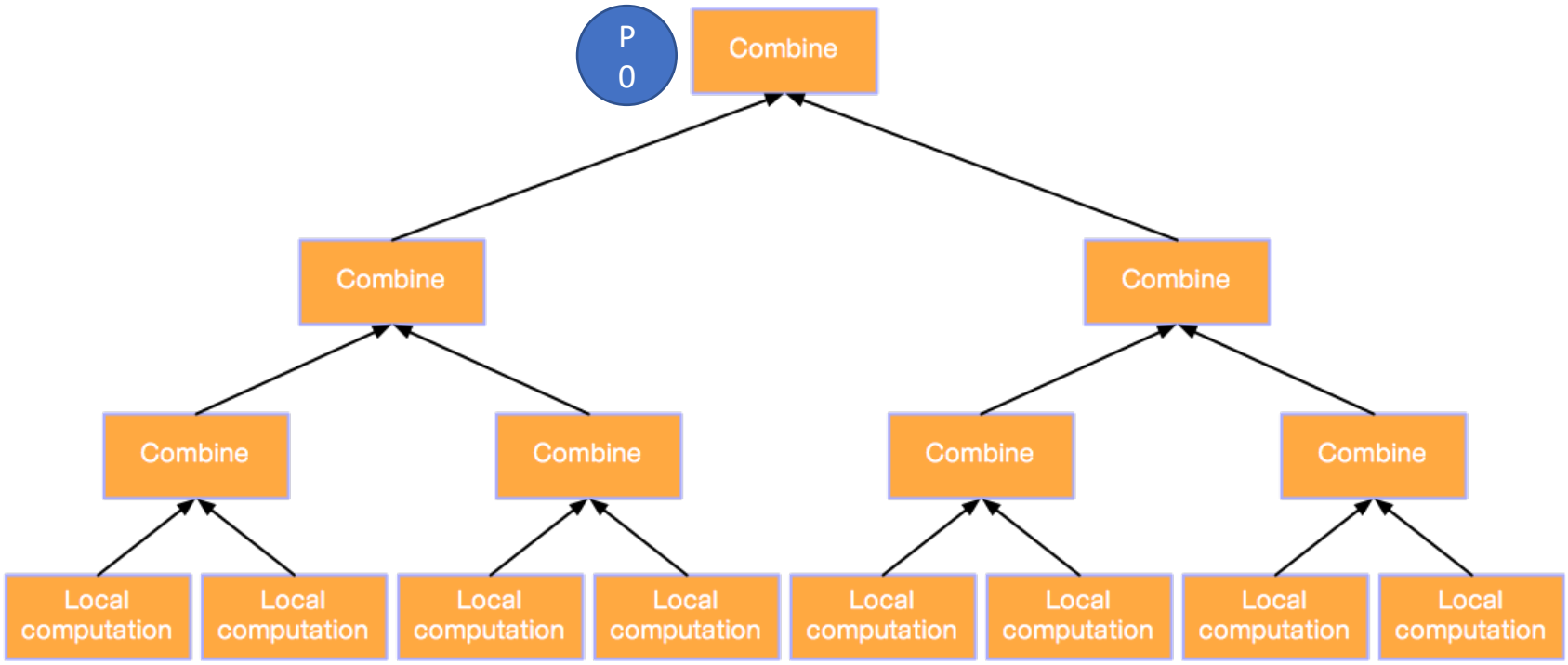
Parallel Reduction의 구조 (cont'd)



Parallel Reduction의 구조 (cont'd)

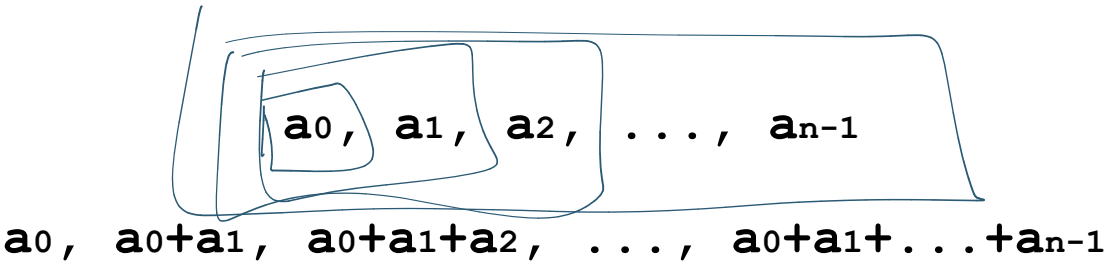


Parallel Reduction의 구조 (cont'd)



Scan

- 주어진 연산의 결과로 얻는 열의 위치 i 에 있는 원소는, 원래 열의 위치 0 부터 위치 i 까지 위치한 원소들에 주어진 연산을 적용하여 얻음
  - 연산에 대하여 결합법칙이 성립해야 함
- 대표적인 병렬 컴퓨팅 패턴
- 예) prefix sum



Prefix Sum

- 루프 캐리드 디펜던스 때문에 for 루프를 순차적으로 실행할 수 밖에 없음
- N 개의 데이터 아이템이 있다면 모두 $N-1$ 번의 덧셈 연산이 필요함

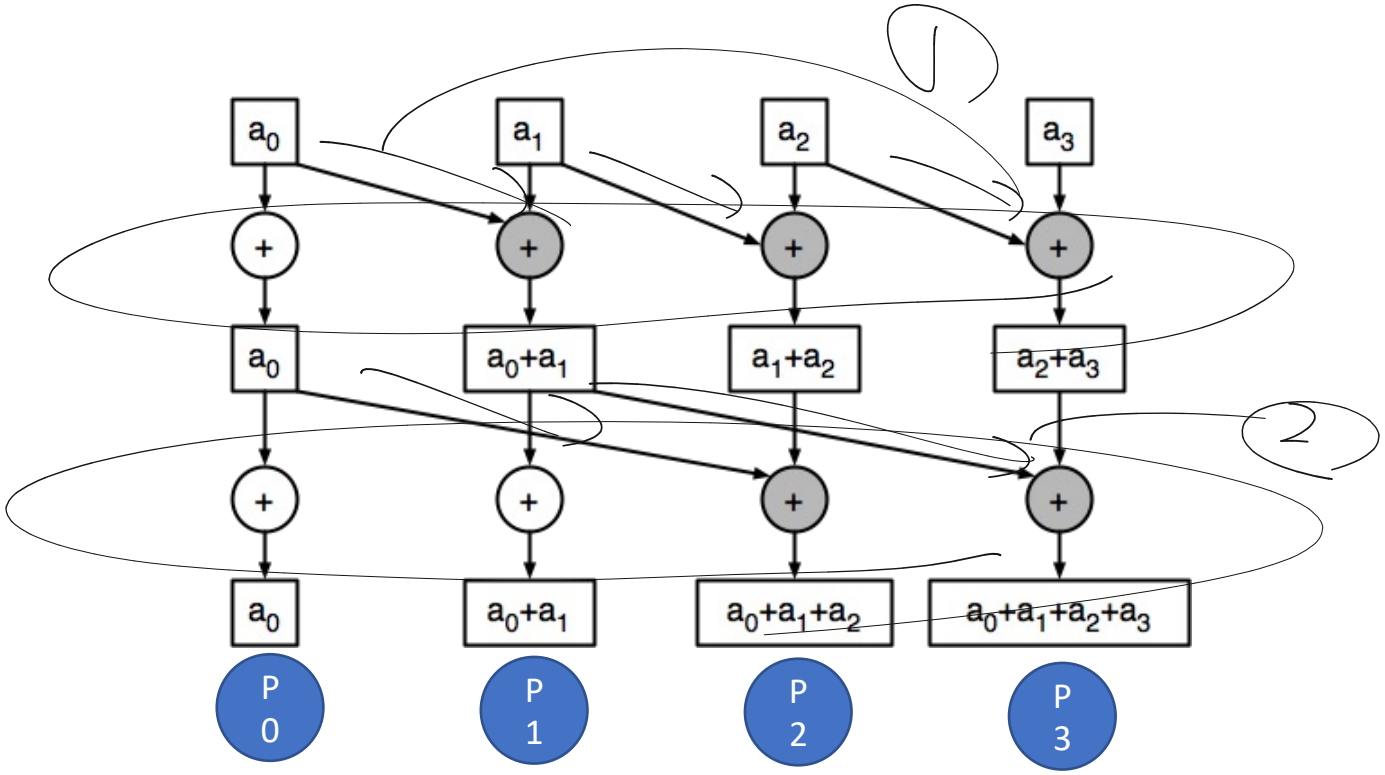
$$a_0,\ a_1,\ a_2,\ \ldots,\ a_{n-1}$$

$$a_0,\ a_0+a_1,\ a_0+a_1+a_2,\ \ldots,\ a_0+a_1+\ldots+a_{n-1}$$

```
prefix_sum[0] = a[0];
for(i = 1; i < 8; i = i+1){
    prefix_sum[i] = prefix_sum[i-1] + a[i];
}
```

Parallel Prefix Sum

- Parallel scan을 적용하면 $\log N$ 번의 덧셈 스텝만 필요
  - 연산의 순서를 바꾸는 것



Parallel Scan의 응용

- 다항식의 계산
- 점화식의 계산
- Sorting
- Histogram → histogram reduction
- ...