Постановка задачи

Зададим начальные и граничные условия, которые определяются следующим образом:

$$\begin{cases} \rho_0(x) = 1, & x \in [0, 10] \\ u_0(x) = 0, & x \in [0, 10] \\ u(t, 0) = \tilde{v}, & t \in [0, T] \\ \rho(t, 0) = \tilde{\rho}, & t \in [0, T] \\ \left.\dfrac{\partial u}{\partial x}\right|_{x=10} = 0, & t \in [0, T] \end{cases}$$

Область $\Omega = [0, T] \times [0, 10]$, а функции f и f_0 тождественно равны 0. Параметры $\tilde{v}$ ($\tilde{v} > 0$) и $\tilde{\rho}$ ($\tilde{\rho} \geq 1$) задают скорость и плотность "набегающего" потока.

Вычисления будут проводиться до времени $N_0\tau$, при котором решение перестанет зависеть от времени (выйдет на стационар). Критерием выхода на стационар будем считать выполнение условия:

$$\|(H^n, V^n) - (H^{n_{st}}, V^{n_{st}})\|_C \leq 10^{-4}$$

при $n_{st} \leq n \leq n_{st} + 50$, где n_{st} — момент выхода на стационар.

Численные эксперементы

$\mu = 0.1, p(\rho) = 1 \cdot \rho, h = 0.001, \tau = 0.0001$							
ρ \ u	1	2	3	4	5	6	7
1	37.183600	26.667400	13.384900	9.111800	7.107800	6.110700	5.749400
2	10.595000	16.015400	10.927700	8.008400	6.983100	6.265400	5.426400
3	8.813600	13.246400	10.036900	7.750200	6.617700	5.981800	5.529400
4	23.592000	11.984800	9.547400	7.660900	6.436000	6.046600	5.408500
5	43.565500	11.279400	9.218700	7.616400	6.857500	6.254500	5.327200
6	49.194500	10.787800	8.979100	7.805500	6.756500	6.202600	5.267000
7	53.766400	10.948400	8.425100	7.693000	6.873800	6.147400	5.215200

$\mu = 0.1, p(\rho) = 10 \cdot \rho, h = 0.001, \tau = 0.0001$							
ρ \ u	1	2	3	4	5	6	7
1	20.626700	16.714200	14.732000	27.878700	22.600000	10.640000	8.016000
2	16.710700	18.382500	18.432000	26.318500	9.215900	6.922300	5.618400
3	14.478200	18.423700	5.172500	14.001300	6.841500	4.542300	4.833900
4	18.470000	5.255900	22.310200	12.787900	5.980400	4.062800	4.487400
5	21.764400	9.460100	25.728500	12.860300	5.894600	4.025300	3.798800
6	17.536300	22.318000	28.387000	12.871400	5.885200	4.111500	3.727500
7	6.306200	24.779700	30.805300	12.864200	5.874900	4.046200	3.669300

$\mu = 0.1, p(\rho) = 100 \cdot \rho, h = 0.001, \tau = 0.0001$							
ρ \ u	1	2	3	4	5	6	7
1	18.218600	14.554200	10.947700	9.338300	8.560500	8.270700	7.106900
2	12.600700	9.325000	8.206000	7.432200	8.218500	8.642300	7.598500
3	9.166400	7.082900	7.809600	8.355900	8.555100	12.317600	18.367800
4	7.981800	7.641600	6.752500	9.938100	16.406800	14.936100	10.225800
5	7.070600	8.240000	9.617600	17.391900	3.603800	22.016300	22.348600
6	7.950700	8.096600	15.271800	6.192100	22.117500	22.353000	22.368400
7	6.428800	11.653100	21.126300	20.950800	22.349800	22.366800	22.371700

$\mu = 0.01, p(\rho) = 1 \cdot \rho, h = 0.001, \tau = 0.0001$							
ρ \ u	1	2	3	4	5	6	7
1	40.814600	31.269500	13.932800	10.037600	8.413100	6.958400	6.254100
2	14.824300	20.504500	11.711300	9.365800	7.744100	6.803000	5.742500
3	13.660800	16.460900	10.745800	8.661200	7.469700	6.539400	5.527800
4	29.475600	11.916900	9.954400	8.272600	7.304600	6.386700	5.403800
5	44.587400	11.454600	9.599100	8.030100	7.188900	6.284800	5.321100
6	49.482700	11.698700	9.440100	7.867100	7.131400	6.207900	5.258700
7	53.910000	11.958100	9.419400	8.789800	7.071200	6.146900	5.206600

$\mu = 0.01, p(\rho) = 10 \cdot \rho, h = 0.001, \tau = 0.0001$							
ρ \ u	1	2	3	4	5	6	7
1	24.221000	17.921900	15.077300	30.039100	23.725200	12.437000	8.548800
2	17.704700	19.026000	22.102600	26.204600	9.433200	6.139400	4.604200
3	15.389300	22.107700	4.140500	13.480100	6.691300	4.527300	4.081300
4	22.605400	3.994200	22.793700	12.261000	5.748500	4.099900	3.940600
5	22.669200	13.033600	25.757600	12.279100	5.629500	4.245400	3.845100
6	6.743500	23.042900	28.382000	12.281500	5.624800	4.161100	3.776100
7	4.129600	25.114800	30.789700	12.280100	5.622100	4.094500	3.719800

$\mu = 0.01, p(\rho) = 100 \cdot \rho, h = 0.001, \tau = 0.0001$							
ρ \ u	1	2	3	4	5	6	7
1	24.293400	16.700300	12.004800	10.424800	9.996200	9.162000	9.039700
2	5.357100	5.971500	6.527700	7.551200	8.369700	9.496200	8.597500
3	6.930700	4.857300	6.297500	8.476700	11.902200	17.409800	22.851000
4	7.537800	10.062400	14.068200	20.311800	22.909500	9.312800	15.592000
5	17.899300	22.177800	22.938600	8.608400	14.288000	22.354700	22.365600
6	58.459000	3.218900	12.636700	22.343300	22.364400	22.366800	22.369600
7	10.953200	16.589500	22.359200	22.365500	22.368600	22.370300	22.372100

Картинки

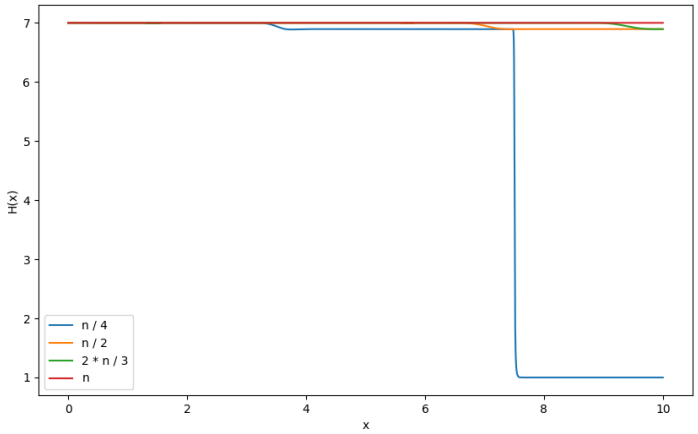

Figure 1: $\mu = 0.1, p(\rho) = 10\rho, u = 7, \rho = 7$, Плотность

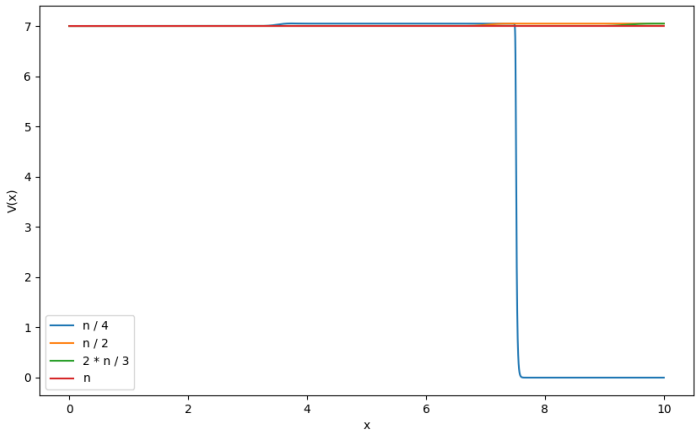

Figure 2: $\mu = 0.1, p(\rho) = 10\rho, u = 7, \rho = 7$, Скорость

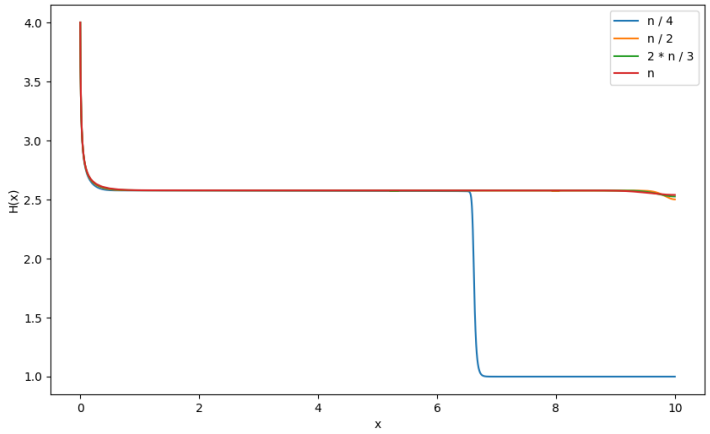

Figure 3: $\mu = 0.1, p(\rho) = 10\rho, u = 2, \rho = 4$, Плотность

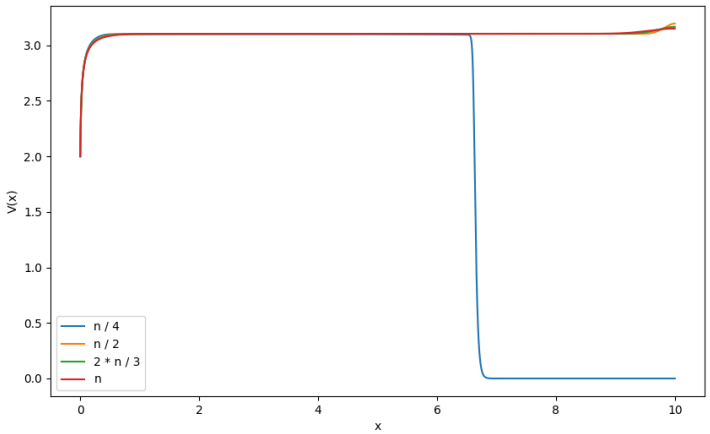

Figure 4: $\mu = 0.1, p(\rho) = 10\rho, u = 2, \rho = 4$, Скорость

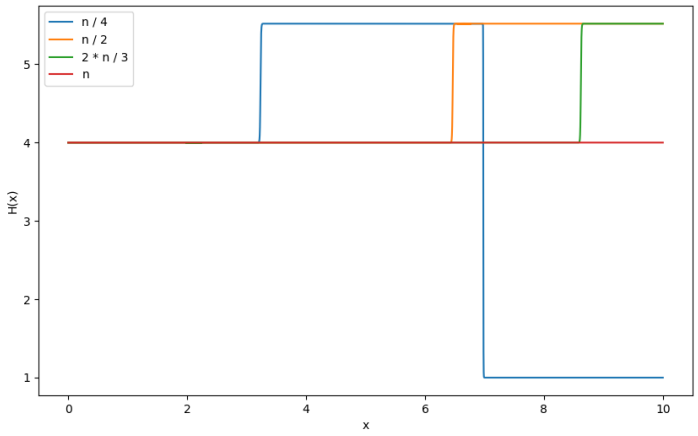

Figure 5: $\mu = 0.01, p(\rho) = 10\rho, u = 7, \rho = 4$, Плотность

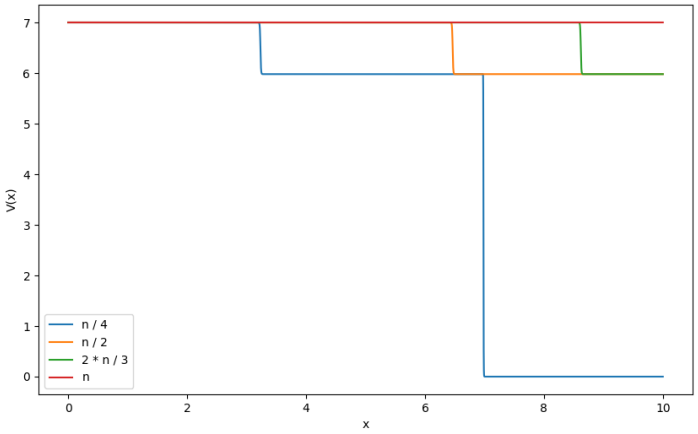

Figure 6: $\mu = 0.01, p(\rho) = 10\rho, u = 7, \rho = 4$, Скорость

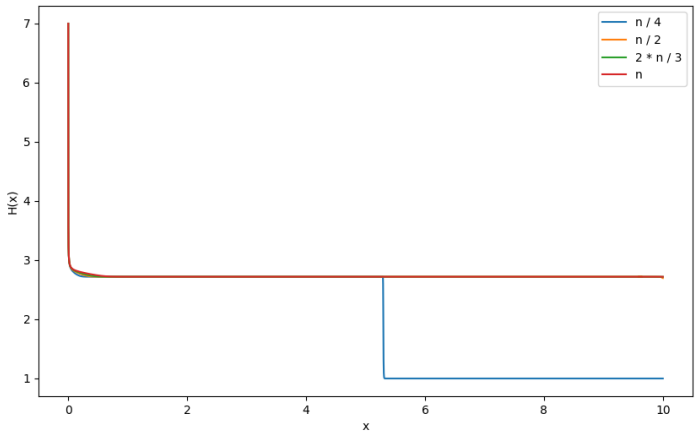

Figure 7: $\mu = 0.01, p(\rho) = 10\rho, u = 1, \rho = 7$, Плотность

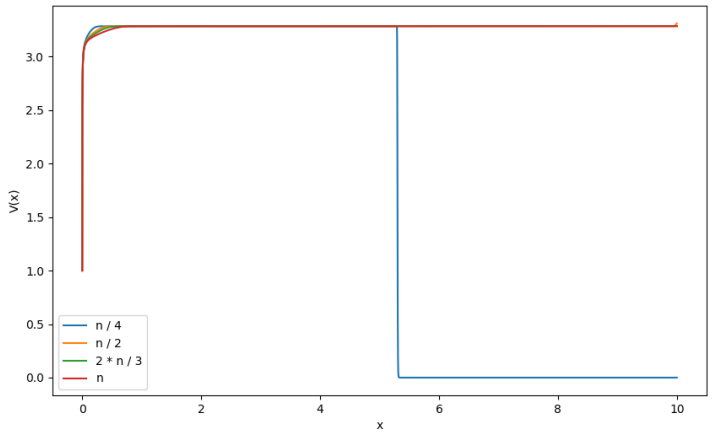

Figure 8: $\mu = 0.01, p(\rho) = 10\rho, u = 1, \rho = 7$, Скорость

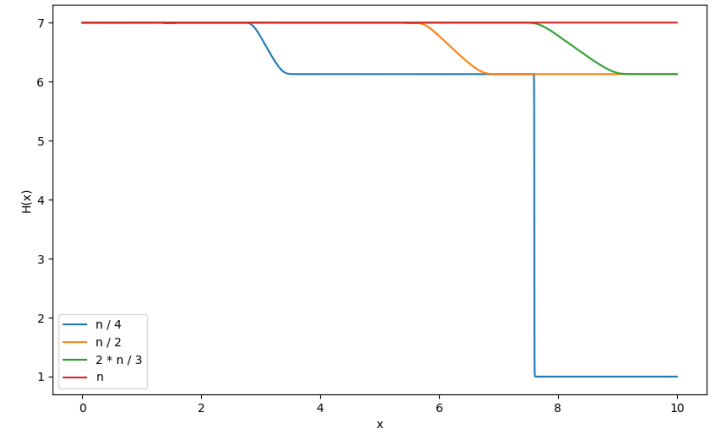

Figure 9: $\mu = 0.01, p(\rho) = 10\rho, u = 6, \rho = 7$, Плотность

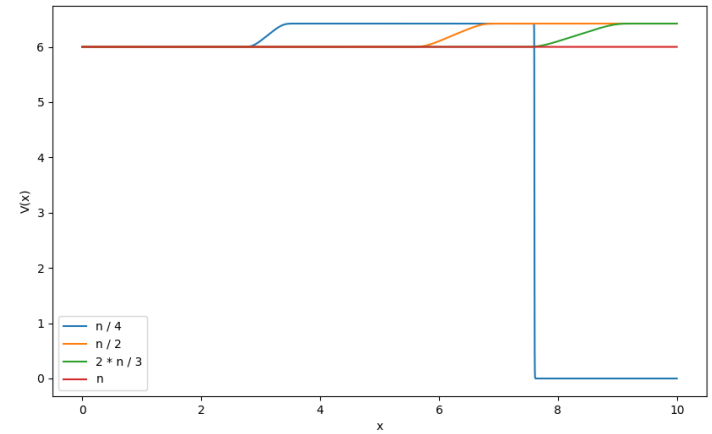

Figure 10: $\mu = 0.01, p(\rho) = 10\rho, u = 6, \rho = 7$, Скорость