

1 Постановка задачи

Решается система дифференциальных уравнений

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0 \quad (1)$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial x} + \frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial x^2} + \rho f \quad (2)$$

Для численного решения используется схема с центральными разностями $(\rho, \rho u)$ вида:

$$H_t + 0.5(V\hat{H}_{\bar{x}} + (V\hat{H})_{\hat{x}} + HV_{\bar{x}}) = 0 \quad (3)$$

$$(HV)_t + \frac{2}{3}(\hat{H}V\hat{V})_{\hat{x}} + \frac{2}{3}\hat{H}V\hat{V}_{\hat{x}} + \frac{V^2}{3}\hat{H}_{\hat{x}} + p(\hat{H})_{\hat{x}} = \mu\hat{V}_{x\bar{x}} + \hat{H}f \quad (4)$$

С граничными условиями:

$$H_{t,0} + 0.5((V\hat{H})_{x,0} + H_0V_{x,0}) - 0.5h((HV)_{x\bar{x},1} - 0.5(HV)_{x,\bar{x},2} + H_0(V_{x\bar{x},1}) - 0.5V_{x\bar{x},2}) = 0 \quad (5)$$

$$H_{t,M} + 0.5((V\hat{H})_{\bar{x},M} + H_MV_{\bar{x},M}) + 0.5h((HV)_{x\bar{x},M-1} - 0.5(HV)_{x,\bar{x},M-2} + H_M(V_{x\bar{x},M-1}) - 0.5V_{x\bar{x},M-2}) = 0 \quad (6)$$

Расписанная схема имеет вид:

$$\frac{\tau}{4h}(V_m^n + V_{m+1}^n)H_{m+1}^{n+1} + H_m^{n+1} + \frac{\tau}{4h}(-V_m^n - V_{m-1}^n)H_{m-1}^{n+1} = H_m^n - \frac{\tau}{4h}H_m^n(V_{m+1}^n - V_{m-1}^n) \quad (7)$$

$$\begin{aligned} & (\frac{\tau}{3h}H_{m+1}^{n+1}V_{m+1}^n + \frac{\tau}{3h}H_m^{n+1}V_m^n - \frac{\mu\tau}{h^2}V_{m+1}^{n+1} + (H_m^{n+1} + \frac{2\mu\tau}{h^2})V_m^{n+1} \\ & + (-\frac{\tau}{3h}H_{m-1}^{n+1}V_{m-1}^n - \frac{\tau}{3h}H_m^{n+1}V_m^n - \frac{\mu\tau}{h^2}V_{m-1}^{n+1} = H_m^nV_m^n - \frac{\tau}{6h}(V_m^n)^2(H_{m+1}^{n+1} - H_{m-1}^{n+1}) \\ & - \frac{\tau}{2h}(p(H_{m+1}^{n+1} - p(H_{m-1}^{n+1}))) \end{aligned} \quad (8)$$

2 Задание 1

Зададим функции давления и скорости:

$$\rho(t, x) = e^t(\cos(3\pi x) + 1.5) \quad (9)$$

$$u(t, x) = \cos(2\pi t)\sin(4\pi x) \quad (10)$$

Вычислим правые части f_0 и f исходных уравнений:

Для функции $f_0(t, x)$:

$$f_0(t, x) = \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x}$$

где

$$\frac{\partial \rho}{\partial t} = e^t(\cos(3\pi x) + 1.5)$$

$$\frac{\partial(\rho u)}{\partial x} = e^t \cos(2\pi t) [4\pi \cos(3\pi x) \cos(4\pi x) - 3\pi \sin(3\pi x) \sin(4\pi x)] + 6\pi e^t \cos(2\pi t) \cos(4\pi x)$$

В случае линейной зависимости давления от плотности $p = C\rho$:

$$f_{\text{lin}}(t, x, C, \mu) = \frac{1}{\rho} \left(\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial p}{\partial x} - \mu \frac{\partial^2 u}{\partial x^2} \right)$$

где $\rho = e^t(\cos(3\pi x) + 1.5)$ и

$$\begin{aligned} \frac{\partial(\rho u)}{\partial t} &= (\cos(3\pi x) \sin(4\pi x) + 1.5 \sin(4\pi x))e^t(\cos(2\pi t) - 2\pi \sin(2\pi t)) \\ \frac{\partial(\rho u^2)}{\partial x} &= e^t \cos^2(2\pi t) [-3\pi \sin(3\pi x) \sin^2(4\pi x) + 8\pi \sin(4\pi x) \cos(4\pi x) \cos(3\pi x)] \\ &\quad + 12\pi e^t \cos^2(2\pi t) \sin(4\pi x) \cos(4\pi x) \\ \frac{\partial^2 u}{\partial x^2} &= -16\pi^2 \mu \cos(2\pi t) \sin(4\pi x) \\ \frac{\partial p}{\partial x} &= Ce^t(-3\pi) \sin(3\pi x) \end{aligned}$$

В случае степенной зависимости давления от плотности $p = \rho^\gamma$:

$$f_{\text{pow}}(t, x, \gamma, \mu) = \frac{1}{\rho} \left(\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial p}{\partial x} - \mu \frac{\partial^2 u}{\partial x^2} \right)$$

где все компоненты аналогичны линейному случаю, кроме производной давления:

$$\frac{\partial p}{\partial x} = \gamma [e^t(\cos(3\pi x) + 1.5)]^{\gamma-1} e^t(-3\pi) \sin(3\pi x)$$

2.1 Численные эксперименты

Измелченный шаг сетки, плотность

$\mu = 0.1, p(\rho) = 1\rho$						
$\tau \setminus h$	0.005	0.0025	0.00125	0.000625	0.0003125	0.00015625
0.005	$2.893845e-01$	$2.915237e-01$	$2.913509e-01$	$2.913015e-01$	$2.913385e-01$	$2.913374e-01$
	$8.611497e-02$	$8.622999e-02$	$8.623645e-02$	$8.623747e-02$	$8.623915e-02$	$8.623975e-02$
	$3.137688e+00$	$3.131871e+00$	$3.129518e+00$	$3.128874e+00$	$3.128766e+00$	$3.128744e+00$
	$7.407000e-03$	$1.487000e-02$	$2.914100e-02$	$5.815800e-02$	$1.157880e-01$	$2.318680e-01$
0.0025	$1.524219e-01$	$1.543208e-01$	$1.547432e-01$	$1.549055e-01$	$1.549308e-01$	$1.549417e-01$
	$4.202855e-02$	$4.260850e-02$	$4.275977e-02$	$4.279980e-02$	$4.281009e-02$	$4.281264e-02$
	$1.483760e+00$	$1.490807e+00$	$1.492612e+00$	$1.493119e+00$	$1.493253e+00$	$1.493286e+00$
	$1.476000e-02$	$2.921000e-02$	$5.818000e-02$	$1.157410e-01$	$2.311380e-01$	$4.626090e-01$
0.00125	$7.691631e-02$	$7.913400e-02$	$7.966958e-02$	$7.980628e-02$	$7.984871e-02$	$7.985736e-02$
	$2.066607e-02$	$2.136305e-02$	$2.156384e-02$	$2.161647e-02$	$2.162970e-02$	$2.163300e-02$
	$7.220191e-01$	$7.325832e-01$	$7.357691e-01$	$7.366206e-01$	$7.368342e-01$	$7.368874e-01$
	$2.937800e-02$	$5.823900e-02$	$1.160410e-01$	$2.312140e-01$	$4.619780e-01$	$9.264950e-01$
0.000625	$3.733308e-02$	$3.972090e-02$	$4.030919e-02$	$4.047760e-02$	$4.051570e-02$	$4.052520e-02$
	$9.988930e-03$	$1.061650e-02$	$1.083062e-02$	$1.088772e-02$	$1.090215e-02$	$1.090576e-02$
	$3.524930e-01$	$3.620059e-01$	$3.655893e-01$	$3.665641e-01$	$3.668107e-01$	$3.668725e-01$
	$5.863100e-02$	$1.163470e-01$	$2.317190e-01$	$4.624440e-01$	$9.230660e-01$	$1.846046e+00$
0.0003125	$1.710448e-02$	$1.957835e-02$	$2.019278e-02$	$2.036015e-02$	$2.039974e-02$	$2.041011e-02$
	$4.860735e-03$	$5.194855e-03$	$5.401525e-03$	$5.459803e-03$	$5.474727e-03$	$5.478480e-03$
	$1.745043e-01$	$1.783389e-01$	$1.817985e-01$	$1.828096e-01$	$1.830702e-01$	$1.831359e-01$
	$1.171520e-01$	$2.324070e-01$	$4.710840e-01$	$9.240540e-01$	$1.852106e+00$	$3.692200e+00$
0.00015625	$7.498453e-03$	$9.400183e-03$	$1.002532e-02$	$1.018984e-02$	$1.023049e-02$	$1.024057e-02$
	$2.823541e-03$	$2.494873e-03$	$2.669442e-03$	$2.726722e-03$	$2.741789e-03$	$2.745601e-03$
	$9.624038e-02$	$8.738025e-02$	$9.017108e-02$	$9.116381e-02$	$9.142896e-02$	$9.149631e-02$
	$2.341350e-01$	$4.651290e-01$	$9.357880e-01$	$1.846729e+00$	$3.734593e+00$	$7.423286e+00$
$\mu = 0.01, p(\rho) = 1\rho$						
$\tau \setminus h$	0.005	0.0025	0.00125	0.000625	0.0003125	0.00015625
0.005	$4.900888e+00$	$5.133649e+00$	$5.604197e+00$	$5.981867e+00$	$2.363684e+10$	$2.299583e+09$
	$7.799330e-01$	$7.761854e-01$	$7.659045e-01$	$7.751129e-01$	$8.904598e+08$	$1.176841e+08$
	$8.058803e+01$	$9.244111e+01$	$1.213865e+02$	$3.685580e+02$	$4.015233e+12$	$1.063727e+12$
	$7.417000e-03$	$1.463800e-02$	$2.910300e-02$	$5.809800e-02$	$1.271220e-01$	$2.331790e-01$
0.0025	$1.929420e+00$	$1.990965e+00$	$2.006569e+00$	$2.007622e+00$	$2.008172e+00$	$2.008361e+00$
	$3.631623e-01$	$3.718577e-01$	$3.740268e-01$	$3.745687e-01$	$3.747041e-01$	$3.747380e-01$
	$2.510589e+01$	$2.578269e+01$	$2.596047e+01$	$2.600609e+01$	$2.601758e+01$	$2.602046e+01$
	$1.473900e-02$	$2.951600e-02$	$5.800400e-02$	$1.229880e-01$	$2.557010e-01$	$4.616250e-01$
0.00125	$8.367617e-01$	$9.074929e-01$	$9.228006e-01$	$9.275400e-01$	$9.285267e-01$	$9.287728e-01$
	$1.688927e-01$	$1.783897e-01$	$1.808039e-01$	$1.814097e-01$	$1.815613e-01$	$1.815992e-01$
	$1.022621e+01$	$1.065398e+01$	$1.076655e+01$	$1.079507e+01$	$1.080222e+01$	$1.080401e+01$
	$2.934500e-02$	$5.819200e-02$	$1.161320e-01$	$2.417220e-01$	$4.806590e-01$	$9.331000e-01$
0.000625	$3.573832e-01$	$4.231343e-01$	$4.396450e-01$	$4.434951e-01$	$4.444552e-01$	$4.447130e-01$
	$7.670919e-02$	$8.591451e-02$	$8.831782e-02$	$8.892395e-02$	$8.907580e-02$	$8.911379e-02$
	$4.531714e+00$	$4.866648e+00$	$4.961996e+00$	$4.986508e+00$	$4.992677e+00$	$4.994222e+00$
	$5.853700e-02$	$1.161800e-01$	$2.315130e-01$	$4.676090e-01$	$9.246050e-01$	$1.860994e+00$
0.0003125	$1.388926e-01$	$1.970203e-01$	$2.123008e-01$	$2.160633e-01$	$2.169896e-01$	$2.172297e-01$
	$3.255612e-02$	$4.096086e-02$	$4.331812e-02$	$4.391886e-02$	$4.406971e-02$	$4.410746e-02$
	$2.046380e+00$	$2.291703e+00$	$2.377804e+00$	$2.400630e+00$	$2.406414e+00$	$2.407865e+00$
	$1.279320e-01$	$2.326540e-01$	$4.740430e-01$	$9.290540e-01$	$1.858447e+00$	$3.705711e+00$
0.00015625	$8.227427e-02$	$8.819948e-02$	$1.024888e-01$	$1.061607e-01$	$1.070592e-01$	$1.072839e-01$
	$1.293381e-02$	$1.887575e-02$	$2.115212e-02$	$2.174663e-02$	$2.189661e-02$	$2.193419e-02$
	$9.978122e-01$	$1.077601e+00$	$1.154051e+00$	$1.175836e+00$	$1.181434e+00$	$1.182842e+00$
	$2.337100e-01$	$4.761070e-01$	$9.252340e-01$	$1.852444e+00$	$3.689680e+00$	$7.388481e+00$

$\mu = 0.001, p(\rho) = 1\rho$						
$\tau \setminus h$	0.005	0.0025	0.00125	0.000625	0.0003125	0.00015625
0.005	0.000000e+00 -nan -nan 1.867200e-02	0.000000e+00 -nan -nan 2.443300e-02	0.000000e+00 nan nan 4.807400e-02	0.000000e+00 -nan -nan 9.570900e-02	0.000000e+00 -nan -nan 1.218770e-01	0.000000e+00 nan nan 2.403440e-01
0.0025	7.054475e+00 1.272566e+00 3.913050e+02 2.434300e-02	0.000000e+00 nan nan 4.807900e-02	0.000000e+00 nan nan 9.575400e-02	0.000000e+00 -nan -nan 1.908370e-01	0.000000e+00 nan nan 2.317490e-01	0.000000e+00 nan nan 4.713370e-01
0.00125	2.564928e+00 3.803479e-01 8.464561e+01 4.857800e-02	3.857608e+00 4.227302e-01 1.810421e+02 9.697500e-02	6.197531e+00 6.064705e-01 3.853700e+02 1.914690e-01	6.093398e+00 1.902754e+00 5.101565e+02 3.812200e-01	0.000000e+00 -nan -nan 4.617600e-01	0.000000e+00 -nan -nan 9.231580e-01
0.000625	1.419521e+00 1.741238e-01 3.484272e+01 9.678000e-02	2.216273e+00 1.796006e-01 5.115149e+01 1.918480e-01	1.726352e+00 1.566479e-01 6.773710e+01 3.828750e-01	1.039263e+00 1.331874e-01 3.161886e+01 7.618870e-01	1.020661e+00 1.377094e-01 3.094928e+01 9.278520e-01	0.000000e+00 -nan -nan 1.869128e+00
0.0003125	6.877637e-01 7.852395e-02 1.596951e+01 1.930050e-01	9.627884e-01 8.556558e-02 1.935555e+01 3.830890e-01	9.843942e-01 8.595773e-02 2.256076e+01 7.640110e-01	8.376073e-01 8.565856e-02 2.207361e+01 1.524760e+00	7.923757e-01 8.570666e-02 2.177715e+01 1.847773e+00	7.828591e-01 8.572498e-02 2.170835e+01 3.690688e+00
0.00015625	3.794978e-01 4.937137e-02 9.650137e+00 3.859240e-01	4.203847e-01 3.918362e-02 8.248641e+00 7.835370e-01	4.428989e-01 4.355501e-02 9.051167e+00 1.534377e+00	4.227906e-01 4.488731e-02 9.249589e+00 2.812675e+00	4.185418e-01 4.523790e-02 9.292250e+00 3.699041e+00	4.174648e-01 4.532674e-02 9.302605e+00 7.386080e+00
$\mu = 0.1, p(\rho) = 10\rho$						
$\tau \setminus h$	0.005	0.0025	0.00125	0.000625	0.0003125	0.00015625
0.005	3.470153e-02 1.707447e-02 9.329289e-02 7.434000e-03	4.027325e-02 1.698463e-02 5.473767e-01 1.483100e-03	5.609978e+06 1.135779e+06 1.668156e+08 2.914000e-02	1.312251e+09 2.292786e+08 1.646350e+10 5.816100e-02	2.278166e+29 6.143380e+27 2.803154e+31 1.158330e-01	1.270990e+101 4.612069e+100 4.174368e+104 2.315760e-01
0.0025	1.777109e-02 8.674932e-03 4.809207e-02 1.478900e-02	1.716100e-02 8.513864e-03 4.575653e-02 2.927800e-02	1.701379e-02 8.475284e-03 4.521750e-02 5.808800e-02	1.697756e-02 8.465746e-03 4.508570e-02 1.158190e-01	1.696851e-02 8.463368e-03 4.505294e-02 2.311240e-01	1.696625e-02 8.462774e-03 4.504476e-02 4.626720e-01
0.00125	9.314353e-03 4.460908e-03 2.575681e-02 2.945500e-02	8.682180e-03 4.290377e-03 2.321783e-02 5.836000e-02	8.533851e-03 4.251016e-03 2.266651e-02 1.162190e-01	8.497537e-03 4.241390e-03 2.253454e-02 2.314260e-01	8.488502e-03 4.238996e-03 2.250193e-02 4.642230e-01	8.486244e-03 4.238399e-03 2.249380e-02 9.255960e-01
0.000625	5.111654e-03 2.361180e-03 1.487860e-02 5.870700e-02	4.443472e-03 2.174334e-03 1.199482e-02 1.256800e-01	4.292020e-03 2.133745e-03 1.141523e-02 2.326680e-01	4.255482e-03 2.124019e-03 1.128166e-02 4.621770e-01	4.246453e-03 2.121615e-03 1.124902e-02 9.229790e-01	4.244198e-03 2.121015e-03 1.124091e-02 1.854221e+00
0.0003125	3.039179e-03 1.330538e-03 9.775877e-03 1.176900e-01	2.328490e-03 1.116735e-03 6.430955e-03 2.325720e-01	2.170933e-03 1.073935e-03 5.798238e-03 4.672030e-01	2.133966e-03 1.064053e-03 5.660945e-03 9.239240e-01	2.124906e-03 1.061636e-03 5.628101e-03 1.846479e+00	2.122652e-03 1.061035e-03 5.619986e-03 3.691511e+00
0.00015625	2.022007e-03 8.424085e-04 7.510906e-03 2.342660e-01	1.277785e-03 5.907288e-04 3.716549e-03 4.748030e-01	1.110956e-03 5.439327e-04 2.997016e-03 9.263840e-01	1.073117e-03 5.337635e-04 2.852353e-03 1.848054e+00	1.064001e-03 5.313269e-04 2.819031e-03 3.693649e+00	1.061746e-03 5.307245e-04 2.810890e-03 7.383658e+00

$\mu = 0.01, p(\rho) = 10\rho$						
$\tau \setminus h$	0.005	0.0025	0.00125	0.000625	0.0003125	0.00015625
0.005	0.000000e+00 nan nan 7.439000e-03	0.000000e+00 -nan -nan 1.470000e-02	0.000000e+00 nan nan 2.918800e-02	0.000000e+00 nan nan 5.805700e-02	0.000000e+00 nan nan 1.160020e-01	0.000000e+00 nan nan 2.352330e-01
0.0025	1.447342e-02 8.086155e-03 4.273009e-02 1.478700e-02	0.000000e+00 nan nan 2.923400e-02	0.000000e+00 nan nan 5.811500e-02	0.000000e+00 nan nan 1.244220e-01	0.000000e+00 nan nan 2.313330e-01	0.000000e+00 nan nan 4.616980e-01
0.00125	7.618541e-03 4.144107e-03 2.334309e-02 3.214200e-02	7.017173e-03 4.004375e-03 2.020443e-02 5.830400e-02	0.000000e+00 nan nan 1.164190e-01	0.000000e+00 nan nan 2.410050e-01	0.000000e+00 nan nan 4.624120e-01	0.000000e+00 nan nan 9.353200e-01
0.000625	4.238767e-03 2.192574e-03 1.421848e-02 5.868300e-02	3.598201e-03 2.024360e-03 1.052328e-02 1.164040e-01	3.458471e-03 1.993195e-03 9.849092e-03 2.318090e-01	0.000000e+00 -nan -nan 4.722120e-01	0.000000e+00 -nan -nan 9.238610e-01	0.000000e+00 nan nan 1.861632e+00
0.0003125	2.573644e-03 1.252290e-03 1.014084e-02 1.174780e-01	1.899704e-03 1.036750e-03 5.788703e-03 2.337210e-01	1.750393e-03 1.001742e-03 5.019055e-03 4.641840e-01	1.717285e-03 9.944316e-04 4.865621e-03 9.349300e-01	1.709393e-03 9.926978e-04 4.830143e-03 1.880127e+00	1.707448e-03 9.922703e-04 4.821461e-03 3.721943e+00
0.00015625	1.748549e-03 8.280625e-04 8.420904e-03 2.342320e-01	1.058703e-03 5.483113e-04 3.537173e-03 4.658310e-01	8.985621e-04 5.062590e-04 2.622240e-03 9.261650e-01	8.637468e-04 4.984515e-04 2.454994e-03 1.868094e+00	8.557285e-04 4.966845e-04 2.418613e-03 3.735896e+00	8.537762e-04 4.962545e-04 2.409885e-03 7.422385e+00
$\mu = 0.001, p(\rho) = 10\rho$						
$\tau \setminus h$	0.005	0.0025	0.00125	0.000625	0.0003125	0.00015625
0.005	0.000000e+00 -nan -nan 7.415000e-03	0.000000e+00 nan nan 1.471200e-02	0.000000e+00 -nan -nan 2.917400e-02	0.000000e+00 nan nan 5.801400e-02	0.000000e+00 -nan -nan 1.158030e-01	0.000000e+00 nan nan 2.319420e-01
0.0025	0.000000e+00 nan nan 1.478000e-02	0.000000e+00 -nan -nan 2.922900e-02	0.000000e+00 -nan -nan 5.809700e-02	0.000000e+00 -nan -nan 1.169840e-01	0.000000e+00 nan nan 2.311160e-01	0.000000e+00 -nan -nan 4.629510e-01
0.00125	0.000000e+00 -nan -nan 2.987100e-02	0.000000e+00 nan nan 5.827000e-02	0.000000e+00 nan nan 1.159390e-01	0.000000e+00 -nan -nan 2.419660e-01	0.000000e+00 nan nan 4.621510e-01	0.000000e+00 -nan -nan 9.237800e-01
0.000625	4.804786e-03 2.244008e-03 1.552917e-02 5.870300e-02	0.000000e+00 -nan -nan 1.164210e-01	0.000000e+00 nan nan 2.322940e-01	0.000000e+00 -nan -nan 4.629840e-01	0.000000e+00 -nan -nan 9.238000e-01	0.000000e+00 nan nan 1.855162e+00
0.0003125	2.989562e-03 1.303740e-03 1.149007e-02 1.175890e-01	2.095558e-03 1.054842e-03 6.072177e-03 2.324700e-01	0.000000e+00 -nan -nan 4.636670e-01	0.000000e+00 -nan -nan 9.242390e-01	0.000000e+00 nan nan 1.847257e+00	0.000000e+00 -nan -nan 3.695794e+00
0.00015625	2.087142e-03 8.908657e-04 9.827234e-03 2.346110e-01	1.186917e-03 5.604874e-04 3.797889e-03 4.654460e-01	9.794936e-04 5.146432e-04 2.719082e-03 9.268990e-01	9.309543e-04 5.073829e-04 4.829216e-02 1.847471e+00	4.063144e+00 2.123764e+00 4.842522e+02 3.695550e+00	0.000000e+00 -nan -nan 7.389896e+00

$\mu = 0.1, p(\rho) = 100\rho$						
$\tau \setminus h$	0.005	0.0025	0.00125	0.000625	0.0003125	0.00015625
0.005	0.000000e+00 -nan -nan 7.429000e-03	0.000000e+00 nan nan 1.467000e-02	0.000000e+00 nan nan 2.915400e-02	0.000000e+00 -nan -nan 5.800300e-02	0.000000e+00 -nan -nan 1.158500e-01	0.000000e+00 nan nan 2.314530e-01
0.0025	0.000000e+00 nan nan 1.476100e-02	0.000000e+00 -nan -nan 2.927700e-02	0.000000e+00 nan nan 5.808600e-02	0.000000e+00 nan nan 1.159690e-01	0.000000e+00 nan nan 2.311340e-01	0.000000e+00 -nan -nan 4.622800e-01
0.00125	8.204304e+00 2.358930e+00 4.319501e+02 2.939800e-02	0.000000e+00 nan nan 5.825500e-02	0.000000e+00 nan nan 1.160600e-01	0.000000e+00 -nan -nan 2.315200e-01	0.000000e+00 -nan -nan 4.621620e-01	0.000000e+00 -nan nan 9.234320e-01
0.000625	3.926593e-03 2.231145e-03 1.349392e-02 5.864200e-02	4.352413e+00 3.236816e-01 1.420743e+02 1.163990e-01	0.000000e+00 nan nan 2.317330e-01	0.000000e+00 nan nan 4.619150e-01	0.000000e+00 nan nan 9.275290e-01	0.000000e+00 -nan -nan 1.845479e+00
0.0003125	2.461054e-03 1.316312e-03 9.633628e-03 1.171160e-01	1.714661e-03 1.031651e-03 5.335081e-03 2.326350e-01	1.529123e-03 9.783640e-04 4.310425e-03 4.630930e-01	1.482896e-03 9.665211e-04 4.061458e-03 9.239220e-01	1.471353e-03 9.636590e-04 3.999812e-03 1.845893e+00	1.468469e-03 9.629497e-04 3.984441e-03 3.702063e+00
0.00015625	1.728140e-03 8.971344e-04 7.748414e-03 2.361940e-01	9.813768e-04 5.577447e-04 3.371825e-03 4.647520e-01	7.954354e-04 4.975614e-04 2.323685e-03 9.261820e-01	7.491536e-04 4.852107e-04 2.072023e-03 1.848197e+00	7.376099e-04 4.823159e-04 2.010185e-03 3.691257e+00	7.347257e-04 4.816046e-04 1.994804e-03 7.378840e+00
$\mu = 0.01, p(\rho) = 100\rho$						
$\tau \setminus h$	0.005	0.0025	0.00125	0.000625	0.0003125	0.00015625
0.005	0.000000e+00 -nan -nan 1.678600e-02	0.000000e+00 -nan -nan 1.472000e-02	0.000000e+00 -nan -nan 3.843300e-02	0.000000e+00 nan nan 5.802500e-02	0.000000e+00 -nan -nan 1.250840e-01	0.000000e+00 -nan -nan 2.317480e-01
0.0025	0.000000e+00 nan nan 1.476500e-02	0.000000e+00 nan nan 2.920600e-02	0.000000e+00 nan nan 6.833100e-02	0.000000e+00 -nan -nan 1.253970e-01	0.000000e+00 -nan -nan 2.317490e-01	0.000000e+00 -nan -nan 4.619390e-01
0.00125	0.000000e+00 nan nan 2.938900e-02	0.000000e+00 -nan -nan 5.825900e-02	0.000000e+00 -nan -nan 1.160580e-01	0.000000e+00 nan nan 2.312040e-01	0.000000e+00 nan nan 4.625210e-01	0.000000e+00 nan nan 9.251190e-01
0.000625	3.875203e-03 2.219355e-03 1.329300e-02 5.860400e-02	0.000000e+00 -nan -nan 1.163060e-01	0.000000e+00 nan nan 2.319200e-01	0.000000e+00 nan nan 4.625050e-01	0.000000e+00 -nan -nan 9.242360e-01	0.000000e+00 -nan -nan 1.847788e+00
0.0003125	2.417445e-03 1.306728e-03 9.494630e-03 1.171230e-01	1.707263e-03 1.029704e-03 5.296081e-03 2.325260e-01	0.000000e+00 nan nan 4.735570e-01	0.000000e+00 -nan -nan 9.242640e-01	0.000000e+00 nan nan 1.847724e+00	0.000000e+00 nan nan 3.701977e+00
0.00015625	1.687110e-03 8.879987e-04 7.633719e-03 2.343890e-01	9.719095e-04 5.558151e-04 3.338353e-03 4.650250e-01	7.957610e-04 4.971985e-04 2.315496e-03 9.262090e-01	8.344639e+00 2.806930e+00 3.998717e+03 1.849222e+00	0.000000e+00 nan nan 3.695493e+00	0.000000e+00 nan nan 7.387029e+00

$\mu = 0.001, p(\rho) = 100\rho$						
$\tau \setminus h$	0.005	0.0025	0.00125	0.000625	0.0003125	0.00015625
0.005	0.000000e+00 -nan -nan 7.449000e-03	0.000000e+00 -nan -nan 1.469300e-02	0.000000e+00 -nan -nan 2.923800e-02	0.000000e+00 nan nan 5.807400e-02	0.000000e+00 nan nan 1.162890e-01	0.000000e+00 nan nan 2.408650e-01
0.0025	0.000000e+00 nan nan 1.474600e-02	0.000000e+00 nan nan 2.925700e-02	0.000000e+00 -nan -nan 5.809300e-02	0.000000e+00 nan nan 1.163280e-01	0.000000e+00 nan nan 2.313050e-01	0.000000e+00 -nan -nan 4.620320e-01
0.00125	0.000000e+00 -nan -nan 2.939900e-02	0.000000e+00 -nan -nan 5.829500e-02	0.000000e+00 -nan -nan 1.160300e-01	0.000000e+00 -nan -nan 2.312500e-01	0.000000e+00 -nan -nan 4.622240e-01	0.000000e+00 -nan -nan 9.230520e-01
0.000625	0.000000e+00 nan nan 5.864900e-02	0.000000e+00 nan nan 1.216810e-01	0.000000e+00 -nan -nan 2.320680e-01	0.000000e+00 nan nan 4.625580e-01	0.000000e+00 -nan -nan 9.257180e-01	0.000000e+00 nan nan 1.845128e+00
0.0003125	0.000000e+00 -nan -nan 1.172640e-01	0.000000e+00 -nan -nan 2.328580e-01	0.000000e+00 nan nan 4.637990e-01	0.000000e+00 nan nan 9.243740e-01	0.000000e+00 nan nan 1.859363e+00	0.000000e+00 -nan -nan 3.698847e+00
0.00015625	1.641701e-03 8.844172e-04 7.643047e-03 2.343140e-01	0.000000e+00 nan nan 4.685050e-01	0.000000e+00 nan nan 9.270240e-01	0.000000e+00 nan nan 1.849776e+00	0.000000e+00 -nan -nan 3.693884e+00	0.000000e+00 nan nan 7.389611e+00
$\mu = 0.1, p(\rho) = \rho^{1.4}$						
$\tau \setminus h$	0.005	0.0025	0.00125	0.000625	0.0003125	0.00015625
0.005	1.706045e-01 7.077821e-02 1.296421e+00 9.653000e-03	1.695805e-01 7.043296e-02 1.287694e+00 1.919000e-02	1.693242e-01 7.034590e-02 1.285382e+00 3.814700e-02	1.692604e-01 7.032429e-02 1.284789e+00 7.587800e-02	1.692473e-01 7.031900e-02 1.284648e+00 1.515670e-01	1.692432e-01 7.031769e-02 1.284614e+00 3.025240e-01
0.0025	8.457763e-02 3.529897e-02 6.388418e-01 1.925800e-02	8.351026e-02 3.494365e-02 6.297914e-01 3.815900e-02	8.324401e-02 3.485697e-02 6.275862e-01 7.616900e-02	8.317756e-02 3.483581e-02 6.270476e-01 1.519610e-01	8.316295e-02 3.483063e-02 6.269165e-01 3.024310e-01	8.315886e-02 3.482933e-02 6.268839e-01 6.041500e-01
0.00125	4.257291e-02 1.779484e-02 3.217010e-01 3.828100e-02	4.144679e-02 1.742509e-02 3.121911e-01 7.645000e-02	4.119084e-02 1.733935e-02 3.100425e-01 1.517220e-01	4.112669e-02 1.731859e-02 3.095263e-01 3.126680e-01	4.111053e-02 1.731348e-02 3.093995e-01 6.042710e-01	4.110650e-02 1.731220e-02 3.093679e-01 1.207063e+00
0.000625	2.210761e-02 9.146072e-03 1.671480e-01 7.670700e-02	2.077249e-02 8.742080e-03 1.564907e-01 1.521230e-01	2.050818e-02 8.655232e-03 1.543099e-01 3.031910e-01	2.044370e-02 8.634570e-03 1.538001e-01 6.108930e-01	2.042758e-02 8.629485e-03 1.536753e-01 1.217862e+00	2.042355e-02 8.628218e-03 1.536443e-01 2.412986e+00
0.0003125	1.220562e-02 4.896447e-03 9.239549e-02 1.527590e-01	1.054251e-02 4.425231e-03 7.955691e-02 3.038470e-01	1.026370e-02 4.334201e-03 7.723327e-02 6.063870e-01	1.019846e-02 4.313348e-03 7.671819e-02 1.218437e+00	1.018244e-02 4.308263e-03 7.659387e-02 2.428881e+00	1.017842e-02 4.307000e-03 7.656305e-02 4.825812e+00
0.00015625	7.265290e-03 2.862024e-03 5.766995e-02 3.060650e-01	5.511580e-03 2.279748e-03 4.154228e-02 6.077000e-01	5.167952e-03 2.179642e-03 3.891035e-02 1.211955e+00	5.101333e-03 2.158221e-03 3.837610e-02 2.428425e+00	5.085185e-03 2.153107e-03 3.825096e-02 4.836315e+00	5.081202e-03 2.151843e-03 3.822020e-02 9.653870e+00

$\mu = 0.01, p(\rho) = \rho^{1.4}$						
$\tau \setminus h$	0.005	0.0025	0.00125	0.000625	0.0003125	0.00015625
0.005	$2.044482e-01$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$
	$7.467224e-02$	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>
	$1.946596e+00$	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>
	$9.635000e-03$	$2.819300e-02$	$3.620100e-02$	$7.167000e-02$	$1.427330e-01$	$2.853330e-01$
0.0025	$7.794318e-02$	$7.704660e-02$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$
	$3.414842e-02$	$3.378200e-02$	<i>-nan</i>	<i>nan</i>	<i>-nan</i>	<i>-nan</i>
	$7.570298e-01$	$7.418792e-01$	<i>-nan</i>	<i>nan</i>	<i>-nan</i>	<i>-nan</i>
	$2.915900e-02$	$4.877900e-02$	$7.516800e-02$	$1.488110e-01$	$2.940430e-01$	$5.965210e-01$
0.00125	$3.626583e-02$	$3.592626e-02$	$3.606035e-02$	$3.609067e-02$	$3.609859e-02$	$3.610112e-02$
	$1.655812e-02$	$1.621748e-02$	$1.615747e-02$	$1.614402e-02$	$1.614076e-02$	$1.613995e-02$
	$3.570426e-01$	$3.450495e-01$	$3.431007e-01$	$3.426782e-01$	$3.425766e-01$	$3.429257e-01$
	$3.829500e-02$	$7.656800e-02$	$1.616030e-01$	$3.076930e-01$	$6.117060e-01$	$1.215711e+00$
0.000625	$1.820844e-02$	$1.734556e-02$	$1.744792e-02$	$1.748683e-02$	$1.749701e-02$	$1.749958e-02$
	$8.406248e-03$	$7.979580e-03$	$7.921240e-03$	$7.909751e-03$	$7.907073e-03$	$7.906415e-03$
	$1.821730e-01$	$1.675986e-01$	$1.658908e-01$	$1.655887e-01$	$1.655209e-01$	$1.655045e-01$
	$8.487800e-02$	$1.520680e-01$	$3.124030e-01$	$6.141140e-01$	$1.219736e+00$	$2.412807e+00$
0.0003125	$1.016166e-02$	$8.542526e-03$	$8.571057e-03$	$8.606491e-03$	$8.616719e-03$	$8.619327e-03$
	$4.629786e-03$	$3.998062e-03$	$3.928132e-03$	$3.916765e-03$	$3.914311e-03$	$3.913721e-03$
	$1.055329e-01$	$8.382995e-02$	$8.174155e-02$	$8.146274e-02$	$8.140845e-02$	$8.139584e-02$
	$1.620620e-01$	$3.139090e-01$	$6.063460e-01$	$1.209040e+00$	$2.415086e+00$	$4.823369e+00$
0.00015625	$6.932529e-03$	$4.341762e-03$	$4.237283e-03$	$4.264879e-03$	$4.274843e-03$	$4.277615e-03$
	$3.009020e-03$	$2.061902e-03$	$1.963085e-03$	$1.950225e-03$	$1.947780e-03$	$1.947217e-03$
	$7.542591e-02$	$4.393059e-02$	$4.076177e-02$	$4.043259e-02$	$4.038083e-02$	$4.036981e-02$
	$3.052210e-01$	$6.081100e-01$	$1.212473e+00$	$2.418077e+00$	$4.828728e+00$	$9.648475e+00$
$\mu = 0.001, p(\rho) = \rho^{1.4}$						
$\tau \setminus h$	0.005	0.0025	0.00125	0.000625	0.0003125	0.00015625
0.005	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$
	<i>-nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>
	<i>-nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>
	$1.967600e-02$	$1.807600e-02$	$3.566100e-02$	$7.094300e-02$	$1.413970e-01$	$2.818480e-01$
0.0025	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$
	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>
	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>
	$1.885800e-02$	$3.699900e-02$	$7.102900e-02$	$1.410750e-01$	$2.812710e-01$	$5.619000e-01$
0.00125	$7.983929e-02$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$
	$2.342631e-02$	<i>-nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>
	$5.068147e+00$	<i>-nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>	<i>nan</i>
	$4.740000e-02$	$7.542100e-02$	$1.500660e-01$	$2.820150e-01$	$5.613340e-01$	$1.121492e+00$
0.000625	$2.201153e-02$	$1.687576e-02$	$1.686784e-02$	$0.000000e+00$	$0.000000e+00$	$0.000000e+00$
	$8.655093e-03$	$7.970136e-03$	$7.975823e-03$	<i>nan</i>	<i>nan</i>	<i>nan</i>
	$4.381088e-01$	$1.950966e-01$	$1.948290e-01$	<i>nan</i>	<i>nan</i>	<i>nan</i>
	$8.666200e-02$	$1.520430e-01$	$3.037280e-01$	$6.023120e-01$	$1.165194e+00$	$2.242980e+00$
0.0003125	$1.251618e-02$	$8.661415e-03$	$8.202282e-03$	$8.341972e-03$	$1.700649e+00$	$5.168797e+00$
	$5.092691e-03$	$3.943722e-03$	$3.906231e-03$	$3.914970e-03$	$3.232768e-01$	$1.394496e+00$
	$1.776548e-01$	$9.670960e-02$	$9.440672e-02$	$9.458416e-02$	$3.165305e+02$	$4.645061e+03$
	$1.527140e-01$	$3.038850e-01$	$6.059600e-01$	$1.210741e+00$	$2.415941e+00$	$4.826810e+00$
0.00015625	$1.066272e-02$	$4.914564e-03$	$4.004981e-03$	$4.111847e-03$	$4.150815e-03$	$4.161059e-03$
	$3.871962e-03$	$2.056291e-03$	$1.933495e-03$	$1.937061e-03$	$1.940142e-03$	$1.941048e-03$
	$1.270728e-01$	$5.176541e-02$	$4.664213e-02$	$4.659634e-02$	$4.666816e-02$	$4.669131e-02$
	$3.054590e-01$	$6.078590e-01$	$1.211678e+00$	$2.419923e+00$	$4.831023e+00$	$9.660136e+00$

Вложенная сетка, плотность

$\mu = 0.1, p(\rho) = 1\rho$		
	tau=h=0.01	tau=h=0.001
$h - h^1$	$5.223002e-01$	$3.166982e-02$
	$1.350436e-01$	$8.600421e-03$
	$5.334825e+00$	$2.949122e-01$
$h - h^2$	$6.128827e-01$	$4.780349e-02$
	$1.743737e-01$	$1.294066e-02$
	$6.692537e+00$	$4.414830e-01$
$h - h^3$	$6.404276e-01$	$5.594455e-02$
	$1.893355e-01$	$1.512291e-02$
	$7.216256e+00$	$5.146063e-01$
$h - \rho$	$6.525722e-01$	$6.413494e-02$
	$2.022396e-01$	$1.731402e-02$
	$7.680877e+00$	$5.876434e-01$

$\mu = 0.01, p(\rho) = 1\rho$		
	tau=h=0.01	tau=h=0.001
$h - h^1$	6.002904e + 00 1.303480e + 00 1.736705e + 02	3.913667e - 01 7.384558e - 02 4.606453e + 00
$h - h^2$	5.342923e + 00 1.448873e + 00 1.683045e + 02	5.661443e - 01 1.092706e - 01 6.571359e + 00
$h - h^3$	5.475949e + 00 1.494066e + 00 1.646203e + 02	6.485901e - 01 1.266163e - 01 7.480678e + 00
$h - \rho$	5.780942e + 00 1.548715e + 00 1.641333e + 02	7.270385e - 01 1.437217e - 01 8.345515e + 00
$\mu = 0.1, p(\rho) = 10\rho$		
	tau=h=0.01	tau=h=0.001
$h - h^1$	0.000000e + 00 -nan -nan	3.417283e - 03 1.700900e - 03 9.083366e - 03
$h - h^2$	0.000000e + 00 -nan -nan	5.120371e - 03 2.550624e - 03 1.359994e - 02
$h - h^3$	0.000000e + 00 -nan -nan	5.970523e - 03 2.975303e - 03 1.585202e - 02
$h - \rho$	0.000000e + 00 -nan -nan	6.819746e - 03 3.399861e - 03 1.810001e - 02
$\mu = 0.1, p(\rho) = \rho^{1,4}$		
	tau=h=0.01	tau=h=0.001
$h - h^1$	1.810754e - 01 7.431081e - 02 1.464890e + 00	1.653108e - 02 6.948234e - 03 1.244371e - 01
$h - h^2$	2.661235e - 01 1.096692e - 01 2.116489e + 00	2.472064e - 02 1.040477e - 02 1.860476e - 01
$h - h^3$	3.056367e - 01 1.268425e - 01 2.423717e + 00	2.879527e - 02 1.212853e - 02 2.167010e - 01
$h - \rho$	3.458994e - 01 1.436457e - 01 2.718655e + 00	3.285604e - 02 1.384925e - 02 2.472530e - 01
$\mu = 0.01, p(\rho) = \rho^{1,4}$		
	tau=h=0.01	tau=h=0.001
$h - h^1$	0.000000e + 00 nan nan	1.460322e - 02 6.516847e - 03 1.391208e - 01
$h - h^2$	0.000000e + 00 nan nan	2.162859e - 02 9.692793e - 03 2.057426e - 01
$h - h^3$	0.000000e + 00 nan nan	2.507996e - 02 1.126091e - 02 2.383676e - 01
$h - \rho$	0.000000e + 00 nan nan	2.849158e - 02 1.281607e - 02 2.705524e - 01

Выводы

На основе анализа таблиц с нормами разности точного решения и численного, можно сделать вывод о том, что сходимость имеет порядок $O(\tau + h^2)$. Так же сходимость является условной и имеет вид $\tau \leq \gamma * h$, где коэффициент γ прямо пропорционален μ и обратно пропорционален C