

Group Project 2

Percolation

Yuanyuan Xu, Matthew Epland, Xiaqing Li, Wesley Cohen

Duke University

April 8, 2016



Introduction – Part A

Percolation transition on a $N \times N$ lattice:

- Sites are subsequently and randomly occupied
- A cluster – a collection of interconnection occupied sites
 - A **spanning cluster** touches all four edges of the lattice
 - Percolation transition – when the spanning cluster occurs
- Occupation probability:

$$p = \frac{\text{number of occupied sites}}{N^2} \quad (1)$$

- At the **critical concentration** p_c percolation transition occurs

Extract p_c of infinitely large 2D square lattice

- Determine the value of p_c for 2D square lattice of different lengths
 - $N = 5, 10, 15, 20, 30, 50, 80$
- For each lattice size: average the results for 50 different simulations
- Plot $p_c(N^{-1})$ to extrapolate to the infinite lattice limit
 - $N^{-1} \rightarrow 0$, $p_c \rightarrow p_c$ of infinitely large lattice

Union-Find Algorithms

- Data structure
 - Integer array `label[i]` of size $N \times N$.
 - Interpretation: `p` and `q` are connected if they have the same label.
- Find: Check if `p` and `q` have the same label.
- Union: To merge components containing `p` and `q`, change all entries with `label[p]` to `label[q]`.

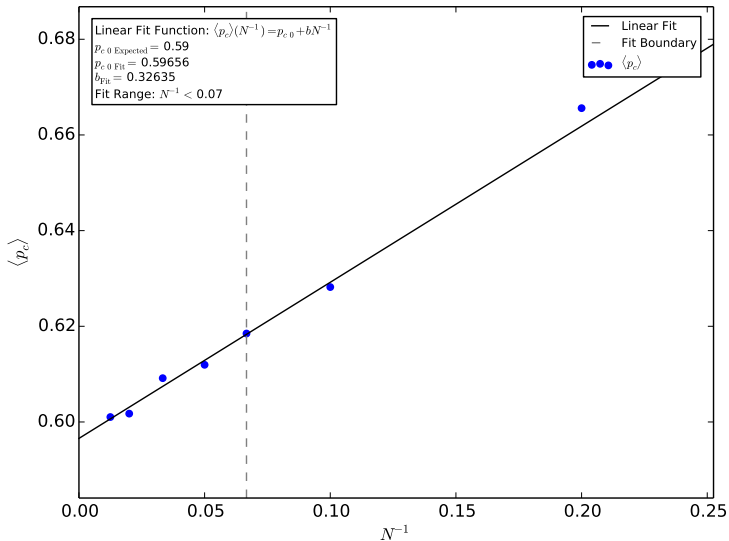
Percolation

- Initialization

0	0	0
0	0	0
0	0	0

- Generate a random sequence from 0 to $N^2 - 1$:
`arr = shuffle([0, 1, 2, ..., N*N-1])`
- Occupy a site given by `arr[i]`.
- Union: Choose a common unique label and update `label`.
- Percolation
 - Data structure: `TreeSet`.
 - $S = \{\text{Edge}_1\} \cap \{\text{Edge}_2\} \cap \{\text{Edge}_3\} \cap \{\text{Edge}_4\} - \{0\}$
 - $S = \begin{cases} \emptyset & \text{unconnected,} \\ \{x\} & \text{connected, } x \text{ is the same label of the spanning cluster.} \end{cases}$
- Animation

Result



Introduction – Part B

Fraction of sites in percolating cluster

- Definition:

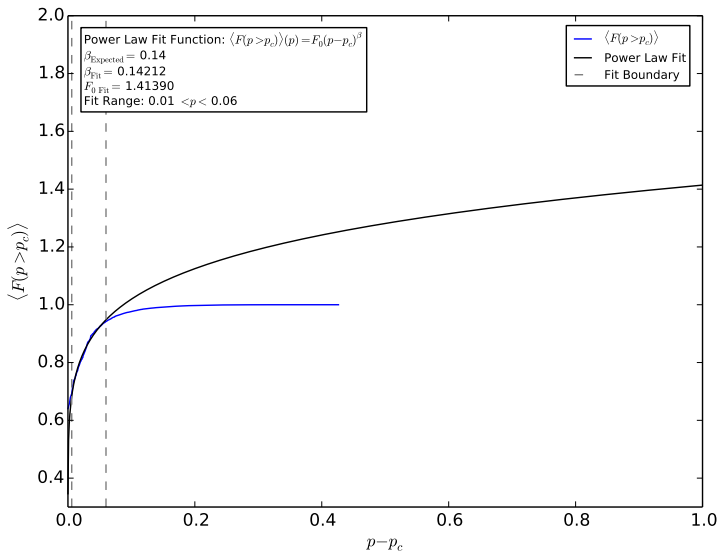
$$F(p > p_c) = \frac{\text{number of sites in spanning cluster}}{\text{number of occupied sites}} \quad (2)$$

- F near p_c satisfies power law:

$$F = F_0(p - p_c)^\beta \quad (3)$$

- Extract β
 - Linear fitting on the log-log scale plot

Result



Result

