Group Project 2
Percolation

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Introduction – Part A

Percolation transition on a $N \times N$ lattice:

- Sites are subsequently and randomly occupied
- A cluster – a collection of interconnection occupied sites
  - A **spanning cluster** touches all four edges of the lattice
  - Percolation transition – when the spanning cluster occurs
- Occupation probability:

$$p = \frac{\text{number of occupied sites}}{N^2} \tag{1}$$

  - At the **critical concentration** p_c percolation transition occurs

Extract p_c of infinitely large 2D square lattice

- Determine the value of p_c for 2D square lattice of different lengths
  - $N = 5, 10, 15, 20, 30, 50, 80$
- For each lattice size: average the results for 50 different simulations
- Plot $p_c(N^{-1})$ to extrapolate to the infinite lattice limit
  - $N^{-1} \to 0,\ p_c \to p_c$ of infinitely large lattice

Union-Find Algorithms

- Data structure
  - Integer array `label[i]` of size $N \times N$.
  - Interpretation: `p` and `q` are connected if they have the same label.
- Find: Check if `p` and `q` have the same label.
- Union: To merge components containing `p` and `q`, change all entries with `label[p]` to `label[q]`.

Percolation

- Initialization

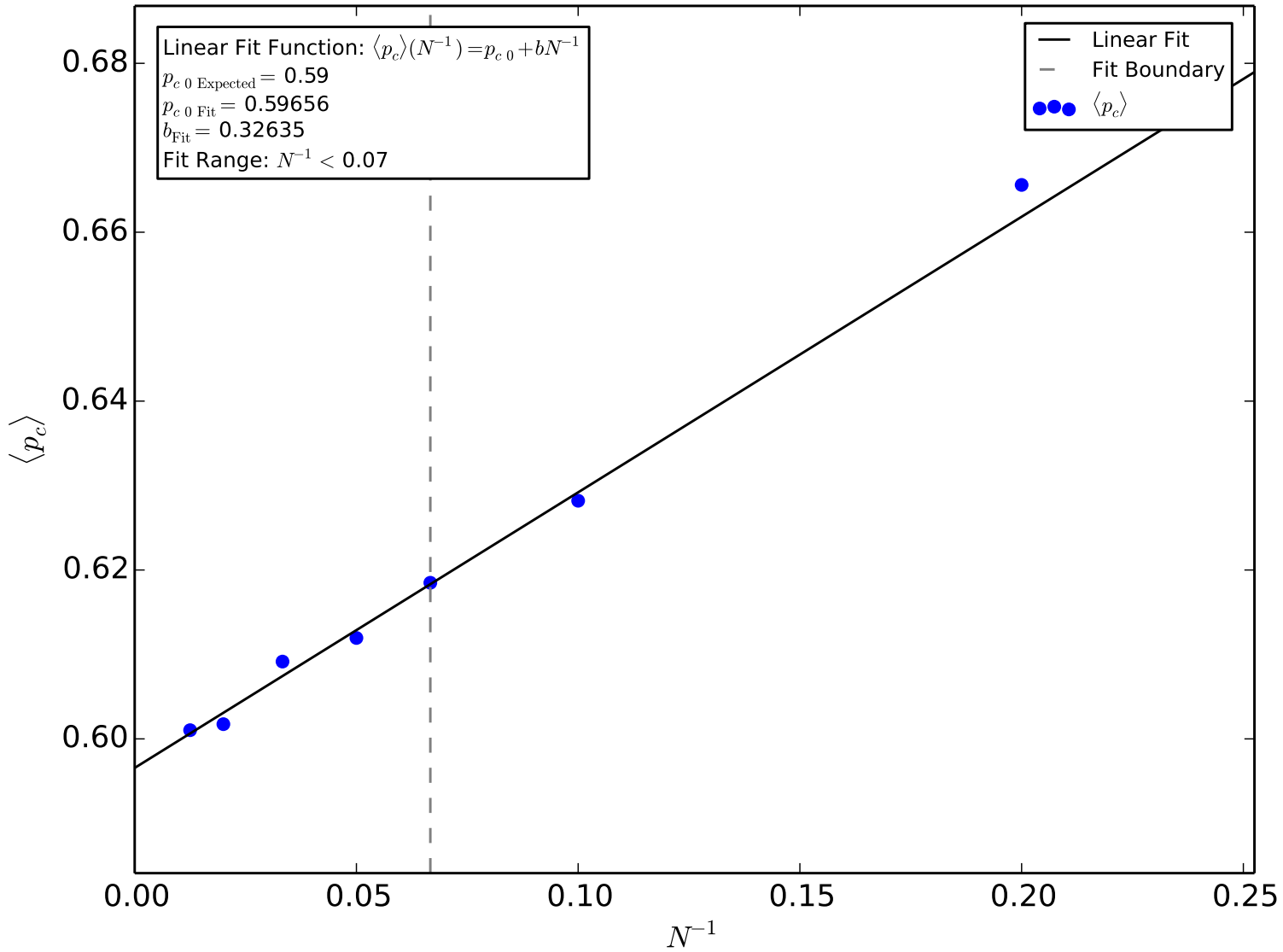
0	0	0
0	0	0
0	0	0

- Generate a random sequence from 0 to $N^2 - 1$:

```
arr = shuffle([0, 1, 2,..., N*N-1])
```

- Occupy a site given by `arr[i]`.
- Union: Choose a common unique label and update `label`.
- Percolation
  - Data structure: TreeSet.
  - $S = \{\text{Edge}_1\} \cap \{\text{Edge}_2\} \cap \{\text{Edge}_3\} \cap \{\text{Edge}_4\} - \{0\}$
  - $S = \begin{cases} \emptyset & \text{unconnected,} \\ \{x\} & \text{connected, } x \text{ is the same label of the spanning cluster.} \end{cases}$
- Animation

Result


Linear Fit Function: $\langle p_c \rangle (N^{-1}) = p_{c\,0} + bN^{-1}$
$p_{c\,0\ \mathrm{Expected}} = 0.59$
$p_{c\,0\ \mathrm{Fit}} = 0.59656$
$b_{\mathrm{Fit}} = 0.32635$
Fit Range: $N^{-1} < 0.07$
Linear Fit
Fit Boundary
$\langle p_c \rangle$
0.68
0.66
0.64
0.62
0.60
$\langle p_c \rangle$
0.00
0.05
0.10
0.15
0.20
0.25
N^{-1}

Introduction – Part B

Fraction of sites in percolating cluster

- Definition:

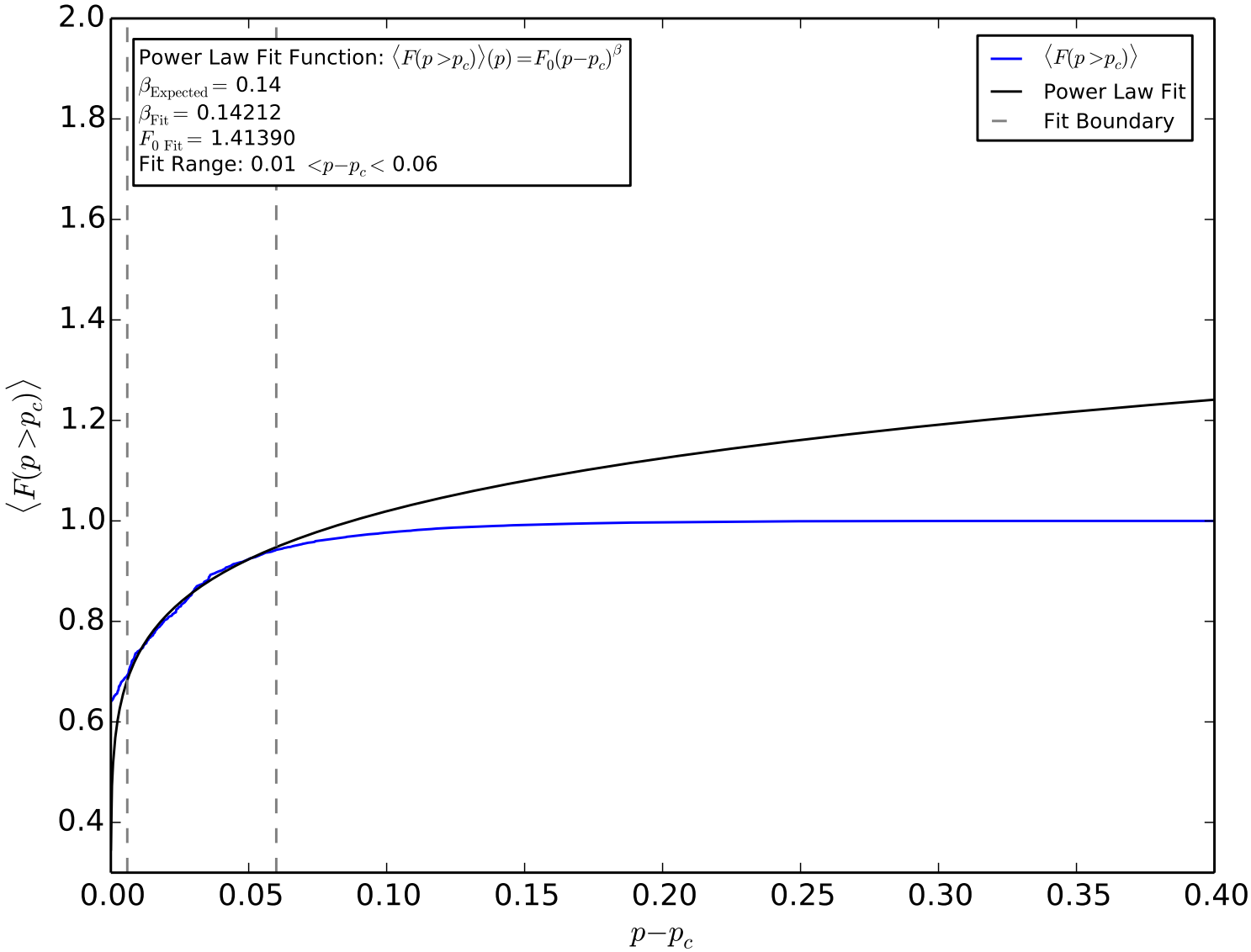
$$F(p > p_c) = \frac{\text{number of sites in spanning cluster}}{\text{number of occupied sites}} \tag{2}$$

- F near p_c satisfies power law:

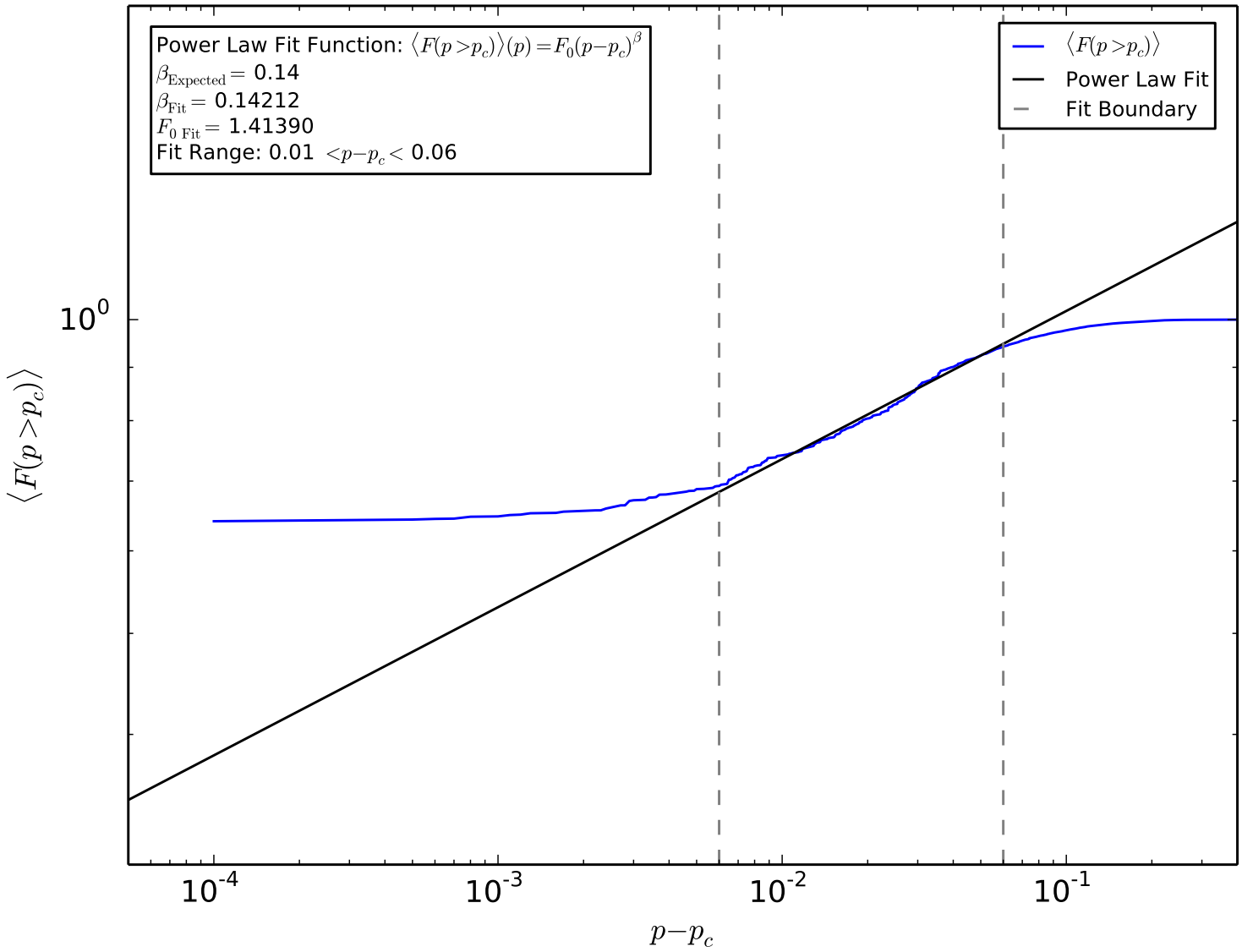
$$F = F_0(p - p_c)^{\beta} \tag{3}$$

- Extract β
  - Linear fitting on the log-log scale plot

Result


Power Law Fit Function: $\langle F(p>p_c)\rangle(p) = F_0(p-p_c)^\beta$
$\beta_{\text{Expected}} = 0.14$
$\beta_{\text{Fit}} = 0.14212$
$F_{0\ \text{Fit}} = 1.41390$
Fit Range: 0.01 $< p-p_c <$ 0.06
$\langle F(p>p_c)\rangle$
Power Law Fit
Fit Boundary
$\langle F(p>p_c)\rangle$
$p-p_c$
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

Result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Fit Range: 0.01 $< p-p_c <$ 0.06
$\langle F(p>p_c)\rangle$
Power Law Fit
Fit Boundary
10^0
$\langle F(p>p_c)\rangle$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
$p-p_c$