

# PHY566 Homework #2

Due Date: Feb. 4th, 5:00pm via Sakai

## *Carbon Dating*

The Carbon isotope  ${}^{14}_6C$  is widely used for dating of ancient artifacts containing biological material. It undergoes  $\beta^-$  decay with a half-life time of  $T_{1/2} = 5700$  years. Suppose an ancient artifact originally contained  $10^{-12}$  kg of  ${}^{14}_6C$  (recall that 1 *mol* of a substance, which has a mass corresponding to the atomic mass-number in *grams*, contains  $N_A = 6.022 \cdot 10^{23}$  particles).

- a) Derive analytically the relation between half-life time,  $T_{1/2}$  and decay constant  $\tau$  as defined in class. [1 point]
- b) write a computer code to numerically calculate the activity of the sample, defined as  $R(t) = -dN/dt$ , over a duration of 20,000 years. Use numerical time-step widths of 10 and 100 years. Plot the results in appropriate units together with the exact (analytical) solution in the same graph. [6 points]
- c) Increase the time-step width to 1,000 years and replot. Is the accuracy of the numerical solution still acceptable? (e.g. what is the percentage deviation from the exact result after 2 half-lives?) Is the deviation from the exact result as large as you would expect from the neglected 2nd order term? [3 points]

Your homework submission should consist of:

- a document, created in  $\text{\LaTeX}$ , outlining the problem, detailing your solution and discussing your results - the document should include the requested figures. The document should be in pdf format and you should use colors and different marker symbols to enhance the readability of your figures.
- the Python code that you used to solve the problem and generate the figures

All files should be named in the following way:

[PHY566\_your\_name\_homework#2\_filedescriptor]