PHY566 Homework #4

Due Date: Feb. 22nd, 5:00pm via Sakai

Oscillatory Motion and Chaos

Consider the linear, damped, driven pendulum, defined by the differential equation:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta - 2\gamma\frac{d\theta}{dt} + \alpha_D \sin(\Omega_D t) \tag{1}$$

(use g= 9.8 m/s^2, l= 9.8 m, $\gamma = 0.25\ s^{-1}$, $\alpha_D = 0.2$ rad/s^2).

a) [1 point] Calculate analytically at what (approximate) value of Ω_D the resonance occurs. Do you expect the small-angle (linear) approximation to be good?

b) [2 points] Write a program to numerically calculate $\theta(t)$ using the Euler-Cromer and the Runge-Kutta 4^{th} order methods. Plot $\theta(t)$ and $\omega(t) = d\theta/dt$ over a sufficiently long time to reach the steady-state solution. From the latter, extract the amplitude $\theta_0(\Omega_D)$ and phase shift $\phi(\Omega_D)$ for at leasts 10 different driving frequencies mapping out the resonance structure and plot $\theta_0(\Omega_D)$ and $\phi(\Omega_D)$. Numerically extract the full-width at half maximum (FWHM) of the resonance curve and compare it to γ.

c) [2 points] For a driving frequency close to resonance, compute potential, kinetic and total energies and plot them in the same graph over ca. 10 periods.

d) [2 poins] Switch on non-linear effects by replacing θ with $\sin(\theta)$ in the restoring force and plot and compare to your previous results for $\theta(t)$ and $\omega(t)$ using Ω_D close to resonance. Now increase α_D to 1.2 rad/s^2 and redo the calculation.

e) [3 points] Use the non-linear pendulum of part (d) with Ω_D=0.666 s^{-1} and values of $\alpha_D = 0.2$, 0.5 and 1.2 rad/s^2 to compute $|\Delta\theta(t)|$ for several trajectories with slightly different initial angle ($\Delta\theta_{\text{in}} \approx 0.001$ rad). Plot the results and estimate the Lyapunov exponent λ of the system.

Your homework submission should consist of:

- a document outlining the problem, detailing your solution and discussion of your results - the document should include the requested figures. The document should be in pdf format and you should use colors and different marker symbols to enhance the readability of your figures.
- the source code of your program