PHY566 Take-Home Final Exam

Due Date: April 20th, 10:00pm via Sakai

The Ising Model

Write a program to numerically study the 2D Ising Model on a $n \times n$ lattice with periodic boundary conditions. ($N = n^2$: total number of spins). The lattice has a nearest neighbor interaction strength $J = 1.5$ and there is no external magnetic field. Use the Metropolis algorithm to relax the system of spins to the desired temperature(s).

a) Choose $n = 50$ and calculate the magnetization $M = N\langle s \rangle$ as a function of temperature (allowing for enough Monte Carlo sweeps to reach equilibrium) and determine the critical temperature T_C. Plot M vs. T. [10 points]

b) Calculate the specific heat per spin C/N for 10 different lattice sizes, $n = 5, 10, 20, 30, 40, 50, 75, 100, 200, 500$, using the fluctuation-dissipation theorem $C = (\Delta E)^2/(k_B T^2)$, and verify the approximate finite-size scaling relation $C_{max}/N \sim \log(n)$. (Hint: make sure to use sufficient temperature resolution when determining the maximum in C/N as you increase n. The relation may yield better results for the smaller values of n). Show figures for $C(T)$ for a few sample cases as well as C_{max}/N vs. n [10 points]

Your homework submission should consist of:

- a document outlining the problem, detailing your solution and discussion of your results - the document should include the requested figures. The document should be in pdf format and you should use colors and different marker symbols to enhance the readability of your figures.
- the source code of your program

Both files should be submitted via Sakai