PHY566 Take-Home Midterm Exam

Due Date: Mar. 2nd, 10:00am via Sakai

Potentials and Fields

1. *Poisson Equation for Dipole:* [total: 25 points]
   Write a program to numerically calculate the electric potential of a static electric dipole, i.e. two point charges $Q/\epsilon_0 = \pm 1$ separated by a distance of $a = 0.6$. Solve the Poisson equation in Cartesian coordinates, but impose a spherical boundary condition, $V(R) = 0$, at a large distance (e.g. $R = 10$).

   a) Starting from an initial condition of zero potential use the Jacobi relaxation algorithm with appropriate numerical tolerance and grid density to obtain the (converged) solution. Plot the equipotential lines. Plot your result for $V(r)$ (r is the distance from the origin) and compare it to the expected large-distance behavior of the dipole potential (which you can find in the literature). [10 points]

   b) Investigate how the number of required iteration steps, N_{iter}, increases with reducing the tolerance (error) limit ϵ and plot it, i.e. $N_{iter}(\epsilon)$ [5 points]

   c) write a second program by modifying the algorithm to use the *Simultaneous Over-Relaxation Method* (SOR), and allowing for different grid densities in different relaxation runs. For fixed accuracy of the solution (not total tolerance, i.e. accuracy would be defined as largest relative change of a single grid point from one iteration to the next), investigate (and plot) how the number of iteration steps N_{iter}, depends on the number n of grid points. You should find $N_{iter} \propto n^2$ and $N_{iter} \propto n$ for the Jacobi and SOR methods respectively. [10 points]

2. *Biot-Savart Law*: [total: 20 points]
Use Biot-Savart's Law for the magnetic field induced by a thin-wire current,

$$d\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \frac{d\vec{r}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \quad , \tag{1}$$

to construct a program calculating the $\vec{B}$-field for a quadratic current loop (side-length 1m, $\mu_0 I = 8$ in SI units, running counter-clockwise) lying centered in the x-y plane. Calculate and plot the x-, y- and z-components of the magnetic field for

a) $x = y = 0$ m as a function of z and compare to (i.e. plot on the same graph) the analytic result for a circular current loop with the same area as the square [10 points],

b) $z = 1$ m, $y = 0$ m as a function of x [5 points],

c) $y = 0$ m, $x = 0.5$ m as a function of z [5 points].

Your exam submission should consist of:

- a document outlining the problem, detailing your solution and discussion of your results - the document should include the requested figures. The document should be in pdf format and you should use colors and different marker symbols to enhance the readability of your figures.
- the source code of your program