

## Research Article

# Full Positional Accuracy Analysis of Spatial Data by Means of Circular Statistics

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### Abstract

The positional error in spatial data is defined as a vector by comparing the coordinates between the true position and the measured position. The standard tests to assess the positional accuracy use only the magnitude of the vector and omit the azimuth. This article suggests that the use of both values allows a much more complete analysis of the positional error. A set of tests is proposed that are relevant for this purpose and demonstrate that some important features are not identified by the common procedures. The test samples come from two datasets. The first is obtained from the comparison of 100 homologous points in two conventional maps, and the second one comes from the geometric calibration of a photogrammetric scanner. The results are analyzed and discussed, showing that important issues such as error anisotropy are detected only by means of the circular statistics tests and density maps of distribution. Therefore, tests that assess the goodness of fit for uniform distribution in azimuths, such as Rayleigh and Rao tests, give low probabilities ( $P = 0$  and  $P > 0.01$ ). Moreover, density maps working with both magnitude and angle can locate the outlier candidate and offer more information about the spatial distribution of error.

## 1 Introduction

Analysis of uncertainty is an essential task before using any spatial database (Zhang and Goodchild 2003), especially concerning the positional accuracy of data. There has been a fair amount of research into this topic in recent decades (Maune 2007, Shi 2009, Veregin 1989) and several statistical methods have become accepted for widespread use,

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such as the National Map Accuracy Standard (USBB 1947), the Accuracy Standards for large-scale maps (ASPRS 1989), the Engineering Map Accuracy Standards (ASCI 1983) and the National Standards for Spatial Data Accuracy (FGDC 1998). These statistical tests have a similar operating pattern: (a) assessment of the quality of the database is performed by comparing it with a more accurate data source; (b) a minimum of 20 points to be used are identified in both databases (to obtain 20 pairs of homologous points); (c) distances or displacements are calculated for each pair of points; and (d) statistical tests are calculated assuming Gaussian distributions of errors. Some features of this general procedure are that these tests deal only with a scalar magnitude (the distance between homologous points), and that the statistics are applicable to the overall study area, assuming that the error is uniform (ASPRS 1989), although certain studies disagree (Zandbergen 2008). Furthermore, the analysis is intrinsically non-spatial because the spatial distribution of the errors is not considered.

Alternatively, we can interpret the comparison between points as an operation that generates a vector having the initial node in the true position and the end-node in the measured position. The error vector therefore has two metric properties, displacement or magnitude and azimuth, and a topological property, the direction (from-to). It is proposed that this vectorial approach allows a more in-depth analysis and that it can clarify some error properties that may go unnoticed in the classical analysis. This approach can be applied to both two- and three-dimensional vectors. This article deals only with the two-dimensional case.

In brief, the two main differences between classical tests for positional accuracy analysis and our proposal are as follows: (a) introduction of a specific analysis of the angular properties of the errors; and (b) the proposal of a non-parametric method that allows detection of outliers and analysis of the error spatial patterns. Thus, the aim of this work is to use both magnitudes of the error vector (displacement and azimuth) for a complete analysis of the positional error and to present the relevant methods to achieve this goal.

## 2 Methods

The term “spatial data” refers to data defined by their coordinates in a reference system. In this context, the error of a point is defined by the vector connecting the location at the true coordinates and that at the measured coordinates. To define this error vector, we consider the initial node to be at the true point and the end node to be at the measured point. Thus, the displacement ( $d$ ) and azimuth ( $\theta$ ) of each vector can be calculated from the respective coordinates.

Using these values, the positional error analysis is performed in three stages: (a) analysis of displacements or distances (scalar values); (b) analysis of azimuths (angular values); and (c) joint study of displacements and azimuths. Classical tests only analyze the first stage by means of the estimation of statistics such as arithmetic mean, standard deviation, and root mean square error (RMSE). This is a known problem, which will not be discussed here but is explored in a later section.

The method presented in this article is an extension of the one presented in Spanish (Polo and Felicísimo 2008). An upgraded version is presented here with different applications and a more extensive discussion.

## 2.1 Analysis of Azimuths

Circular data are a type of directional data used in many disciplines, such as meteorology (winds) (Jammalamadaka and Lund 2006), geology (dip slopes) (Dey and Ghosh 2008), biology (animal movements) (Tracey et al. 2005), environmental sciences (Arnold and SenGupta 2006), and geography (Corcoran et al. 2009).

Statistical analysis of circular data requires managing certain elements that are different from analysis of linear data (Batschelet 1981, Chrisman 1998) requiring the use of specific statistical methods: (a) the origin in circular data is an arbitrary direction (North, X axis, etc.); (b) relationships between circular data are not the same as between linear data (for example,  $180^\circ$  is not “bigger” than  $120^\circ$ ); and (c) operations on circular data are within the  $0^\circ$ – $360^\circ$  interval, in case of using angles (or 0–24 interval when using hours, for example). In this article, we consider North as the origin and values to be increasing clockwise.

Circular statistics have some limitations for our proposals because they are designed to deal with angular values but consider only unitary vectors. Some useful results can be obtained from this analysis, but it is possible to improve the analysis by introducing non-unitary displacements, as shown in Section 2.2.

The use is proposed of the following statistical methods and tests, which are sufficient to characterize the azimuth distribution and properties.

### 2.1.1 Basic circular statistics

Considering a sample of  $n$  circular data or azimuths ( $\theta_1, \theta_2, \dots, \theta_n$ ), the vectorial sum of all  $n$  data to obtain the resultant length ( $R$ ) can be calculated. The basic circular statistics for the sample of this circular data are as follows (Fisher 1995):

- The mean direction ( $\bar{\theta}$ ) is the azimuth of the vector sum ( $R$ ) of the  $n$  data.
- The resultant length ( $R$ ) allows us to calculate the mean resultant length ( $\bar{R}$ ) by:

$$\bar{R} = \frac{R}{n}$$

- As we work only with unit vectors, the mean resultant length is observed in the range (0, 1).  $\bar{R} = 1$  implies that all azimuths are coincident, but  $\bar{R} = 0$  does not signifies an angular uniform distribution.
- The circular standard deviation ( $v$ ) is a similar statistic to the standard deviation for linear data.
- The von Mises parameter ( $\kappa$ ) is a measure of the concentration of the data around a preferred orientation. If the azimuth distribution is uniform, then  $\kappa = 0$ , increasing with the data concentration.

### 2.1.2 Testing the data distribution

In this section, a brief account is given of the main probability distributions for circular data (Jammalamadaka and SenGupta 2001, Mardia and Jupp 2000). The more useful probability distributions for circular data are the following:

- Uniform distribution, where all directions from 0 to  $360^\circ$  are equally probable and the distribution is spread uniformly around the circle. Consequently, there is no mean direction and the mean resultant length is 0.

- The von Mises distribution is a symmetric unimodal distribution that can be considered as a circular analog of the Gaussian distribution in linear data. Two parameters define this distribution: the mean direction ( $\bar{\theta}$ ) and the von Mises parameter ( $\kappa$ ).

Several tests can be applied to determine whether the distribution of the data conforms to a uniform or a von Mises distribution. The uniform distribution can be tested by means of the Rayleigh test or the Rao test. The von Mises distribution can be studied by the Watson test or the Kuiper test (Batschelet 1981, Fisher 1995, Mardia and Jupp 2000).

## 2.2 Joint Analysis of Displacements and Azimuths

The independent analysis of displacements and azimuths provides useful but incomplete information, and several tests are proposed for a more complete error characterization, dealing with both displacement and azimuth data. The first test is an estimation of the circular-linear correlation coefficient (Mardia and Jupp 2000). This parameter gives the correlation between a circular variable and a linear one, and ranges from 0 to 1. This coefficient works under the assumption of the Gaussian distribution of linear data; therefore, it should be used and interpreted with caution. To deal with this problem, a complementary analysis is necessary, for which a graphical procedure called the “density map” is proposed. The density map was built as follows: (a) error vectors are moved to a common origin without changing its azimuth and displacement; (b) thus, end-nodes perform a point cloud of varying density that depends on the accumulation of errors with similar properties; (c) end-nodes are used to calculate the “density map” as a surface with values depending on the end-node accumulation per area unit. The density map provides information about the joint distribution of displacements and azimuths and allows the detection of errors or outliers. The harmonic mean method (Dixon and Chapman 1980) is used as the density algorithm. The harmonic mean ( $Hm$ ) of  $n$  real numbers is defined by:

$$Hm = \frac{n}{\sum_{i=1}^n \frac{1}{r_{iX}}} \quad r_i > 0$$

As  $r_{iX}$  is the distance between the  $x$  point and the remaining  $i$  points, an  $Hm$  value is calculated for each point. The density map represents the true error distribution in the common space and provides tools for analyzing the particular cases without any assumption about theoretical distributions or statistical properties. Furthermore, points with the largest  $Hm$  value which will be considered as outliers can be detected, as will be seen later.

## 3 Study Cases

The proposed methodology is applicable to any set of vector data and two very different datasets are used to explain and comment on the analysis.

Dataset 1: data were collected for an area ( $9 \times 15$  km) located in southwestern Spain. Two digital maps of the same area were used to evaluate the correspondence between homologous points at different scales. Points were compared from the

1:25,000 digital national topographic map and the 1:50,000 digitized national topographic map. The current standards suggest a sample size of at least 20 points, although some authors recommend a larger size (Ariza and Atkinson 2005, Li 1991). To avoid potential problems derived from an insufficient sample size, 100 homologous points were identified, the coordinates of which were extracted from both maps. As expected, the coordinates do not match exactly, and the displacement vectors were calculated, using them to characterize the error of correspondence between the maps. Measurement units in this case were meters.

Dataset 2: for the second test error vectors were provided by the geometric calibration procedure of a flatbed photogrammetric scanner designed for digitalizing aerial imagery (negatives or paper photographs) (UltraScan 5000 by Vexcel). The geometric calibration is a measurement and adjustment process aimed to reduce the spatial errors in the scanning process to obtain the required nominal accuracy (Baltsavias 1994). The geometric calibration is performed by scanning a template with crosses ( $46 \times 46$  in this case) over the entire scan area. The crosses are automatically detected and measured and compared with the known reference values. An affine transformation was calculated that allows computing the displacements between true (theoretical) and measured locations, providing a sample of 2,116 error vectors. Measurement units in this case were micrometers.

## 4 Results

### 4.1 Results for Dataset 1: The Need for Azimuth and Azimuth/displacement Analysis

Dataset 1 reflects the differences between two maps of different scales. It is expected as the null hypothesis that the errors treated as vectors are random and isotropic. As a consequence, the analysis of displacement should show null arithmetic means, Gaussian distributions in both X and Y vector components and a moderate RMSE value, according to the uncertainty specifications of the map scales. Consequently, it is expected that the azimuth analysis shows an isotropic (random) distribution, with marginal points distributed without preferred direction. These hypotheses can be verified using the previously proposed tests.

The NSSDA test was calculated and the results revealed 43.973 m horizontal accuracy at 95% confidence level for dataset 1.

#### 4.1.1 Analysis of distances

The results of the analysis of the X and Y coordinates and the displacement (arithmetic mean, maximum and minimum values and RMSE) are shown in Table 1. The frequency distribution of the displacement is displayed in Figure 1. The Kolmogorov–Smirnov test provides a *P*-value of 0.37; thus, the hypothesis of normality cannot be rejected despite the distribution being bimodal. This issue emphasizes the need for large samples if robust results are needed. Furthermore, the errors show a systematic component in both in X and Y axes.

#### 4.1.2 Analysis of azimuths

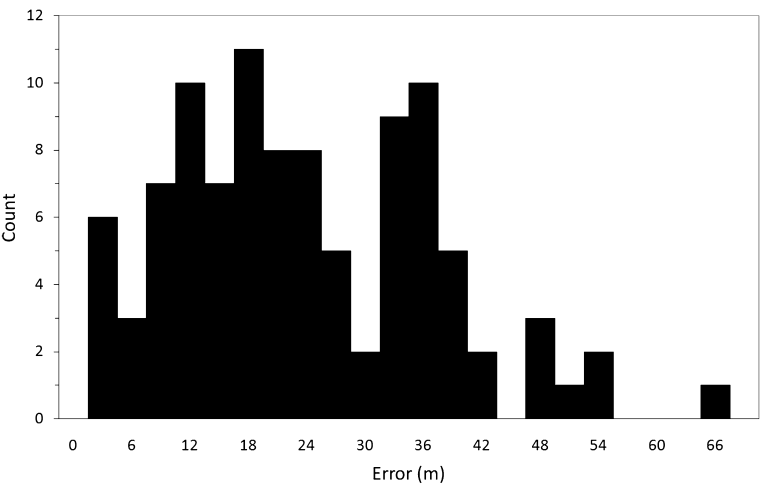
The basic statistics for circular data (mean direction  $\bar{\theta}$ , mean resultant length  $\bar{R}$ , circular standard deviation  $\upsilon$ , and von Mises parameter  $\kappa$ ) are shown in Table 2. The mean

**Table 1** Basic linear statistics for dataset 1 ( $n = 100$ )

	$X$ axis	$Y$ axis	Module
Arithmetic mean	13.0 m	−3.5 m	22.3 m
Maximum value	65.0 m	33.9 m	65.0 m
Minimum value	−22.6 m	−39.5 m	2.1 m
RMSE	17.3 m	14.0 m	13.2 m

**Table 2** Basic circular statistics for dataset 1 ( $n = 100$ )

Mean direction ( $\bar{\theta}$ )	108°
Mean resultant length ( $\bar{R}$ )	0.45
Circular standard deviation ( $\nu$ )	73°
Von Mises parameter ( $\kappa$ )	1.00



**Figure 1** Histogram of the displacements in dataset 1

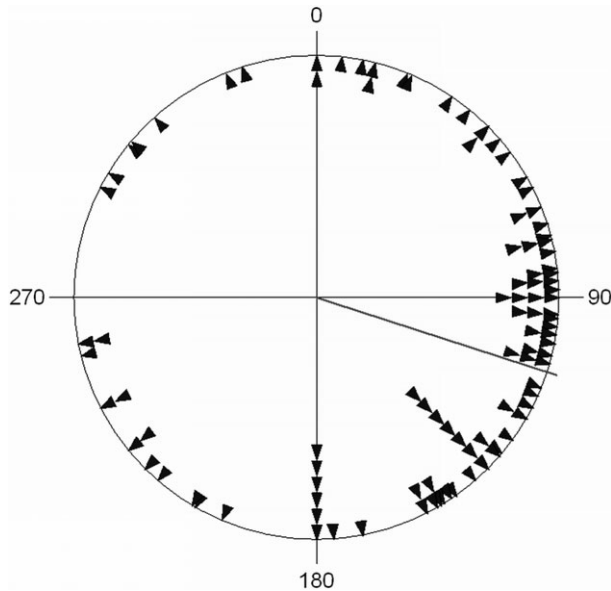
direction is 108°, and the mean resultant length  $\bar{R}$  is 0.45, a large value, as  $\bar{R}$  is observed in the range (0, 1). The circular standard deviation has a value of 73°, showing a moderate dispersion around the mean value. Finally, the von Mises parameter has a value of 1.0, showing some aggregation.

The uniformity and von Mises tests provide more information as the Rayleigh and Rao tests confirm that azimuths are not uniformly distributed (see Table 3). As azimuths do not follow a random distribution, we can look for a preferred direction; therefore, two tests for von Mises distribution were performed. The  $P$ -values do not allow rejection of the null hypothesis:  $0.15 < P < 0.25$  and  $P < 0.15$ , supporting the possibility of fitting a von Mises distribution.

The plot of raw data in Figure 2 shows an arrowhead for each azimuth in a unit circle. This figure allows assessment of the size of the sample and the distribution of the

**Table 3** Tests for uniform and von Mises distributions in dataset 1 ( $n = 100$ )

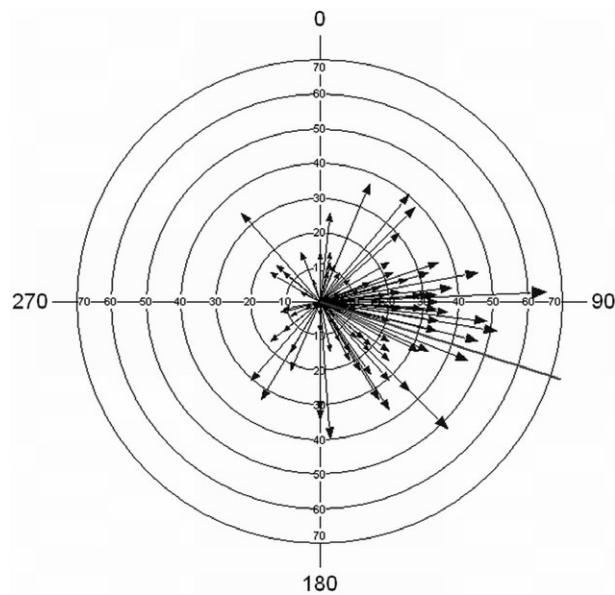
Tests for uniform distribution	Rayleigh	$P < 0.001$
	Rao	$P < 0.01$
Test for von Mises distribution	Watson	$0.15 < P < 0.25$
	Kuiper	$P < 0.15$

**Figure 2** Circular histogram of azimuths for dataset 1. Every arrowhead represents an azimuth. The mean direction is  $108^\circ$ 

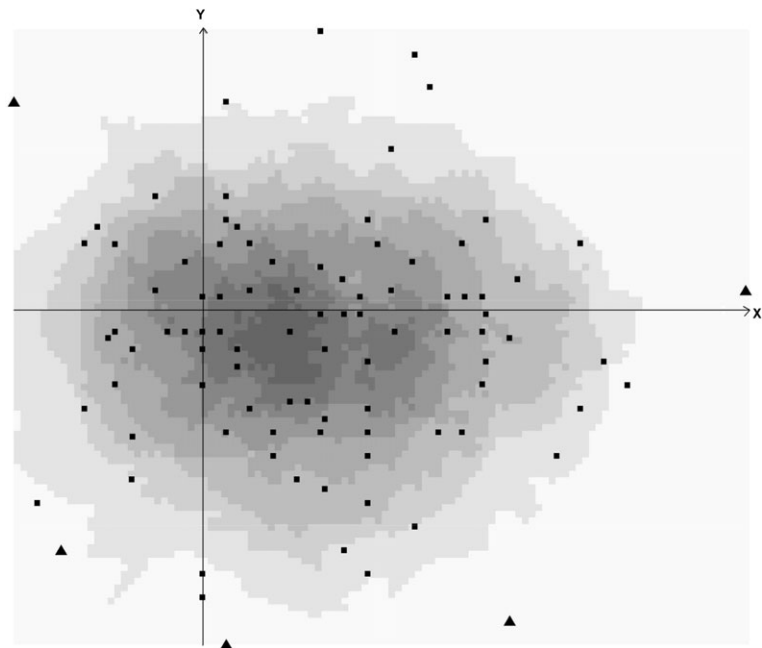
data. The mean direction ( $108^\circ$ ) is plotted with a line. From these tests it can be concluded that the error distribution is not isotropic; moreover there is a significant data concentration around the mean direction. This is a relevant result that cannot be inferred from a single distance analysis.

#### 4.1.3 Joint analysis of displacement and azimuths

At this stage, it is known that the error displacement is statistically significant and that the azimuths are not randomly distributed. From Figure 3 a correlation is expected between the azimuth and error displacement. Nevertheless, the circular-linear correlation coefficient is not an appropriate statistic in this case because it works under the assumption of a Gaussian distribution of linear data, a condition that is not satisfied here. Figures 3 and 4 (density map) clearly show a strong relation between displacement and azimuth: the mean direction agrees with the larger vector errors, and from the density map, it can be observed that the biggest arrow concentration is not centered at the origin



**Figure 3** Vector diagram from dataset 1. Every arrow corresponds to a single error vector. The mean direction is 108°



**Figure 4** Density map from dataset 1. The density map is created by placing every error vector on a common origin (0, 0) and calculating the end-node density. Outliers (5%) are plotted as triangles



of coordinates but is obviously biased. The density map allows locating the outlier candidates that do not clearly show a preferred direction (see dataset 2).

These results indicate that the difference between the maps is not just due to the scales and their respective uncertainties, but that there is a bias that shifts the maps along the mean direction, introducing a complementary error.

#### 4.2 Results for Dataset 2: The Need for the Density Map

The photogrammetric scanner provides internal methods to calibrate the hardware and to optimize the accuracy of the measurements. If these methods work well, the residuals would be expected to show a Gaussian pattern for both  $X$  and  $Y$  axis components of displacement, a non-Gaussian distribution for the displacement (the zero left limit tends to generate asymmetric distributions), null mean displacement values and values of RMSE according to the scanner accuracy specifications (uncertainty of about  $2\text{ }\mu\text{m}$ ). Azimuths should be randomly and isotropically distributed.

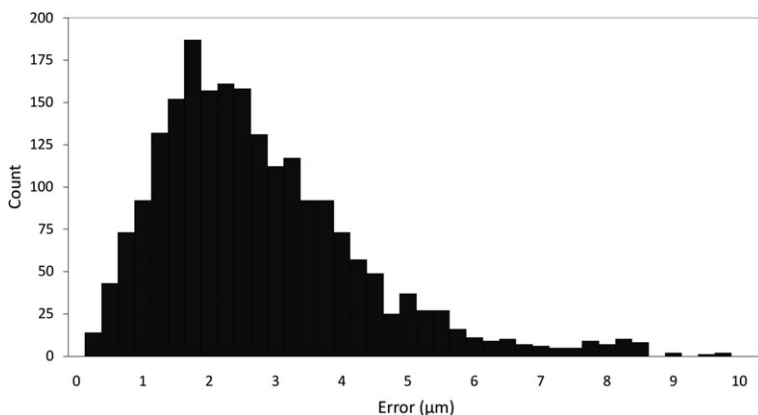
##### 4.2.1 Analysis of displacements

Figure 5 shows the distribution of the errors treated as a linear magnitude. This distribution is not Gaussian ( $P < 0.001$ ); consequently the mean value is not an unbiased and suitable statistic. The test for gamma distribution gives a  $P$ -value of 0.42.

The results for the  $X$  and  $Y$  error components are shown in Table 4. The mean errors are null for both the  $X$  and  $Y$  axis; these results and the very low value of RMSE ( $1.6\text{ }\mu\text{m}$ ) reflect an apparently optimal accuracy, as the spatial resolution that the provider guarantees for the template is about  $2\text{ }\mu\text{m}$ . Further analysis revealed that this conclusion is not accurate.

##### 4.2.2 Analysis of azimuths

The basic statistics for circular data are shown in Table 5. The mean direction is  $180^\circ$  and the mean length has a very low value of  $0.08\text{ }\mu\text{m}$ . As  $\bar{R}$  is observed in the range  $(0, 1)$ , this is evidence of a uniform distribution of azimuths. As expected, the circular standard



**Figure 5** Histogram of the displacements in dataset 2

**Table 4** Basic linear statistics for dataset 2 ( $n = 2,116$ )

	X axis	Y axis	Module
Arithmetic mean	0.0 $\mu\text{m}$	0.0 $\mu\text{m}$	2.6 $\mu\text{m}$
Maximum value	4.8 $\mu\text{m}$	9.6 $\mu\text{m}$	9.6 $\mu\text{m}$
Minimum value	-4.7 $\mu\text{m}$	-6.7 $\mu\text{m}$	0.1 $\mu\text{m}$
RMSE	1.5 $\mu\text{m}$	2.7 $\mu\text{m}$	1.6 $\mu\text{m}$

**Table 5** Basic circular statistics for dataset 2 ( $n = 2,116$ )

Mean direction ( $\bar{\theta}$ )	180°
Mean resultant length ( $\bar{R}$ )	0.08
Circular standard deviation ( $\psi$ )	129°
Von Mises parameter ( $\kappa$ )	0.16

**Table 6** Tests for uniform and von Mises distributions in dataset 2 ( $n = 2,116$ )

Tests for uniform distribution	Rayleigh	$P < 0.001$
	Rao	$P < 0.01$
Test for von Mises distribution	Watson	$P < 0.005$
	Kuiper	$P < 0.01$

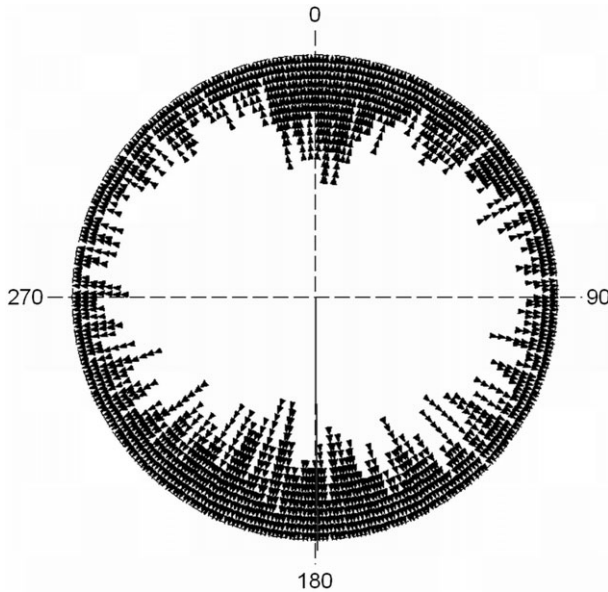
deviation is a very high value, 129°; therefore there is no evidence of preferred direction. The von Mises parameter may be considered significant for  $\kappa \geq 2$ ; here, its value is 0.16. All of the basic circular statistics suggest a circular uniform distribution, but this must be confirmed by applying the complementary tests.

The uniformity and von Mises tests were applied to dataset 2 (see Table 6). The  $P$ -values are  $P < 0.005$  (Watson test) and  $P < 0.01$  (Kuiper test), and the null hypothesis can be rejected (adjusting to a von Mises distribution) at these significance levels. Nevertheless, both Rayleigh and Rao tests for uniform distribution show that azimuths are not uniformly distributed ( $P < 0.001$ ). Partial results show a distribution with no preferred direction but not randomly distributed.

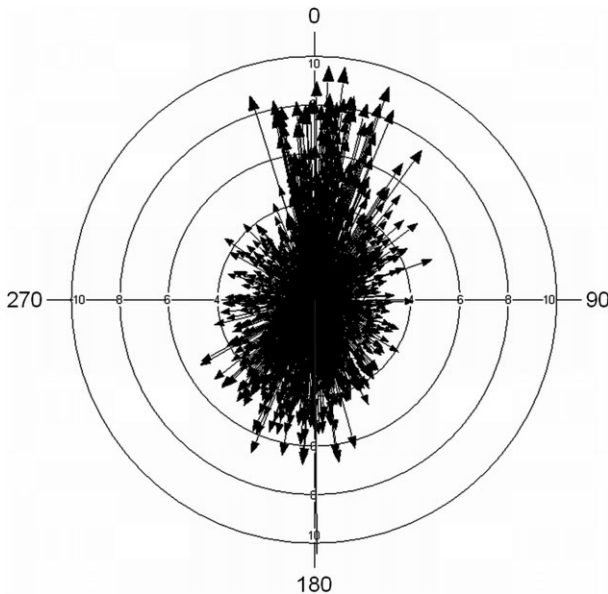
The plot of raw data in Figure 6 shows an arrowhead symbol for each azimuth. The mean direction (180°) is plotted with a line. The distribution is not isotropic; moreover, there is a bimodal distribution with a 0°–180° preferred direction. This relevant feature is not apparent from the displacement analysis and in the basic circular tests.

#### 4.2.3 Joint analysis of displacements and azimuths

Figures 7 and 8 more clearly show the error pattern in this dataset. The density map allows the location of most of the outlier candidates in the upper sector of the map: the largest errors are in the 0° direction. The 2% outliers were calculated by means of the

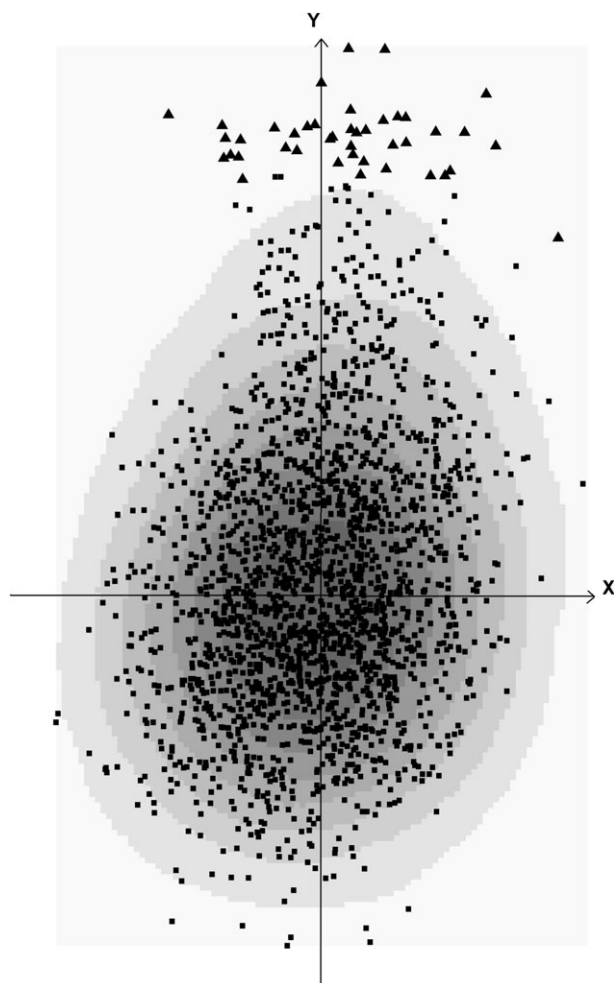


**Figure 6** Circular histogram of azimuths for dataset 2. Every arrowhead represents an azimuth. The mean direction is  $180^\circ$ [Q2]



**Figure 7** Vector diagram from dataset 2. Every arrow corresponds to a single error vector. The mean direction is  $180^\circ$

harmonic mean (plotted as triangles) which cannot be performed using methods based on the error magnitudes. As a practical result, it can be confirmed that the scanner has a serious mechanical problem that is hidden if you use the qualification methods proposed by the manufacturer.



**Figure 8** Density map from dataset 2. The density map is created by placing every error vector on a common origin (0, 0) and calculating the end-node density. Outliers (2%) are plotted as triangles and are located in the upper sector of the density map

## 5 Discussion and Conclusions

Linear statistics provide an incomplete description of error characterization. The first example shows an anisotropic pattern that is not described by conventional statistics. This pattern is suggested by the von Mises aggregation parameter and evidenced by the displacement/azimuth analysis. Error characterization reveals that the two maps are affected by a bias or a systematic error because the vectors have a statistically significant preferred direction.

The second example suggests a mechanical problem that affects the scanner, introducing a bias in the direction of sensor movement. While this issue is not revealed by linear statistics, it is suggested by the circular statistics (showing a non-von Mises and a non-circular uniform distribution), and it is verified by the density map procedure. The

cause remains unknown, but the present authors believe it to be a consequence of the acceleration of the sensor head, generating errors at the beginning and end of scanning lines.

The complementary procedures for the analysis of positional accuracy have a low cost in practice. The core of the procedure is the introduction of statistical analysis of circular data and density maps and the use of tests to verify the uniformity and isotropy of errors. The study of the azimuths allows detection of the concentration of errors. Because linear data are not always Gaussian-distributed, non-parametric methods such as density maps should be a solution for these cases. The density map allows objective detection of errors or outliers and reflects the real data patterns without assumptions about the data distribution. The joint analysis of displacements and azimuths by means of a density map is due to the limitation of circular statistics in analyzing vectors with non-unitary displacements.

Therefore, it is suggested that the accuracy/quality spatial metadata could be improved by adding records to cover these features, providing more complete information about the quality of the data. Future work will deal with spherical data for a complete accuracy analysis of spatial databases.

## References

- Ariza F J and Atkinson A J 2005 Sample size and confidence when applying the NSSDA. In *Proceedings of the Twenty-second International Cartographic Conference*, La Coruña, Spain
- Arnold B C and SenGupta A 2006 Recent advances in the analyses of directional data in ecological and environmental sciences. *Environmental and Ecological Statistics* 13: 253–56
- ASCI 1983 *Map Uses, Scales and Accuracies for Engineering and Associated Purposes*. New York, American Society of Civil Engineers, Committee on Cartographic Surveying, Surveying and Mapping Division
- ASPRS 1989 Accuracy standards for large scale maps. *Photogrammetric Engineering and Remote Sensing* 56: 1038–40
- Baltsavias E P 1994 Test and calibration procedures for image scanners. *International Archives of Photogrammetry and Remote Sensing* 30: 163–70
- Batschelet E 1981 *Circular Statistics in Biology*. London, Academic Press
- Chrisman N R 1998 Rethinking levels of measurement for cartography. *Cartography and Geographical Information Systems* 25: 231–42
- Corcoran J, Chhetri P, and Stimson R 2009 Using circular statistics to explore the geography of the journey to work. *Papers in Regional Science* 88: 119–32
- Dey S and Ghosh P 2008 GRDM: A digital field-mapping tool for management and analysis of field geological data. *Computers and Geosciences* 34: 464–78
- Dixon K R and Chapman J A 1980 Harmonic mean measure of animal activity areas. *Ecology* 61: 1040–44
- FGDC 1998 *National Standards for Spatial Data Accuracy*. Washington, D.C., Federal Geographic Data Committee
- Fisher N I 1995 *Statistical Analysis of Circular Data*. Cambridge, Cambridge University Press
- Jammalamadaka S R and Lund U J 2006 The effect of wind direction on ozone levels: A case study. *Environmental and Ecological Statistics* 13: 287–98
- Jammalamadaka S R and SenGupta A 2001 *Topics in Circular Statistics*. Singapore, World Scientific Publishing
- Li Z 1991 Effects of check points on the reliability of DTM accuracy estimates obtained from experimental tests. *Photogrammetric Engineering and Remote Sensing* 57: 1333–40
- Mardia K V and Jupp P E 2000 *Directional Statistics*. Chichester, John Wiley and Sons
- Maune D 2007 *Digital Elevation Model Technologies and Applications: The DEM Users Manual*. Falls Church, VA, American Society of Photogrammetry and Remote Sensing

- Polo M E and Felicísimo Á M 2008 Propuesta de metodología para el análisis del error de posición en bases de datos espaciales mediante estadística circular y mapas de densidad. *Geofocus* 8: 281–96
- Shi W 2009 *Principles of Modeling Uncertainties in Spatial Data and Spatial Analyse*. Kowloon, The Hong Kong Polytechnical University
- Tracey J A, Zhu J, and Crooks K 2005 A set of nonlinear regression models for animal movement in response to a single landscape feature. *Journal of Agricultural, Biological and Environmental Statistics* 10: 1–18
- USBB 1947 *United Stated National Map Accuracy Standards*. Washington, D.C., U.S. Bureau of the Budget
- Veregin H 1989 *A Taxonomy of Error in Spatial Databases*. Santa Barbara, CA, National Center for Geographic Information and Analysis Technical Paper No. 89-12
- Zandbergen P A 2008 Positional accuracy of spatial data: Non-normal distributions and a critique of the National Standard for Spatial Data Accuracy. *Transaction in GIS* 12: 103–30
- Zhang J and Goodchild M F 2003 *Uncertainty in Geographical Information*. London, Taylor and Francis