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Assignment 1

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Task 1

Output for degree = 1, $\lambda = 0$,

w0=-6.3872
w1=0.0276
w2=0.0432
w3=0.0126
w4=0.0176
w5=0.0080
w6=-0.0058
w7=-0.0081
w8=0.0714
w9=-0.0153
w10=-0.0190
w11=0.0117
w12=0.0222
w13=-0.0018
w14=-0.0013
w15=0.0091
w16=0.0382

ID= 3498, output= 3.8514, target value = 4.0000, squared error = 0.0221

Output for degree = 1, $\lambda = 1$,

w0=-6.2611
w1=0.0275
w2=0.0428
w3=0.0126
w4=0.0172
w5=0.0078
w6=-0.0059
w7=-0.0081
w8=0.0713
w9=-0.0154
w10=-0.0191
w11=0.0116
w12=0.0221
w13=-0.0018
w14=-0.0017
w15=0.0090
w16=0.0383

ID= 3498, output= 3.8528, target value = 4.0000, squared error = 0.0217

Output for degree = 2, $\lambda = 0$,

w0=-7.5608
w1=0.0223
w2=0.0001
w3=0.0352
w4=0.0000
w5=0.0049
w6=-0.0000
w7=-0.0299
w8=0.0002
w9=0.0327
w10=-0.0001
w11=0.0694
w12=-0.0004
w13=0.0079
w14=-0.0002
w15=0.0596

w16=-0.0003
w17=-0.0184
w18=-0.0000
w19=0.0093
w20=0.0002
w21=0.0162
w22=-0.0000
w23=0.0398
w24=-0.0002
w25=-0.0041
w26=0.0001
w27=0.0538
w28=-0.0007
w29=-0.0149
w30=0.0002
w31=0.1215
w32=-0.0007

ID= 3498, output= 3.6074, target value = 4.0000, squared error = 0.1542

Output for degree = 2, $\lambda = 1$,

w0=-7.0384
w1=0.0219
w2=0.0001
w3=0.0310
w4=0.0001
w5=0.0043
w6=-0.0000
w7=-0.0345
w8=0.0002
w9=0.0315
w10=-0.0001
w11=0.0678
w12=-0.0004
w13=0.0077
w14=-0.0002
w15=0.0574

w16=-0.0003
w17=-0.0192
w18=-0.0000
w19=0.0091
w20=0.0002
w21=0.0156
w22=-0.0000
w23=0.0401
w24=-0.0002
w25=-0.0050
w26=0.0001
w27=0.0536
w28=-0.0007
w29=-0.0155
w30=0.0002
w31=0.1208
w32=-0.0007

ID= 3498, output= 3.6001, target value = 4.0000, squared error = 0.1599

Task 2

$$\phi(x) = \begin{bmatrix} 1 & 5.3 \\ 1 & 7.1 \\ 1 & 6.4 \end{bmatrix}, t = \begin{bmatrix} 9.6 \\ 4.2 \\ 2.2 \end{bmatrix}$$

We know, w = $(\lambda I + \phi^T \phi)^{-1} \phi^T t$ minimizes the regularized least square

so, $(\lambda I + \phi^T \phi) = \begin{bmatrix} 3+\lambda & 18.8 \\ 18.8 & 119.46+\lambda \end{bmatrix}$,

$(\lambda I + \phi^T \phi)^{-1} = \frac{1}{\lambda^2 + C} \begin{bmatrix} 119.46+\lambda & -18.8 \\ -18.8 & 3+\lambda \end{bmatrix}$, where C is some constant

here, ignoring the lower order terms of λ,

$(\phi^T \phi)^{-1} = \begin{bmatrix} \frac{k}{\lambda} & \frac{l}{\lambda^2} \\ \frac{m}{\lambda^2} & \frac{n}{\lambda} \end{bmatrix}$, where k, l, m, n are constants

As, λ approaches ∞, $\frac{constant}{\lambda}$ terms become 0. Multiplying $\phi^T t$ by 0 would be 0. so w approaches 0.

Task 3

To minimize $E_D(w)$, we need to minimize $\sum_{n=1}^{N}[(t_n - (\phi(x_n)w)^2]$

Given: $\phi(x) = \begin{bmatrix} 1 & 5.3 \\ 1 & 7.1 \\ 1 & 6.4 \end{bmatrix}, t = \begin{bmatrix} 9.6 \\ 4.2 \\ 2.2 \end{bmatrix}$

Also, we have two w's from two f(x) equations (let's suppose w_1 and w_2), then

$w_1 = \begin{bmatrix} 4.2 \\ 3.1 \end{bmatrix}$, and $w_2 = \begin{bmatrix} -1.5 \\ 2.4 \end{bmatrix}$

for w_1, t_n - $\phi w_1 = \begin{bmatrix} 9.6 \\ 4.2 \\ 2.2 \end{bmatrix} - \begin{bmatrix} 20.63 \\ 26.21 \\ 24.04 \end{bmatrix}$

$\sum_{n=1}^{N}[(t_n - (\phi(x_n)w)^2] = 999.7266$

for w_2, t_n - $\phi w_2 = \begin{bmatrix} 9.6 \\ 4.2 \\ 2.2 \end{bmatrix} - \begin{bmatrix} 11.22 \\ 15.54 \\ 13.86 \end{bmatrix}$

$\sum_{n=1}^{N}[(t_n - (\phi(x_n)w)^2] = 224.5356$

Therefore, using sum-of-squares, f(x) = 2.4x - 1.5 would be the best line.