Date

Normal Distribution: $X \sim \mathcal{N}(\mu, \sigma^2)$ where,
(continuous R.V.)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \; ; \; \forall x \in (-\infty, \infty)$$

Remember

$$\left[\begin{array}{c} E(X) = \mu \\ Var(X) = E((X-\mu)^2) = E[X^2] - (E(X))^2 = \sigma^2 \end{array} \right]$$

Now, to operate under above conditions is tough.
So, we transform X to $Z \sim \mathcal{N}(0,1)$.

$Z \sim \mathcal{N}(0,1)$ is 'standard normal'

$E(Z) = 0$; $V(Z) = 1$.

Distribution fn $\Phi(z)$ is calculated for values saved in the Table (A1 of book).

Results

(*) if $X \sim \mathcal{N}(\mu, \sigma^2)$ and $Z \sim \mathcal{N}(0,1)$
then,

(i) $P(X \le b) = \Phi\left(\frac{b-\mu}{\sigma^2}\right)$

(ii) $\Phi(-x) = 1 - \Phi(x)$

* $E(aX+b) = aE(X) + b$
$V(aX+b) = a^2\sigma^2$

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sum & difference of indep. $X_1, X_2, \ldots X_n$ are also, normal.

$$X_1 \pm X_2 \pm X_3 \pm \cdots X_n \sim \mathcal{N}\left((\mu_1 \pm \mu_2 \pm \cdots \mu_n), (\sigma_1^2 + \sigma_2^2 + \cdots \sigma_n^2)\right)$$

no -ve sign here)

* Percentile of standard normal distb^n

$Z_\alpha = 100(1-\alpha)$ percentile of $N(0,1)$ if

$$P(Z > Z_\alpha) = \alpha$$

so, α is given, use table to find Z_α.

NOTES | IDEAS | DOODLES

Exponential R.V

$X \sim Exp.(\lambda > 0)$ if

$$f(x) = \begin{cases} \lambda \cdot e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \quad \text{p.d.f}$$

$$F(x) = \int_0^x f(x)\,dx = 1 - e^{-\lambda x}$$

Use cases → Amount of time until some specific event occurs.

Ex – time until a phone call
" " " war breaks out

$E(X) = 1/\lambda$, $V(X) = 1/\lambda^2$

Key property : Memoryless property.

if $X \sim Exp(\lambda)$ then,

$\cancel{P((X > t_1 + t_2 \mid X > t_1)) \neq P(}$

$P(X > t_1 + t_2) = P(X > t_1) \cdot P(X > t_2)$

or

$P(X > t_1 + t_2 \mid X > t_1) = P(X > t_2)$

$X_1, X_2, \ldots X_n$ are indep. exp. variables

$\Rightarrow \min(X_1, \ldots X_n)$ is $exp(\sum_{i=1}^{n} \lambda_i)$

↳ it is used if a machin fns @ n componts

$P(\text{system life} > t) = e^{-\sum \lambda_i t}$