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Confidence Interval Practice

From S and P Roys

8) An electric scale gives a reading equal to the true weight plus a random error that is normally distributed with the mean 0 and standard deviation $\sigma = .1$ mg. Suppose that the results of 5 successive ~~readings~~ ~~weighed~~ weighings of the same object are as follows: 3.142, 3.163, 3.155, 3.150, 3.141

a) Determine a 95% confidence interval estimate of the true weight.

b) Determine a 99% percent confidence interval estimate of the true weight

Ans 8) The sample mean $\bar{x}$ is given by,

$$\bar{x} = \frac{3.142 + 3.163 + 3.155 + 3.150 + 3.141}{5}$$

$$\therefore \bar{x} = \cancel{3.7812}\ 3.1502$$

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{(.1)}{\sqrt{5}} \approx \cancel{4.472136 \times 10^{-3}}\ 0.044721$$

The $100(1-\alpha)\%$ confidence interval for the mean is,

$$\bar{x} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

lookup $\left(1 - \left(\frac{1-0.95}{2}\right)\right) = 0.975$

in the middle of Z table

$$= 3.7812 \pm 1.96 \times \cancel{4.72136 \times 10^{-3}}\ 0.044721$$

$$= \cancel{(3.7812 - 9.25387 \times 10^{-3},\ 3.7812 + 9.25387 \times 10^{-3})}$$

$$= \left(\cancel{3.771447}^{\,3.0626}, \cancel{3.790453}^{\,3.23185} \right)$$

b) Determine the 99% confidence interval estimate of the true weight

$\bar{X} = 3.1502$

$$Z_{\alpha/2} = \left[1 - \left(\frac{1 - 0.99}{2}\right)\right] = 0.995$$

$\therefore$ ~~$Z_{\alpha/2}$~~ From looking up ~~Z~~ 0.995 in the middle of the ~~Z tab~~ standard Normal table ~~The closest clo~~

The closest value to 0.995 in the Middle of the ~~Z~~ standard Normal table Correspond to row 2.5 and column 0.06

$$Z^*_{\alpha/2} = 2.56$$

$\therefore$ The 99% confidence interval is given by,

$$\bar{x} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 3.1502 \pm 2.56\,(0.04472)$$

$$= (3.1502 - 0.11449, 3.1502 + 0.11449)$$

$$= (3.03571, 3.26469)$$

9) The PCB concentration of a fish caught in lake michigan was measured by a technique that is known to result in an error of measurement that is normally distributed with a standard deviation of 0.08 ppm. Suppose the results of 10 independent measurements of this fish are,

11.2, 12.4, 10.8, 11.6, 12.5, 10.1, 11.0,
12.2, 12.4, 10.6

a) Give a 95% confidence interval for the PCB level of this fish.

b) Give a 95% lower confidence interval

c) Give a 95% upper confidence interval

9a)
$$\bar{x} = \frac{[11.2+12.4+10.8+11.6+12.5+10.1+11.0+12.2+12.4+10.6]}{10}$$

$$\bar{x} = 11.48$$

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{0.08}{\sqrt{10}} \approx 0.025299$$

For 95% confidence interval, First compute

$$\left[1-\left(\frac{1-\alpha}{2}\right)\right] = 1-\left(\frac{1-0.95}{2}\right) = 0.975$$

check the Middle of the normal table to find the Value closest to 0.975, and adding the row and Column value corresponding to the value closest to 0.0975 in the standard Normal table. We get

$$Z_{\alpha/2} = 1.96$$

$$\Rightarrow \cancel{x}\ 11.48 \pm 1.96\,(0.025299)$$

$$\Rightarrow (11.48-0.049587,\ 11.48+0.049587)$$

$$= (11.430413,\ 11.529587)$$

For a 95% lower Confidence interval (one-sided)

9b) For a 95% ~~lower~~ lower Confidence interval provides an interval that has A lower confidence interval provides an interval that has a lower bound but No upper bound (i.e, the upper bound is infinity). For a 95% confidence level, we use the Critical value Z_α, where $\alpha = 0.05$. We need the z-score that cuts off $\alpha = 0.05$ in the upper tail, which is nothing but $Z_{0.05} = 1.645$ (From the standard table area to the left of z)

$\therefore$ The 95% lower confidence interval is given by, $\left(\bar{x} - Z_\alpha \frac{\sigma}{\sqrt{n}}, \infty\right)$

$= \left(11.48 - 1.64\left(0.025299\right), \infty\right)$

$= \left(11.48 - 0.041490, \infty\right)$

$= \left(11.4385, \infty\right)$

9c) ~~The~~ Similarly, the 95% upper confidence interval is given by, $\cancel{\left(\bar{x} + Z_\alpha \frac{\sigma}{\sqrt{n}}, \infty\right)}$

$= \left(-\infty, \bar{x} + Z_\alpha \frac{\sigma}{\sqrt{n}}\right)$

$= \left(-\infty, 11.48 + 0.041490\right]$

$= \left(-\infty, 11.52149\right)$

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Distribution of sampling statistics

Central limit theorem

Let $X_1, X_2, \ldots, X_n$ be a sequence of independent and ~~iden~~ identically distributed random variables each having mean μ and variance σ^2, Then for large, the distribution of

$$X_1 + \ldots + X_n$$

is approximately normal with mean $n\mu$ and variance $n\sigma^2$

It follows from the central limit theorem that

$$\frac{X_1 + \ldots + X_n - n\mu}{\sigma\sqrt{n}}$$

is approximately a standard normal random variable; thus, for n large,

$$P\left\{\frac{X_1 + \ldots + X_n - n\mu}{\sigma\sqrt{n}} < x\right\} \approx P\{Z < x\}$$

where Z is a standard normal variable

Ex 1) An insurance company has 25000 automobile policy holders. If the yearly claim of a policy holder is a random variable with mean 320 and standard deviation 540, approximate the probability that the total yearly claim exceeds 8.3 million

Ans 1) Let X denote the total yearly claim. Number the policy holders & let X_i denote the yearly claim of policy holder i. With $n = 25000$, we have from the central limit theorem that $X = \sum_{i=1}^{n} X_i$ will have approximately a normal distribution with mean $320 \times 25000 = 8 \times 10^6$ and standard deviation $540\sqrt{25000} = 8.5381 \times 10^4$.

Therefore,

$$P\{X > 8.3 \times 10^6\} = P\left\{\frac{X - 8 \times 10^6}{8.5381 \times 10^4} > \frac{8.3 \times 10^6 - 8 \times 10^6}{8.5381 \times 10^4}\right\}$$

$$= P\left\{\frac{X - 8 \times 10^6}{8.5381 \times 10^4} > \frac{.3 \times 10^6}{8.5381 \times 10^4}\right\}$$

$\approx P[Z > 3.51]$, where Z is a standard normal

$\approx .00023$

Thus, there are only 2.3 chances out of 10,000 that the total yearly claim will exceed 8.3 million

One of the most important applications of the central limit theorem is in regard to binomial random variables. Since such a random variable X having parameters (n, p) represents the number of successes in n independent trials when each trial is a ~~Stst~~ success with probability p, we can express it as,

$$X = X_1 + \dots + X_n,$$

where,

$$X_i = \begin{cases} 1 & \text{if the } i\text{th trial is a success} \\ 0 & \text{otherwise} \end{cases}$$

Because, $E[X_i] = p$, $Var(X_i) = p(1-p)$

it follows from the central limit theorem that for 'n' large.

$$\frac{X - np}{\sqrt{np(1-p)}},$$ will be approximately be a standard Normal Variable

~~Ex~~ Example 1) The ideal size of a first-year class at a particular college is 150 students, the college, knowing from ~~the~~ past experience that, on the average, only 30% of those accepted for admission will actually attend, uses a policy of approving the applicants of 450 students. Compute the probability that more than 150 first-year students attend this college.

Ans

Solution 1) Let X denote the number of students that attend; then assuming that each accepted applicant will independently attend, it follows that X is a binomial random variable with parameters $n = 450$ and $p = .3$. Since the binomial ~~random variable with~~ is a discrete and the normal a continuous distribution, it's best to compute

$$P\{X = i\} \text{ as } P\{i - .5 < X < i + .5\}$$

when applying the normal approximation (this is referred to as a continuity correction).

This yields the approximation

$$P\{X > 150.5\} = P\left\{\frac{X-(450)(.3)}{\sqrt{450(.3)(.7)}} \geq \frac{150.5-(450)(.3)}{\sqrt{450(.3)(.7)}}\right\}$$

$$\approx P\{Z > 1.59\} = 0.06$$

Hence, only 6 percent of the time do more than 150 of the first 450 accepted actually attend

Approximate distribution of the sample mean

Let $X_1, \ldots, X_n$ be a sample from a population having mean μ and variance σ^2. The central limit theorem can be used to approximate the distribution of the sample mean,

$$\overline{X} = \sum_{i=1}^{n} \frac{X_i}{n}$$

Since a constant multiple of a normal random variables is also normal, it follows from the central limit theorem that $\overline{X}$ will be approximately normal when sample size n is large.

Since the sample mean has expected value μ and standard deviation $\frac{\sigma}{\sqrt{n}}$, it then follows that $\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$, has approximately a standard normal distribution

EX1) The weights of a population of workers have mean 167 and standard deviation 27

a) If a sample of 36 workers is chosen, approximate the probability that the sample mean of their weights lies between 163 and 170

b) Repeat part (a) when the sample is of size 144

Solution 1) Let Z be a standard normal random variable

a) It follows from the Central limit theorem that $\bar{X}$ is approximately normal with mean 167 and standard deviation $\frac{27}{\sqrt{36}} = 4.5$.

Therefore, with Z being a standard normal random variable,

$$P\{163 < \bar{X} < 170\} = P\left\{\frac{163-167}{4.5} < \frac{\bar{X}-167}{4.5} < \frac{170-167}{4.5}\right\}$$

$$\approx P\{-.8889 < Z < .8889\}$$

$$= P\{Z < .8889\} - P\{Z < -.8889\}$$

$$= 2P\{Z < .8889\} - 1$$

$\approx 2 * pnorm(.8889) - 1$

$= 0.6259432$

b) For a sample of size 144, the sample mean will be approximately normal with mean 167 and standard deviation $\frac{27}{\sqrt{144}} = 2.25$. Therefore,

$$P\left\{163 < \bar{X} < 170\right\} = P\left\{\frac{163-167}{2.25} < \frac{\bar{X}-167}{2.25} < \frac{170-167}{2.25}\right\}$$

$$= P\left\{-1.7778 < \frac{\bar{X}-167}{4.5} < 1.7778\right\}$$

$$\approx 2P\{Z < 1.7778\} - 1$$

$$= 2 * pnorm(1.7778) - 1$$

$$= 0.9245633$$

Thus increasing the sample size from 36 to 144 increases the probability from .6259 to .9246

The Sample Variance

Let $X_1, \ldots, X_n$ be a random sample from a distribution with mean μ and variance σ^2. Let $\overline{X}$ be the sample mean, and recall the following definition

Definition: The statistic S^2, is defined by,

$$S^2 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n-1}$$

is called the sample variance $S = \sqrt{S^2}$ is called the sample standard deviation

Conclusion, $E[S^2] = \sigma^2$, That is, the expected value of the sample variance S^2 is equal to the population variance σ^2.

Sampling distributions from a normal population

Let $X_1, X_2, \ldots, X_n$ be a sample from a normal population having mean μ and variance σ^2. That is, they are independent and $X_i \sim N(\mu, \sigma^2)$, $i = 1, \ldots, n$. Also let

$$\bar{X} = \sum_{i=1}^{n} \frac{X_i}{n}$$

and

$$S^2 = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{n-1}$$

denote the sample mean and sample variance, respectively.
We would like to compute their distributions.

Distribution of the Sample Mean

Since the sum of independent normal random variables is normally distributed, it follows that $\bar{X}$ is normal with mean

$$E[\bar{X}] = \sum_{i=1}^{n} \frac{E[X_i]}{n} = \mu$$

and variance

$$\mathrm{Var}(\bar{X}) = \frac{1}{n^2} \sum_{i=1}^{n} \mathrm{Var}(X_i) = \frac{\sigma^2}{n}$$

That is, $\bar{X}$, the average of the sample, is normal with a mean equal to the population mean but with a variance reduced by a factor of $1/n$. It follows from this that $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ is a standard normal random Variable.

Theorem 6.5.1 : If $X_1, \dots, X_n$ is a sample from a normal population having mean μ and Variance σ^2, then $\bar{X}$ and S^2 are independent random variables, with $\bar{X}$ being normal with mean μ and Variance $\frac{\sigma^2}{n}$ and $\frac{(n-1)S^2}{\sigma^2}$ being chi-square with $n-1$ degrees of freedom

Example 1) The time it takes a central processing unit to process a certain type of job is normally distributed with mean 20 seconds and standard deviation 3 seconds. If a sample of 15 such jobs is observed, what is the probability that the sample variance will exceed 12?

Solution 1) since the sample is of size $n = 15$ and $\sigma^2 = 9$, write,

$$P(S^2 > 12) = P\left\{\frac{14S^2}{9} > \frac{14}{9} \cdot 12\right\}$$

$$\Rightarrow 1 - \text{pchisq}(56/3, 14)$$

$$= 0.1780811$$

Corollary 6.5.2. Let $X_1, \ldots, X_n$ be a sample from a normal population with mean μ. If $\overline{X}$ denotes the sample mean and S the sample standard deviation then,

$$\frac{\sqrt{n}\,(\overline{X}-\mu)}{S} \sim t_{n-1}$$

That is, $\sqrt{n}\,(\overline{X}-\mu)/S$ has a t-distribution with $n-1$ degrees of freedom.

Confidence Intervals

T- test

1] The average weight of 20 students in a certain school was found to be 165 lbs with a standard deviation of 4.5

a) Construct a 95% confidence Interval for the population mean

b) Determine the EBM for the population Mean.

Ans 1)

Given,

$\bar{x} = 165$

$n = 20$

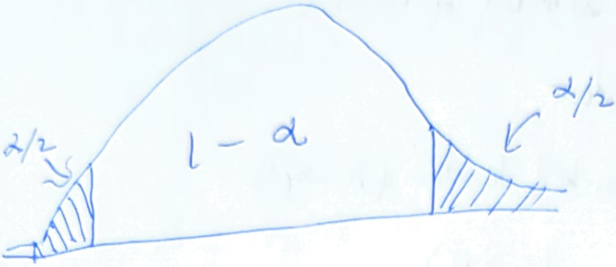
Sample standard deviation $= S = 4.5$

∴ Since the population standard deviation or Variance is unknown and the number of samples is ~~is~~ less than 30 i.e $n < 30$ then use T- distribution

$$\therefore \mu \Rightarrow \bar{x} \pm t_{r, \alpha/2} \frac{S}{\sqrt{n}} \quad \ldots (1)$$

$r = n-1$ (degrees of freedom) $= 20 - 1 = 19$

and $\frac{\alpha}{2} = \frac{1 - \text{Confidence Interval}}{2} = \frac{1-0.95}{2} = \frac{0.05}{2} = 0.025$



$\therefore \alpha = 0.05$ and $\alpha/2 = 0.025$

$\Rightarrow t_{19, 0.025} =$

In the t - table find the row corresponding to 19 and ~~Column~~ Column corresponding to 0.025, that ~~cell is the~~ ~~Kepai~~ intersection is the cell required for the desired t- value.

$\therefore t_{19, 0.025} = 2.093$

using equation ① and substituting the values of $\bar{x}$, $t_{r, \alpha/2}$ and $\frac{s}{\sqrt{n}}$

$\mu = \cancel{\alpha}\ 165 \pm 2.093\left(\frac{4.5}{\sqrt{20}}\right) = 165 \pm 2.106$

The EBM $= t_{r, \alpha/2} \frac{s}{\sqrt{n}} = 2.106$

95% CI $= (\cancel{\alpha}\,\bar{x} - \text{EBM}, \bar{x} + \text{EBM})$

$= (162.894, 167.106)$

2) A chemistry class at a certain university has 500 students. The scores of 10 students were selected at random and are shown in the table below

a) Calculate the mean and standard deviation of the sample

b) Calculate the margin of error (EBM)

c) Construct a 90% confidence interval for the mean score of all the students in the chemistry class

76	84	69	92	58
89	73	97	85	77

Ans 2 a) $\bar{x} = \frac{\text{Sum}}{n}$ ~~$= \frac{76+84+69+92+58}{10}$~~

$$= \frac{76+84+69+92+58+89+73+97+85+77}{10}$$

$$= \frac{800}{10} = 80$$

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$s = \sqrt{\frac{(76-80)^2+(84-80)^2+(69-80)^2+\dots+(77-80)^2}{10-1}}$$

$$s = \sqrt{\frac{4^2+4^2+11^2+12^2+22^2+9^2+7^2+17^2+5^2+3^2}{9}}$$

$\therefore \overline{x} = 80, \; s = 11.709$

Ans 2b) Now WKT, $\overline{x} = 80, \; s = 11.709, \; n = 10$

$\therefore EBM = t_{r, \alpha/2} \frac{s}{\sqrt{n}} \quad \ldots \text{(1)}$

$\therefore r = n-1 = 10-1 = 9$

$$\alpha/2 = \frac{1-CL}{2} = \frac{1-0.90}{2} = \frac{0.10}{2} = 0.05$$

$\therefore t_{9, 0.05} = 1.8331$ (row $= 9$, Column $= 0.05$)

using eqn (1)

$$EBM = 1.8331\left(\frac{11.709}{\sqrt{10}}\right) = 6.787$$

Ans 2c)

$\therefore \mu = \bar{x} \pm t_{r, \alpha/2} \frac{s}{\sqrt{n}} = 80 \pm 6.787$

90% CI $\Rightarrow (\bar{x} - EBM, \bar{x} + EBM)$

$\Rightarrow (80 - 6.787, 80 + 6.787)$

$\Rightarrow (73.213, 86.787)$

$\frac{\omega}{3)}$

Hypothesis Testing

1) A factory has a machine that dispenses 80 mL of fluid in a bottle. An employee believes the average amount of fluid is not 80 mL. Using 40 samples, he measures the average amount dispensed by the machine to be 78 mL with a standard deviation of 2.5

a) state the null and alternative hypothesis

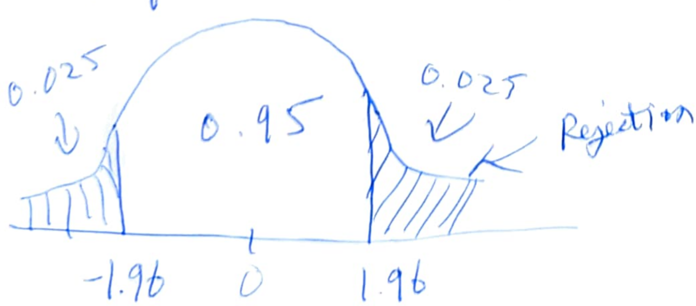
b) At a 95% confidence level, is there enough evidence to support the idea that the machine is not working properly?

Ans 1)

a)
$H_0 : \mu = 80$

$H_a : \mu \neq 80$

b) Since $\mu > 80$ or $\mu < 80$ for H_a, we are going to perform a 2-tailed test



Given,

$\bar{x} = 78$

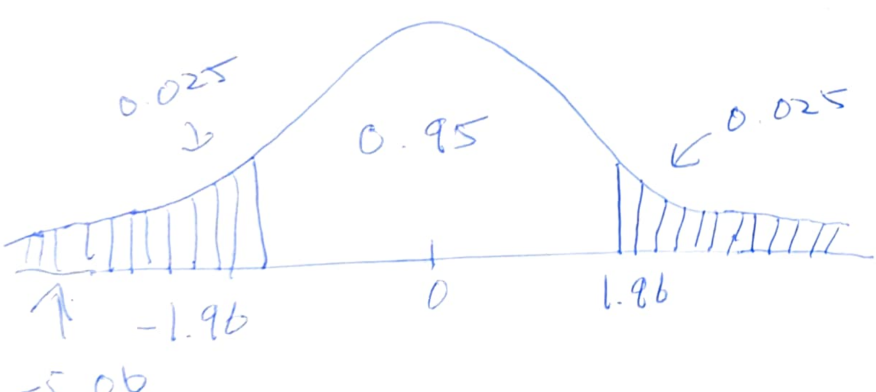
$s = 2.5$

$n = 40$

$$Z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$ $\Rightarrow$ Test statistic

$$= \frac{78 - 80}{2.5/\sqrt{40}} = \frac{-2}{0.39528}$$

$$Z_c \approx -5.06$$



$Z_c \approx 5.06$ is inside the rejection ~~regi~~ region

$\Rightarrow$ reject the Null Hypothesis with 95% Confidence

2) A company manufactures car batteries with an average life span of 2 or more years. An engineer believes this value to be less. Using 10 samples, he measures the average life span to be 1.8 years with a standard deviation of 0.15

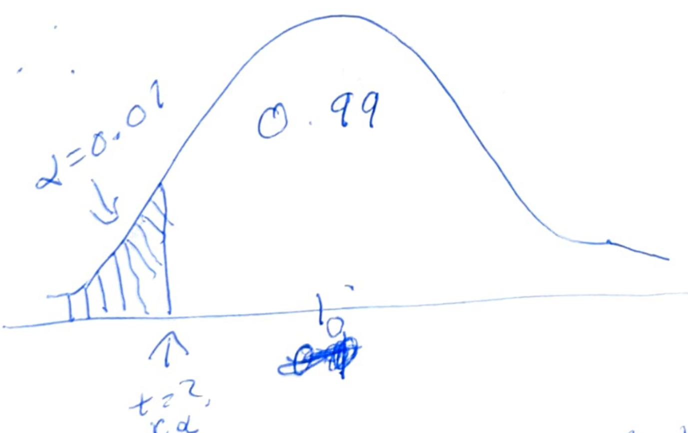
a) state the null and alternative hypothesis.

b) At a 99% confidence level, is there enough evidence to discard the null hypothesis?

Ans 2a) $H_0 : \mu \geq 2$

$H_a : \mu < 2$

2b) Since this an one-sided t-test with $H_a : \mu < 2$



Given, $n = 10$, $\bar{X} = 1.8$, $S = 0.15$, $\mu_0 = 2$

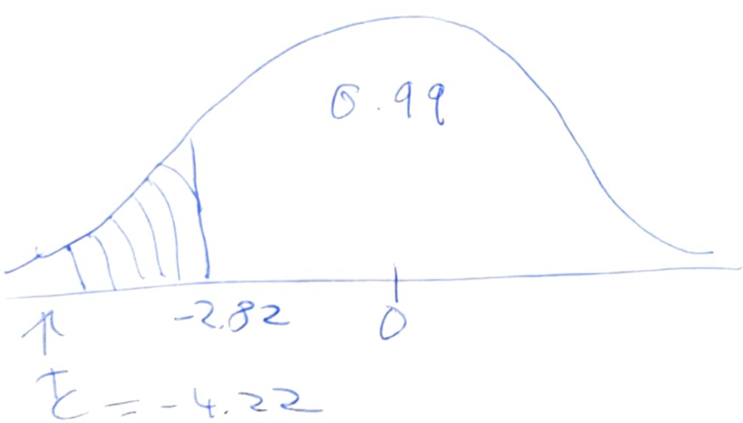
$\therefore df = n - 1 = 10 - 1 = 9$, $\alpha = 0.01 \Rightarrow t_{9,\,0.01}$

$\Rightarrow t_{9,\,0.01} = -2.82$ (with the left side)

$$\therefore T_c = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= \frac{\cancel{10}\ 1.8 - 2}{0.15/\sqrt{10}} = \frac{-0.2}{0.047434}$$

$$\cancel{t_c}\ T_c \approx -4.22$$



$\therefore$ with 99% ~~CoI~~ confidence we can ~~discard~~ reject the Null hypothesis

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Comparing 2 Means or proportions

EX1) A tech company believes that the percentage of residents in town XYZ that own a cell phone is 70%. A marketing manager believes this value to be different. He conducts a survey of 200 individuals and found that 130 responded yes to owning a cell phone.

(a) state the null and alternative hypothesis

(b) At a 95% confidence level, is there enough evidence to reject the null hypothesis?

Ans (a) ~~$H_0 : \mu = 0.7$~~

~~$H_0 : \mu = 0.70$~~

~~$H_a : \mu \neq 0.70$~~

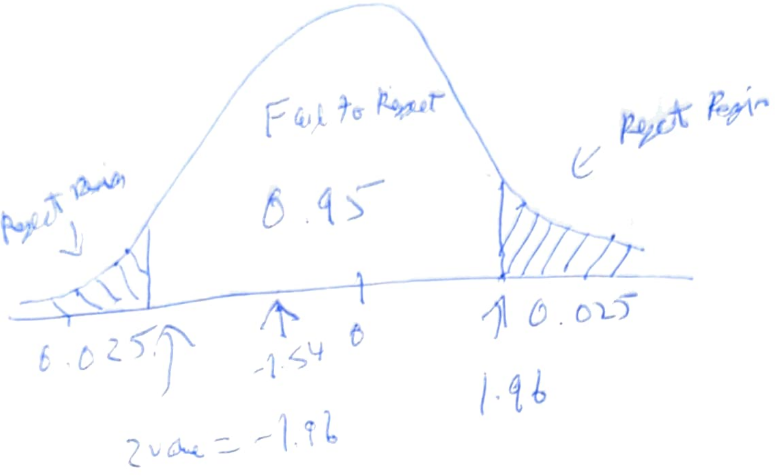
$H_0 : p = 0.70$

$H_a : p \neq 0.70$

(b) Given, $n = 200$, $x = 130$

$\therefore$ Sample proportion $\hat{p} = \frac{x}{n} = \frac{130}{200} = 0.65$

The proportion that is associated with the null hypothesis P_0 is $P_0 = 0.70 \Rightarrow q_0 = 0.30$



$$A_c = 0.975 \Rightarrow Z_c = 1.96$$

$$Z_c = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.65 - 0.70}{\sqrt{\frac{0.7(0.3)}{200}}}$$

$$Z_c = \frac{-0.05}{\sqrt{0.00105}} = \frac{-0.05}{0.0324} \approx -1.54$$

At a 95% confidence level there's ~~not~~ Not enough evidence to reject the Null hypothesis

Ex2] A Car Company believes that the percentage of residents in city ABC that own a vehicle is 60% or less. A sales Manager disagrees with this. He conducts a hypothesis test surveying 250 residents and found that 170 responded yes to owning a vehicle.

a) state the Null and alternative ~~hypothesis~~ hypotheses

b) At a 10% significance level, is there enough evidence to support the idea that the vehicle ownership in city ABC is 60% or less?

Ans 2a) $H_0 : p \leq 0.60$

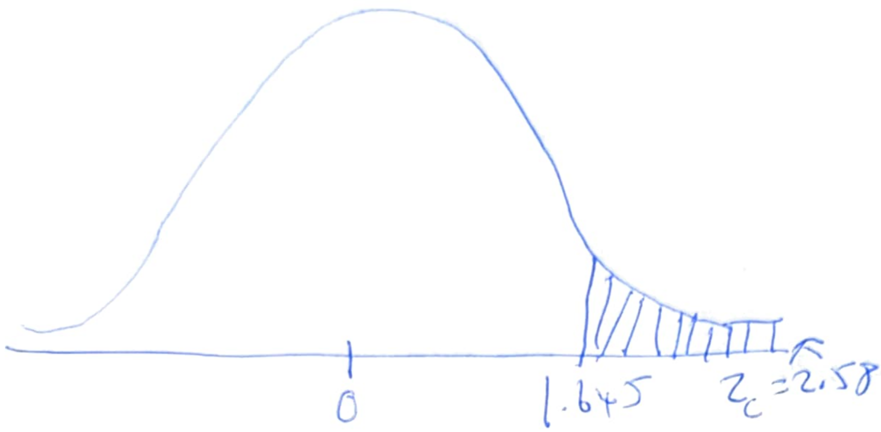
$H_a : \cancel{p \neq 0.6} \; p > 0.60$

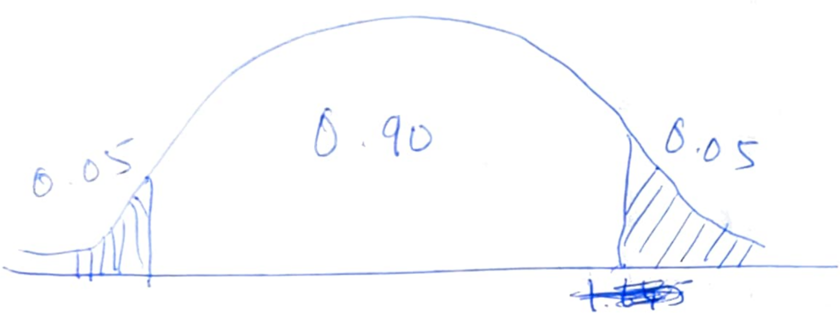
Ans 2b) Given, $n = 250$, $X = 170$

$\therefore \hat{p} = \frac{X}{n} = \frac{170}{250} = 0.68$, $P_0 = 0.60$ (Null hypothesis)

$\therefore q_0 = 0.40$

$C = 1 - \alpha = 0.90$ ($\alpha = 0.10$ 10% significance)





$A_L = 0.05 + 0.90 = 0.95$

In the z-table 0.950 is between 0.94950 and 0.95053

$\downarrow$ 1.64 $\quad \downarrow$ 1.65

$\Rightarrow \dfrac{1.64 + 1.65}{2} = 1.645$

$$\therefore Z_c = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.68 - 0.60}{\sqrt{\frac{0.60(0.40)}{250}}}$$

$$Z_c = \frac{0.08}{0.030984} \approx 2.58$$

Since 2.58 is in the Rejection ~~Rejection~~ Region, we can say with a 10% significance level ~~thes~~ there is enough evidence to reject the Null hypothesis

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Ex1) A test was conducted on two different classes to see if there was any significant difference between the performance of two teachers. The final exam scores of 15 students were sampled in the 1st class yielding a mean score of 82 with a standard deviation of 2.4. The mean final exam score of the 2nd class was 84 with a standard deviation of 1.7 using a sample of 12 students. Determine, if there is any major difference at a 5% significance level.

Ans 1) For the 1st class: $n_1 = 15$, $\bar{x}_1 = 82$, $s_1 = 2.4$

For the 2nd class: $n_2 = 12$ $\bar{x}_2 = 84$, $s_2 = 1.7$

$\alpha = 0.05$

$\therefore \mu_1 - \mu_2 = 0 \Rightarrow H_0$

$H_0: \mu_1 = \mu_2 \Rightarrow \mu_1 - \mu_2 = 0$

$H_a: \mu_1 \neq \mu_2 \Rightarrow \mu_1 - \mu_2 \neq 0$

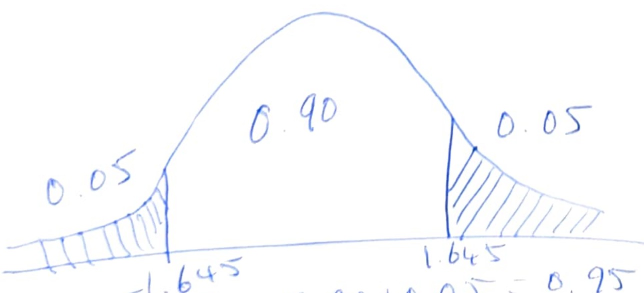
2) A business owner is in the process of deciding whether or not to invest in a new factory that refines oil at a mean rate of 3.1L per second at a standard deviation of 1.0 using a sample size of 40. The new factory was measured to refine oil at a mean rate of 3.8L per second at a standard deviation of 1.5 using a sample size of 36. Determine if there is any major difference at a 10% significance level.

~~Ans~~ Ans 2) ~~F~~ Old factory: $n_1 = 40$, $\bar{x}_1 = 3.1$, $S_1 = 1$

New factory: $n_2 = 36$, $\bar{x}_2 = 3.8$, $S_2 = 1.5$

$H_0 = \mu_1 = \mu_2$ or $H_0 = \mu_1 - \mu_2 = 0$

$H_a = \mu_1 \neq \mu_2$ $\quad$ $H_a = \mu_1 - \mu_2 \neq 0$



$A_L = 0.90 + 0.05 = 0.95$

$$\frac{1.64 + 1.65}{2} = 1.645$$

$$Z_c = \frac{(\bar{X}_1 - \bar{X}_2) - (\bar{\mu}_1 - \bar{\mu}_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$= \frac{3.1 - 3.8}{\sqrt{\frac{1^2}{40} + \frac{1.5}{36}}} = \frac{-0.7}{0.2958}$$

$Z_c = -2.366 < -1.645$ lies in the Rejection region

$\therefore$ We can Reject the Null hypothesis with a 90% Confidence level

P- Test

1) The average weight of all residents in town XYZ is 168 lbs. A nutritionist believes the true mean to be different. She measured the weight of 36 individuals and found the mean to be 169.5lbs with a standard deviation of 3.9 (a) state the null and alternative hypotheses

b) At a 95% Confidence level is, there enough evidence to discard the ~~~~ null hypothesis? (use the p-value)

1a) $H_0 : \mu = 168$

$H_a : \mu \neq 168$

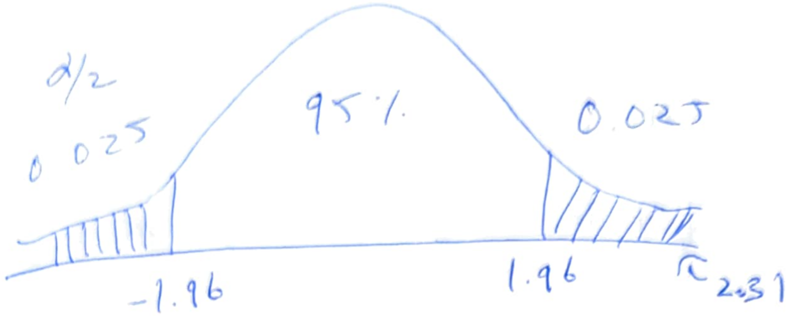
1b) Given, $n = 36$, $\bar{x} = 169.5$, $s = 3.9$

$CI = 0.95$ $\alpha = 1 - C = 0.05$

significance level

If $p\text{-value} < \alpha \Rightarrow$ reject Ho

$p\text{-value} \geq \alpha \Rightarrow$ Fail to reject Ho



$A_L \Rightarrow$ Cumulative Area to the left

$\Rightarrow A_L = 0.95 + 0.025 = 0.975$

$\Rightarrow \cancel{1.96}\ Z_c = 1.96$

$$Z_c = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{169.5 - 168}{3.9/\sqrt{36}}$$

$$= \frac{1.5}{0.65} = 2.31 = Z_{cv}$$

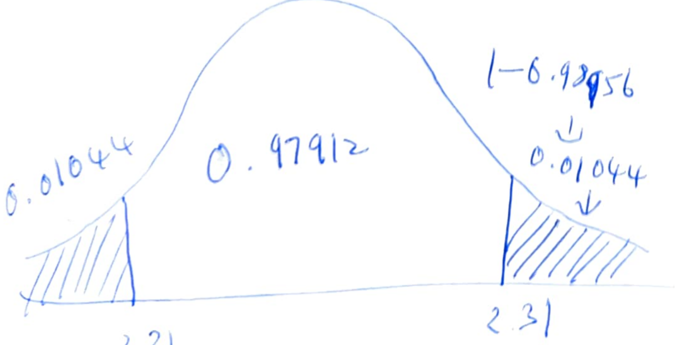
$\Rightarrow Z_c > Z_{cv} \Rightarrow$ reject H_0

Using p-Value Method

If p-Value $< \alpha \Rightarrow$ Reject H_0

p-Value $\geq \alpha \Rightarrow$ Fail to reject H_0

$Z \Rightarrow A_L$



α

P-Value $= 0.01044 + 0.01044 = 0.02088 < 0.5$

$\Rightarrow$ reject H_0 with 95% Confidence

2) A factory manufactures cars with a warranty of 5 years on the engine and transmission. An engineer believes that the engine or transmission will Malfunction in less than 5 years. He tests a sample of 48 cars and finds the average time to be 4.8 years with a standard deviation of 0.50

a) State the null and alternative hypotheses

b) At a 2% significance level, is there enough evidence to support the idea that the warranty should be revised?

Ans 2a) $H_0 : \mu \geq 5$

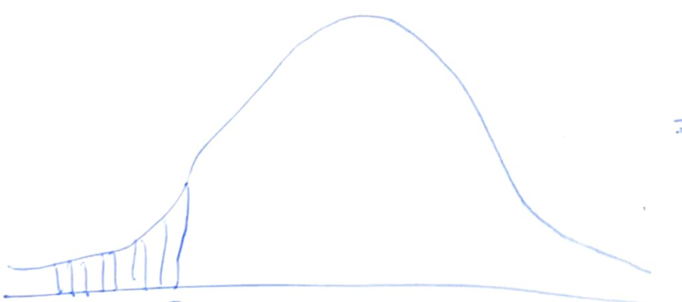
$H_a : \mu < 5$

2b) $n = 40$, $\bar{x} = 4.8$, $s = 0.50$, $\mu_0 = 5$

$\alpha = 0.02$, $c = 0.98$

$$z_c = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{4.8 - 5}{0.50/\sqrt{40}}$$

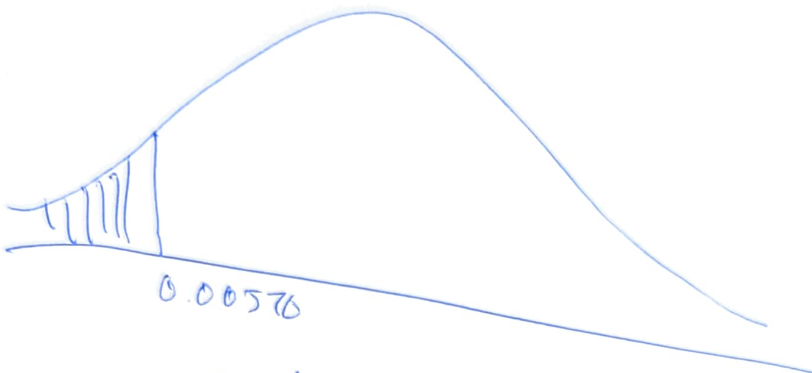
$$z_c = \frac{-0.2}{0.079} \approx -2.53$$



$z_c = -2.53$

$Z \Rightarrow A_L = p\text{-value}$

$-2.53 = 0.00570 = p\text{-value}$



$\Rightarrow p\text{-value} = 0.0057 < 0.02$

$P\text{-value} < \alpha \Rightarrow$ reject H_0 with 98% Confidence