Bernoulli Random Variable

1. $X = \begin{cases} 1 & \text{if success} \\ 0 & \text{if failure} \end{cases}$ || This is R.V.

2. P.M.F $\quad P(X=1) = p \qquad (0 \leq p \leq 1)$

   $\qquad\qquad P(X=0) = 1-p$

3. $E(X) = p$

4. $V(X) = p(1-p)$

5. its distribution function is not relevant as only 2 values are possible. $X=0$ or $X=1$ .... so, $P(X \leq x)$ is of no use...

$X \sim (n, p)$ Binomial Random Variable (n, p) → called parameters

1. X = number of successes in n independent trials, each trial having p as probability of success.

2. p.m.f $\quad P(X=i) = {}^nC_i\,(p)^i\,(1-p)^{n-i}$

   ≈ i successes in n trials.

3. $E(X) = np$

4. $V(X) = np(1-p)$

5. Distribution - $\quad P(X \le i) = \sum_{k=0}^{i} {}^nC_k\, p^k (1-p)^{n-k}$

   $i = 0, 1, \ldots n$

Note
$$\left[\begin{array}{l} X_1 \sim (n_1, p),\; X_2 \sim (n_2, p) \\ \Rightarrow (X_1 + X_2) \sim (n_1 + n_2, p) \end{array}\right]$$

Something like seq. sampling I

⇒ we can ↑ no. of indep. trials if prob. remains same

Poisson Random Variable (parameter λ)

1. X = defined via p.m.f.
i.e a variable that follows p.m.f given in 2. is a P.R.V.

2. p.m.f $\quad P(X=i) = \dfrac{e^{-\lambda}\lambda^{i}}{\lfloor i}$; $i = 0, 1, \ldots$

where $\lambda\ (>0)$ is parameter.

3. $E(X) = \lambda$

$$\boxed{\frac{P(X=i+1)}{P(X=i)} = \frac{\lambda}{i+1} \ : \text{just a result}}$$

4. $V(X) = \lambda$

5. Distribution Function : $P(X \leq i) = \sum_{k=0}^{i} e^{-\lambda} \dfrac{\lambda^{k}}{\lfloor k}$; $i = 0, 1, \ldots.$

Note → when n is large and p is small, binomial R.V goes close to Poisson.... mathematically, it is easy to calculate Poisson. So, we use it.

Example – (i) no. of failures of bank transactions
(ii) no. of meteors reaching earth
(iii) no. of failed downloads over a website etc.

Poisson reproductive property →

$X_1 \sim Poisson(\lambda_1)$ & X_1, X_2 are independent.
$X_2 \sim Poisson(\lambda_2)$

$\Rightarrow X_1 + X_2 \sim Poisson(\lambda_1 + \lambda_2)$
↳ basically avg adds

Very useful in factory production analysis.

Ex- Daily defect items ~ Poisson (4)
Find possibilities of defects over a week.

$\Rightarrow \quad X_1 =$ defects on day 1 $\sim Poisson(4)$
⋮
$X_7 =$ " " " 7 ~ " (4)

Total weekly $(X_1 + \cdots X_7) = Poisson(28)$
So, instantly we can analyze weekly thing.

Note - There are 2 more R.V's given in book - Hypergeometric → I think not needed.
and uniform → very simple

Normal Random Variable $\mathcal{N}(\mu, \sigma^2)$

continuous

1. $X \sim \mathcal{N}(\mu, \sigma^2)$ is said to be normally distributed.

2. p.d.f $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} : (x \in \mathbb{R})$

cdf can be found using integration

$\left(\text{Note} - \dfrac{1}{\sqrt{2\pi}} = 0.399\right)$

3. $E(X) = \mu$

$[E(aX+b) = a\mu + b]$

4. $V(X) = \sigma^2$

$[V(aX+b) = a^2\sigma^2]$

Property - each $\mathcal{N}(\mu, \sigma^2)$ can be reduced to $\mathcal{N}(0,1)$. This eases the maths...

$$\text{cdf standard} = \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2}\, dy \quad (x \in \mathbb{R})$$

Note - if $X \sim \mathcal{N}(\mu, \sigma^2) \Rightarrow \left(\dfrac{X-\mu}{\sigma}\right) \sim \mathcal{N}(0,1)$

$$P(X \leq b) = \Phi\left(\frac{b-\mu}{\sigma}\right)$$

$$P(a \leq X \leq b) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$$