30/11/2025

Pravin Patil. Stats End Exam pg. 1

भारतीय प्रौद्योगिकी संस्थान मुंबई
INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

उत्तर पुस्तिका/ Answer Book-8

रोल नं./Roll No.: X2ptp512

पाठ्यक्रम नाम/Course Name: Stats

शाखा/प्रभाग/Branch/Div. शैक्षणिक बैच /Tutorial Batch: Aug-25

अनुभाग/Section:

Q.1.
$n = 100$ (samples)
$\bar{x} = 45$ (sample mean)
$s = 8$

(a) Coefficient of variation $CV = \frac{s}{\bar{x}} = \frac{8}{45} = 0.178$

(b) $y_i = 2x_i + 5$
mean $= (0.178) \times 45$
$=$
std. dev $= \sqrt{s^2}$

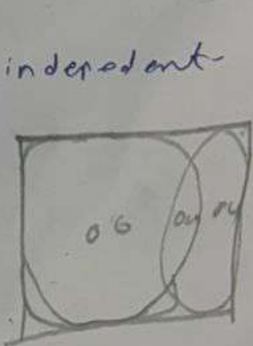
Q.2.
$P(A) = 0.6$
$P(B) = 0.4$
$P(A \cap B) = 0.24$

(a) for independence :- $P(A \cap B) = P(A) \times P(B)$
$= 0.6 \times 0.4$
$\boxed{P(A \cap B) = 0.24}$

Since $P(A) \cdot P(B) = P(A \cap B)$, A & B are independent

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.6 + 0.4 - 0.24$

$\boxed{P(A \cup B) = 0.76}$



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Q.3 $x \to PDB \to f(x) = \begin{cases} kx^2 & 0 \le x \le 2 \\ 0 \end{cases}$

Formula for PDF $\int_0^2 f(x)\,dx = 1$

$= \int_0^2 kx^2\,dx$

$= k\int_0^2 x^2\,dx = k\left[\frac{x^3}{3}\right]_0^2$

$1 = k\left[\frac{8}{3} - 0\right]$

$1 = k\left(\frac{8}{3}\right)$

$\frac{3}{8} = k$, so, $\boxed{k = \frac{3}{8}}$

$P(x>1) = \int_1^2 f(x)\,dx = \frac{3}{8}\int_1^2 x^2\,dx$

$= \frac{3}{8}\left[\frac{x^3}{3}\right]_1^2$

$= \frac{3}{8}\left[\frac{8}{3} - \frac{1}{3}\right] = \frac{3}{8} \times \frac{7}{3}$

$\boxed{P(x>1) = \frac{7}{8} = 0.875}$

Q.4.

p - critical value = 0.032

(a) if $\alpha = 0.05$

then if p value $< \alpha \to$ Reject $H_0 \to$ Null hypothesis.

$(0.032) < 0.05)$

(b) $\alpha = 0.01$, p value = 0.032

pvalue $> \alpha \to$ Fail to Reject H_0

Q.5. $Y = B_{01} + B_1 X + e$

(a) R^2 = change in Y with respect to change in ~~X~~
since 0.85 is significant, x affects Y significantly.

(b) $B_1 = 0$
if $B_1 = 0$ → that mean there is no linear relationship.

Section B starts here

Q.6

(a) $\text{mean} = \frac{\sum x_i}{n} = \frac{15+18+22+25+28+30+32+35+38+42+45+50}{12}$
$= 31.66$

median = since n is even & $\text{avg}\left(\frac{n}{2}, \frac{n}{2}+1\right)$
$= \frac{30+32}{2} = 31$

(b) Q1 ⇒ first half → 15, 18, 22, 25, 28, 30
$Q1 = \text{median} = \frac{22+25}{2} = 23.5$
Q3 = second half = 32, 35, 38, 42, 45, 50
$Q3 = \frac{38+42}{2} = 40$

(c) $IQR = Q3 - Q1 = 40 - 23.5 = 16.5$

outliers $[Q1 - 1.5 IQR], [Q3 + 1.5 IQR]$
$= [23.5 - (1.5 \times 16.5)], [40 + 1.5(16.5)]$
$= [-1.25, 64.75]$

There are no outliers.

Ans:-	mean: 31.66	Q1 = 23.5
	median = 31	Q3 = 40

Q.7. Cricket application

(a) $p(x)$ = prob. of hitting six = 0.15

It is Binonimal distribution

$$P(X=x) = \binom{n}{x} p^x (1-p)^{(n-x)}$$

n = no of balls

p = probability of hitting six

x = random variable

(b) $$P(X=2) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$= ({}^6C_2) \times 0.15 \times (0.85)^4$$

=

(c) $E(\bar{x}) = n \times p = 6 \times 0.15 = 0.9$

(d) variance $V(x) = np(1-p)$

$$= 0.9(1-0.15) = \boxed{0.765}$$

Q.8 (a) Sample mean $\bar{x} = \sum_{n=1}^{n} \frac{x_i}{n}$

$$= \frac{2.1 + (-1.3) + 3.2 + (-0.8) + 1.4}{5}$$

$$\boxed{\bar{x} = \frac{4.4}{5} = 0.88}$$

Q -> 8 contine -> PTO

sample variance $s^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}$

=

S.deviation σs: $\boxed{\sqrt{s^2} = 0.061}$

Q.8.

(b) variance is unknown, we have to go for t test

$$95\%\ CI = \bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$$

$$= 0.88 \pm 2.776 \times \frac{0.061}{\sqrt{5}}$$

$$= 0.88 \pm 0.027$$

$$\boxed{95\%\ CI = (0.907, 0.8773)}$$

(c) stock returns
on daily with el 0.88 It is
chance that 95% of times the
stock price will be between
0.907, 0.8773

Q.9 $\mu = 500 \rightarrow$ mean weight assume/claim

$\bar{x} = 497$

$n = 25$

$s = 12$

steps

(a) state hypothesis

$H_0 = \mu = 500$

$H_1 \neq 500$

(b) calc. test statistic $\rightarrow n > 30$, normals, t stat

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{497 - 500}{12/\sqrt{25}} = \frac{-3}{5.36}$$

$$= -0.559$$

(c)

$\alpha = 0.05$ & $t_{0.05, 24} = 2.064$

since p value $-0.559 < 2.064$

reject H_0

(d) This means based on α and test type mean weight of ceral box is not exactly 500

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Q.10

(a) ma

$P_x(t)$

(b) E

(c)

Q.10

(a) mangiral distributions.

$p_x(i) = i=\phi$ ~~0.30~~ 0.35

$i=2$ ~~0.45~~

$i=3$ ~~0.45~~ 0.35

0.30

$= 1$

$p_y(y_i) = i=0 \quad 0.30$

$=1 \quad 0.45$

$=2 \quad 0.25$

(b) E(X) and E(Y)

$E(X) = \sum_i x_i \, p(X = x_i)$

$= p(X=\phi) + p(X=2) + p(X=3)$

$= 1$

$E(Y) = \sum_i y_i \, p(X = y_i)$

$= 1$

(c) check independence.

$F(X,Y) = F_X(x) \cdot F_Y(y)$

$f_X(x) = 1$

$f_Y(y) = 1$

X and Y are indepent.

Q.11 $x = 5 \quad 10 \quad 15 \quad 20 \quad 25 \quad 30$

$y = 45 \quad 55 \quad 65 \quad 75 \quad 80 \quad 90$

Find $\bar{x} = \sum x_i/n = 17.5$

$\bar{y} = \sum y_i/n = 68.33$

$\boxed{r = \frac{S_{xy}}{S_x S_y}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{(n-1) S_x S_y}$

$\boxed{S_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}}, \quad \boxed{S_y = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n-1}}}$

$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 * \sum_{i=n}^{n} (y_i - \bar{y})^2}}$

$= \frac{775}{\sqrt{(437.5) \times (1383.34)}} = \frac{}{605211}$

(a) $\boxed{r = 0.9962}$

$b_0 = \bar{y} - b_1 x, \quad b_1 = r \frac{S_y}{S_x}$

$b_1 = 0.9962 \times \frac{276}{20.79} =$

$\boxed{b_1 = 13.256}$

$b_0 = 68.33 - 13.256 \times 17.5$

$\boxed{b_0 = -163.65}$

(d) $\bar{y} = b_0 + b_1 x$

$= -163.65 + 13.256 (18)$

$\boxed{\bar{y} = 74.89}$ → Studies 18 hours.

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Part B Pg. 9

Q.12

Given $\mu = 10$

$n = 16$

$\bar{x} = 11.5$

$s = 3$

(a) Hypothesis

H_0 = Null hypothesis = $\mu = 10$

H_1 = Alternative hypothesis = $\mu > 10$ (upper tailed alternative)

(b) Test statics.

$n = 16$, σ unknown → go for t stat.

t-test

(c) p value = $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

$= \dfrac{11.5 - 16}{3/\sqrt{16}} = \dfrac{-4.5}{0.75}$

pvalue $= -6$

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(d) $\alpha = 0.05$

$t(0.05, 15) = 1.753$

p value

$-6 < 1.753$

Reject Null hypothesis.

Q.13 Given $P(F) = 0.20 \rightarrow$ High risk

$P[F^c] = 1 - 0.20 = 0.8$

$P[E|F] = 0.12$

$P[E|F^c] = 0.03$

(a) $P(E) = P(E|F)\,P(F) + P(E|F^c)\,[1 - P(F$

$P(E) = 0.12 \times 0.20 + 0.03\,(0.8)$

(b) $P(E) = (0.12 \times 0.20) + 0.03\,(0.8)$

$= 0.024 + 0.024$

$P(E) = 0.048$

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(c) find $P(F|E)$

Bayes theorem

$$P(F|E) = \cancel{P(E|F)\,/\,P(F)}$$

$$\cancel{\Sigma}$$

$$P(F|E) = \frac{P(F \cap E)}{P[E]}$$

which means

$$\boxed{P(F|E) \cdot P(E) = \frac{P(E|F) \cdot P[F]}{\cancel{P[E]}}}$$

$$P[F|E] = \frac{P(E|F] \cdot P[F]}{P[E]}$$

$$= \frac{0.12 \times 0.2}{0.048} = \frac{0.024}{0.048}$$

$$P[F|E) = 0.5$$

(d) Since $P[F|E) = 0.5$

$P(E)$ is disease

there is 0.5 % chance of high risk if there is disease..

1- P(F])

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