28-9-25

Pravin Pasi
midterm stats.

Pg No:-1

Q-11) Probability question

$P(B_1) = 0.27$
$P(A_2) = 0.27$
$P(A_2 \cap B_1) = 0.10$

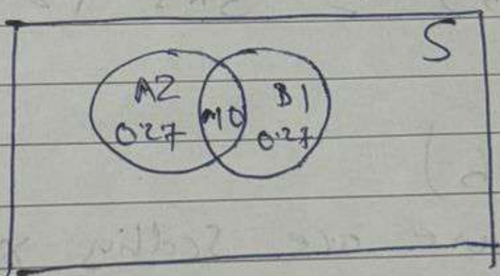
Compute $P(A_2 | B_1)$
Conditional probability says

$$P(A_2 | B_1) = \frac{P(A_2 \cap B_1)}{P(B_1)}$$

$$= \frac{0.10}{0.27}$$

$$P(A_2 | B_1) = 0.370$$

Independence of $A_2$ and $B_1$




$P(A_2 \cap B_1) = 0.10$
$P(A_2) = 0.27$
$P(B_1) = 0.27$

$P(A_2 \cap B) \neq P(A_2) * P(B_1)$
$0.10 \neq 0.27 \times 0.27$

So, $A_2$ and $B_1$ not independent
they are dependent.

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Pravin Patil PG NO:-2

Stats midterm

Q.12 Random Variable :- $X$

Mean $\rightarrow \mu$

Variance $\rightarrow \sigma^2$

find $E[aX+b]$

$Var(aX+b)$

$\Rightarrow$ (i) $E[aX+b]$

Here we are scaling $x$ to $ax$, so we can pull out a

$$E[aX+b] = a\cdot[E(X)+b]$$

e.g.

$a=3,\ b=2,\ E[X]=5$

$$E[3X+b] = 3*5+2 = 17$$

(ii) $Var(aX+b)$

Here also we are scaling $X$, this will change the variance.

$$Var(aX+b) = a^2\,Var(X)$$

e.g. $a=3,\ b=2,\ Var X\ 4$

$$Var(aX+b) = (3)^2 \times 4$$

$$= 9\times 4$$

$$= 36$$

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Pravin Patil PG.NO:- 3
Stats midterm

Q.13) overbooking

$X \sim Bin(n, 0.8)$

$n = 20$

$p = 0.8$

This is Binomial distribution

formula

$P(X=k) = \boxed{nC_k \, (p)^k \, (1-p)^{n-k}}$

(a) $P(X \geq 17) = P(17) + P(18) + P(19) + P(20)$

$\boxed{nC_r = \frac{n!}{r!(n-r)!}}$

$P(17) = 20C_{17} \, (0.8)^{17} \, (0.2)^3 = 1140 = 0.20$

$P(18) = 20C_{18} \, (0.8)^{18} \, (0.2)^2 = 190 = 0.136$

$P(19) = 20C_{19} \, (0.8)^{19} \, (0.2)^1 = 20 = 0.05$

$P(20) = 20C_{20} \, (0.8)^{20} \, (0.2)^0 = 1 = 0.011$

$P(X \geq 17) = 0.3897$

(b) $P(X \leq 15)$

$= 1 - (P(16) + P(17) + P(18) + P(19) + P(20))$

$P(16) = {}^{20}C_{16} + (0.8)^{16} (0.2)^4$

$= 4845 + (0.8)^{16} (0.2)^4 = 0.21$

$+ (0.028) \; 0.016$

$= 1 - (0.217 \; 0.389)$

$P(X \leq 15) = 0.393$

Q.14

$X \sim (\cancel{\mu}\mu, \sigma^2)$

then

$$Z = \frac{X-\mu}{\sigma}$$

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-x^2/2} \, dx \quad -\infty < x < \infty$$

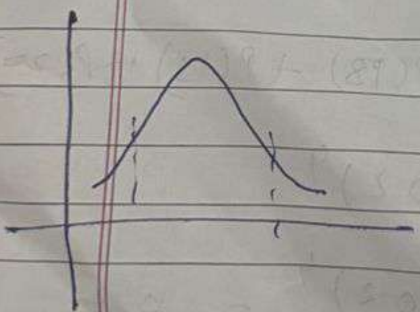
$P(a \le X \le b)$

$$P(a < X < b) = P\left\{\frac{a-\mu}{\sigma} < \frac{X-\mu}{\sigma} \le \frac{b-\mu}{\sigma}\right\}$$

$$= P\left\{\frac{a-\mu}{\sigma} \le Z \le \frac{b-\mu}{\sigma}\right\}$$

$$= P\left\{Z \le \frac{b-\mu}{\sigma}\right\} - P\left\{Z \le \frac{a-\mu}{\sigma}\right\}$$

$$= \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$$



$$\phi(-x) = P\{Z < -x\}$$

$$= P\{Z > x\}$$

$$= 1 - \phi(x)$$

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Pg NO:- 5

Q.15 part 1

| x | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| y | 80 | 60 | 70 | 40 | 0 |

$$r = \frac{\sum_{i=1}^{n} (x_i-\bar{x})(y_i-\bar{y})}{(n-1) S_x S_y}$$

$$= \frac{\sum_{i=1}^{n} (x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i-\bar{x})^2 \sum_{i=1}^{n} (y_i-\bar{y})^2}}$$

Now

$$\bar{x} = \frac{6+7+8+9+10}{5} = 8$$

$$\bar{y} = \frac{80+60+70+40+0}{5} = 50$$

$$= \frac{((6-8)+(7-8)+(8-8)+(9-8)(10-8))}{((80-50)+(60-50)+(70-50)+(40-50)+(0-50)}$$

$$\sum (x_i-\bar{x})^2 = 4+1+0+1+4 = 10$$

$$\sum (y_i-\bar{y})^2 = 900+100+400+100+2500$$
$$= 4000$$

$$= \frac{-180}{\sqrt{40,000}}$$

$$r = \frac{-180}{200} = -0.9$$

Pg No:- 6

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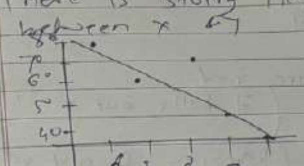
Q.15 part 2

Ans:- There is strong negative correlation between x & y




(b) if duplicate each paired entry

or

if any of the x or y scales or shifts the origin, the coralation cofficent doesnt change.

So, r will not change

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Q.16 Given

$P(D|M1) = 0.02$ $P(M1) = 0.3$
$P(D|M2) = 0.03$ $P(M2) = 0.45$
$P(D|M3) = 0.05$ $P(M3) = 0.25$

(a) $P(D) = P(M1) \times P(D|M1) +$
$P(M2) \times P(D|M2) +$
$P(M3) \times P(D|M3$
$= 0.02 \times 0.3 +$ $= 0.006$
$0.03 \times 0.45 +$ $= 0.0135$
$0.05 \times 0.25$ $= 0.0125$

$P(D) = 0.032$

(b) $P(M2|D) = \dfrac{P(M2 \cap D)}{P(D)}$

Associate law
$P(M2|D) = \dfrac{P(D \cap M2)}{P(D)}$

$= \dfrac{\cancel{0.032}}{} \quad \dfrac{0.03}{0.032}$

$= 0.\cancel{4375}\ 0.42$

(c) machine 1 $= 0.18$
machine 2 $= 0.42$
machine $= 0.39$

Machine 2 will produe defective.

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midterm Stats.

Q.17

| Red | Blue |
|---|---|
| 7 | 4 |

(a) Both balls are red

$P(R_1) = \frac{7}{11}$ (7 balls out of 11)

$P(R_2) = \frac{6}{10}$ (Remaining 6 out of 10)

$P(R_1 \cap R_2) = \frac{7}{11} \times \frac{6}{10}$

$= \frac{42}{110}$ $\boxed{= \frac{21}{55}}$

(b) P(Second Red | First Red) → conditional prob.

$P(R_2 | R_1) = \frac{P(R_2 \cap R_1)}{P(R_1)}$

as per solution a

$P(R_1 \cap R_2) = 21/55$

$P(R_1) = 7/11$

$P(R_2 | R_1) = \frac{21/55}{7/11} = \boxed{\frac{3}{5}}$

(c) $P(R_2 | B_1) = \frac{P(R_2 \cap B_1)}{P(B_1)}$

$P(B_1) = \cancel{4/11}$ $\frac{4}{11}$

$P(R_2 | B_1) = \boxed{\frac{7}{10}}$

28-9-

Q.16

28-09-25 Pravin Patil Pg No:

midterm Stats

Q. 18) mean = $\mu$

S.dev = $\sigma$

① $B = |Y|$, $H = \cancel{\phi}\ 3|Y|$

Area of triangle $= \frac{1}{2} \times$ Base $\times$ height

$$= \frac{1}{2} \times B \times H$$

$$A = \frac{1}{2} \times BH$$

$$E[A] = E\left[\frac{1}{2} \times B \times H\right]$$

$$= \frac{1}{2} \times E[B \cdot H]$$

$$= \frac{1}{2} \times E[Y \cdot 3Y]$$

$$= \frac{1}{2} \times E[3Y^2]$$

$$= \frac{1}{2} \times 3\ E[Y^2]$$

$$\boxed{E[A] = \frac{3}{2} E[Y^2]}$$