7-Nov

Pravin Patil
Stats Quiz 8

PG NO:

Q.17)

x and y joint PDF

$f_{x,y}(x,y) = cxy \quad 0 < x < y < 1$

PDF is interval

$$f(x,y) = \int_0^1 \int_0^y cxy \, dx \, dy$$

$$= cy \int_0^y x \, dx = cy \left[\frac{x^2}{2}\right]_0^y$$

$$= cy \cdot \frac{y^2}{2}$$

$$\int_0^1 \frac{cy^3}{2} dy = \frac{c}{2}\left(\frac{y^4}{4}\right)_0^1 = \frac{c}{2} \times \frac{1}{4} = \frac{c}{8}$$

$c = 8$

Check independence =

$f(x,y) = f_x(x) \cdot f_y(y)$

$$f(x) = \int_{y=x}^{1} 8xy \, dx \, dy$$

$$= \left(\frac{1}{2} - \frac{x^2}{2}\right)$$

$$= 4x(1-x^2)$$

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Stats

$$f(y) = \int_x^y 8xy \, dx$$

$$= 8y\left[\frac{x^2}{2}\right]_0^y = 4y^3$$

$16xy^3(1-x^2) \neq 8xy$

x & y are not independent

Marginal of

$$f_y(y) = 8y \cdot \frac{y^2}{2}$$

$f_y y = 4y^3$

$f_x(x) = 4x(1-x^2)$

19 Nov Pravin Pahl
Stats Quiz 2 [Pg No:- 3]

[Q.18]

Television picture tube → 90% CI

1. Sample size $n = 20$
Sample mean $\bar{x} = 9000$
pop. sd. deviation $= \sigma = 400$

Calc. Z score

$Z_{\alpha/2} = 1.645$

90% CI $= \bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$= 9000 \pm 1.645 \times \frac{400}{\sqrt{20}}$

$= 9000 \pm 1.645 \times \frac{400}{4.47}$

$\approx 9000 \pm 147.1$

$= (8852.9, 9147.1)$

90% C.I. $= [8852.9, 9147.1]$

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Stats Quiz 2

9-Nov

Pg No:- 4

Q.19

Test hypothesis

$H_0 : \mu = 105$

$H_1 = \mu \neq 105$

$n = 9$

$\bar{x} = 100$

$\sigma = 15$

Step 1 :- Compute z stat

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{100 - 105}{15/\sqrt{9}} = \frac{-5}{5} = -1$$

find the pvalue, two tailed

$p = 2 \cdot P(Z < -1)$

$P(Z < -1) \approx 0.1587$

$p = 2 \times 0.1587$

$$p = 0.3174$$

9-11-25

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Stats Quiz 2

Pg No: 5

Q.20

$\mu = 400$

$\sigma^2 = 1600 \Rightarrow \sigma = 40$

$n = 35$

(a) mean and standard of sampling distribution

$\mu_{\bar{x}} = \mu = 400$

$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} = \frac{1600}{35} = 45.71$

(b) $P(\bar{x} > 412)$

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{412 - 400}{40/\sqrt{35}} = 1.775$

$P(Z > 1.775)$

→ By looking at z table

$= 0.038$

$\boxed{P(\bar{x} > 412) = 0.038}$

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Stats Quiz 2

Pg No: 6

Q.20 Part 2

(c) $\bar{x} \rightarrow 393 \rightarrow 407$

$\bar{x} = 393$

$z = \frac{393 - 400}{6.76} \simeq 1.036$

$\bar{x} = 407$

$= \frac{407 - 400}{6.76} \simeq 1.036$

$P(393 \leq \bar{x} \leq 407)$

$= 0.8507 - 0.1493$

$\boxed{(393, 407) = 0.7014}$

(d) $P(\bar{x} \leq 389)$

$z = \frac{389 - 400}{6.76} = -1.628$

$= 0.0519$

$\boxed{P(\bar{x} \leq 389) = 0.0159}$

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Stats Quiz 2

Pg No: 7

classmate

Date

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Q.21

$n = 50$ = sample size

$\mu = 0.9$

$\sigma = 9.7$

(a) 95% confidence interval

$$= \mu \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

for 95% $Z_{\alpha/2}\left[1-\left(\frac{1-0.95}{2}\right)\right] = 1.96$

$$= 9.7 \pm 1.96\left(\frac{6.2}{\sqrt{50}}\right)$$

$$= 9.7 \pm 1.718$$

95% CI = (7.982, 11.41)

(b) 90%

$$= \mu \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$= 9.7 \pm 2.57 \times \left(\frac{6.2}{\sqrt{50}}\right)$$

$$= 9.7 \pm 2.2599$$

90% = (7.440, 11.959)

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Stats Quiz 2

Pg No: 8

Q.16

Sample : $n = 5$

(a) at least 80
total fav. outcomes : $\frac{21}{100}$

$= 0.21$

$= (1-p)^5$

$= (1-0.21)^5$

$= (0.79)^5$

$P(X \geq 80) = 0.307$

(b) $(1 \leq j \leq 5)$

$P(1 \leq j \leq 5) = \frac{1}{5} = 0.20$

(c) $P(100) =$

$= (1 - 0.01)$