Discrete → Bernoulli, Binomial, Poisson,
→ D Uniform

Continuous → Normal, Uniform, exponential

$$\text{Experiment} \begin{cases} \nearrow 0 \\ \searrow 1 \end{cases}$$

Bernoulli trials

$X \sim \text{Bern}(p)$

$\underline{P(X=1) = p}$, $P(X=0) = 1-p$

$$X = X_1 + X_2 + \cdots + X_n$$

$$P(X_1 = 1) = p$$

$$P(X_2 = 1) = p$$

$$i = 1, 2, \ldots, n$$

$$X \rightarrow 0, 1, 2, \ldots, n$$

$$X \sim Bin(n, p)$$

$\downarrow$

# of success in n bernoulli trials where p is the prob of success at

each trial.

Prob : A lottery ticket → 'p' ← win

Gambler → 3 ticket →

↳ thought as this will triple the chance of wining

? The $\boxed{\text{distribution}}$ of how many of the 3 tickets are winning tickets.

X → # of tickets out of 3 that came as winning

↳ 0, 1, 2, 3

$X = 0 \longrightarrow$ Gambler didnot win anything

$X = 1 \longrightarrow$ wins 1 out of 3

$X = 2 \longrightarrow$ " 2 out of 3

$X = 3 \longrightarrow$ won all lottery

$X \sim Bin(3, p)$ P(winning a lottery) $= p$

Q: What P(atleast one of the 3 tickets)

$\sim Bin(n, k)$

$X \sim$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$Bin(3, p)$

$$P(X=k) = \binom{3}{k} p^k (1-p)^{3-k}$$

$X \rightarrow$ atleast one

$P(\text{atleast one lottery } \ldots)$

$= P(X \geq 1)$

$= 1 - P(X < 1)$ $\quad X \rightarrow 0, 1, 2, 3$

$= 1 - P(X=0)$ $\quad X < 1 \Rightarrow X = 0$

$$= 1 - \binom{3}{0} p^0 (1-p)^{3-0}$$

$$= 1 - (1-p)^3$$

$$= 1 - (1 - 3p + 3p^2 - p^3)$$

$$= 3p - 3p^2 + p^3$$

Prob : (Poisson)

Book → typographical error on a

single → 0, 1, 2, 3, ...

$\sim$ Poisson with parameter $\lambda = \frac{1}{2}$

P(at least one error in this page)

$X \sim \text{Poisson}(\lambda = \frac{1}{2})$

$\hookrightarrow 0, 1, 2, 3, \ldots$

$\hookrightarrow$ # of errors in this page

P(atleast one error)

$= \boxed{P(X \geq 1)}$

$= 1 - P(X < 1)$

$\underline{X < 1 \Rightarrow X = 0}$

$$= 1 - P(X=0)$$

$$X \sim Pois(\lambda) \quad P(X=k) = e^{-\lambda}\frac{\lambda^k}{k!}$$

$$= 1 - e^{-1/2}$$

Prob :- A website on average receive $\lambda = 6$ visits / min. Find the probability that in 5 minutes, the site receives between 25 and 35 visits (Inclusive).

Sol^n :- $X \rightarrow$ # of visits in 5 minutes

since the average was $\lambda = 6/\text{min}$

the average for 5 minutes $= 6 \times 5$

$= 30$

$$X \sim \text{Pois}(30)$$

$$P(25 \le X \le 35)$$

$$= P(X=25) + P(X=26) + \cdots + P(X=35)$$

$$P(X=k) = \frac{e^{-\lambda} \lambda^k}{k!}$$

$$= e^{-30}\frac{30^{25}}{25!} + e^{-30}\frac{30^{26}}{26!} + \cdots + e^{-30}\frac{30^{35}}{35!}$$

$$= \sum_{k=25}^{35} e^{-30}\frac{30^{k}}{k!}$$

Discrete $\rightarrow$ PMF | Cont $\rightarrow$ PDF

$$\boxed{X \sim N(10, 36)}$$

$$\underline{\mu = 10} \quad \sigma^2 = 36$$

(a) $P(X > 5)$

$X \sim N(\mu, \sigma^2)$

$\Rightarrow X - \mu \sim N(0, \sigma^2)$

$\Rightarrow \dfrac{X-\mu}{\sigma} \sim N(0,1)$

(Z)

$= P\left(\dfrac{X-10}{6} > \dfrac{5-10}{6}\right)$

$= P\left(Z > -5/6\right)$

$\sim N(0,1)$

$X \sim N(\mu, \sigma^2)$

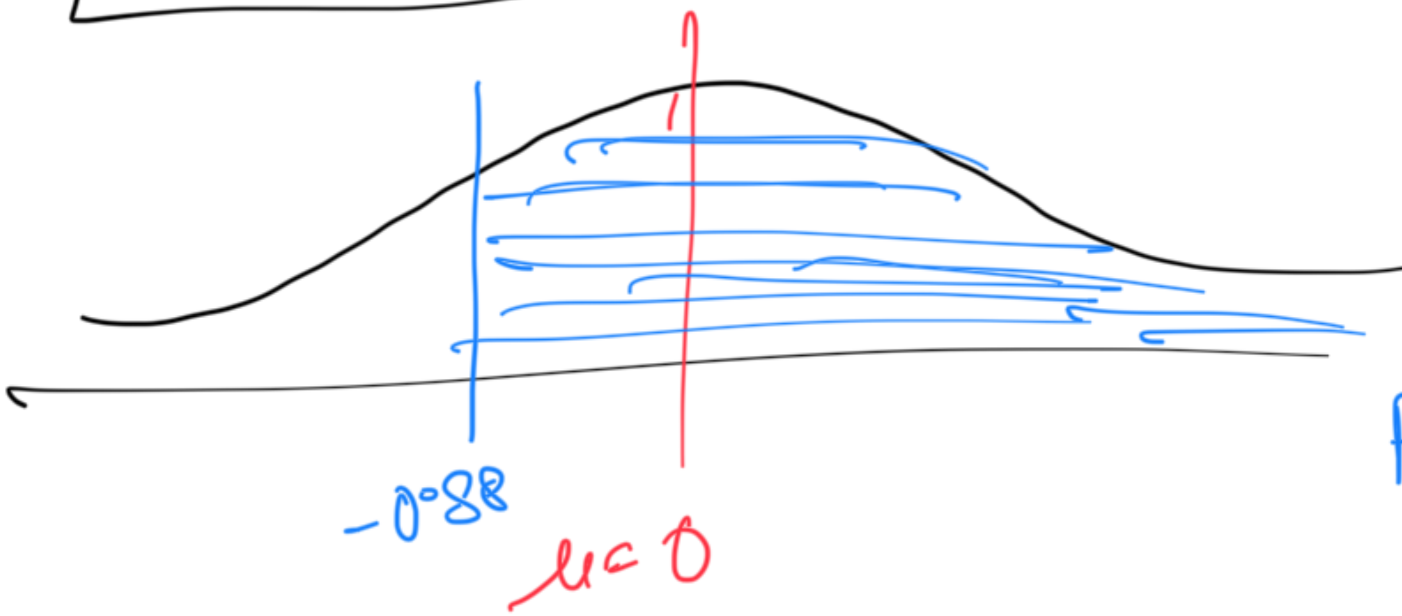
$\Rightarrow$ convert this X ist standard normal

$N(0,1)$

$$Z = \frac{X - \mu}{\sigma} \sim N(0,1)$$

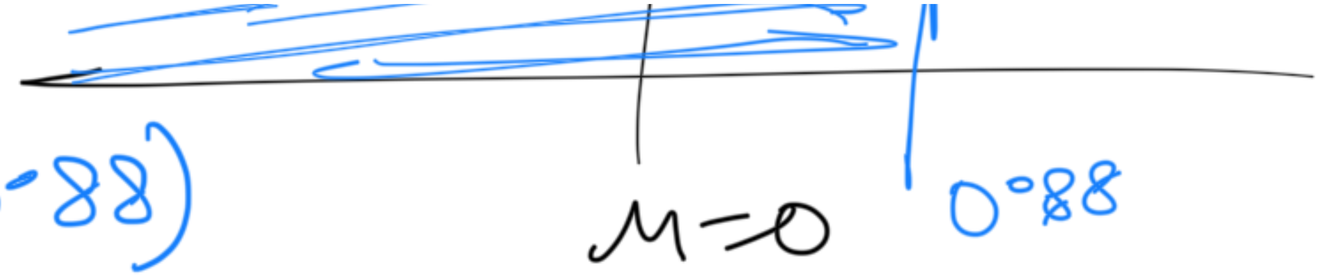
$$= P\left(Z > -5/6\right)$$

$$= \boxed{P\left(Z > -0.833\right)}$$



$P(Z > -0.88)$

$P(Z < 0.88)$



$$= P(Z < 0.833)$$

$$= \Phi(0.833) = .7967$$

Q:- $X \sim$ Lognormal distⁿ

if $\boxed{\log X} \sim$ Normally distribution

$E[\log X] = \mu$ and $Var(\log X) = \sigma^2$

Find the distribution function X.

Soln.

CDF of X

$F_X(x) = P(X \le x)$

$F_X(x)$ CDF of r.v X

what we have → information on $\log X$

$= P(\underline{\log X} \le \log x)$

$\log X \sim$ mean $= \mu$ and variance $= \sigma^2$

$$= P\left(\boxed{\frac{\log X - \mu}{\sigma}} \le \frac{\log x - \mu}{\sigma}\right)$$

$$\log X \sim N(\mu, \sigma^2) \Rightarrow \frac{\log X - \mu}{\sigma} \sim N(0,1)$$

$$= P\left(Z < \frac{\log x - \mu}{\sigma}\right)$$

CDF of std normal

ϕ

CDF

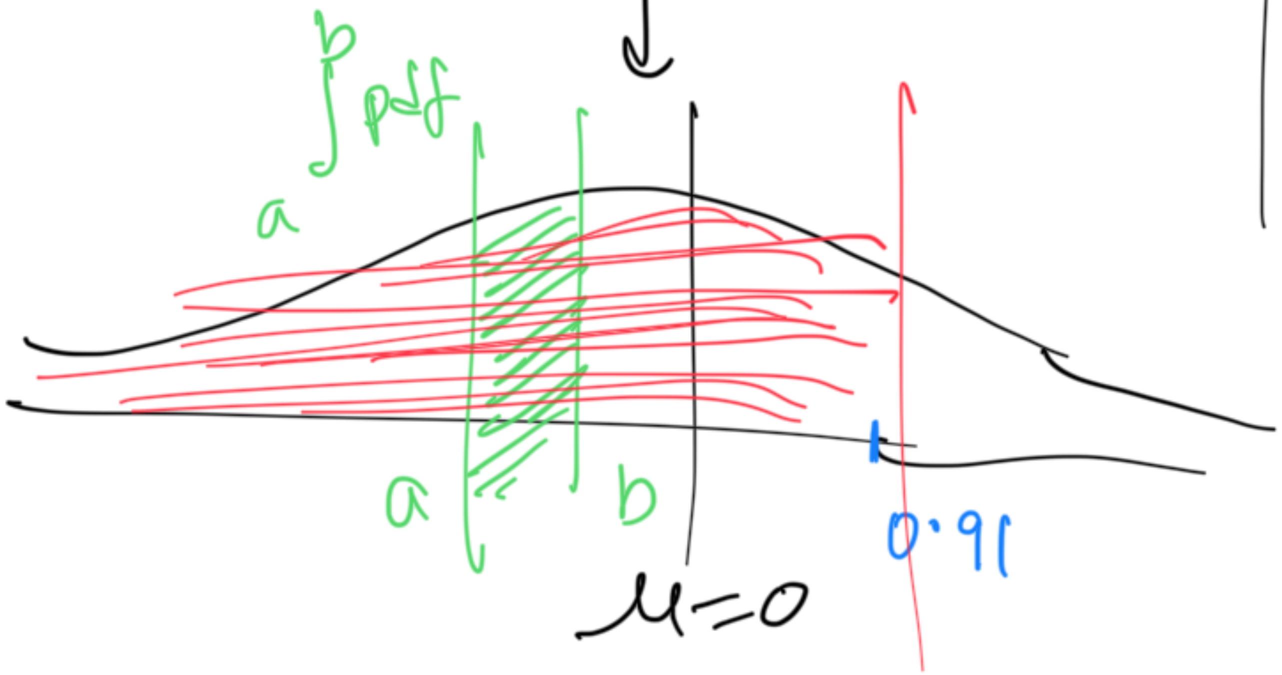
$$= \phi\left(\frac{\log x - \mu}{\sigma}\right)$$

$$= \int^{\frac{\log x - \mu}{\sigma}} \frac{1}{\sqrt{\quad}} e^{-\frac{1}{2}u^2} \, du$$

$\sqrt{2\pi}$

$-\infty$

CDF of a r.v.



pdf → $f_X(x)$

probability of any events associated with the r.v.

$$\int_a^b f_X(x) = P(X \in (a,b))$$

$$F_X(0.91) \quad P(X < 0.91)$$

$$\int^{0.91} f_X(x)\,dx$$

$-\infty$

<u>CDF</u> :- let X be a continuous with PDF f. Then the CDF of X is given by

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

accumulated area under the PDF.

till a

$$P(a < X \leq b) = P(-\infty < X \leq b) - P(-\infty < X \leq a)$$

$$= F(b) - F(a)$$

$$= \int_a^b f(x)\, dx$$