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THEORY AND APPLICATIONS OF SOFT COMPUTING METHODS

Development of prediction models for shear strength of SFRCB using a machine learning approach

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Abstract In this study, new design equations were derived for the assessment of shear resistance of steel fiber-reinforced concrete beams (SFRCB) utilizing multi-expression programming (MEP). The superiority of MEP over conventional statistical techniques is due to its ability in modeling of mechanical behavior without a need to predefine the model structure. The MEP models were developed using a comprehensive database obtained through an extensive literature review. New criteria were checked to verify the validity of the models. A sensitivity analysis was carried out and discussed. The MEP models provide good estimations of the shear strength of SFRCB. The developed models significantly outperform several equations found in the literature.

Keywords SFRCB · Multi-expression programming · Shear strength · Prediction

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1 Introduction

Fiber-reinforced concrete (FRC) is made of portland cement, aggregate and discrete discontinuous fibers. Adding steel fibers into the concrete mix increases its shear strength and enhances the ductile behavior [1, 2]. Using fiber reinforcement may remarkably reduce construction time and costs as conventional stirrups require relatively high labor input to bend and fix in place. Adding fibers to high-strength concrete mix overcomes the relative brittleness and lack of ductility of high-strength concrete [3]. Strength of steel FRC is dependent on several parameters [4]. The fibers have different influence on concrete properties because of their different mechanical properties [5]. Numerous studies have been conducted on rectangular, steel fiber-reinforced concrete beams (SFRCB) without stirrups. Several researchers have considered the possibility of using FRC reinforced by the fibers. Table 1 presents some of the empirical relationships developed to evaluate the ultimate shear strength of SFRCB [2].

Machine learning techniques such as artificial neural networks (ANNs) are widely used for modeling civil engineering problems [13–18]. Recently, ANNs have been applied to the evaluation of the SFRCB [19, 20]. A limitation of ANNs is that the user should define their structure and parameters before modeling which may be a time-consuming procedure.

Genetic programming (GP) [21] is a fairly new machine learning approach for developing nonlinear regression equations [22–25]. Multi-expression programming (MEP) [26] is a new variant of GP. Oltean and Grosan [27] showed that the advantage of MEP over other modeling techniques. There are few applications of this powerful tool in civil engineering domain [28, 29]. It is worth mentioning that MEP possesses obvious advantage over traditional

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Table 1 Empirical relationships for predicting the ultimate shear strength of SFRCB

No.	References	Equations
1	Khuntia et al. [3]	$v_{\mathrm{frc}} = (0.167\alpha + 0.25F)\sqrt{f'_{\mathrm{c}}}$
2	Li et al. [4]	$v_{\mathrm{frc}} = \begin{cases} 1.25 + 4.68\left(f_{\mathrm{f}} f_{\mathrm{spfc}}\right)^{\frac{3}{4}}\left(\rho\frac{d}{a}\right)^{\frac{1}{3}} d^{-\frac{1}{3}} & \text{For FRC} \\ 0.53 + 5.47\left(f_{\mathrm{f}} f_{\mathrm{spfc}}\right)^{\frac{3}{4}}\left(\rho\frac{d}{a}\right)^{\frac{1}{3}} d^{-\frac{1}{3}} & \text{For FRM} \end{cases}$
3	Kwak et al. [6]	$v_{\mathrm{frc}} = 2.1 e {f_{\mathrm{spfc}}}^{0.7}\left(\rho\frac{d}{a}\right)^{0.22} + 0.8{v_{\mathrm{b}}}^{0.97}$
4	Kwak et al. [6]	$v_{\mathrm{frc}} = 3.7 e {f_{\mathrm{spfc}}}^{\frac{2}{3}}\left(\rho\frac{d}{a}\right)^{\frac{1}{3}} + 0.8 v_{\mathrm{b}}$
5	Swamy et al. [7]	$v_{\mathrm{frc}} = 0.37\tau V_{\mathrm{f}}\frac{L_{\mathrm{f}}}{D_{\mathrm{f}}} + v_{\mathrm{c}} \times \begin{cases} 1 & \frac{a}{d} \geq 2 \\ 2\frac{d}{a} & \frac{a}{d} < 2 \end{cases}$
6	Sharma [8]	$v_{\mathrm{frc}} = k f_{\mathrm{t}}\left(\frac{d}{a}\right)^{\frac{1}{4}}$ where $k = \begin{cases} 1 & \text{If } \mathrm{f}_t \text{ is obtained by direct tension test} \\ 2/3 & \text{If } f_{\mathrm{t}} \text{ is obtained by indirect tension test} \\ 4/9 & \text{If } f_{\mathrm{t}} \text{ is obtained by modulus of rupture} \end{cases}$
7	Narayanan and Darwish [9]	$v_{\mathrm{frc}} = e\left(0.24 f_{\mathrm{spfc}} + 80\rho\frac{d}{a}\right) + v_{\mathrm{b}} \qquad e = \begin{cases} 1 & \frac{a}{d} > 2.8 \\ 2.8\frac{d}{a} & \frac{a}{d} \leq 2.8 \end{cases}$
8	Ashour et al. [10]	$v_{\mathrm{frc}} = \begin{cases} \left(2.11\sqrt[3]{f_{\mathrm{c}}} + 7F\right)\left(\rho\frac{d}{a}\right)^{\frac{1}{3}} & \text{For } \frac{a}{d} \geq 2.5 \\ \left(2.11\sqrt[3]{f_{\mathrm{c}}} + 7F\right)\left(\rho\frac{d}{a}\right)^{\frac{1}{3}}\frac{5d}{2a} + v_{\mathrm{b}}\left(2.5 - \frac{a}{d}\right) & \text{For } \frac{a}{d} < 2.5 \end{cases}$
9	Ashour et al. [10]	$v_{\mathrm{frc}} = \left(0.7\sqrt{f'_{\mathrm{c}}} + 7F\right)\frac{d}{a} + 17.2\rho\frac{d}{a}$
10	Shin et al. [11]	$v_{\mathrm{frc}} = \begin{cases} 0.19 f_{\mathrm{spfc}} + 93\rho\frac{d}{a} + 0.34\tau F & \text{For } \frac{a}{d} \geq 3 \\ 0.22 f_{\mathrm{spfc}} + 217\rho\frac{d}{a} + 0.34\tau F & \text{For } \frac{a}{d} < 3 \end{cases}$
11	Mansur et al. [12]	$v_{\mathrm{frc}} = 0.41\left(\tau V_{\mathrm{f}}\frac{L_{\mathrm{f}}}{D_{\mathrm{f}}}\right) + \left(0.16\sqrt{f_{\mathrm{c}}} + 17.2\frac{\rho V d}{M}\right)$

v_{frc} (MPa): ultimate shear strength; V_{f} (%): fiber volume fraction; $L_{\mathrm{f}}/D_{\mathrm{f}}$: fiber aspect ratio; ρ (%): flexural steel reinforcement ratio; f'_{c} (MPa): cylinder compressive strength of concrete; *a/d*: shear span–depth ratio. τ: average fiber matrix interfacial bond stress = 4.15 MPa; v_{c}: concrete contribution to shear strength and calculated according to ACI design code $= 0.167\sqrt{f'_{\mathrm{c}}}$; f_{spfc} (MPa) computed value of split-cylinder strength of fiber concrete; f_{t} (MPa): splitting tensile strength; f_{f} (MPa) modulus of rupture; α_1 = coefficient representing the fraction of bond mobilized at first matrix cracking = 0.5 and α_2: efficiency factor of fiber orientation in the uncracked state of the composite = 1; V (N): shear force at section; M (N mm): bending moment at section; v_{b} (MPa): 0.41 τF; F: fiber factor ($d_{\mathrm{f}}(L_{\mathrm{f}}/D_{\mathrm{f}})\,V_{\mathrm{f}}$); d_{f}: bond factor

statistical modeling techniques because it does not need a preliminary definition of the form of the existing relationships. Moreover, optimal MEP-based models are usually obtained after developing numerous linear and highly nonlinear models [24].

In this research, MEP is utilized to derive empirical prediction models for the shear capacity of SFRCB without stirrups. The derived models relate the shear strength to a couple of influencing parameters. The models were established based on several published shear tests on SFRCB.

2 Multi-expression programming

GP is a branch of evolutionary algorithms (EAs). It uses the natural selection concept to evolve computer models. GP uses many of the operators of genetic algorithms (GAs). The solutions created by GP are encoded in a functional programming language and have a tree-shape structure [2]. Linear and graph are other types of GP [2]. MEP is a linear division of GP developed by Oltean and Dumitrescu [26]. The MEP solutions are presented as linear strings [27]. Several programs can be evolved in a single MEP. The algorithm first randomly creates a number of computer programs and then uses genetic operators such as mutation and crossover to find the optimal model [28]. An example of a program generated by MEP is as given below:

0: *l*
1: *m*
2: ×0, 1
3: *n*
4: +2, 3
5: *o*
6: /4, 5
7: ×1, 6

In this example, genes 0, 1, 3 and 5 encode simple expressions: $\mathrm{Program}_0 = l$, $\mathrm{Program}_1 = m$, $\mathrm{Program}_3 = n$, $\mathrm{Program}_5 = o$. The operation × is applied to positions 0 and 1 ($\mathrm{Program}_2 = l \times m$). The operation + is later applied to genes 2 and 3 ($\mathrm{Program}_4 = (l \times m) + n$). This process continues until the last operation, and finally, gene 7 provides the final model ($\mathrm{Program}_7 = l \times ((l \times m) + n)/o$. For k genes, the fitness of each program can be evaluated using the following equation [27, 28]:

Table 2 Descriptive statistics of the independent variables used in the modeling process

Parameter	a (mm)	d (mm)	f_c' (MPa)	ρ (%)	L_f/D_f	V_f (%)	d_f	v_u (MPa)
Mean	633.142	203.061	47.221	2.811	79.427	0.827	0.728	3.929
Standard error	39.344	7.358	1.377	0.089	1.677	0.030	0.010	0.178
SD	574.203	107.390	20.100	1.305	24.478	0.435	0.139	2.600
Sample variance	329,709.5	11532.5	404.0	1.7	599.2	0.19	0.019	6.76
Range	3517.870	490.000	90.900	9.500	108.333	1.780	0.500	13.610
Minimum	120	80	20.6	1	25	0.22	0.5	1.44
Maximum	3637.87	570	111.5	10.5	133.333	2	1	15.05

$$f = \min_{i=1,k}\left\{\sum|P - O|\right\} \tag{12}$$

where P is the measured value and O is the predicted value.

3 Deriving simplified design equations for shear strength of SFRCB

The shear strength of SFRCB without stirrups is influenced by several parameters. These parameters were determined based on previous studies [2–12]. Accordingly, the formulation of the shear strength (v_u) in MPa was taken:

$$v_u = f\left(\frac{a}{d}, \rho, v_b, f_c', f_t', f_{spfc}'\right) \tag{13}$$

where a/d = shear span–depth ratio; d = effective depth of beam, mm; f_c' = cylinder compressive strength of concrete, MPa; ρ = flexural steel reinforcement ratio, %; $v_b = 0.41\ \tau F$, MPa; τ = average fiber matrix interfacial bond stress, taken as 4.15 MPa [30]; F = fiber factor $(d_f$ (L_f/D_f) $V_{f)}$; L_f/D_f = fiber aspect ratio; L_f = fiber length, mm; D_f = fiber diameter, mm; V_f = volume fraction of steel fibers, %; d_f = bond factor: 0.50 for round fibers, 0.75 for crimped fibers and 1.00 for indented fibers [9]; f_t' = splitting tensile strength ($0.79\ \sqrt{f_c'}$), MPa; f_{spfc} = split-cylinder strength of fiber concrete:

$$f_{spfc} = \frac{f_{cuf}}{(20-F)} + 0.7 + 1.0\sqrt{F}\ \text{(MPa)} \tag{14}$$

where f_{cuf} (MPa) is cube strength of fiber concrete. As it is seen, the formulation of v_u takes into account the contributions of various geometrical and mechanical factors.

3.1 Experimental database

The comprehensive database used for developing the models was gathered through an extensive literature review. The collected data included 208 test results for the SFRCB without shear reinforcement [2–12, 31–47]. The ranges and statistics of the variables are given in Table 2. The histograms of the data are presented in Fig. 1. As can be observed in this figure, the database contains experimental results for both normal-strength concrete (NSC) and high-strength concrete (HSC) beams.

3.2 MEP-based formulation for shear strength of SFRCB

Two MEP models were obtained for the shear strength of the NSC and HSC beams. Also, a combined model was developed for both of these concrete types. To avoid overfitting, the data sets were randomly divided into learning, validation and testing subsets [2]. The statistical properties of the parameters in each category were consistent. Out of the 104 data sets for each of the NSC and HSC types, 84 values were taken for the training process (learning 70 records, validation 14 records). Twenty records were used for the testing of the models. Of the total of 208 data sets for NSC and HSC, 168 values were taken for the training process (learning 140 records, validation 28 records). Also, 40 sets were taken for the model testing.

Table 3 shows the parameter settings for the MEP algorithm. Different parameters were taken for the algorithm to explore the optimal model. This was based on the values recommended by other researchers [24, 28, 29], as well as a number of preliminary runs. As can be seen in Table 3, 3 values were checked for the population size and number of generation. The chromosome length was set to 50. There are $3 \times 3 = 9$ parameter combinations. Five replications were done for each combination. Therefore, the overall number of runs was equal to 45×5 (number of the input combinations) $\times$ 3 (generic, NSC and HSC formulations) = 675. The program was run until it terminated automatically. The source code of MEP [48] in C++ was used for the analysis.

Three criteria were considered for the selection of the optimal mode: simplicity, good performance of the learning data and good performance of the validation data. Parameters like chromosome length can control the first criterion. For the performance criteria, an objective function (OBJ) was used [49]:

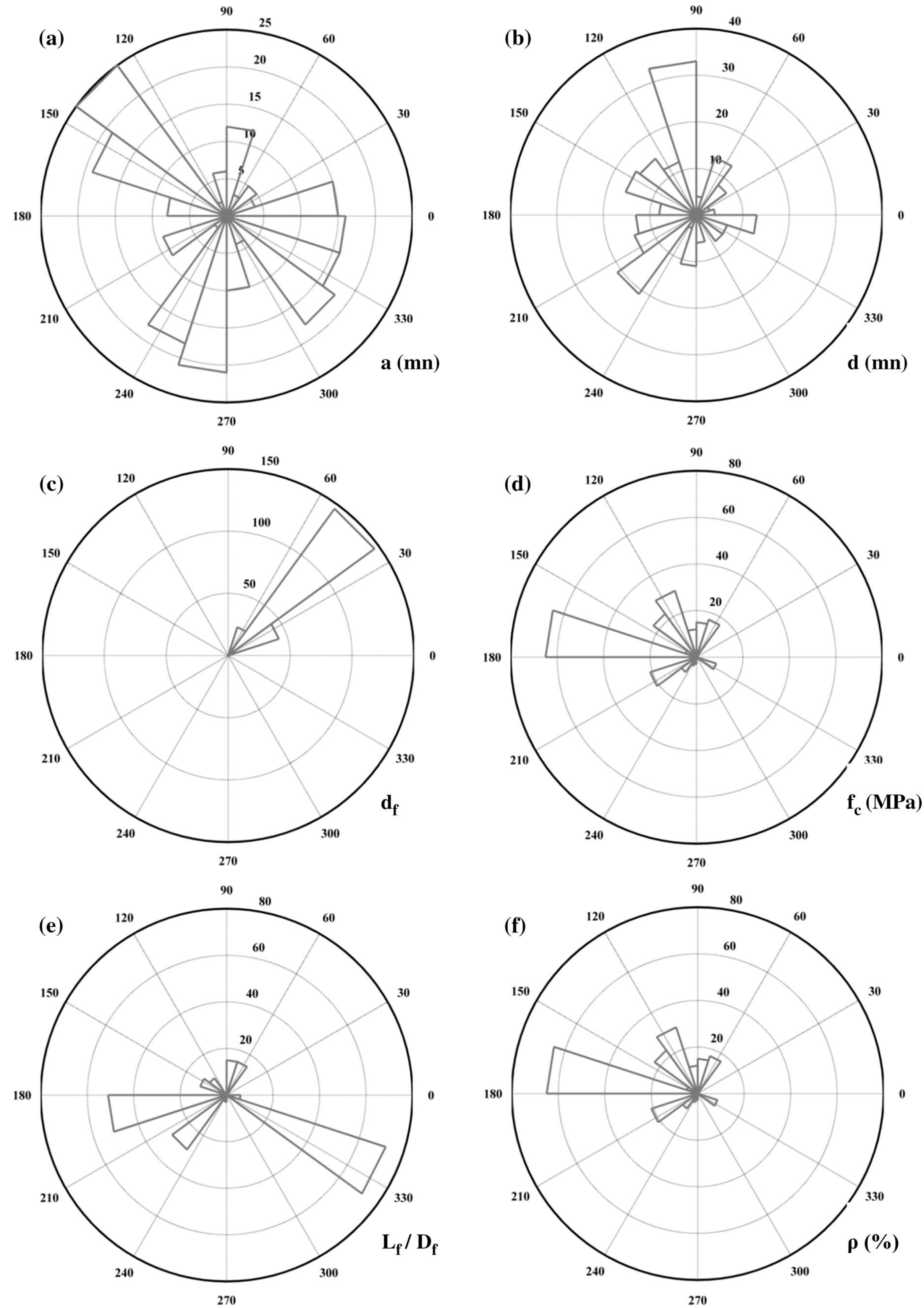


Fig. 1 Rose histograms of the variables

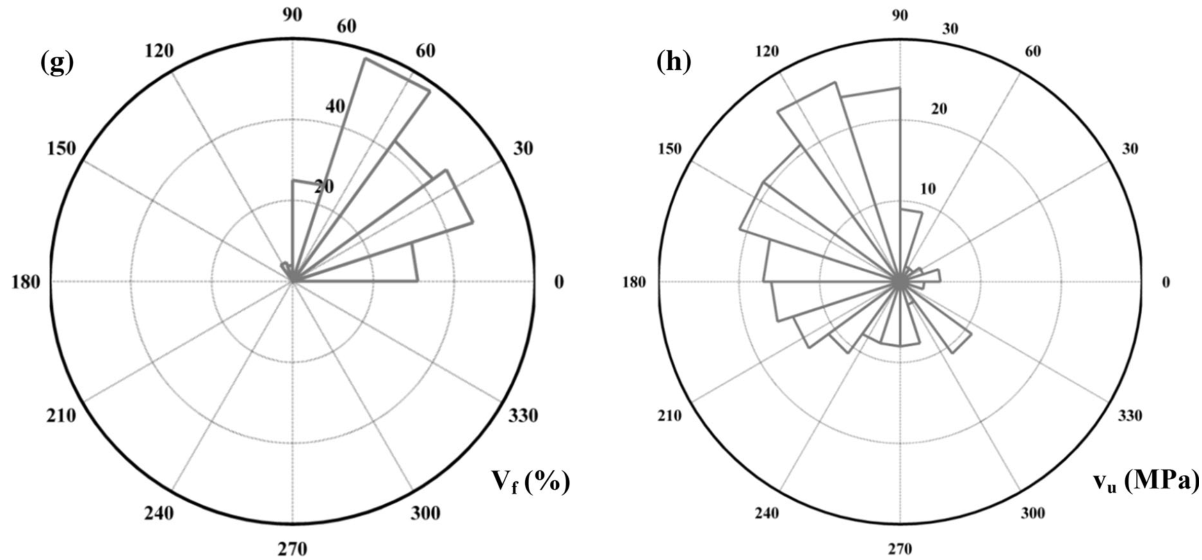


Fig. 1 continued

Table 3 Parameter settings for the MEP algorithm

Parameter	Setting
Population size	100, 250, 500
Number of generations	250, 500, 1000
Crossover probability (%)	90
Crossover type	Uniform
Mutation probability (%)	10
Chromosome length	50 genes
Function set	+, −, ×, /
Terminal set	Problem input
Replication number	5

$$\mathrm{OBJ} = \left(\frac{\mathrm{No.}_{\mathrm{Learning}} - \mathrm{No.}_{\mathrm{Validation}}}{\mathrm{No.}_{\mathrm{Training}}}\right)\frac{\mathrm{MAE}_{\mathrm{Learning}}}{R^2_{\mathrm{Learning}}} + \frac{2\mathrm{No.}_{\mathrm{Validation}}}{\mathrm{No.}_{\mathrm{Training}}}\frac{\mathrm{MAE}_{\mathrm{Validation}}}{R^2_{\mathrm{Validation}}} \tag{15}$$

where No. means the number of data in that category. Correlation coefficient (R) and mean absolute error (MAE) were defined as follows:

$$R = \frac{\sum_{i=1}^{n}\left(h_i - \overline{h}\right)\left(t_i - \overline{t}\right)}{\sqrt{\sum_{i=1}^{n}\left(h_i - \overline{h}\right)^2 \sum_{i=1}^{n}\left(t_i - \overline{t}\right)^2}} \tag{16}$$

$$\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n}|h_i - t_i| \tag{17}$$

in which h_i and t_i are, respectively, the measured and predicted outputs for the ith output, $\overline{h}$ and $\overline{t}$ are, respectively, the averages of the measured and predicted values, and n is the number of samples. Evidently, higher R and lower MAE values lead into a lower OBJ and a more precise model. The best MEP models for predicting the shear strength of NSC and HSC beams were built using a/d, ρ, v_b and f_t'. These four parameters seem to be sufficient representatives of the geometrical parameters and mechanical properties of SFRCB. The MEP-based formulations of the shear strength of SFRCB, v_u, is as follows:

$$v_u(\mathrm{MPa}) = \begin{cases} \dfrac{f_t' + v_b}{\frac{a}{d} - +\rho + \frac{3\rho}{v_b}\left(v_b + 2 + \frac{a}{d} - f_t + 4\rho f_t\right)} + v_b, & f_c' < 41.4(\mathrm{NSC}) \\ \dfrac{1}{\frac{a}{d}}\left(2f_t' + \dfrac{\frac{a}{d}}{\rho + \rho(4 + v_b)\left(\frac{a}{d} + \rho\right)\left(-1 - \frac{a}{d}\right)} - 2\right) + v_b, & f_c' > 41.4(\mathrm{HSC}) \end{cases} \tag{18}$$

$$v_u(\mathrm{MPa}) = \rho + \frac{\rho}{v_b} + \frac{1}{\frac{a}{d}}\left(\frac{\rho f_t(\rho + 2)\left(f_t'\frac{a}{d} - \frac{3}{v_b}\right)}{\frac{a}{d}} + f_t'\right) + v_b, \quad (\mathrm{NSC}\quad \mathrm{and}\quad \mathrm{HSC}) \tag{19}$$

Figures 2 and 3 show the predictions made by MEP.

4 Discussion

According to Smith [50], for $|R| > 0.8$, the correlation between the predicted and measured values is good.

Referring to Figs. 2 and 3, the MEP models with high R and low MAE, and therefore low OBJ, provide good predictions of the shear strength. The models separately developed for the NSC and HSC beams (Eq. 18) provide

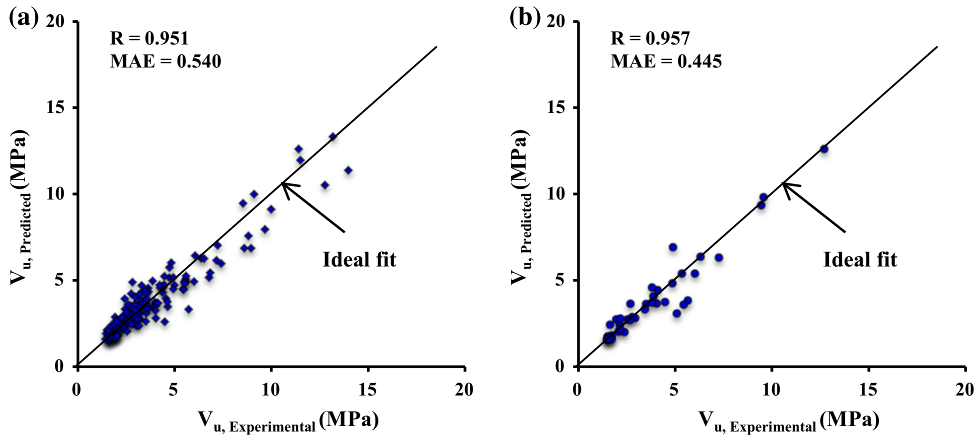


Fig. 2 Performance of the MEP model, Eq. (18): **a** training data, **b** testing data

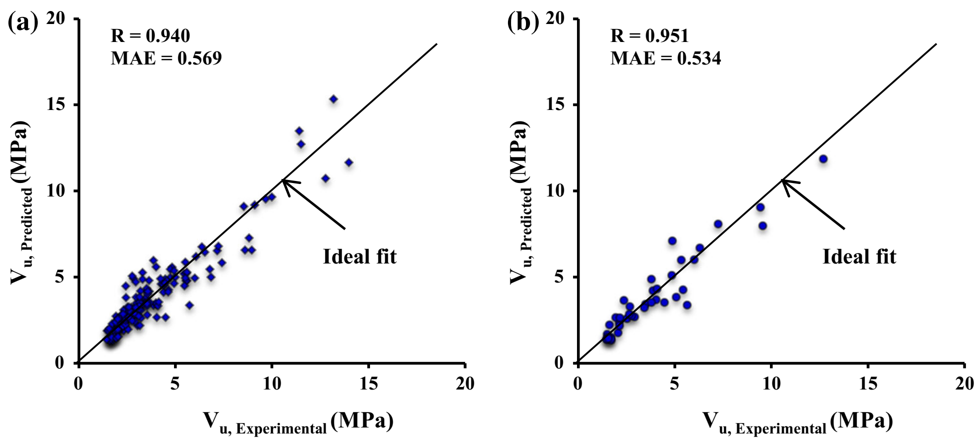


Fig. 3 Performance of the MEP model, Eq. (19): **a** training data, **b** testing data

Table 4 External validation of the MEP models

Item	Formula	Conditions	MEP, Eq. (18)	MEP, Eq. (19)
1	R	$R > 0.8$	0.951	0.957
2	$k = \frac{\sum_{i=1}^{n}(h_i \times t_i)}{h_i^2}$	$0.85 < K < 1.15$	1.001	1.000
3	$k' = \frac{\sum_{i=1}^{n}(h_i \times t_i)}{t_i^2}$	$0.85 < K' < 1.15$	0.971	0.975
4	$m = \frac{R^2 - Ro^2}{R^2}$	$\lvert m \rvert < 0.1$	−0.106	−0.091
5	$n = \frac{R^2 - Ro'^2}{R^2}$	$\lvert n \rvert < 0.1$	−0.103	−0.089
6	$R_m = R^2 \times \left(1 - \sqrt{\lvert R^2 - Ro^2 \rvert}\right)$	$R_m > 0.5$	0.623	0.651
where	$Ro^2 = 1 - \frac{\sum_{i=1}^{n}(t_i - h_i^o)^2}{\sum_{i=1}^{n}(t_i - \overline{t_i})^2}, \quad h_i^o = k \times t_i$		1.000	1.000
	$Ro'^2 = 1 - \frac{\sum_{i=1}^{n}(h_i - t_i^o)^2}{\sum_{i=1}^{n}(h_i - \overline{h_i})^2}, \quad t_i^o = k' \times h_i$		0.997	0.998

k or k': slope of regression lines; m and n: performance indexes; R_m: confirm indicator; Ro^2: squared correlation coefficient (through the origin) between predicted and experimental values, Ro'^2: squared correlation coefficient coefficient between experimental and predicted values

slightly better results than the MEP generic model obtained for both NSC and HSC (Eq. 19).

Table 4 shows new criteria recommended by Golbraikh and Tropsha [51] and Roy and Roy [52] for model verification (on testing data). As can be seen in this table, the derived models satisfy the required conditions. The exception is for the first MEP model (Eq. 18) which marginally fails to satisfy the n and m conditions.

Figure 4 shows the ratios of the experimental shear strength values to the values predicted by the MEP models

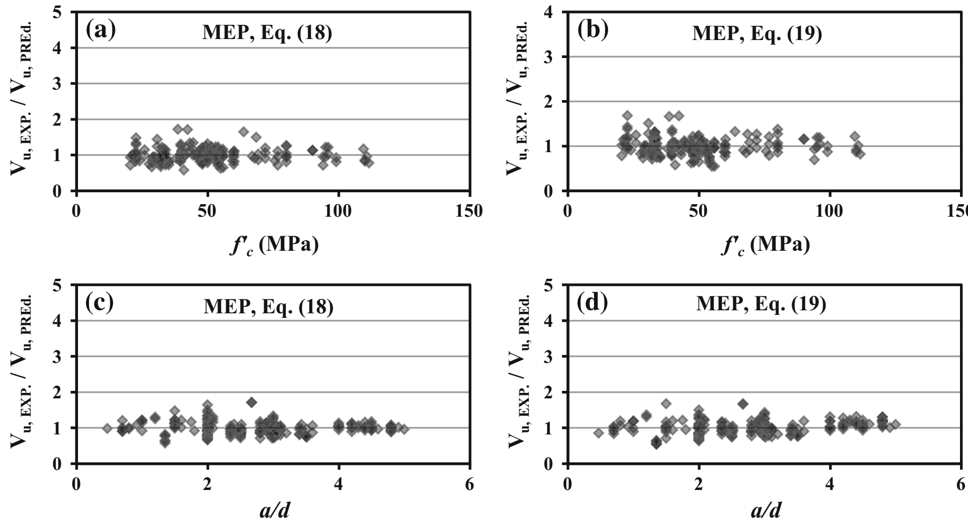


Fig. 4 Ratio of the experimental to the predicted shear strength values

Table 5 Overall performances of the shear strength prediction models

Model	$v_{Exp.}$ versus $v_{Per.}$			$v_{Exp.}/v_{Per.}$		
	R^2	MAE	RMSE	SD	Ave.	Cov.
Khuntia et al. [3], Eq. (1)	0.8972	1.5417	2.1557	0.4047	1.6485	0.2455
Li et al. [4], Eq. (2)	0.6169	1.1759	1.9308	0.4595	1.1853	0.5077
Kwak et al. [6], Eq. (3)	0.9030	0.8038	1.6795	0.2075	0.9867	0.2103
Kwak et al. [6], Eq. (4)	0.8890	0.9594	2.2906	0.2265	1.0057	0.2252
Swamy et al. [7], Eq. (5)	0.7458	1.5015	2.2140	0.4679	1.5882	0.4762
Sharma [8], Eq. (6)	0.5963	1.2135	2.0841	0.5167	1.2552	0.5036
Narayanan and Darwish [9], Eq. (7)	0.7946	1.3638	4.0000	0.2949	1.1432	0.2580
Ashour et al. [10], Eq. (8)	0.7500	1.2465	2.5579	0.3302	1.1914	0.3405
Ashour et al. [10], Eq. (9)	0.8154	0.9440	1.5691	0.2861	1.0443	0.3544
Shin et al. [11], Eq. (10)	0.6861	0.9714	2.0949	0.2816	1.0932	0.2733
Mansur et al. [12], Eq. (11)	0.3744	1.5804	2.5528	0.7169	1.6540	0.5504
MEP, Eq. (18)	0.9063	0.5198	0.7333	0.1834	0.9925	0.1848
MEP (Generic), Eq. (19)	0.8875	0.5601	0.8042	0.1953	1.0039	0.1954

RMSE root mean squared error, *SD* standard deviation, *Cov* covariance

with respect to a/d and f'_c. As it is seen, the models have a good performance.

Besides, Table 5 presents a comparison of the shear strength predictions made by the MEP models and various empirical models found in the literature [3, 4, 6–12] for the entire database. Figure 5 visualizes the histogram plots of the ratio of the experimental to the predicted shear strength values. Referring to Table 5 and Fig. 5, the MEP models have a much better performance than the existing models.

Most of the existing empirical models were developed using statistical regression techniques after controlling a limited number of equations, while the solutions evolved by MEP were selected from numerous preliminary models [2]. However, the MEP approach depends on the data to provide better generalization. Hence, the proposed MEP models are mostly applicable for the validity verification of the laboratory test results or for cases where testing is not possible. It is worth mentioning that the MEP algorithm is sensitive to the parameter setting. The performance of MEP can be improved using optimization algorithms, such as GAs, simulated annealing, to optimize the run parameters [53].

5 Sensitivity analysis

A sensitivity analysis was carried out using "frequency values." If an input is appearing in the best 30 models evolved by MEP, its frequency values will be equal to 100 % [28]. The sensitivity analysis results are presented in Fig. 6. As it is seen, the shear strength is more influenced by *a*/*d* and f'_t in comparison with the other variables.

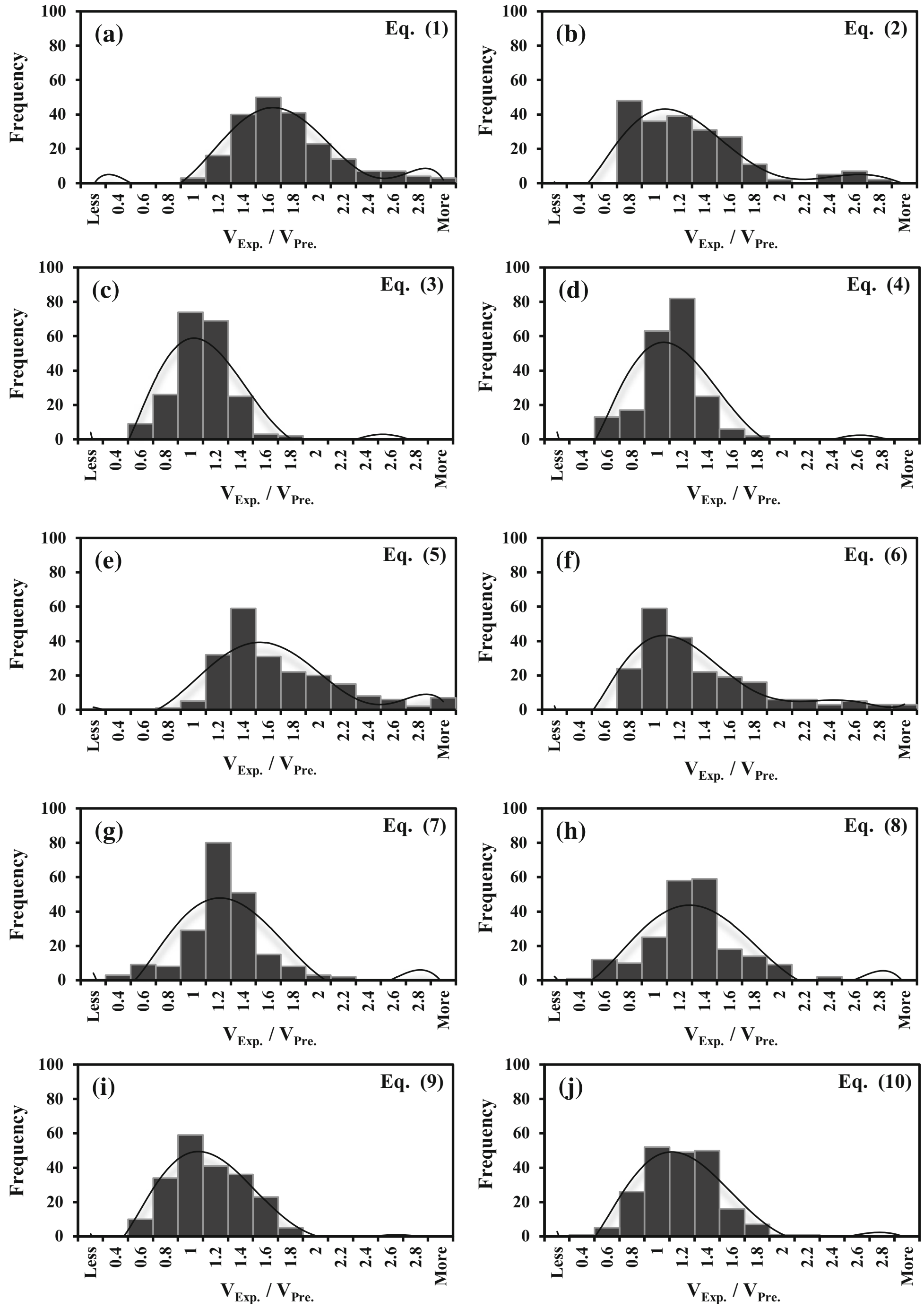


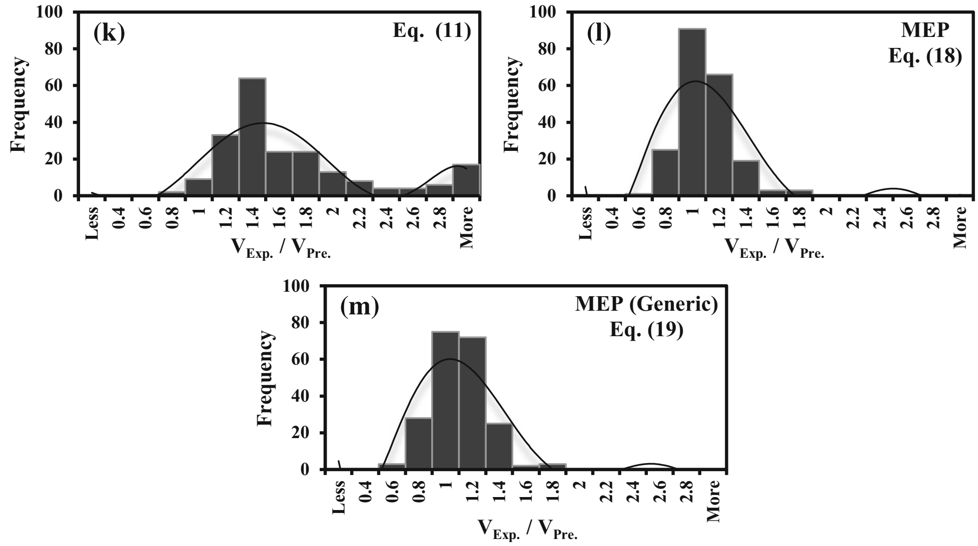
Fig. 5 Histogram of experimental/predicted shear strength values using different models

Further, the shear strength exhibits less sensitivity to the ρ values in both of the models.

6 Conclusions

In this research, new design equations were developed to assess the shear resistance of SFRCB without stirrups using MEP. The proposed models accurately predicted the shear strength of SFRCB. Furthermore, the MEP prediction models efficiently satisfy different external validation phases. The final explanatory variables (a/d, ρ, v_b, f_t') were selected after developing different models with different combinations of the input parameters. The model can be used for practical design purposes since it was derived from tests on beams with a wide range of geometrical and mechanical properties, including the HSC and NSC beams. The proposed models provide better results than the prediction equations

Fig. 5 continued



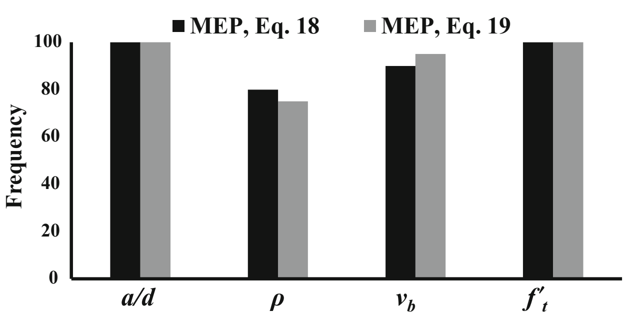


Fig. 6 Sensitivity analysis results

found in the literature. Future research can be devoted to the development of new prediction models for the shear resistance of SFRCB using linear/nonlinear and nonparametric regression methodologies such as multivariate adaptive regression splines (MARS), support vector regression (SVR) or response surface methodology (RSM).

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References

1. Chanh VN (2004) Steel fiber-reinforced concrete. Faculty of Civil Engineering Ho chi minh City University of Technology. Seminar Material, pp 108–116
2. Gandomi AH, Alavi AH, Yun GJ (2011) Nonlinear modeling of shear strength of SFRC beams using linear genetic programming. Struct Eng Mech 38(1):1–25
3. Khuntia M, Stojadinovic B, Goel SC (1999) Shear strength of normal and high-strength fiber reinforced concrete beams without stirrups. ACI Struct J 96(2):282–289
4. Li VC, Ward R, Hamza AM (1992) Steel and synthetic fibers as shear reinforcement. ACI Mater J 89(5):499–508
5. Wang C (2006) Experimental investigation on behavior of steel fiber-reinforced concrete. MSc Thesis. University of Canterbury, New Zealand
6. Kwak YK, Eberhard MO, Kim WS, Kim J (2002) Shear strength of steel fiber-reinforced concrete beams without stirrups. ACI Struct J 99(4):530–538
7. Swamy RN, Jones R, Chiam ATP (1993) Influence of steel fibers on the shear resistance of lightweight concrete I-beams. ACI Struct J 90(1):103–114
8. Sharma AK (1986) Shear strength of steel fiber reinforced concrete beams. ACI J 83(4):624–628
9. Narayanan R, Darwish IYS (1987) Use of steel fibers as shear reinforcement. ACI Struct J 84(3):216–227
10. Ashour SA, Hasanain GS, Wafa FF (1992) Hear behavior of high-strength fiber reinforced concrete beams. ACI Struct J 89(2):176–184
11. Shin SW, Oh J, Ghosh SK (1994) Shear behavior of laboratory-sized high-strength concrete beams reinforced with bars and steel fibers. Fiber reinforced concrete developments and innovations, SP-142. American Concrete Institute, Farmington Hills, pp 181–200
12. Mansur MA, Ong KCG, Paramsivam P (1986) Shear strength of fibrous concrete beams without stirrups. J Struct Eng 112(9):2066–2079
13. Tanyildizi H, Çevik A (2010) Modeling mechanical performance of lightweight concrete containing silica fume exposed to high temperature using genetic programming. Constr Build Mater 24(12):2612–2618
14. Ozbay E, Gesoglu M, Güneyisi E (2008) Empirical modeling of fresh and hardened properties of self-compacting concretes by genetic programming. Constr Build Mater 22(8):1831–1840
15. Sarıdemir M (2010) Genetic programming approach for prediction of compressive strength of concretes containing rice husk ash. Constr Build Mater 24(10):1911–1919
16. Alqedra MA, Ashour AF (2005) Prediction of shear capacity of single anchors located near a concrete edge using neural networks. Comput Struct 83(28–30):2495–2502
17. Sakla SSS, Ashour AF (2005) Prediction of tensile capacity of single adhesive anchors using neural networks. Comput Struct 83(21–22):1792–1803

18. Cabalar AF, Cevik A (2009) Modelling damping ratio and shear modulus of sand-mica mixtures using neural networks. Eng Geol 104:31–40
19. Adhikary BB, Mutsuyoshi H (2006) Prediction of shear strength of steel fiber RC beams using neural networks. Constr Build Mater 20:801–811
20. Ahn N, Jang H, Park DK (2007) Presumption of shear strength of steel fiber reinforced concrete beam using artificial neural network model. J Appl Polym Sci 103:2351–2358
21. Koza J (1992) Genetic programming, on the programming of computers by means of natural selection. MIT Press, Cambridge
22. Sarıdemir M (2011) Empirical modeling of splitting tensile strength from cylinder compressive strength of concrete by genetic programming. Exp Syst Appl 38(11):14257–14268
23. Azamathulla HMd, Guven A, Demir YK (2011) Linear genetic programming to scour below submerged pipeline. Ocean Eng 38(8):995–1000
24. Alavi AH, Gandomi AH (2011) A robust data mining approach for formulation of geotechnical engineering systems. Eng Comput 28(3):242–274
25. Azamathulla HMd, Zahiri A (2012) Flow discharge prediction in compound channels using linear genetic programming. J Hydrol 454–455C:203–207
26. Oltean M, Dumitrescu D (2002) Multi expression programming. Technical report, UBB-01-2002, Babeş-Bolyai University, Cluj-Napoca, Romania
27. Oltean M, Grosan C (2003) A comparison of several linear genetic programming techniques. Complex Syst 14(4):1–29
28. Alavi AH, Gandomi AH, Sahab MG, Gandomi M (2010) Multi expression programming: a new approach to formulation of soil classification. Eng Comput 26(2):111–118
29. Gandomi AH, Alavi AH, Yun GJ (2011) Formulation of uplift suction caissons using multi expression programming. KSCE J Civil Eng 15(2):363–373
30. Swamy RN, Mangat PS, Rao CVSK (1974) The mechanics of fiber reinforcement of cement matrices, fiber reinforced concrete, SP-44. American Concrete Institute, Farmington Hills, pp 1–28
31. Batson G, Jenkins E, Spatney R (1972) Steel fibers as shear reinforcement in beams. ACI J 69(10):640–644
32. Roberts TM, Ho NL (1982) Shear failure of deep fiber reinforced concrete beams. Int J Cement Comp Lightweight Concr 4(3):145–152
33. Jindal RL (1984) Shear and moment capacities of steel fiber reinforced concrete beams. In: Hoff GC (ed) Fiber reinforced concrete, SP-81. American Concrete Institute, Farmington Hills, pp 1–16
34. Swamy RN, Bahia HM (1985) The effectiveness of steel fibers as shear reinforcement. Concr Int 7(3):35–40
35. Uomoto T, Weerarathe RK, Furukoshi H, Fujino H (1986) Shear strength of reinforced concrete beams with fibre reinforcement. In: Proceedings of the third international on RILEM symposium on developments in fibre reinforced cement and concrete, Sheffield, 1986. RILEM Technical Committee 49-TFR, Sheffield University Press Unit, Sheffield, 553–562
36. Kadir MRA, Saeed JA (1986) Shear strength of fibre reinforced concrete beams. J Eng Technol 4(3):98–112
37. Kaushik SK, Gupta VK, Tarafdar NK (1987) Behavior of fiber reinforced concrete beams in shear. In: Proceedings of the international symposium on fiber reinforced concrete, Madras, India, pp 1.133–1.149
38. Murty DSR, Venkatacharyulu T (1987) Fiber reinforced concrete beams subjected to shear force. In: Proceedings of the international symposium on fiber reinforced concrete, Madras, India, pp 1125–1132
39. Lim TY, Paramasivam P, Lee SL (1987) Shear and moment capacity of reinforced steel-fiber-concrete beams. Mag Concr Res 39(140):148–160
40. Narayanan R, Darwish IYS (1988) Fiber concrete deep beams in shear. ACI Struct J 85(2):141–149
41. Tan KH, Murugappan K, Paramasivam P (1992) Shear behavior of steel fiber reinforced concrete beams. ACI Struct J 89(6):3–11
42. Imam M, Vandewalle L, Mortelmans F (1994) Shear capacity of steel fiber high-strength concrete beams. In: Malhotra VM (ed) High-performance concrete, SP-149. American Concrete Institute, Farmington Hills, pp 227–241
43. Adebar P, Mindess SSt, Pierre D, Olund B (1997) Shear tests of fiber concrete beams without stirrups. ACI Struct J 94(1):68–76
44. Oh BH, Lim DH, Yoo SW, Kim ES (1998) Shear behavior and shear analysis of reinforced concrete beams containing steel fibers. Mag Concr Res 50(4):283–291
45. Casanova P, Rossi P (1999) High-strength concrete beams submitted to shear: steel fibers versus stirrups. In: Banthia N, MacDonald C, Tatnall P (eds) Structural applications of fiber reinforced concrete, SP-182. American Concrete Institute, Farmington Hills, pp 53–67
46. Noghabai K (2000) Beams of fibrous concrete in shear and bending: experiment and model. J Struct Eng ASCE 126(2):243–251
47. Cucchiara C, Mendola LL, Papia M (2004) Effectiveness of stirrups and steel fibres as shear reinforcement. Cem Concr Comp 26:777–786
48. Oltean M (2004) Multi expression programming source code. http://www.mep.cs.ubbcluj.ro/
49. Gandomi AH, Alavi AH, Mousavi M, Tabatabaei SM (2011) A hybrid computational approach to derive new ground-motion attenuation models. Eng Appl Artif Intell 24(4):717–732
50. Smith GN (1986) Probability and statistics in civil engineering. Collins, London
51. Golbraikh A, Tropsha A (2002) Beware of q2. J Mol Graph Model 20(4):269–276
52. Roy PP, Roy K (2008) On some aspects of variable selection for partial least squares regression models. QSAR Comb Sci 27:302–313
53. Dimopoulos C, Zalzala AMS (2001) Investigating the use of genetic programming for a classic one-machine scheduling problem. Adv Eng Softw 32(6):489–498