A Lattice Boltzmann Method for Electromagnetic Wave Propagation in Medium

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***Abstract*—A detailed measurement of rainfall is an important input to various climate models. Accuracy of rainfall measurements also has a socio-economic impact since it forms an important constituent in the region-wise and seasonal water budget. Radar measurements are most promising in this direction since they provide the detailed spatio-temporal data than any conventional method. However, it also has the disadvantage that rainfall estimation using radar is an indirect measurement and often involves the use of a number of correlations. The amount of scattered radiation is directly related to the raindrop size and shape distribution. Therefore, this project is aimed at solving Maxwell's equations of electromagnetism using the lattice Boltzmann method with the intention of accurately relating the size and shape distribution of falling raindrops to scattered intensity and thus improving the rainfall measurement. Towards this objective, a D3Q7 code for a pseudo vector discrete Boltzmann distribution function has been developed following the work of Hauser and Verhey (2017) [1]. Standard validations involving scattering of electromagnetic waves by a dielectric interface and a circular scatterer have been performed. Error in the solutions was found to be minimal on comparing with analytical solutions. The applicability of the proposed numerical method towards analyzing scattering of electromagnetic waves by a three-dimensional scatterer, modelled on realistic hydrometeors, will be discussed..**

***Index Terms*—Electromagnetic Wave, Lattice Boltzmann Method, Rainfall Rate, Scattering, Reflectivity.**

I. Introduction

The equation of lattice Boltzmann was developed for fluid mechanics simulation. This numerical method is known as lattice Boltzmann method, like other conventional method finite difference time domain, LBM is efficient and alternate method and for implementation of boundary condition for complex geometry is easy in LBM [2].Later on studies of LBM applied to simulate the heat transfer, multi component fluids, acoustics problems.

Lattice Boltzmann model for electrodynamics was firstly presented by the Dellar et al.(2005) [3] in which they presented the kinematics of ferro fluids by using LBM and this model is also used for magneto-hydrodynamics simulations. Mendoza and Muñoz (2010), they simulate the EM wave propagation in homogeneous medium by taking different approach of LBM [4], but their simulation shows instabilities for sharp medium and which can be avoided by using the smooth transition of medium interface. A. Hauser and J. L. Verhey (2017) they present stable method for simulating EM wave propagation in medium when the medium transitions are sharp [1].

EM Wave propagation in dielectric medium such as raindrop shows scattering phenomena of wave. By using the LBM we can calculate the backscatter amplitude and radar cross section of the dielectric medium. This backscatter data are related to the physical quantity called reflectivity (Z). This reflectivity (Z) is important parameter for estimation of rainfall rate (R in mm/h, or rainfall accumulation RA in mm) by using Z–R relationship. This relationship can also be written in terms of a power law of the form $Z = aR^b$ as discussed in [5].

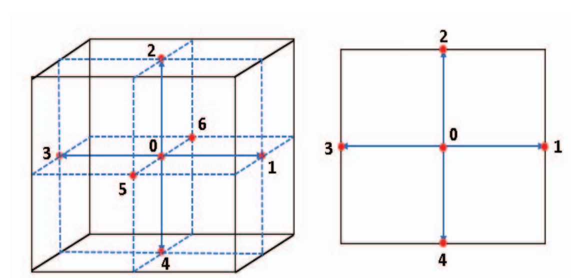

Fig. 1. Lattice cell arrangement of D3Q7 lattice model. This lattice model is able to simulate the EM wave propagation

II. Method

A. Lattice Boltzmann Method

Lattice Boltzmann equation originated from kinetic theory of gases and it is the approximation of the Boltzmann equation. The domain is divided by lattice cell which has the lattice velocity vector in $DdQq$ (d = dimension, q = lattice direction) form as shown in fig (1). The Boltzmann equation can be written as:

$$\frac{\partial f}{\partial t} + c \cdot \frac{\partial f}{\partial r} + \frac{F}{m} \cdot \frac{\partial f}{\partial c} = \Omega \tag{1}$$

In the LBM, the above Boltzmann equation is discretized and it is assumed that it is valid in the all lattice streaming direction, where Ω is the collision operator governed by the BGK collision model:

$$\Omega = \frac{1}{\tau}(f^{eq} - f) \tag{2}$$

Using a discrete lattice with i directions as shown in figure (1), equation (1) assumes the following discrete form:

$$f_i(r + c_i\Delta t, t + \Delta t) = f_i(r,t) + \frac{\Delta t}{\tau}\left[f_i^{eq}(r,t) - f_i(r,t)\right] \tag{3}$$

The above equation (3) is discrete form of the lattice Boltzmann equation by using Bhatnagar–Gross–Krook (BGK) collision operator. Where τ is relaxation time assign as a fixed value which represent the macroscopic transport properties, like thermal diffusivity and kinematic viscosity. In the Maxwell equation we don't have this diffusive properties so that $\tau = 1/2$ is fixed for this model.

$$\sum_i \left(\partial_t f_i^{eq} + \sum_\alpha c_{\alpha i}\partial_\alpha f_i^{eq}\right) = 0 \tag{4}$$

The expansion of equation (3) gives the second order accuracy and this continuity equation (4) is suitable for solving the Maxwell's equation for EM wave propagation.

B. Maxwell Equations

The Maxwell equations in differential form can be written as -

$$\sum_\alpha \partial_\alpha B_\alpha = 0 \quad \sum_{\beta,\gamma} \varepsilon_{\alpha\beta\gamma}\partial_\beta E_\gamma = -\partial_t B_\alpha \tag{5a}$$

$$\sum_\alpha \partial_\alpha D_\alpha = 0 \quad \sum_{\beta,\gamma} \varepsilon_{\alpha\beta\gamma}\partial_\beta H_\gamma = -\partial_t D_\alpha + j_\alpha \tag{5b}$$

Where B is magnetic induction, E is electric field, D is displacement field, ρ is charge density, H is magnetic field, and j is current density. The Displacement field and magnetic field can be written as in terms of material property -

$$D_\alpha = \epsilon_0 E_\alpha + P_\alpha \rightarrow \epsilon_r\epsilon_0 E_\alpha \tag{6a}$$

$$B_\alpha = \mu_0(H_\alpha + M_\alpha) \rightarrow \mu_r\mu_0 H_\alpha \tag{6b}$$

C. Macroscopic Equation

The vector component of distribution function is used to calculate the electric and magnetic field component (E_α, H_α) [6]. The vector distribution function f_α and g_α was introduced for macroscopic field-

$$E_\alpha(r,t) = \frac{1}{\epsilon_r(r)}\sum_i f_{\alpha,i}(r,t) \tag{7a}$$

$$H_\alpha(r,t) = \frac{1}{\mu_r(r)}\sum_i g_{\alpha,i}(r,t) \tag{7b}$$

Using the discrete Boltzmann equation (3) for the streaming and collision, we get the time evolution equation -

$$f_{\alpha,i}(r + c_i\Delta t, t + \Delta t) = 2f_{\alpha,i}^{eq}(r,t) - f_{\alpha,i}(r,t) \tag{8a}$$

$$g_{\alpha,i}(r + c_i\Delta t, t + \Delta t) = 2g_{\alpha,i}^{eq}(r,t) - g_{\alpha,i}(r,t) \tag{8b}$$

For homogeneous medium the LBM solution of EM wave propagation remain stable but for sharp interface of medium become unstable [7]. So by taking the additional component of equilibrium distribution function for centre population as suggested in [1] gives the stable solution for sharp interface, this separated model of equilibrium distribution function is given as -

$$f_{\alpha,i}^{eq} = \begin{cases} \frac{1}{6}\Big(E_\alpha - \sum_{\beta,\gamma}\varepsilon_{\alpha\beta\gamma}c_{\beta,i}H_\gamma\Big) & \text{if } i \neq 0 \\ \Big(\epsilon_r - 1\Big)E_\alpha & \text{if } i = 0 \end{cases} \tag{9a}$$

$$g_{\alpha,i}^{eq} = \begin{cases} \frac{1}{6}\Big(H_\alpha + \sum_{\beta,\gamma}\varepsilon_{\alpha\beta\gamma}c_{\beta,i}E_\gamma\Big) & \text{if } i \neq 0 \\ \Big(\mu_r - 1\Big)H_\alpha & \text{if } i = 0 \end{cases} \tag{9b}$$

The same scaling properties is used for both the equilibrium distribution function and by using the equilibrium distribution function given in equation (9), Maxwell equations can be recovered by Chapman-Enskog expansion.

III. Results and Discussion

LBM simulation has been performed for electromagnetic wave propagation and it is able to solve the Maxwells equations. The main focus of the present work is to calculate the magnitude of the back-scatter electric and magnetic field intensity also the reflected power from the dielectric media.

The EM wave propagation in dielectric medium with plane interface is shown in fig (2). The dashed line is the initial condition of the wave which is in the form of -

$$E_y(x) = H_z(x) = \begin{cases} \sin\left(\frac{2\pi x}{\lambda}\right) & \text{if } x = x_0 \ to \ x_1 \\ 0 & \text{if } x \neq x_0 \ to \ x_1 \end{cases} \tag{10a}$$

where $\lambda = 60$ (wavelength). This wave is propagating from low dielectric medium to high dielectric medium. At the interface some part of the incident wave is transmitting in the second medium and some part is reflected back to first medium. The intensity of the transmitted and reflected wave

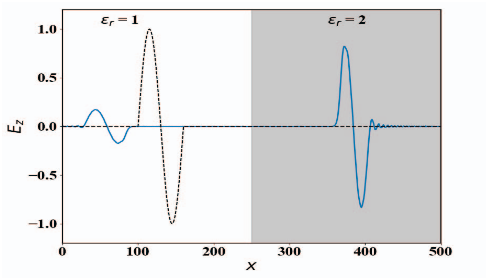

Fig. 2. Wave propagating in dielectric medium. At particular instance of time Reflected and transmitted wave magnitude has been recorded. Dashed line represent the initial condition on wave.

recorded numerically is $E_t/E_i = 0.8349,\ E_r/E_i = 0.17304$ and analytically $E_t/E_i = 0.8284,\ E_r/E_i = 0.17157$, which has the error of magnitude less than 1%.

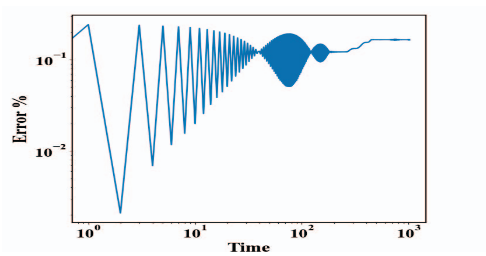

Fig. 3. Relative $\%$ Error of theoretical and numerical total energy for 1D wave propagation in dielectric media.

In order to validate the total energy, the comparison of the theoretical and numerical total energy has shown in fig (3), which shows the relative percentage error of the theoretical and numerical total energy as a function of time. It is found that the magnitude of error is less than 1%.

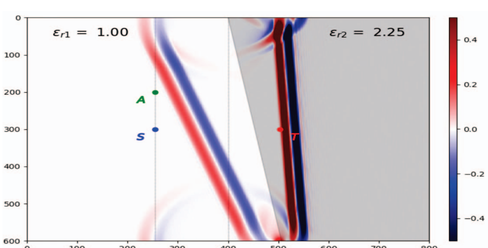

Fig. 4. Oblique Wave propagation in dielectric medium with incidence angle $\theta_i = 10^o$.Boundary conditions are periodic and contour plot shows the reflection from the oblique interface.

Simulation of EM wave propagation through the oblique incidence has shown in fig (4). In which the same initial condition has been applied and the boundary conditions are periodic. The incidence angle is $\theta_i = 10^o$. From the simulation it is observed that as the wave strike with the interface the reflection wave propagate with the same angle as incidence but numerical value of the transmitted wave is $\theta_t = 6.85^o$ which match with the analytic value of $\theta_t = 6.65$.

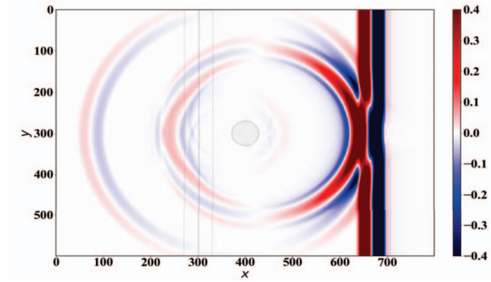

Fig. 5. EM wave scattering by dielectric sphere. Size of the sphere is small as compare to the wavelength.Boundary conditions are periodic, at particular instance of time this contour plot shows scattering of EM wave from sphere.

The same EM wave simulation has been carried out for the dielectric sphere which is shown in the fig (5). Size (a) of the particle is small as compare to the wave length (λ). As the EM wave hits the sphere scattering phenomena starts and wave scatter in all the directions as shown in fig (5). Our main focus was to capture the backscatter wave at the source location.

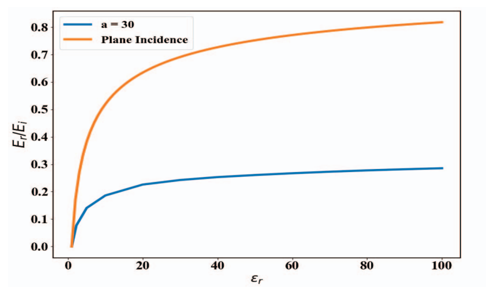

Fig. 6. Particle Size dependency for fix wave length.

For different value of the size and dielectric constant the simulation has been performed. As the size of the sphere increases as compare to the wave length the solution approach toward the plane incidence wave solution as shown in fig (6).

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