Formal Languages and Automata Theory, SS 2019. Homework 9 (due Week 12)

1. Write context-free grammars for the following languages:

   (a) $L = \{w|w \text{ is a binary palindrome}\}$

   (b) $L = \{$ the language of balanced paranthesis $\}$
   Examples: ()((((())))((())), ((((()))()()); *Counterexamples:* ((((()))))((())), ((((())))())

   (c) $L = \{0^m 1^n | m \geq n\}$

   (d) $L = \{0^n 1^n | n \geq 1\}$ *Examples:* 01, 00001111

   (e) $L = \{0^n 1^n | n \geq 0\}$ *Examples:* λ, 01, 00001111

   (f) $L = \{$ The set of all strings with an equal number of a's and b's$\}$
   Examples: λ, *aabb*, *bbaa*, *abbababa*,*bbababaa*.

   (g) $L = \{$ Binary words with even length$\}$

   (h) $L = \{0^i 1^j 2^k | i = j \text{ or } j = k, \text{ where } i, j, k \geq 0\}$

   (i) $L = \{a^i b^j c^k | i + j = k, i, j, k \geq 0\}$.

   (j) $L = \{a^i b^j c^k | \neg(i = j) \text{ or } \neg(j = k)\}$

   (k) The language of all binary strings that are not of the form ww, that is, not equal to any repeated string.

   (l) The language of all strings with twice as many 0's as 1's.

2. The following grammar generates the language of regular expression $0^0 1(0|1)^*$. $S \to A1B, A \to 0A|\varepsilon, B \to 0B|1B|\varepsilon$. Give the leftmost and rightmost derivations of the following strings: (a) 00101, (b) 1001, (c) 00011.

3. Consider the CFG G defined by productions: $S \to aS|Sb|a|b$.

   (a) Prove by induction on the string length that no string in $L(G)$ has ba as a substring.

   (b) Describe $L(G)$ informally. Justify your answer using point (a).

4. Consider the CFG G defined by productions: $S \to aSbS|bSaS|\varepsilon$. Prove that $L(G)$ is the set of all strings with an equal number of a's and b's.

5. Consider the grammar: $S \to aS|aSbS|\varepsilon$. This grammar is ambiguous. Show in particular that the string aab has two: (a) parse trees, (b) leftmost derivations, (c) rightmost derivations.

6. Find an unambigous grammar for the language generated by the grammar above.