

Formal Methods in Software Development

Decidability and Decision Procedures

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Based on slides of the lecture Satisfiability Checking (Erika Ábrahám), RTWH Aachen

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FO theories and their decidability

Some first-order theories:

Logic	decidability	algorithm

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Propositional logic		

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