**Tehnici de baza in activitatea stiintifica**
**Techniques for Scientific Work (WS 2016-2017)**
**Homework 4 (due January 18, 2017)**

1. In the following formula:

$$\forall_f \forall_a C[f,a] \Leftrightarrow \forall_{\epsilon>0}\ \exists_{\delta>0}\ \forall_x\ (|x-a| < \delta\ \Rightarrow\ |f[x]-f[a]| < \epsilon)$$

identify the type of each symbol (logical, function, predicate, etc.) and indicate whether it is constant or variable.

2. In the previous formula, $f$ denotes a real function of real argument, $a, \epsilon, \delta, x$ denote real numbers, and $C[f,a]$ denotes continuity of function $f$ in point $a$. Prove that the sum of two continuous functions in the same point is continuous in this point.

3. Prove that the formula "$(A \vee B) \Rightarrow P$" is equivalent to the formula "$(A \Rightarrow P) \wedge (B \Rightarrow P)$".

4. Prove that the formula "$(\exists_x P[x]) \Rightarrow Q$" is equivalent to the formula "$\forall_x (P[x] \Rightarrow Q)$".

5. In the following formulae, $n$ and $s$ stand for natural numbers:

$$f[0] = 0$$
$$\forall_n\ f[n+1] = n+1+f[n]$$
$$\forall_s\ g[0,\ s] = s$$
$$\forall_n\ \forall_s\ G[n+1,\ s] = G[n,\ s+n+1]$$

Use these equalities as rewrite rules in order to compute the expressions: $F[3]$ and $G[3,\ 0]$.

6. Using the formulae above, prove $\forall_n\ F[n] = G[n,\ 0]$.
Hint: prove first $\forall_n\ \forall_s\ G[n,\ s] = s+F[n]$ by induction on $n$. For proving the latter, consider the predicate $P[n]$ defined as $\forall_s\ G[n,\ s] = s + F[n]$ and use an induction principle for natural numbers in order to prove $\forall_n\ P[n]$. (One must prove $P[0]$ and $\forall_n\ (P[n] \Rightarrow P[n+1])$.) Note that for proving equalities it is enough to transform both sides by using known equalities as rewrite rules and the appropriate properties of natural numbers.

7. The 5-pages version of your paper.