Automated Theorem Proving, SS 2017. Homework 2 (due April 26, 2017)

0. Remaining exercises from the previous homework.

1. Write the tables of the boolean functions corresponding to $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$. Using them, determine the truth value of:

   - The formula $(A \wedge (A \Rightarrow B)) \Rightarrow B$ under the interpretation $I = \{A \to \mathbb{T}, B \to \mathbb{F}\}$.
   - The formula $(P \Rightarrow Q) \iff (\neg Q \Rightarrow \neg P)$ under the interpretation $I = \{P \to \mathbb{F}, Q \to \mathbb{F}\}$.
   - The formula $((A \vee B) \Rightarrow C) \iff ((A \Rightarrow C) \wedge (B \Rightarrow C))$ under the interpretation $I = \{A \to \mathbb{T}, B \to \mathbb{T}, C \to \mathbb{F}\}$.

   (*Hint: The tables of boolean functions corresponding to $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ correspond to the tables which we have outlined in the first lab and used for defining the semantics of logical connectives.*)

2. Is it possible to have a formula that is both in conjunctive and disjunctive normal form. If so, give 5 examples.

3. Construct the sequent proof (proof tree) for $(A \vee B) \Rightarrow C \models (A \Rightarrow C) \wedge (B \Rightarrow C)$.

4. In the "$\neg\wedge$- calculus", prove the correctness of the following inference rules:

$$\text{(a)} \quad \frac{\Phi, \varphi_1, \varphi_2 \ \vdash \ \Psi}{\Phi, \varphi_1 \wedge \varphi_2 \ \vdash \ \Psi} \ (\wedge \vdash)$$

$$\text{(b)} \quad \frac{\Phi, \varphi \ \vdash \ \Psi}{\Phi \ \vdash \ \neg\varphi, \Psi} \ (\vdash \neg)$$

5. Using the "$\neg\wedge$- calculus", derive the sequent rules for

   (a) disjunction in the assumption
   (b) disjunction in the goal
   (c) implication in the assumption
   (d) implication in the goal