Formal Methods in Software Developement

Modeling with propositional logic

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Based on slides of the lecture Satisfiability Checking (Erika Ábrahám), RTWH Aachen

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- There are numerous problems in the industry that are solved via the satisfiability problem of propositional logic
  - Logistics
  - Planning
  - Electronic Design Automation industry
  - Cryptography
  - ...
- For the following examples, use a SAT solver to find a satisfying assignment.

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- Q: Can we assign to each station a frequency, such that no station pairs from E have the same frequency?

Example 1 (continued)

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Close stations are not assigned the same frequency:
For each $(s1, s2) \in E$,

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Example 2: Seminar topic assignment

- n participants
- n topics
- Set of preferences $E \subseteq \{1, \ldots, n\} \times \{1, \ldots, n\}$
  $(p, t) \in E$ means: participant p would take topic t

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- Set of preferences $E \subseteq \{1, \ldots, n\} \times \{1, \ldots, n\}$
  $(p, t) \in E$ means: participant p would take topic t
- Q: Can we assign to each participant a topic which he/she is willing to take?

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  Each participant is willing to take his/her assigned topic:

$$\bigwedge_{p=1}^{n}\bigwedge_{(p,t)\notin E}\neg x_{p,t}$$

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