

# Formal Methods in Software Development

## Decidability and Decision Procedures

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Based on slides of the lecture Satisfiability Checking (Erika Ábrahám), RTWH Aachen

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# FO theories and their decidability

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