Automated Theorem Proving, SS 2015. Homework 1 (due April 2, 2015)

1. For each of the following formulas determine whether is valid/invalid/satisfiable/unsatisfiable or some combination of these. For (a) and (b) use the truth table method, for the rest use equivalent transformations.

   (a) $(P \Rightarrow Q) \Rightarrow (\neg Q \Rightarrow \neg P)$
   (b) $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$
   (c) $P \vee (P \Rightarrow Q)$
   (d) $(P \wedge (Q \Rightarrow P)) \Rightarrow P$
   (e) $P \vee (Q \Rightarrow \neg P)$
   (f) $(P \vee \neg Q) \wedge (\neg P \vee Q)$
   (g) $\neg P \wedge (\neg (P \Rightarrow Q))$
   (h) $P \Rightarrow \neg P$
   (i) $\neg P \Rightarrow P$

2. Transform the following into disjunctive normal form

   (a) $(P \Rightarrow Q) \Rightarrow R$
   (b) $\neg (P \wedge Q) \wedge (P \vee Q)$

3. Transform the following into conjunctive normal form

   (a) $(P \Rightarrow Q) \Rightarrow R$
   (b) $(\neg P \wedge Q) \vee (P \wedge \neg Q)$

4. Verify each of the following pairs of equivalent formulas by transforming the formulas on both sides of the sign $\equiv$ into the same normal form:

   (a) $P \wedge P \equiv P$
   (b) $P \vee P \equiv P$
   (c) $(P \Rightarrow Q) \wedge (P \Rightarrow R) \equiv P \Rightarrow (Q \wedge R)$
   (d) $(P \Rightarrow Q) \Rightarrow (P \wedge Q) \equiv (\neg P \Rightarrow Q) \wedge (Q \Rightarrow P)$
   (e) $P \wedge Q \wedge (\neg P \vee \neg Q) \equiv \neg P \wedge \neg Q \wedge (P \vee Q)$

5. Define the meta-function $\mathrm{Vars}[\varphi]$ which gives set of propositional variables of the propositional formula φ. (Hint: use the induction principle suggested by the definition of propositional logic formulas.) Examples: $\mathrm{Vars}[\mathbb{F}] = \emptyset$, $\mathrm{Vars}[A] = \{A\}$, $\mathrm{Vars}[P \Rightarrow \mathbb{T}] = \{P\}$, $\mathrm{Vars}[(P \Rightarrow Q) \Rightarrow (P \wedge Q)] = \{P, Q\}$, $\mathrm{Vars}[Q \Rightarrow Q] = \{Q\}$