Automated Theorem Proving, SS 2014. Homework 6 (due May 29, 2014)

1. For each symbol occurring in the following formula, specify whether it is a: logical quantifier, logical connective, predicate symbol, function symbol, variable, or constant. (Note that functions and predicates can also be constant or variable.)

$$\underset{f}{\forall}\, C[f] \iff \underset{\epsilon>0}{\forall}\ \underset{\delta>0}{\exists}\ \underset{x,y}{\forall}\ (|x-y|<\delta \implies |f[x]-f[y]<\epsilon|\ )$$

2. Find the truth value of the formula $F \ :\iff\ \underset{x}{\forall}\ (P[x] \implies Q[f[x],a])$, where

$$I: \begin{cases} D=\{1,2\} \\ a_I = 1 \\ f_I : D\to D & \begin{cases} f_I[1]=1 \\ f_I[2]=1 \end{cases} \\ P_I : D \to \{\mathbb{T},\mathbb{F}\} & \begin{cases} P_I[1]=\mathbb{T} \\ P_I[2]=\mathbb{F} \end{cases} \\ Q_I : D^2 \to \{\mathbb{T},\mathbb{F}\} & \begin{cases} Q_I[1,1]=\mathbb{T} & Q_I[1,2]=\mathbb{F} \\ Q_I[2,1]=\mathbb{F} & Q_I[2,2]=\mathbb{T} \end{cases} \end{cases}$$

3. For the interpretation $D=\{a,b\}$, $P[a,a]=\mathbb{T}$, $P[a,b]=\mathbb{F}$, $P[b,a]=\mathbb{F}$, $P[b,b]=\mathbb{T}$, determine the truth value of the following formulas:

   (a) $\underset{x}{\forall}\underset{y}{\exists} P[x,y]$

   (b) $\underset{x}{\exists}\underset{y}{\forall} P[x,y]$

   (c) $\underset{x,y}{\forall}\ (P[x,y] \implies P[y,x])$

   (a) $\underset{x,y}{\forall} P[x,y]$

   (b) $\underset{y}{\exists} \neg P[a,y]$

   (c) $\underset{x}{\forall} P[x,x]$

4. Transform the following formulas into prenex normal form:

   (a) $\underset{x}{\forall}\left(P[x] \implies \underset{y}{\exists} Q[x,y]\right)$

   (b) $\underset{x}{\exists}\left(\neg\left(\underset{y}{\exists}P[x,y]\right) \implies \left(\left(\underset{z}{\exists}Q[z]\right) \implies R[x]\right)\right)$

   (c) $\underset{x,y}{\forall}\left(\underset{z}{\exists}P[x,y,z] \wedge \left(\underset{u}{\exists}Q[x,u] \implies \underset{v}{\exists}Q[y,v]\right)\right)$

5. Transform the following formulas into Skolem normal form:

   (a) $\neg\left(\underset{x}{\forall}P[x] \implies \underset{y}{\exists}\underset{z}{\forall}Q[y,z]\right)$

   (b) $\neg\left(\underset{x}{\forall}P[x] \implies \underset{y}{\exists}P[y]\right)$

   (c) $\underset{x,y}{\forall}\ \underset{z}{\exists}P[x,y,z]$

(d)

$$\left(\forall_{x,y,z,u,v,w} (P[x,y,u] \;\wedge\; P[y,z,v] \;\wedge\; P[u,z,w] \;\;\Rightarrow\;\; P[x,v,w])\right)$$
$$\wedge$$
$$\left(\forall_{x,y,z,u,v,w} (P[x,y,u] \;\wedge\; P[y,z,v] \;\wedge\; P[x,v,w] \;\;\Rightarrow\;\; P[u,z,w])\right)$$

(e) $\forall_x P[x,e,x] \wedge \forall_x P[e,x,x]$

(f) $\forall_x P[x,i[x],e] \wedge \forall_x P[i[x],x,e]$

(g) $\left(\forall_x P[x,x,e]\right) \;\Rightarrow\; \left(\forall_{u,v,w} (P[u,v,w] \;\Rightarrow\; P[v,u,w])\right)$