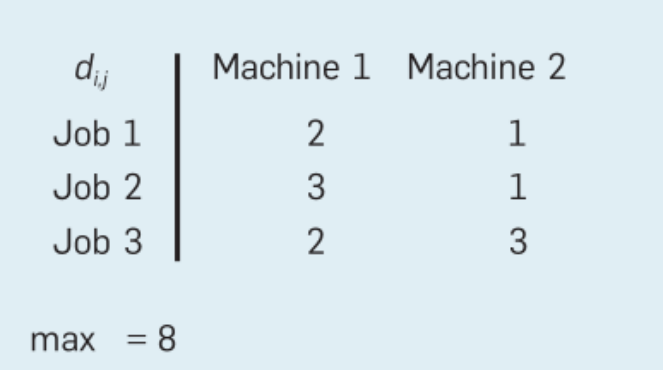
Formal Methods in Software Development, WS 2018. Lab 6

Job shop scheduling Problem. *Job shop scheduling* is an optimization problem in computer science and operations research in which jobs are assigned to resources at particular times. The most basic version is as follows: We are given n jobs J_1, J_2, ..., J_n of varying processing times, which need to be scheduled on m machines with varying processing power, while trying to minimize the *makespan*. The *makespan* is the total length of the schedule (that is, when all the jobs have finished processing). Additionally, the following constraints might be involved:

- *Precedence.* Between two jobs which want to take a machine.
- *Resource.* Machines execute at most one job at a time.

Figure 1: Example of a Job shop scheduling problem



Consider the problem in Figure 1. We consider d_{ij} the duration of Job i on Machine j. For example $d_{11} = 2$ time units (TU), $d_{12} = 1$ TU, etc. The maximal makespan is 8 TU.

The problem can be formalized as follows. We consider the variable t_{ij} representing "time required for job i on machine j." ($i = \overline{1,3}$, $j = \overline{1,2}$)

- *precedence.* For example, for Job 1 we have:

$$(t_{1,1} \geq 0) \wedge (t_{1,2} \geq t_{1,1} + 2) \wedge (t_{1,2} + 1 \leq 8)$$

- *resource.* For example, we have "Job 1 on machine 1 is scheduled either before or after job 2 on the same machine":

$$(t_{1,1} \geq t_{2,1} + 3) \vee (t_{2,1} \geq t_{1,1} + 2)$$

Using Z3 SMT solver, find a suitable schedule. Is it optimal? Can you find the optimal one? Justify your answers.