Tehnici de baza in activitatea stiintifica
Techniques for Scientific Work (WS 2015-2016)
Homework 4 (due January 6, 2016)

(1) Problems 6 and 7 from `http://staff.ieat.ro/~merascu/links/WS2015TSW/HW1.pdf`.

(2a) In the following formulae, t stands for a tuple (i. e. list of elements). Examples of tuples are: $\langle\rangle$ (the empty list), $\langle a, b\rangle$ (a list with two elements). The binary infix function $\smile$ concatenates two tuples. Examples:

$$\langle\rangle \smile \langle a, b\rangle = \langle a, b\rangle$$
$$\langle a, b\rangle \smile \langle b, c\rangle = \langle a, b, b, c\rangle$$

Consider the following definitions:

(a) $F[\langle\rangle] = \langle\rangle$

(b) $\underset{a}{\forall}\ \underset{t}{\forall}\ F[\langle a\rangle \smile t] = F[t] \smile \langle a\rangle$

(c) $\underset{s}{\forall}\ G[\langle\rangle, s] = s$

(d) $\underset{a}{\forall}\ \underset{t,s}{\forall}\ G[\langle a\rangle \smile t, s] = G[t, \langle a\rangle \smile s]$

Use these equalities as rewrite rules in order to compute the expressions: $F[\langle a, b, c\rangle]$, $G[\langle a, b, c\rangle, \langle\rangle]$.

(2b) Using the formulae at (2a), prove: $\underset{t}{\forall}\ F[t] = G[t, \langle\rangle]$.

Hint: prove first $\underset{t}{\forall}\ \underset{s}{\forall}\ F[t] \smile s = G[t, s]$. For proving the later, consider the predicate $P[t]$ defined as $\underset{s}{\forall}\ F[t] \smile s = G[t, s]$ and use the induction principle for tuples in order to prove $\underset{t}{\forall}\ P[t]$. (One must prove $P[\langle\rangle]$ and $\underset{a}{\forall}\ \underset{t}{\forall}\ (P[t] \Rightarrow P[\langle a\rangle \smile s])$). Note that for proving equalities it is enough to transform both sides by using known equalities as rewrite rules, and, of course, if necessary, the appropriate properties of tuples).