Automated Theorem Proving, SS 2015. Homework 3 (due May 14, 2015)

1. Prove the following theorems:
   (1.a) $\varphi \equiv \psi$ iff $\varphi \iff \psi$ is valid.
   (1.b) (optional) $\varphi \equiv \psi$ iff $(\varphi \models \psi$ and $\psi \models \varphi)$.
   (1.c) $\varphi_1, ..., \varphi_n \models \psi$ iff $\varphi_1 \wedge ... \wedge \varphi_n \Rightarrow \psi$ is valid.
   (1.d) (optional) $\varphi_1, ..., \varphi_n \models \psi$ iff $\varphi_1 \wedge ... \wedge \varphi_n \wedge \neg\psi$ is unsatisfiable.
2. Prove that Q is a logical consequence of P and $P \Rightarrow Q$. This is the so-called *modus ponens* rule.
3. Prove that Q is a logical consequence of P and $\neg Q \Rightarrow \neg P$.
4. Read the paper introducing a variant of Propositional Logic (it can be found in the section Additional Material at `http://staff.ieat.ro/~merascu/links/SS2015ATP/ATP2015.html`).
5. Based on the paper mentioned before, solve the followings
   (5.a) Find the truth value of the empty disjunction. (In the paper it is exemplified the truth value of the empty conjunction).
   (5.b) Find the truth value of the disjunction applied to a singleton set. (In the paper it is exemplified the truth value of the conjunction applied to a singleton set.)
   (5.c) Prove $\neg \bigwedge \Phi \equiv \bigvee \bar{\Phi}$. (In the paper it is exemplified the the dual.)