Automated Theorem Proving, SS 2017. Homework 5 (due May 17, 2017)

1. Apply resolution to the following set of clauses:
   (a) (1) $\neg P \vee Q$
       (2) $\neg Q$
       (3) P
   (b) (1) $P \vee Q$
       (2) $\neg P \vee Q$
       (3) $P \vee \neg Q$
       (4) $\neg P \vee \neg Q$
2. Let

$$
\begin{aligned}
A_1 :&\ \ \underset{x,y,u,v}{\forall} T(x,y,u,v) \Rightarrow P(x,y,u,v) \\
A_2 :&\ \ \underset{x,y,u,v}{\forall} P(x,y,u,v) \Rightarrow E(x,y,u,v,y) \\
A_3 :&\ \ T(a,b,c,d)
\end{aligned}
$$

   Prove that A_3 is a logical consequence of A_1 and A_2.
3. Prove by resolution that G is a logical consequence of F_1 and F_2 where

$$
\begin{aligned}
F_1 :&\ \ \underset{x}{\forall}(C[x] \Rightarrow (W[x] \wedge R[x])) \\
F_2 :&\ \ \underset{x}{\exists}(C[x] \wedge O[x]) \\
G :&\ \ \underset{x}{\exists}(O[x] \wedge R[x])
\end{aligned}
$$

4. Prove by resolution that G is a logical consequence of F_1 and F_2 where

$$
\begin{aligned}
F_1 :&\ \ \underset{x}{\exists}\left(P[x] \wedge \underset{y}{\forall}(D[y] \Rightarrow L[x,y])\right) \\
F_2 :&\ \ \underset{x}{\forall}\left(P[x] \Rightarrow \underset{y}{\forall}(Q[y] \Rightarrow \neg L[x,y])\right) \\
G :&\ \ \underset{x}{\forall}(D[x] \Rightarrow \neg Q[x])
\end{aligned}
$$

5. Prove by resolution that G is a logical consequence of F where

$$
\begin{aligned}
F :&\ \ \underset{x}{\forall}\underset{y}{\exists}(S[x,y] \wedge M[y]) \Rightarrow \underset{y}{\exists}(I[y] \wedge E[x,y]) \\
G :&\ \ \neg\underset{x}{\exists} I[x] \Rightarrow \underset{x,y}{\forall}(S[x,y] \Rightarrow \neg M[y])
\end{aligned}
$$

6. Prove by resolution that G is a logical consequence of F_1, F_2, and F_3 where

$$
\begin{aligned}
F_1 :&\ \ \underset{x}{\forall}(Q[x] \Rightarrow \neg P[x]) \\
F_2 :&\ \ \underset{x}{\forall}\left((R[x] \wedge \neg Q[x]) \Rightarrow \underset{y}{\exists}(T[x,y] \wedge S[y])\right) \\
F_3 :&\ \ \underset{x}{\exists}\left(P[x] \wedge \underset{y}{\forall}(T[x,y] \Rightarrow P[y]) \wedge R[x]\right) \\
G :&\ \ \underset{x}{\exists}(S[x] \wedge P[x])
\end{aligned}
$$