Tehnici de baza in activitatea stiintifica
Techniques for Scientific Work (WS 2016-2017)
Homework 3 (due January 11, 2017)

(1a) In the following formulae, t stands for a tuple (i. e. list of elements). Examples of tuples are: $\langle\rangle$ (the empty list), $\langle a, b\rangle$ (a list with two elements). The binary infix function $\smile$ concatenates two tuples. Examples:

$$\langle\rangle \smile \langle a, b\rangle = \langle a, b\rangle$$
$$\langle a, b\rangle \smile \langle b, c\rangle = \langle a, b, b, c\rangle$$

Consider the following definitions:

(a) $F[\langle\rangle] = \langle\rangle$

(b) $\underset{a}{\forall}\ \underset{t}{\forall}\ F[\langle a\rangle \smile t] = F[t] \smile \langle a\rangle$

(c) $\underset{s}{\forall}\ G[\langle\rangle, s] = s$

(d) $\underset{a}{\forall}\ \underset{t,s}{\forall}\ G[\langle a\rangle \smile t, s] = G[t, \langle a\rangle \smile s]$

Use these equalities as rewrite rules in order to compute the expressions: $F[\langle a, b, c\rangle]$, $G[\langle a, b, c\rangle, \langle\rangle]$.

(1b) Using the formulae (a)–(d) above, prove: $\underset{t}{\forall}\ F[t] = G[t, \langle\rangle]$.

Hint: prove first $\underset{t}{\forall}\ \underset{s}{\forall}\ F[t] \smile s = G[t, s]$. For proving the later, consider the predicate $P[t]$ defined as $\underset{s}{\forall}\ F[t] \smile s = G[t, s]$ and use the induction principle for tuples in order to prove $\underset{t}{\forall}\ P[t]$. (One must prove $P[\langle\rangle]$ and $\underset{a}{\forall}\ \underset{t}{\forall}\ (P[t] \Rightarrow P[\langle a\rangle \smile s])$). Note that for proving equalities it is enough to transform both sides by using known equalities as rewrite rules, and, of course, if necessary, the appropriate properties of tuples).

(2) The 1-page version of your paper.