

# Laboratory: *Basic Proof Techniques (2)*

## Objectives

1. Practicing basic proof techniques with **RISC ProofNavigator**

## Problem 1

Take the file `exercise1.pn` and use the **RISC ProofNavigator** to prove the formulas  $A$ ,  $B$ , and  $C$  in this file. The proofs only require the commands `scatter`, `split`, and `instantiate`. The proof of formula  $C$  is essentially a *proof by contradiction*; here the command `flip` can be used to introduce the negation of the goal as an assumption. For developing the proofs, you may also try `auto`; the final proofs, however, must not make use of the `auto` command. Try also the repeated application of the command `flatten` (rather than `scatter`) to see the gradual decomposition of the proof.

## Problem 2

Develop in **RISC ProofNavigator** a theory that formalizes each of the following statements  $F_1$ ,  $F_2$ ,  $F_3$  as boolean constants:

- If there are any taxpayers, then all politicians are taxpayers.
- If there are any philanthropists, then all tax payers are philanthropists.
- So, if there are any tax-paying philanthropists, then all politicians are philanthropists.

For instance,  $F_1$  requires a definition **F1**: `BOOLEAN = ...`; To write the corresponding definitions, first introduce an undefined type **T** of objects **T**: `TYPE`; and, for each required property, an atomic predicate on **T**, e.g., **taxpayer**: `T->BOOLEAN`; You can then denote by the atomic formula **taxpayer**(**x**) the statement “ $x$  is a taxpayer”. Finally, define a formula **F**: `FORMULA F1 AND F2 => F3`; and prove it with the same restrictions as in exercise above (the final result must not contain applications of command `auto`). In addition to the commands needed for the previous exercise, you also need the command `expand`; for easier intuition, you may apply the command `goal` to exchange the goal formula.

## Problem 3

Suppose the stock prices go down if the prime interest rate goes up. Suppose also that most people are unhappy when stock prices go down. Assume that the price interest rate goes up. Conclude that the most people are unhappy.

## Problem 4

Some patients like all doctors. No patient likes any quack. Therefore, no doctor is a quack.