

## Automated Theorem Proving, SS 2014. Homework 1 (due March 27, 2014)

1. Give an example of a mathematical result which has a important impact on real life.
2. Give an example of a software failure which had an important negative impact in real life.
3. For each of the following formulas determine whether is valid/invalid/satisfiable/unsatisfiable or some combination of these. For (a) and (b) use the truth table method, for the rest use equivalent transformations.

(a)  $(P \Rightarrow Q) \Rightarrow (\neg Q \Rightarrow \neg P)$

(b)  $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$

(c)  $P \vee (P \Rightarrow Q)$

(d)  $(P \wedge (Q \Rightarrow P)) \Rightarrow P$

(e)  $P \vee (Q \Rightarrow \neg P)$

(f)  $(P \vee \neg Q) \wedge (\neg P \vee Q)$

(g)  $\neg P \wedge (\neg(P \Rightarrow Q))$

(h)  $P \Rightarrow \neg P$

(i)  $\neg P \Rightarrow P$

4. Transform the following into disjunctive normal form

(a)  $(P \Rightarrow Q) \Rightarrow R$

(b)  $\neg(P \wedge Q) \wedge (P \vee Q)$

5. Transform the following into conjunctive normal form

(a)  $(P \Rightarrow Q) \Rightarrow R$

(b)  $(\neg P \wedge Q) \vee (P \wedge \neg Q)$

6. Verify each of the following pairs of equivalent formulas by transforming the formulas on both sides of the sign  $\equiv$  into the same normal form:

(a)  $P \wedge P \equiv P$

(b)  $P \vee P \equiv P$

(c)  $(P \Rightarrow Q) \wedge (P \Rightarrow R) \equiv P \Rightarrow (Q \wedge R)$

(d)  $(P \Rightarrow Q) \Rightarrow (P \wedge Q) \equiv (\neg P \Rightarrow Q) \wedge (Q \Rightarrow P)$

(e)  $P \wedge Q \wedge (\neg P \vee \neg Q) \equiv \neg P \wedge \neg Q \wedge (P \vee Q)$

7. Define the meta-function  $\text{Vars}[\varphi]$  which gives set of propositional variables of the propositional formula  $\varphi$ . (Hint: use the induction principle suggested by the definition of propositional logic formulas.) Examples:  $\text{Vars}[\mathbb{F}] = \emptyset$ ,  $\text{Vars}[A] = \{A\}$ ,  $\text{Vars}[P \Rightarrow \mathbb{T}] = \{P\}$ ,  $\text{Vars}[(P \Rightarrow Q) \Rightarrow (P \wedge Q)] = \{P, Q\}$ ,  $\text{Vars}[Q \Rightarrow Q] = \{Q\}$