Propositional Logic

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Based on the book: Chin-Liang Chang and Richard Char-Tung Lee. Symbolic Logic and Mechanical Theorem Proving, Chapter 2

Outline

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Syntax

Definition
A *proposition* is a declarative sentence that is either true ($\mathbb{T}$) or false ($\mathbb{F}$), but not both.

Can you give some examples?

We use symbols like P, Q, R, etc. for denoting propositions. They are called atomic formulas or atoms.

Complex propositions are built using logical connectives: $\neg$, $\wedge$, $\vee$, , $\Rightarrow$ (implication), $\iff$ (equivalence).

Definition (Syntax)
Well-formed formulas (formulas) in propositional logic are defined recursively as follows:

1. An *atom* is a formula.
2. If G is a formula, then $\neg G$ is a formula.
3. If G and H are formulas, then $G \wedge H$, $G \vee H$, $G \Rightarrow H$, and $G \iff H$ are formulas.
4. All formulas are generated by applying the rules above.

What is the meaning of 4. ?

Can you give some examples/counterexamples of formulas?

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We introduce the notation $\langle G \rangle_I$ instead of $f_G(I)$ meaning *the truth evaluation of the formula G in the interpretation I.*

Definition (Interpretation)

Given a propositional formula G, let A_1, ..., A_n be the atoms occurring in the formula G. Then an interpretation of G is an assignment of truth values to A_1, ..., A_n in which every A_i is assigned either $\mathbb{T}$ or $\mathbb{F}$, but not both.

Example

Evaluate the truth value of $(A \wedge (A \Rightarrow B)) \Rightarrow B$.

To evaluate it we need to know an interpretation I as well as the semantics of the logical connectives.

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$\mathbb{T}$	$\mathbb{F}$
$\mathbb{F}$	$\mathbb{T}$

$\mathcal{B}_\wedge$	$\mathbb{T}$	$\mathbb{F}$
$\mathbb{T}$	$\mathbb{T}$	$\mathbb{F}$
$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$

$\mathcal{B}_\vee$	$\mathbb{T}$	$\mathbb{F}$
$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$
$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$

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$\mathcal{B}_\iff$	$\mathbb{T}$	$\mathbb{F}$
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Then we have

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For a formula with n distinct atoms, how many distinct interpretation do exist for it?

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Validity/Invalidity, Inconsistency (unsatisfiability) / Consistency (satisfiability)

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A formula F is said to be valid iff it is true under all its interpretations (For any $I \in \mathcal{I} : \langle F \rangle_I = \mathbb{T}$). A formula is said to be invalid iff it is not valid.

Definition (Inconsistent (unsatisfiable) / Consistent (satisfiable))

A formula is said to be inconsistent (unsatisfiable) iff it is false under all its interpretations (For any $I \in \mathcal{I} : \langle F \rangle_I = \mathbb{F}$). A formula is said to be consistent (satisfiable) iff it is not inconsistent.

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3. By bringing the two formulas in the normal form

Equivalent Formulas

Definition (Equivalent Formulas)

Let F, G be two formulas. Then $F = G$ iff for any $I \in \mathcal{I} : \langle F \rangle_I = \langle G \rangle_I$.

Proving the equivalence of two formulas:

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Equivalent transformations

Let $\square$ be the formula which is always false, $\blacksquare$ the formula which is always true (tautology).

We have the followings:

$F \iff G = (F \Rightarrow G) \wedge (G \Rightarrow F)$		
$F \Rightarrow G = \neg F \vee G$		
$F \vee G = G \vee F$	$F \wedge G = G \wedge F$	(comutativity)
$F \vee (G \vee H) = (F \vee G) \vee H$	$F \wedge (G \wedge H) = (F \wedge G) \wedge H$	(associativity)
$F \vee (G \wedge H) = (F \vee G) \wedge (F \vee H)$	$F \wedge (G \vee H) = (F \wedge G) \vee (F \wedge H)$	(distributivity)
$F \vee \square = F$	$F \wedge \blacksquare = F$	
$F \vee \blacksquare = \blacksquare$	$F \wedge \square = \square$	
$F \vee \neg F = \blacksquare$	$F \wedge \neg F = \square$	
$\neg(\neg F) = F$		
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Definition (Literal)

A literal is an atom or the negation of an atom.

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Normal Forms

Definition (Negation Normal Form)

A formula F is in negation normal form (NNF) iff F contains only the connectives $\neg$, $\wedge$, and $\vee$ and that negations appear only in literals.

Definition (Conjunctive Normal Form)

A formula F is in conjunctive normal form (CNF) iff F is in the form $F_1 \wedge \ldots \wedge F_n$, $n \geq 1$, where each F_i is a disjunction of literals.

Definition (Disjunctive Normal Form)

A formula F is in disjunctive normal form (DNF) iff F is in the form $F_1 \vee \ldots \vee F_n$, $n \geq 1$, where each F_i is a conjunction of literals.

A formula can be brought into a normal form by following the next steps:

- Step 1. Use the laws
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  to eliminate $\iff$ and $\Rightarrow$.
- Step 2. Repeatedly use the laws
  - $\neg(\neg F) \quad = \quad F$

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Outline

Syntax

Semantics

Normal Forms in the Propositional Logic

Logical Consequence

Logical Consequences

Definition

Given formulas F_1, F_2, ..., F_n and a formula G, G is a logical consequence of F_1, F_2, ..., F_n iff for all interpretation I in which $F_1 \wedge F_2 \wedge F_n$ is true, G is also true. F_1, F_2, ..., F_n are called axioms/postulates/premises.

Theorem

Given formulas F_1, F_2, ..., F_n and a formula G, G is a logical consequence of F_1, F_2, ..., F_n iff the formula $(F_1 \wedge F_2 \wedge ... \wedge F_n) \Rightarrow G$ is valid.

Theorem

Given formulas F_1, F_2, ..., F_n and a formula G, G is a logical consequence of F_1, F_2, ..., F_n iff the formula $F_1 \wedge F_2 \wedge F_n \wedge \neg G$ is inconsistent.

If G is a logical consequence of F_1, F_2, ..., F_n, the formula $(F_1 \wedge F_2 \wedge ... \wedge F_n) \Rightarrow G$ is called a theorem, and G is also called the conclusion of the theorem.

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