Formal Methods in Software Developement

Propositional Logic on Examples

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Based on slides of the lecture Satisfiability Checking (Erika Ábrahám), RTWH Aachen

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Satisfiability with truth table

Example formula:

$$\phi := \neg(a \rightarrow (b \vee \neg c))$$

Satisfiability with truth table

Example formula:

$$\phi := \neg(a \to (b \lor \neg c))$$

a	b	c	$\neg(a \to (b \lor \neg c))$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
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Satisfiability with semantical algorithm

$$
\begin{array}{lcl}
\text{Eval}(\alpha, p) & = & \alpha(p) \\
\text{Eval}(\alpha, \neg A) & = & \neg\text{Eval}(\alpha, A) \\
\text{Eval}(\alpha, A \vee B) & = & \text{Eval}(\alpha, A) \vee \text{Eval}(\alpha, B) \\
\text{Eval}(\alpha, A \wedge B) & = & \text{Eval}(\alpha, A) \wedge \text{Eval}(\alpha, B) \\
\text{Eval}(\alpha, A \rightarrow B) & = & \text{Eval}(\alpha, \neg A) \vee \text{Eval}(\alpha, B) \\
\text{Eval}(\alpha, A \leftrightarrow B) & = & \text{Eval}(\alpha, A \rightarrow B) \wedge \text{Eval}(\alpha, A \leftarrow B)
\end{array}
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NNF conversion

Only operators $\neg, \vee, \wedge$, negation only in front of atomic propositions.

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CNF conversion: The exponential way

CNF: $\bigwedge_{i=1,\dots,n} \bigvee_{j=1,\dots,m} l_{ij}$

Example formula:

$$\phi := (a \wedge b) \vee (\neg c \wedge (d \vee e))$$

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= \quad & (a \vee (\neg c \wedge (d \vee e))) \wedge (b \vee (\neg c \wedge (d \vee e))) \\
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= \quad & (a \vee \neg c \vee (\neg c \wedge e)) \wedge (a \vee d \vee (\neg c \wedge e)) \wedge \\
& (b \vee \neg c \vee (\neg c \wedge e)) \wedge (b \vee d \vee (\neg c \wedge e)) \\
= \quad & (a \vee \neg c \vee \neg c) \wedge (a \vee \neg c \vee e) \wedge (a \vee d \vee \neg c) \wedge (a \vee d \vee e) \wedge \\
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= & (a \vee \neg c \vee (\neg c \wedge e)) \wedge (a \vee d \vee (\neg c \wedge e)) \wedge \\
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CNF: $\bigwedge_{i=1,\dots,n} \bigvee_{j=1,\dots,m} l_{ij}$

Example formula:

$$\phi := (a \wedge b) \vee (\neg c \wedge (d \vee e))$$

$$
\begin{aligned}
& (a \wedge b) \vee (\neg c \wedge (d \vee e)) \\
= \; & (a \vee (\neg c \wedge (d \vee e))) \wedge (b \vee (\neg c \wedge (d \vee e))) \\
= \; & (a \vee (\neg c \wedge d) \vee (\neg c \wedge e)) \wedge (b \vee (\neg c \wedge d) \vee (\neg c \wedge e)) \\
= \; & (a \vee \neg c \vee (\neg c \wedge e)) \wedge (a \vee d \vee (\neg c \wedge e)) \wedge \\
& (b \vee \neg c \vee (\neg c \wedge e)) \wedge (b \vee d \vee (\neg c \wedge e)) \\
= \; & (a \vee \neg c \vee \neg c) \wedge (a \vee \neg c \vee e) \wedge (a \vee d \vee \neg c) \wedge (a \vee d \vee e) \wedge \\
& (b \vee \neg c \vee \neg c) \wedge (b \vee \neg c \vee e) \wedge (b \vee d \vee \neg c) \wedge (b \vee d \vee e)
\end{aligned}
$$

CNF conversion: Tseitin's encoding

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CNF conversion: Tseitin's encoding

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- $a_2 \leftrightarrow (a \wedge b)$
- $a_3 \leftrightarrow (\neg c \wedge a_4)$

CNF conversion: Tseitin's encoding

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- $a_4 \leftrightarrow (d \vee e)$
- a_1

CNF conversion: Tseitin's encoding

In the previous formula, we have the following templates:

1. $h \leftrightarrow (p_1 \vee p_2)$

2. $h \leftrightarrow (p_1 \wedge p_2)$

CNF conversion: Tseitin's encoding

In the previous formula, we have the following templates:

1. $h \leftrightarrow (p_1 \vee p_2)$
   $= (h \rightarrow (p_1 \vee p_2)) \quad \wedge \quad (h \leftarrow (p_1 \vee p_2))$

2. $h \leftrightarrow (p_1 \wedge p_2)$

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2. $h \leftrightarrow (p_1 \wedge p_2)$

CNF conversion: Tseitin's encoding

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1. $h \leftrightarrow (p_1 \vee p_2)$
$$\begin{aligned} &= (h \rightarrow (p_1 \vee p_2)) &\wedge\quad &(h \leftarrow (p_1 \vee p_2)) \\ &= (\neg h \vee (p_1 \vee p_2)) &\wedge\quad &(h \vee \neg(p_1 \vee p_2)) \\ &= (\neg h \vee p_1 \vee p_2) &\wedge\quad &(h \vee (\neg p_1 \wedge \neg p_2)) \end{aligned}$$

2. $h \leftrightarrow (p_1 \wedge p_2)$

CNF conversion: Tseitin's encoding

In the previous formula, we have the following templates:

1. $h \leftrightarrow (p_1 \vee p_2)$
$$\begin{aligned} &= (h \rightarrow (p_1 \vee p_2)) && \wedge && (h \leftarrow (p_1 \vee p_2)) \\ &= (\neg h \vee (p_1 \vee p_2)) && \wedge && (h \vee \neg(p_1 \vee p_2)) \\ &= (\neg h \vee p_1 \vee p_2) && \wedge && (h \vee (\neg p_1 \wedge \neg p_2)) \\ &= (\neg h \vee p_1 \vee p_2) && \wedge && (h \vee \neg p_1) \quad \wedge \quad (h \vee \neg p_2) \end{aligned}$$

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2. $h \leftrightarrow (p_1 \wedge p_2)$
$$= (h \rightarrow (p_1 \wedge p_2)) \quad \wedge \quad (h \leftarrow (p_1 \wedge p_2))$$

CNF conversion: Tseitin's encoding

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$$\begin{aligned}
&= (h \rightarrow (p_1 \vee p_2)) && \wedge \quad (h \leftarrow (p_1 \vee p_2)) \\
&= (\neg h \vee (p_1 \vee p_2)) && \wedge \quad (h \vee \neg (p_1 \vee p_2)) \\
&= (\neg h \vee p_1 \vee p_2) && \wedge \quad (h \vee (\neg p_1 \wedge \neg p_2)) \\
&= (\neg h \vee p_1 \vee p_2) && \wedge \quad (h \vee \neg p_1) \quad \wedge \quad (h \vee \neg p_2)
\end{aligned}$$

2. $h \leftrightarrow (p_1 \wedge p_2)$
$$\begin{aligned}
&= (h \rightarrow (p_1 \wedge p_2)) && \wedge \quad (h \leftarrow (p_1 \wedge p_2)) \\
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CNF conversion: Tseitin's encoding

$$\phi := (a \wedge b) \vee (\neg c \wedge (d \vee e))$$

$$\boxed{\begin{array}{l} h \leftrightarrow (p_1 \vee p_2) = (\neg h \vee p_1 \vee p_2) \wedge (h \vee \neg p_1) \wedge (h \vee \neg p_2) \\ h \leftrightarrow (p_1 \wedge p_2) = (\neg h \vee p_1) \wedge (\neg h \vee p_2) \wedge (h \vee \neg p_1 \vee \neg p_2) \end{array}}$$

$$\begin{aligned} a_1 \leftrightarrow (a_2 \vee a_3) &= \\ a_2 \leftrightarrow (a \wedge b) &= \\ a_3 \leftrightarrow (\neg c \wedge a_4) &= \\ a_4 \leftrightarrow (d \vee e) &= \end{aligned}$$

CNF conversion: Tseitin's encoding

$$\phi := (a \land b) \lor (\neg c \land (d \lor e))$$

$$\boxed{\begin{array}{l} h \leftrightarrow (p_1 \lor p_2) = (\neg h \lor p_1 \lor p_2) \land (h \lor \neg p_1) \land (h \lor \neg p_2) \\ h \leftrightarrow (p_1 \land p_2) = (\neg h \lor p_1) \land (\neg h \lor p_2) \land (h \lor \neg p_1 \lor \neg p_2) \end{array}}$$

$$\begin{array}{lll} a_1 \leftrightarrow (a_2 \lor a_3) & = (\neg a_1 \lor a_2 \lor a_3) & \land (a_1 \lor \neg a_2) \quad \land (a_1 \lor \neg a_3) \\ a_2 \leftrightarrow (a \land b) & = & \\ a_3 \leftrightarrow (\neg c \land a_4) & = & \\ a_4 \leftrightarrow (d \lor e) & = & \end{array}$$

CNF conversion: Tseitin's encoding

$$\phi := (a \wedge b) \vee (\neg c \wedge (d \vee e))$$

$$\boxed{\begin{array}{l} h \leftrightarrow (p_1 \vee p_2) = (\neg h \vee p_1 \vee p_2) \wedge (h \vee \neg p_1) \wedge (h \vee \neg p_2) \\ h \leftrightarrow (p_1 \wedge p_2) = (\neg h \vee p_1) \wedge (\neg h \vee p_2) \wedge (h \vee \neg p_1 \vee \neg p_2) \end{array}}$$

$$\begin{array}{llll} a_1 \leftrightarrow (a_2 \vee a_3) & = (\neg a_1 \vee a_2 \vee a_3) & \wedge (a_1 \vee \neg a_2) & \wedge (a_1 \vee \neg a_3) \\ a_2 \leftrightarrow (a \wedge b) & = (\neg a_2 \vee a) & \wedge (\neg a_2 \vee b) & \wedge (a_2 \vee \neg a \vee \neg b) \\ a_3 \leftrightarrow (\neg c \wedge a_4) & = & & \\ a_4 \leftrightarrow (d \vee e) & = & & \end{array}$$

CNF conversion: Tseitin's encoding

$$\phi := (a \wedge b) \vee (\neg c \wedge (d \vee e))$$

$$\boxed{\begin{array}{l} h \leftrightarrow (p_1 \vee p_2) = (\neg h \vee p_1 \vee p_2) \wedge (h \vee \neg p_1) \wedge (h \vee \neg p_2) \\ h \leftrightarrow (p_1 \wedge p_2) = (\neg h \vee p_1) \wedge (\neg h \vee p_2) \wedge (h \vee \neg p_1 \vee \neg p_2) \end{array}}$$

$$\begin{array}{llll} a_1 \leftrightarrow (a_2 \vee a_3) & = (\neg a_1 \vee a_2 \vee a_3) & \wedge(a_1 \vee \neg a_2) & \wedge(a_1 \vee \neg a_3) \\ a_2 \leftrightarrow (a \wedge b) & = (\neg a_2 \vee a) & \wedge(\neg a_2 \vee b) & \wedge(a_2 \vee \neg a \vee \neg b) \\ a_3 \leftrightarrow (\neg c \wedge a_4) & = (\neg a_3 \vee \neg c) & \wedge(\neg a_3 \vee a_4) & \wedge(a_3 \vee c \vee \neg a_4) \\ a_4 \leftrightarrow (d \vee e) & = & & \end{array}$$

CNF conversion: Tseitin's encoding

$$\phi := (a \wedge b) \vee (\neg c \wedge (d \vee e))$$

$$\boxed{\begin{array}{l} h \leftrightarrow (p_1 \vee p_2) = (\neg h \vee p_1 \vee p_2) \wedge (h \vee \neg p_1) \wedge (h \vee \neg p_2) \\ h \leftrightarrow (p_1 \wedge p_2) = (\neg h \vee p_1) \wedge (\neg h \vee p_2) \wedge (h \vee \neg p_1 \vee \neg p_2) \end{array}}$$

$$\begin{array}{llll}
a_1 \leftrightarrow (a_2 \vee a_3) & = (\neg a_1 \vee a_2 \vee a_3) & \wedge (a_1 \vee \neg a_2) & \wedge (a_1 \vee \neg a_3) \\
a_2 \leftrightarrow (a \wedge b) & = (\neg a_2 \vee a) & \wedge (\neg a_2 \vee b) & \wedge (a_2 \vee \neg a \vee \neg b) \\
a_3 \leftrightarrow (\neg c \wedge a_4) & = (\neg a_3 \vee \neg c) & \wedge (\neg a_3 \vee a_4) & \wedge (a_3 \vee c \vee \neg a_4) \\
a_4 \leftrightarrow (d \vee e) & = (\neg a_4 \vee d \vee e) & \wedge (a_4 \vee \neg d) & \wedge (a_4 \vee \neg e)
\end{array}$$

CNF conversion: Tseitin's encoding

$$\phi := (a \wedge b) \vee (\neg c \wedge (d \vee e))$$

$\mathrm{CNF}(\phi) =$

$$\begin{array}{lllllll}
(\neg a_1 \vee a_2 \vee a_3) & \wedge & (a_1 \vee \neg a_2) & \wedge & (a_1 \vee \neg a_3) & & \wedge \\
(\neg a_2 \vee a) & \wedge & (\neg a_2 \vee b) & \wedge & (a_2 \vee \neg a \vee \neg b) & & \wedge \\
(\neg a_3 \vee \neg c) & \wedge & (\neg a_3 \vee a_4) & \wedge & (a_3 \vee c \vee \neg a_4) & & \wedge \\
(\neg a_4 \vee d \vee e) & \wedge & (a_4 \vee \neg d) & \wedge & (a_4 \vee \neg e) & & \wedge \\
a_1 & & & & & &
\end{array}$$

where: a_1, a_2, a_3, a_4 are newly introduced variables.