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A Case Study on Model Predictive Control of the VLS-1 Attitude at Max Dynamic Pressure

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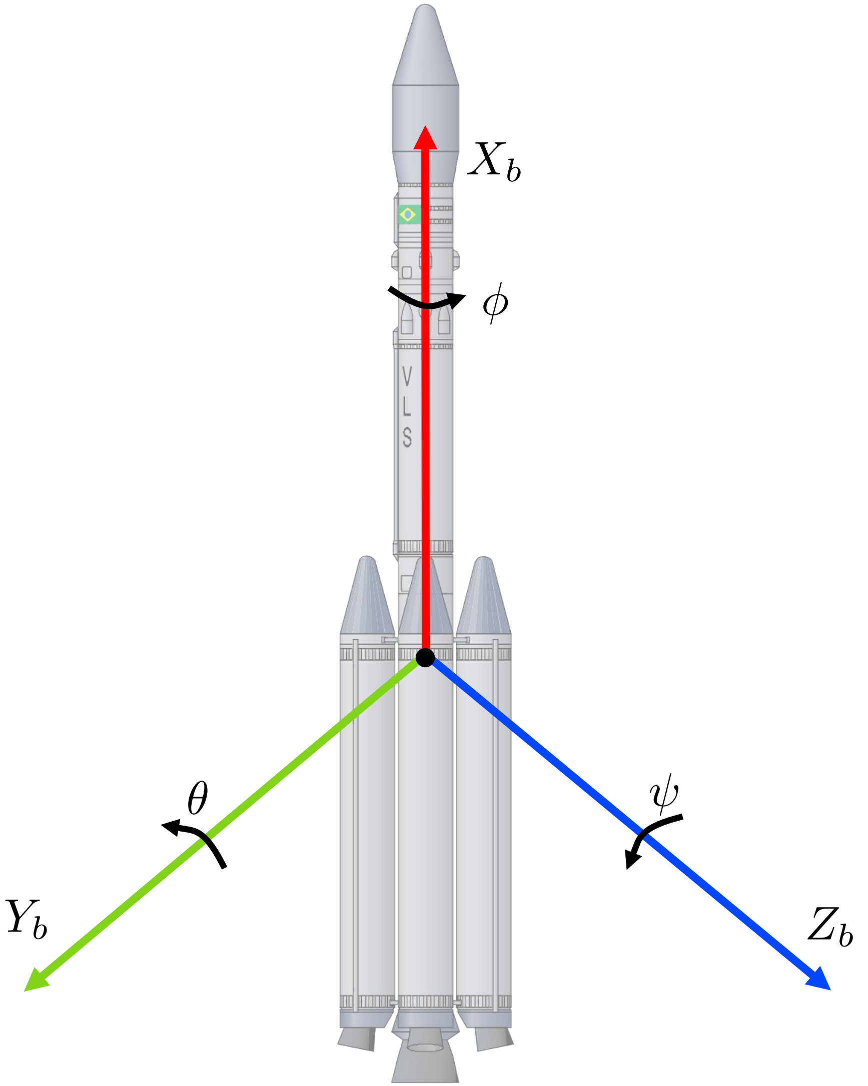
Introduction

- Main topics
  - From the attitude control system perspective: investigate the thesis that the intense vibratory regime caused the VLS-1 V02 failure
  - Case study: VLS-1-based model with bending modes, time delay, control saturation, and sensor noise
  - A functional approximation of the VLS-1 **PI-D** control
  - MPC proposition: Predictive Minimum Variance Control with Full State Feedback (**PMVC**)

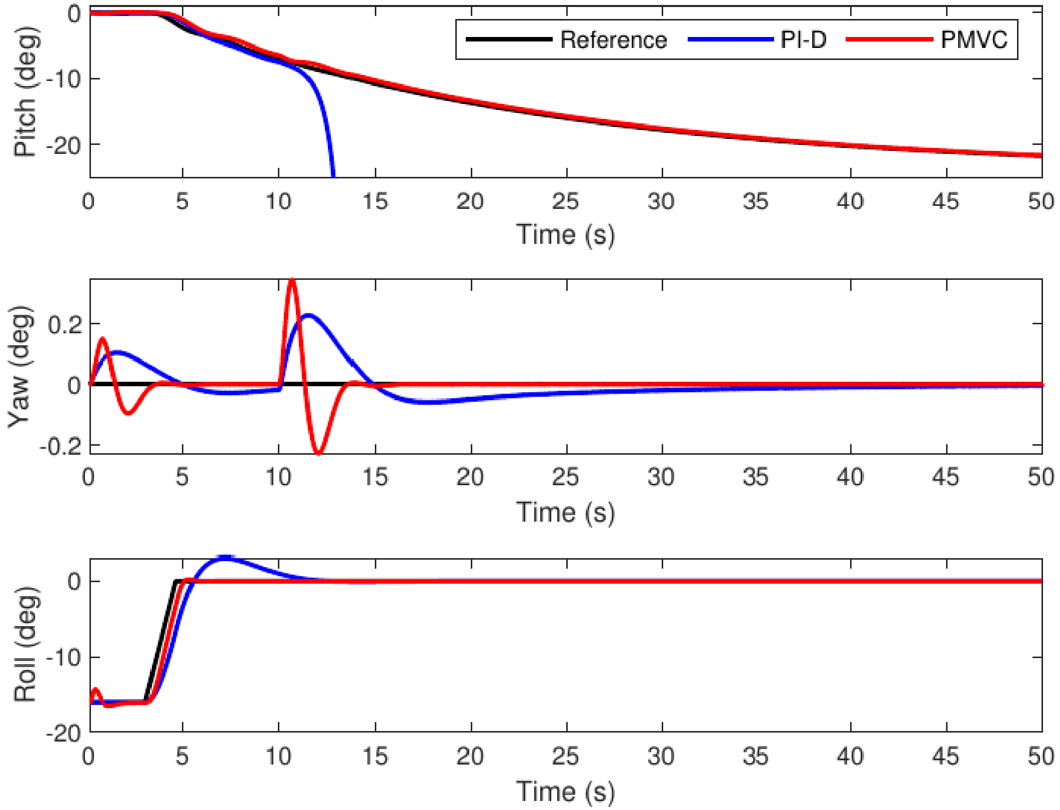
Control Problem Overview

- Pitch (θ)
- Yaw (ψ)
- Roll (ϕ)

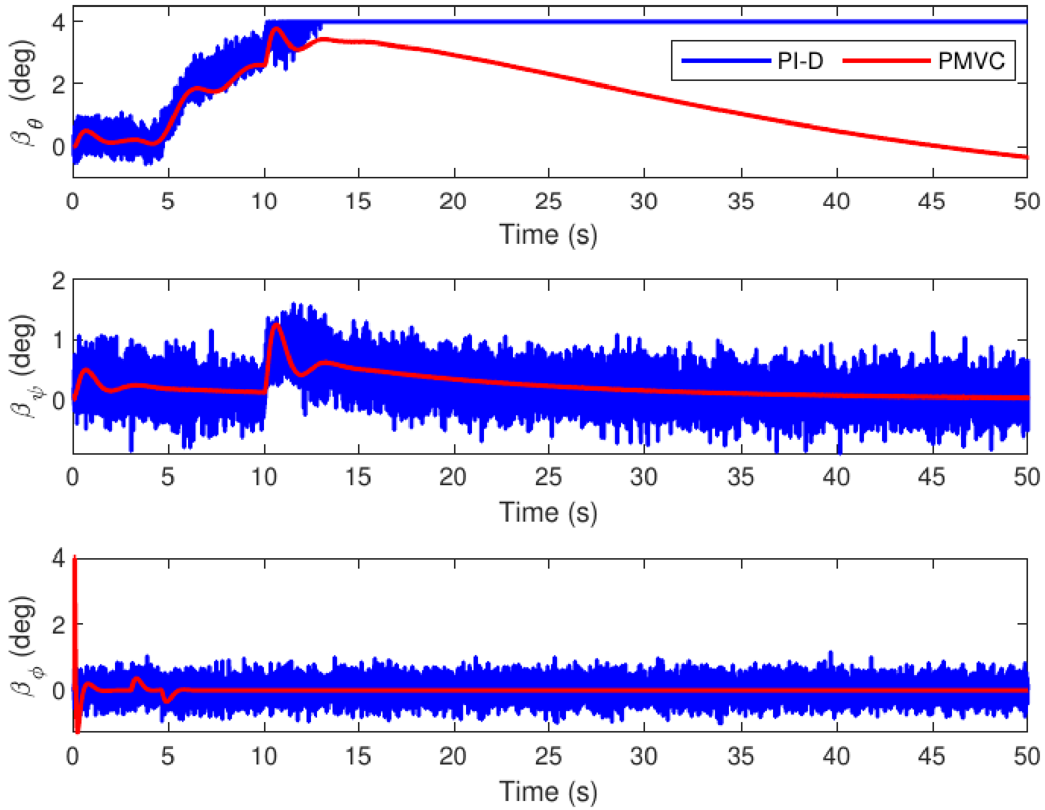


https://en.wikipedia.org/wiki/VLS-1#/media/File:VLS-1.svg

Control Problem Overview

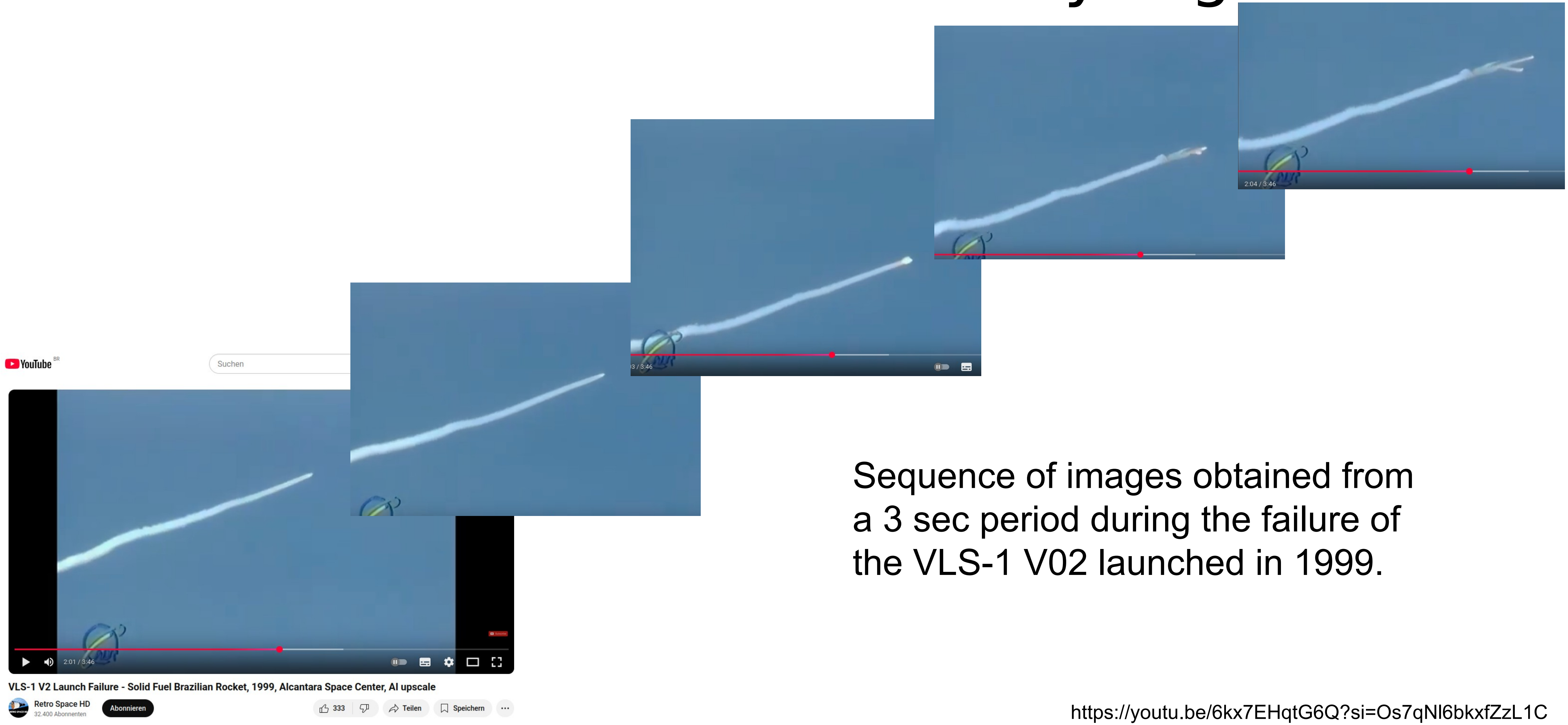


Attitude angles



- Actuators' angular commands

VLS-1 in the intense vibratory regime



Sequence of images obtained from a 3 sec period during the failure of the VLS-1 V02 launched in 1999.

https://youtu.be/6kx7EHqtG6Q?si=Os7qNl6bkxfZzL1C

VLS-1 in the intense vibratory regime

“... em dezembro de 1999, foi lançado o segundo protótipo, o VLS-1 V02. O voo ocorreu dentro do esperado até o momento da ignição do segundo estágio, quando esse motor explodiu. A conclusão técnica relatada pela comissão de investigação é apresentada no capítulo 4. Apesar de não haver sido cumprida a missão, ***o veículo demonstrou ser capaz de vencer as fases mais exigentes do voo atmosférico, tais como a região transônica e a de pressão dinâmica máxima. Há quem questione a afirmação acima, admitindo que o intenso regime vibratório tivesse causado a explosão do segundo estágio. No entanto, não há prova cabal dessa tese até o momento.***”

“Although the mission was not accomplished, ***the vehicle demonstrated its ability to overcome the most demanding phases of atmospheric flight, such as the transonic region and maximum dynamic pressure. Some question the above statement, assuming that the intense vibratory regime caused the second stage to explode. However, there is no conclusive proof of this theory to date.***”

PALMERIO, A. F. Introdução à Tecnologia de Foguetes. 2nd. ed. SindCT, São José dos Campos - SP, 2017. Disponível em: <https://sindct.org.br/sindct/comunicacao/livros-e-cartilhas/introducao-a-tecnologia-de-foguetes/>.

VLS-1 simulation model, and design models

- Parameters from a linearized model at Max Q

  RAMOS, F. de O. Automation of H∞ controller design and its observer-based realization. Tese (Doutorado) — Instituto Nacional de Pesquisas Espaciais (INPE), Institut Supérieur de l'Aéronautique et de l'Espace (ISAE), São José dos Campos (Brasil), Toulouse (França), 2011.

- Simulation of a coupled MIMO model with
  - enabled bending modes
  - atmospheric disturbances to the states
  - delay to simulate non-collocated sensor-actuator
  - sensor noise to simulate vibrations
- Design models: high-order *versus* 2nd-order approx.

VLS-1 Attitude Simulation Model

$$\dot{\mathbf{x}}(t) = \mathbf{A_c}\mathbf{x}(t) + \mathbf{B_c}\mathbf{sat}[\mathbf{u}(t-t_d)] + \mathbf{G_c}\mathbf{w}(t)$$

$$\mathbf{y}(t) = \mathbf{C}x(t) + \mathbf{v}(t)$$

$$\mathbf{sat}\left[\mathbf{u}(t)\right] = \begin{cases} \mathbf{u}_{\max} & \text{if } \mathbf{u}(t) \geqslant \mathbf{u}_{\max} \\ \mathbf{u}(t) & \text{if } \mathbf{u}_{\min} < \mathbf{u}(t) < \mathbf{u}_{\max} \\ \mathbf{u}_{\min} & \text{if } \mathbf{u}(t) \leqslant \mathbf{u}_{\min} \end{cases}$$

$$\mathbf{x^T}(t) = \left[\begin{array}{cccccccccccccccc} w & q & \theta & \theta_{b_{11}} & \theta_{b_{12}} & \theta_{b_{21}} & \theta_{b_{22}} & v & r & \psi & \psi_{b_{11}} & \psi_{b_{12}} & \psi_{b_{21}} & \psi_{b_{22}} & p & \phi \end{array}\right].$$

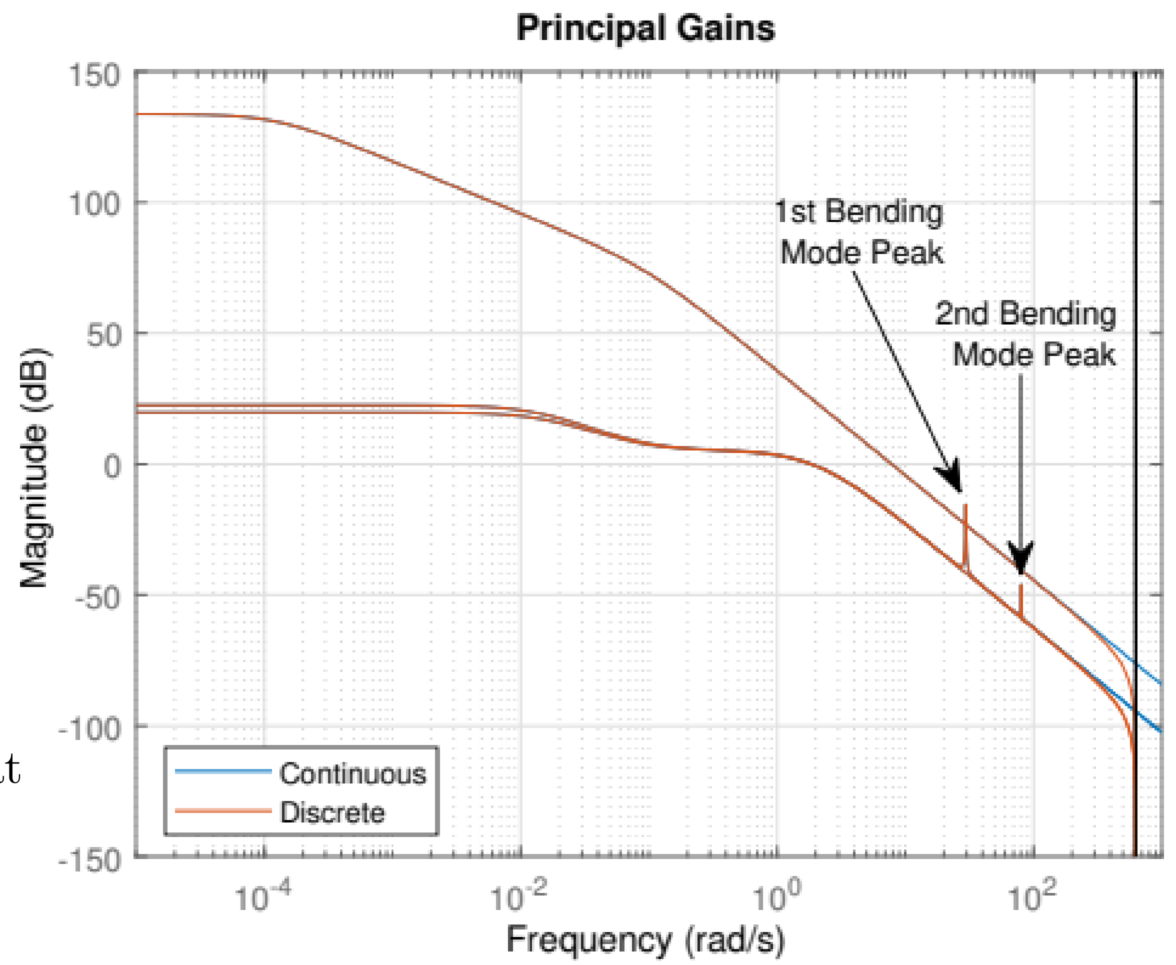
$$\mathbf{u^T}(t) = \left[\begin{array}{ccc} \beta_\theta & \beta_\psi & \beta_\phi \end{array}\right] \qquad \mathbf{w^T}(t) = \left[\begin{array}{cc} w_\theta & w_\psi \end{array}\right]$$

$$\mathbf{y^T}(t) = \left[\begin{array}{ccc} \theta & \psi & \phi \end{array}\right] \qquad \mathbf{v^T}(t) = \left[\begin{array}{ccc} v_\theta & v_\psi & v_\phi \end{array}\right]$$

Control system design models

- **High-order design model in the discrete time domain**

$$\mathbf{x}(k) = \mathbf{A}\mathbf{x}(k-1) + \mathbf{B}\mathbf{sat}\left[\mathbf{u}(k-d)\right] + \mathbf{G}\mathbf{w}(k-1)$$
$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{v}(k)$$
$$d = 1 + \lceil t_d/T_s \rceil$$



VLS-1 principal gains, remarking the two bending modes at 29.5 and 80.4 rad/s.
Discrete-time model using a sampling time of $T_s = 5$ ms.

Control system design models

- **2nd-order approximations (at max Q) for control system design**

$$\frac{\theta(s)}{\beta_\theta(s)} = \frac{-\bar{M}_{\beta_z}}{s^2 - \bar{M}_\alpha} = \frac{-(-7.3734)}{s^2 - (4.077)},$$

$$\frac{\psi(s)}{\beta_\psi(s)} = \frac{\bar{N}_{\beta_y}}{s^2 - \bar{N}_\beta} = \frac{(-7.5648)}{s^2 - (4.0726)},$$

$$\frac{\phi(s)}{\beta_\phi(s)} = \frac{\bar{L}_{\beta_x}}{s^2 + \bar{L}_p s} = \frac{(61.0492)}{s^2 + (0.098859)\, s}.$$

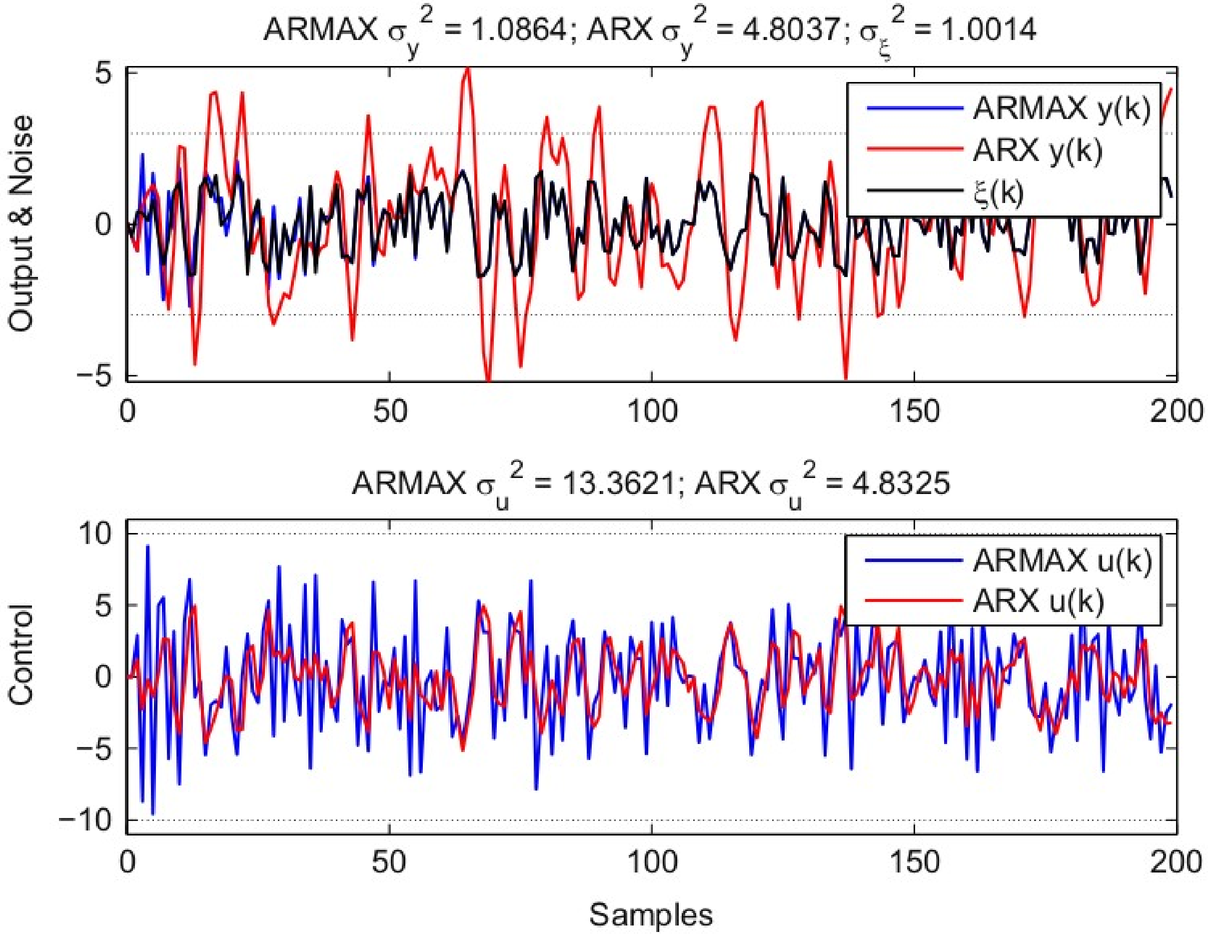
- **A PI-D control structure. Tuning using LQR and a canonical state realization**

$$U(s) = \left(k_p + k_i \frac{1}{s}\right) R(s) - \left(k_p + k_i \frac{1}{s} + k_d s\right) Y(s),$$

SILVA, F. O. et al. Tuning techniques evaluation for satellite launch vehicle attitude controllers. Journal of Aerospace Technology and Management, Dept. of Aerospace Science and Technology, v. 11, p. e2419, 2019. ISSN 2175-9146. Disponível em: <https: //doi.org/10.5028/jatm.v11.1004>.

Minimum variance control vs PMVC

- **Ideal MVC and the high control chattering**



- **Detuned generalized MVC as a solution to the high control signal chattering**

$$\min_{\Delta u(k)} \; \mathbf{E}\left[S(z)y(k+d) - T(z)r(k) + R(z)\Delta u(k)\right]^2$$

$$\text{Solved based on } \Delta u(k) = \frac{T(z)}{R(z)}r(k) - \frac{S(z)}{R(z)}y(k)$$

- **PMVC with Full State Feedback proposition**

Solved based on

$$\mathbf{\Lambda}\Delta\mathbf{u}(k) = \mathbf{K}\left[\mathbf{r}(k) - \hat{\mathbf{x}}_{\mathbf{a}}(k+N_x)\right],$$

PMVC (using MIMO LQG-based tuning)

$$\mathbf{\Lambda}\Delta\mathbf{u}(k) = \mathbf{K}\left[\mathbf{r}(k) - \hat{\mathbf{x}}_{\mathbf{a}}(k+N_x)\right],$$

$$\mathbf{g}(k+N_x) = \mathbf{\Lambda}\Delta\mathbf{u}(k) + \mathbf{K}\hat{\mathbf{x}}_{\mathbf{a}}(k+N_x) - \mathbf{K}\mathbf{r}(k)$$

$$\min_{\Delta\mathbf{u}(k)} J = \mathbf{E}\left[\mathbf{g}^{\mathbf{T}}(k+N_x)\mathbf{g}(k+N_x)\right]$$

- **Augmented state space model to include a discrete integrator**

$$\mathbf{x}_{\mathbf{a}}^{\mathbf{T}}(k) = \begin{bmatrix} \mathbf{y}(k) & \Delta\mathbf{x}(k) \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{y}(k) \\ \Delta\mathbf{x}(k) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{CA} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{y}(k-1) \\ \Delta\mathbf{x}(k-1) \end{bmatrix} + \begin{bmatrix} \mathbf{CB} \\ \mathbf{B} \end{bmatrix} \Delta\mathbf{u}(k-d) + \begin{bmatrix} \mathbf{I} & \mathbf{CG} \\ \mathbf{0} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{v}(k) \\ \Delta\mathbf{w}(k-1) \end{bmatrix},$$

$$\mathbf{y}_{\mathbf{a}}(k) = \begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{y}(k) \\ \Delta\mathbf{x}(k) \end{bmatrix}.$$

PMVC state predictor

$$\hat{\mathbf{x}}_{\mathbf{a}}(k+N_x) = \left(\mathbf{A}_{\mathbf{a}}^{N_x} - \mathbf{A}_{\mathbf{a}}^{(N_x-1)}\mathbf{L}\mathbf{C}_{\mathbf{a}}\right)\bar{\mathbf{x}}_{\mathbf{a}}(k) + \mathbf{B}_{\mathbf{a}}\Delta\mathbf{u}(k)$$

$$+\begin{bmatrix} \mathbf{A}_{\mathbf{a}}^{1}\mathbf{B}_{\mathbf{a}} \\ \vdots \\ \mathbf{A}_{\mathbf{a}}^{(N_x-1)}\mathbf{B}_{\mathbf{a}} \end{bmatrix}^T \underbrace{\begin{bmatrix} \Delta\mathbf{u}(k-1) \\ \vdots \\ \Delta\mathbf{u}(k-N_x+1) \end{bmatrix}}_{\underleftarrow{\Delta\mathbf{u}}(k-1)} + \mathbf{A}_{\mathbf{a}}^{(N_y-1)}\mathbf{L}\mathbf{y}(k)$$

$$\bar{\mathbf{x}}_{\mathbf{a}}(k) = (\mathbf{A}_{\mathbf{a}} - \mathbf{L}\mathbf{C}_{\mathbf{a}})\,\bar{\mathbf{x}}_{\mathbf{a}}(k-1) + \mathbf{B}_{\mathbf{a}}\Delta\mathbf{u}(k-N_x) + \mathbf{L}\mathbf{y}(k-1).$$

SILVEIRA, A. et al. Design and real-time implementation of a wireless autopilot using multivariable predictive generalized minimum variance control in the state-space. Aerospace Science and Technology, v. 105, p. 106053, 2020. ISSN 1270-9638.

PMVC optimal control law

$$\Delta\mathbf{u}(k) = (\mathbf{K}\mathbf{B_a} + \mathbf{\Lambda})^{-1}\left[\mathbf{K}\mathbf{r}(k) - \mathbf{M_x}\bar{\mathbf{x}}_\mathbf{a}(k) - \mathbf{M_u}\underleftarrow{\Delta\mathbf{u}}(k-1) - \mathbf{M_y}\mathbf{y}(k)\right]$$

$$\mathbf{M_x} = \mathbf{K}\left(\mathbf{A_a}^{N_x} - \mathbf{A_a}^{(N_x-1)}\mathbf{L}\mathbf{C_a}\right)$$

$$\mathbf{M_u} = \mathbf{K}\left[\begin{array}{ccc} \mathbf{A_a}^1\mathbf{B_a} & \cdots & \mathbf{A_a}^{(N_x-1)}\mathbf{B_a} \end{array}\right]$$

$$\mathbf{M_y} = \mathbf{K}\mathbf{A_a}^{(N_x-1)}\mathbf{L}$$

$$\mathbf{u}(k) = \mathbf{u}(k-1) + \Delta\mathbf{u}(k)$$

- **Remark: the detuned PMVC solution is now based on MIMO state space dynamic compensators with full estimated state feedback.**

- **Designer tuning parameters:** $N_x, \mathbf{K}, \mathbf{L}, \mathbf{\Lambda}$

Simulations setup

- **PMVC**

$$N_x = 11,\ \mathbf{\Lambda} = \mathbf{I}_3$$
$$\text{LQR } \mathbf{Q} = \mathbf{C_a^T C_a} \text{ and } \mathbf{R} = 10^3 \mathbf{I}_3$$
$$\text{Kalman filter } \mathbf{Q} = \mathbf{I}_{19} \text{ and } \mathbf{R} = \mathbf{I}_3$$

  - **Remark:** with $N_x = 11$, closed-loop stability is guaranteed up to $t_d = 50$ ms, theoretically.

- *** Guidance profile:** 50-second simulation, executing pitch-over and roll program maneuvers.

- *** Wind disturbances:** 6 m/s during liftoff, and 20 m/s 10 seconds later.

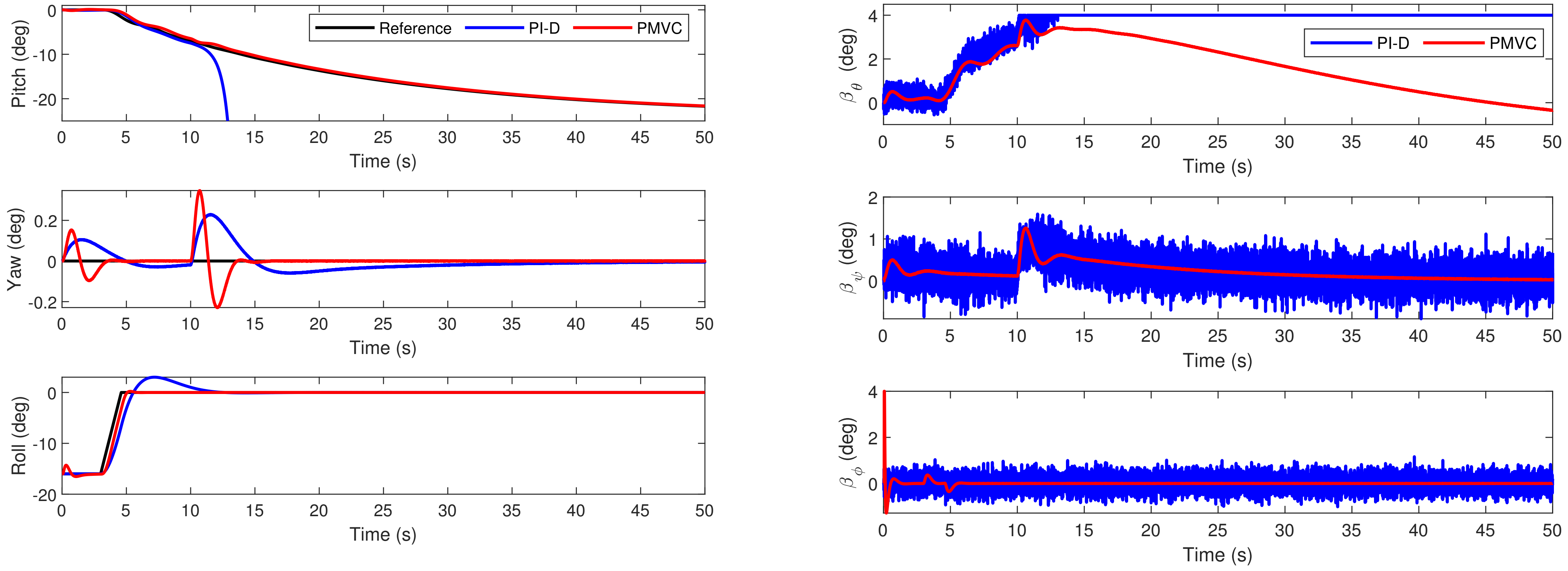
- **Sensor noise power:** $\sigma_{\mathbf{v}}^2 = 10^{-10}\ (\mathrm{rad/h})^2$

  ** Interferometric Fiber Optic Gyroscope noise power: $\sigma_{\mathbf{v}}^2 = 10^{-5}\ (\mathrm{rad/h})^2$

* YAMADA, A. F. C. de S.; KIENITZ, K. H.; RAMOS, F. de O. Robust attitude control of a flexible launch vehicle subjected to wind disturbances. In: XXV Brazilian Congress of Automatics (CBA), Rio de Janeiro, RJ, Brazil, 2024.

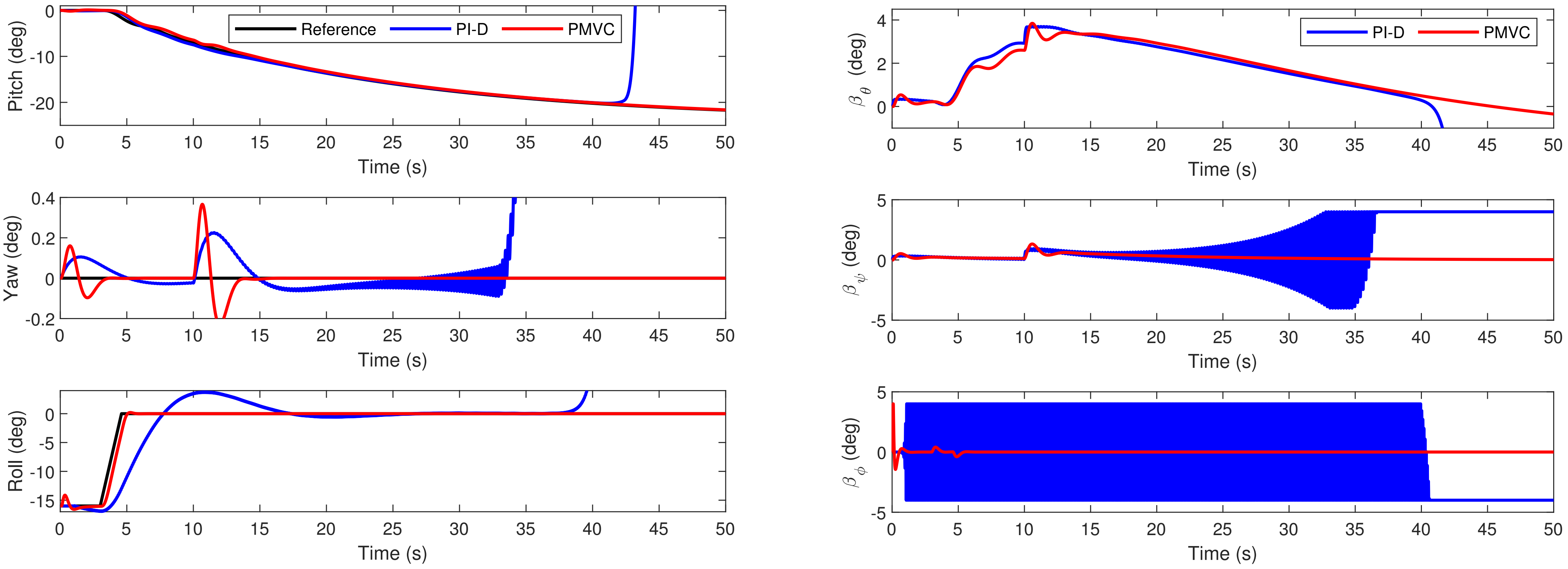
** NUNES, G. et al. Simulation of an interferometric fiber optic gyroscope applied to a rocket model. In: Simpósio de Aplicações Operacionais em Áreas de Defesa (SIGE), 2022.

Results: sensor noise, without delay



Attitude angles on the left graphics and control signals on the right. The vibrations simulated with noises were barely visible, but their effect due to feedback in the PI-D was catastrophic, leading to high control signal chattering, saturation, and instability.

Results: with delay, without noise



Attitude angles on the left graphics and control signals on the right. Instability with the PI-D was observed for $t_d \geq 10$ ms, but this case in the graphics was simulated using $t_d = 20$ ms, to better depict the problem.

Conclusions and beyond...

- Despite not shown, the PMVC handled up to $t_d = 120$ ms with $N_x = 11.$

- Even though the PMVC used a much more complex design model, it solves the control problem at once for all state variables, and runs faster than the three independent PI-D controllers. The PMVC simulated loop-time required 0.23545 microseconds, and the PI-D 5.5674 microseconds. These were the average times after processing 10001 iterations using MATLAB R2021a, using an AMD Ryzen 5 5600 processor, with 32 GB DDR4 at 3200 MHz.

- Despite this work cannot bring conclusive proof regarding the thesis added by Palmerio (2017), that the intense vibratory regime was related to the accident with the VLS-1 in 1999, it does bring evidence that this functional approximation of the PI-D used in the vehicle lacked the robustness to deal with the vibrations.

- In a follow-up work, this lack of robustness will be detailed using gain and phase margins, showing from a closed-loop sensitivity analysis perspective that these margins were smaller than expected and how LQG- and MPC-based techniques are by far more appropriate to handle such problems.

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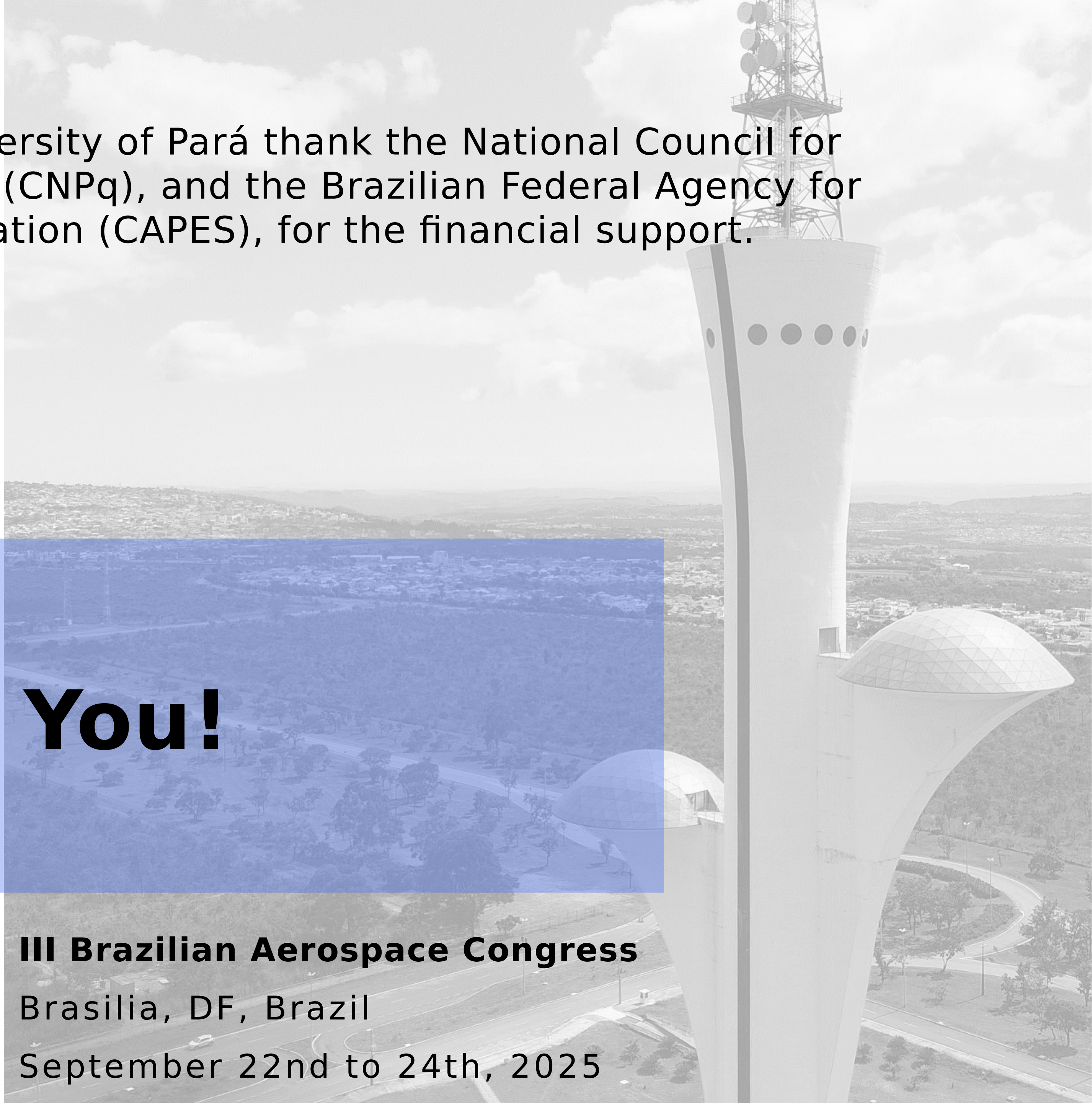
Thank You!



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