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Astro 507: Thermodynamics
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Homework 4

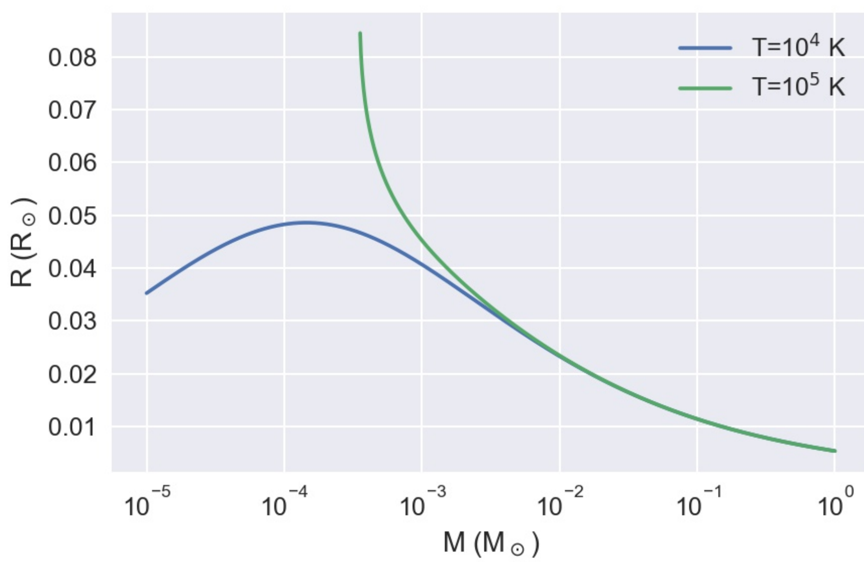
1. (a) In the one-zone model, $P_c \sim GM^2/R^4$. We also assume $\rho \sim M/R^3$.

$$\frac{GM^2}{R^4} = K_{\text{ideal}} \left(\frac{M}{R^3}\right) T + K_e \left(\frac{M}{R^3}\right)^{5/3} - K_C \left(\frac{M}{R^3}\right)^{4/3} \tag{1}$$

$$G = K_{\text{ideal}} M^{-1} RT + K_e M^{-1/3} R^{-1} - K_C M^{-2/3} \tag{2}$$

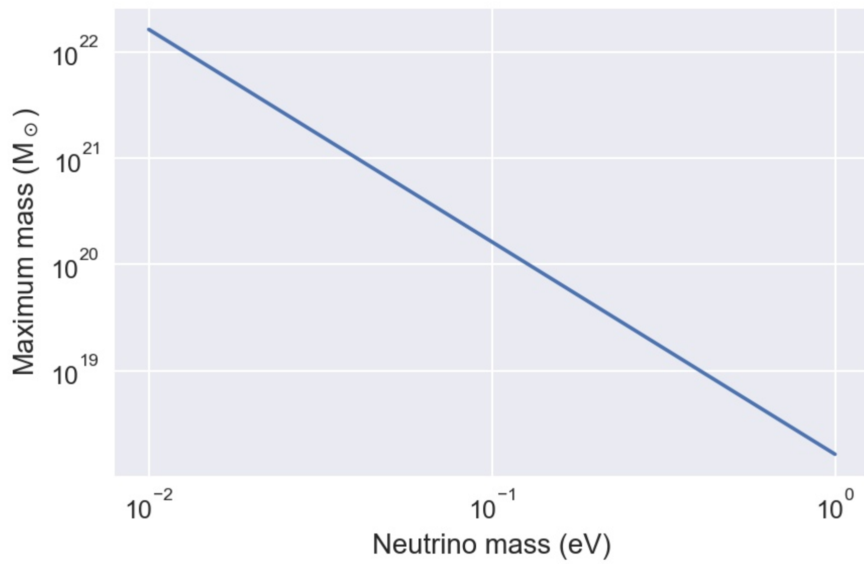
$$R = \frac{GM + K_C M^{1/3} - \sqrt{(K_C M^{1/3} + GM)^2 - 4K_{\text{ideal}} K_e M^{2/3} T}}{2K_{\text{ideal}} T} \tag{3}$$

(b) Plot of radius as a function of mass:



(c) I don't really understand what's meant by branches, but when the temperature is increased to 10^5 K everything blows up below $\sim 10^{-3.5}$ $M_\odot$.

2. The maximum mass of a degenerate neutrino ball is $M_{\max} \approx (\hbar c/G)^{3/2} m_\nu^{-2}$. Plot of $M_{\max}$ as a function of neutrino mass:



For a maximum black hole mass of $4 \times 10^9\ M_\odot$, the requisite neutrino mass is 20.2 keV, which is not at all consistent with the known neutrino mass upper limit. The mass is related to the radius by $R \approx h^2/(m_\nu^{8/3}G)M^{-1/3}$, which for $M_{BH} = 4 \times 10^9\ M_\odot$ and $m_\nu = 0.1$ eV gives a radius of approximately 1×10^{12} pc, which seems somewhat unreasonable. For a 20 keV neutrino, the black hole radius is 1559 AU, which is still on the unreasonable side.