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Astro 507: Thermodynamics
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Homework 5

1. (a) The thermal de Broglie wavelength of a particle is $\lambda = h/(2\pi mkT)^{1/2}$; for an electron at the center of the sun, $\lambda_e = 1.924573 \times 10^{-9}$ cm. This gives a degeneracy parameter of $n_e\lambda_e^3 = 0.713$.

   (b) Assuming the sun is entirely hydrogen and disregarding all states other than the ground state, we can use a simplified form of the Saha equation:

$$\frac{n_p}{n_{HI}} = n_e^{-1}\lambda_e^{-3}e^{-\chi_I/kT} \tag{1}$$
$$= 1.39 \tag{2}$$

   Substituting $X = n_p/(n_p + n_{HI}) = n_p/n_H$, we find:

$$\frac{X^2}{1-X} = 1.39 \tag{3}$$
$$X = 0.67 \tag{4}$$

   (c) Assuming that this question refers to ionized hydrogen, we can apply the concept of a Wigner-Seitz cell and find r_p using $n_p = n_e$:

$$r_p = \left(\frac{3}{4\pi n_p}\right)^{1/3} \tag{5}$$
$$= 1.34 \times 10^{-9} \text{ cm} \tag{6}$$
$$= 0.13 \text{ Å} \tag{7}$$

   The calculation changes only slightly for non-ionized hydrogen, given that $n_H = n_p/0.67$.

   (d) The Saha equation is only valid for relatively low-density gases, where $n_e\lambda_e^3 \ll 1$; we found that this was not so in part (a). One cannot treat atoms at the center of the sun in isolation because the mean separation is well under a Bohr radius, which means that the ionization energy is impacted by degeneracy. I would expect that the hydrogen at the center of the sun would be fully ionized.