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Astro 507: Thermodynamics
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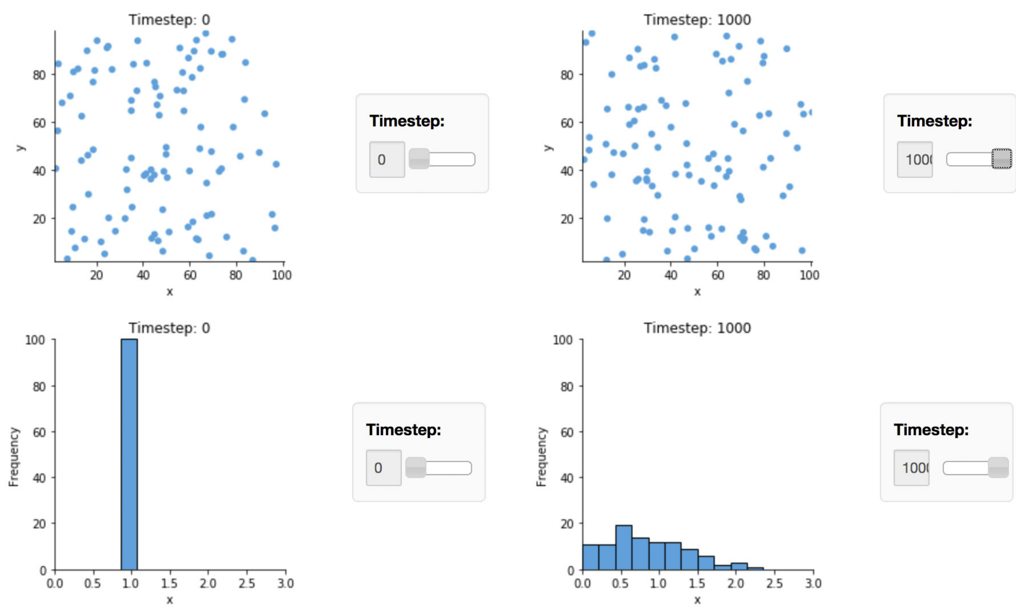
Homework 1

1. (a) For my particles in a box I chose a box side length of 100, particle masses and initial velocities of 1, and a timestep of 0.1. I use the `query_pairs()` method of `scipy.spatial.cKDTree` to find all pairs of particles within twice the particle radius of each other. During a collision, I calculate the normal and tangential velocities during a collision and use those to update the particle velocities. I chose reflecting box boundaries, such that if a particle bumps into a horizontal wall the y-component of its velocity changes sign, and if it bumps into a vertical wall the x-component of its velocity changes sign.

   An interactive notebook (since github won't display it properly) is available at `https://meredith-durbin.github.io/holoviews/particlesinabox`.
   Github: `https://github.com/meredith-durbin/ASTR507/tree/master/HW1`.

   (b) Plots for initial and final positions and velocities:



2. (a) We are given that $f(q, p) \propto e^{-E/k_B T}$.

$$dN \propto f(p)dp \tag{1}$$
$$= e^{-p^2/2mk_B T} dp \tag{2}$$
$$= e^{-(p_x^2+p_y^2)/2mk_B T} dp_x dp_y \tag{3}$$
$$= e^{-m(v_x^2+v_y^2)/2mk_B T} m^2 dv_x dv_y \tag{4}$$
$$= e^{-mv^2/2k_B T} m^2 v dv d\theta \tag{5}$$

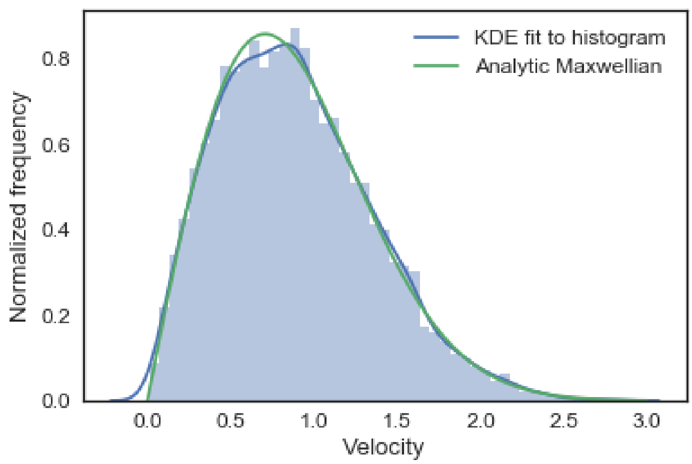
Integrating this over all θ and rearranging, we arrive at $dN/dv \propto 2\pi e^{-mv^2/2k_BT}m^2v$. To find the normalization constant, we want this to integrate to one over all velocities.

$$1 = C\int_0^\infty e^{-mv^2/2k_BT}m^2vdv \tag{6}$$
$$C = \frac{m}{2\pi k_BT} \tag{7}$$

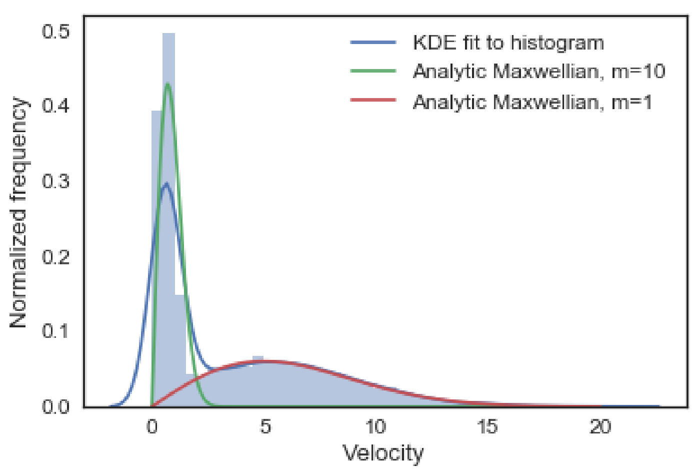
From this we find the Maxwellian velocity distribution, $dN/dv = \frac{m}{2\pi k_BT}ve^{-mv^2/2k_BT}$.

For the energy distribution, we have $dN/dE \propto e^{-E/k_BT}$. It is easy to see that this normalizes simply to $dN/dE = \frac{1}{k_BT}e^{-E/k_BT}$.

(b) Maxwellian velocity distribution overplotted on results coadded from 100 simulation runs:



(c) Two Maxwellian velocity distributions overplotted on results coadded from 100 simulations with half the particles at m=1 and half at m=10:



3. (a) A simple numerical criterion for relaxation would be to calculate the difference between the

50th percentile of the particle velocities and the theoretical 50th percentile of the Maxwellian velocity distribution.

(b) The relaxation time t_{relax} for a given particle is dependent on the mean free path and initial velocity v_0. The mean free path is the inverse of the cross section times the number density, $1/\sigma n$. Therefore, $t_{\text{relax}} = 1/\sigma n v_0$. (For two dimensions, σ is the particle diameter, assuming circular particles.)

(c) Plots of relaxation time by number density and radius: