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## Homework 5

- 1. (a) The thermal de Broglie wavelength of a particle is  $\lambda = h/(2\pi mkT)^{1/2}$ ; for an electron at the center of the sun,  $\lambda_e = 1.924573 \times 10^{-9}$  cm. This gives a degeneracy parameter of  $n_e \lambda_e^3 = 0.713$ .
  - (b) Assuming the sun is entirely hydrogen and disregarding all states other than the ground state, we can use a simplified form of the Saha equation:

$$\frac{n_p}{n_{HI}} = n_e^{-1} \lambda_e^{-3} e^{-\chi_I/kT} \tag{1}$$

$$=1.39\tag{2}$$

Substituting  $X = n_p/(n_p + n_{HI}) = n_p/n_H$ , we find:

$$\frac{X^2}{1-X} = 1.39\tag{3}$$

$$X = 0.67 \tag{4}$$

(c) Assuming that this question refers to ionized hydrogen, we can apply the concept of a Wigner-Seitz cell and find  $r_p$  using  $n_p = n_e$ :

$$r_p = \left(\frac{3}{4\pi n_p}\right)^{1/3} \tag{5}$$

$$= 1.34 \times 10^{-9} \text{ cm} \tag{6}$$

$$= 0.13 \text{ Å}$$
 (7)

The calculation changes only slightly for non-ionized hydrogen, given that  $n_H = n_p/0.67$ .

(d) The Saha equation is only valid for relatively low-density gases, where  $n_e \lambda_e^3 \ll 1$ ; we found that this was not so in part (a). One cannot treat atoms at the center of the sun in isolation because the mean separation is well under a Bohr radius, which means that the ionization energy is impacted by degeneracy. I would expect that the hydrogen at the center of the sun would be fully ionized.