Meredith Durbin
Emily Levesque
Astro 531: Stellar Interiors
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Homework 1

All calculations can be found in the notebook
`https://github.com/meredith-durbin/ASTR531/blob/master/HW1/HW1.ipynb`.

2.3 (a) A distance of 470 ly gives τ Sco a distance modulus of 5.79 mag, which means that its M_V is -2.99 mag.

(b) With a bolometric correction of -3.16 mag, the bolometric magnitude is $M_{\rm bol} = -6.15$ mag, giving a luminosity of 2.28×10^4 $L_\odot$.

(c) From the Stefan-Boltzmann equation, the radius of the star is 5.59 $R_\odot$.

(d) Using the relation $L/L_\odot = 12(M/M_\odot)^{2.9}$, we find a mass of 13.5 $M_\odot$.

(e) The surface gravity of the star is 1.19×10^4 cm s^{-2} ($\log g = 4.07$), and the escape velocity is 9.6×10^7 cm s^{-1}.

(f) The mean density is $\rho = 0.11$ g cm^{-3}.

(g) The surface gravity of τ Sco is about 0.43 of solar, whereas the escape velocity is about 1.55 times solar. τ Sco's mean density is only 0.08 of solar.

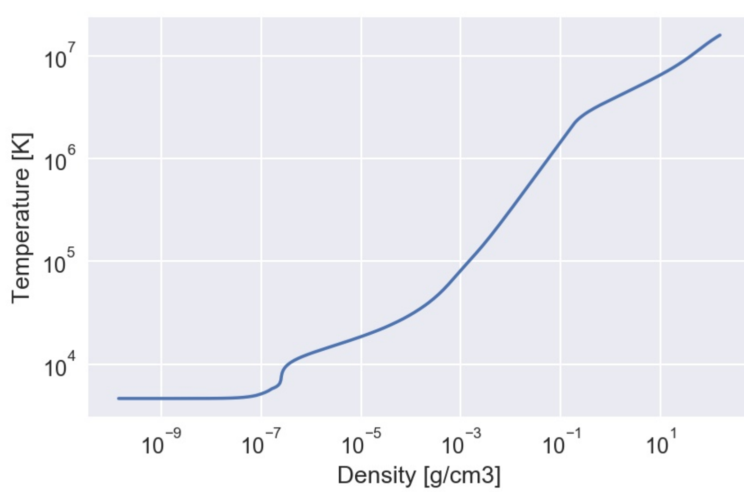
3.4 The virial theorem states that $K = -\frac{1}{2}U$. The potential energy of a star is on the order of $-GM^2/R$, and the kinetic energy is $\frac{3}{2}Nk_B\bar{T}$, where $N = M/\mu m_H$.

$$\frac{3}{2}\frac{M}{\mu m_H}k_B\bar{T} = \frac{1}{2}\frac{GM^2}{R} \tag{1}$$

$$\bar{T} = \frac{\mu m_H}{3k_B}\frac{GM}{R} \tag{2}$$

$$\bar{T} \sim \frac{\mu m_H}{k_B}\frac{GM}{R} \tag{3}$$

4.3 Based on the plot of solar temperature vs. density, it looks as though the sun is largely within the ideal gas regime.

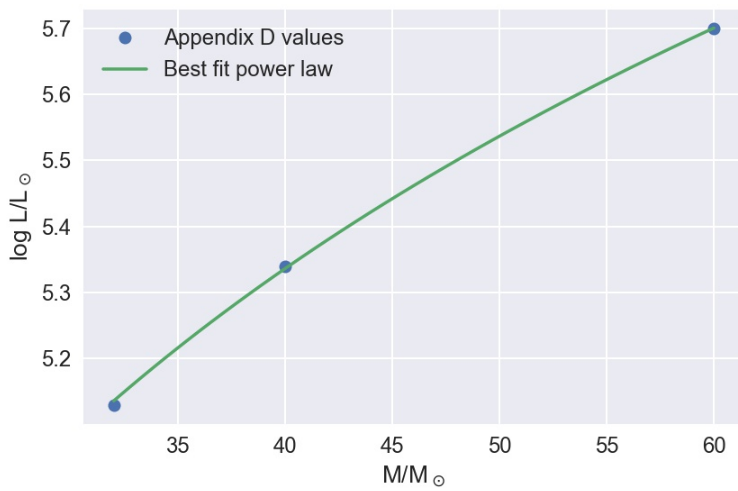


5.2 (a) For a mean free path of $\ell = 1$ cm, it will take a photon 4.8×10^{21} scatterings to travel 1 $R_\odot$.

(b) The total path length ℓN is 4.8×10^{21} cm, or 6.9×10^{10} $R_\odot$. It will take a photon traveling this path 1.6×10^{11} s to exit the sun, or about 5116 years.

(c) This is almost certainly not the same photon.

6.2 (a) Like a true observational astronomer, I fit a power law to three data points to find the high-mass ML relation, and found that the best-fit law is $L/L_\odot = 106(M/M_\odot)^{2.07}$.



(b) For a mass-luminosity relation of the form $L = C_1 M^{C_2}$, we can find $M_{\rm max}$ by substituting the mass-luminosity relation into equation 6.13:

$$\frac{L_E}{L_\odot} = 3.8 \times 10^4 \frac{M}{M_\odot} \tag{4}$$

$$C_1 M_{\rm max}^{C_2} = 3.8 \times 10^4 \frac{M_{\rm max}}{M_\odot} \tag{5}$$

$$\frac{M_{\rm max}}{M_\odot} = \left(\frac{3.8 \times 10^4}{C_1}\right)^{1/(C_2-1)} \tag{6}$$

For $C_1 = 106$ and $C_2 = 2.07$, we find $M_{\rm max} = 248$ $M_\odot$ and $L_{\rm max} = 9.43 \times 10^6$ $L_\odot$.

7.3 (a) The main sequence lifetime can be estimate by comparing the stellar luminosity to the total amount of energy that core fusion can produce. Assuming that all of the hydrogen in the convective core is converted to helium over the MS lifetime, and assuming a hydrogen fusion efficiency factor of 0.007, we can estimate the MS lifetime as $t_{\rm MS} = 0.007 M_{\rm core} c^2 / L_\star$. Assuming a convective core mass fraction of 0.25 for 4 $M_\odot$ and 0.5 for 20 $M_\odot$, we find MS lifetimes of 4.4×10^8 and 2.5×10^7 years respectively.

(b) According to Appendix D, the MS lifetimes of 4 and 20 $M_\odot$ stars are 1.5×10^8 and 7.8×10^6 years respectively. Our derived lifetimes are slight overestimates.