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Homework 2

All calculations can be found in the notebook
`https://github.com/meredith-durbin/ASTR531/blob/master/HW2/HW2.ipynb`.

8.2 (a) The helium-burning core must be less massive than the hydrogen-burning core for a given stellar mass; for a given temperature/density profile, less of the enclosed mass will be at the temperature required to kick off helium vs. hydrogen burning.

(b) The star will be more chemically stratified at the end of the helium-burning phase, with the helium fusion products at the core and a shell of unfused helium surrounding them.

9.1 I used the following equations for timescales:

$$\tau_{\rm dyn} = (G\rho)^{-0.5} \tag{1}$$

$$\tau_{\rm KH} = \frac{GM^2}{RL} \tag{2}$$

$$\tau_{\rm nucl} = 10^{10}\left(\frac{M/\mathrm{M}_\odot}{L/\mathrm{L}_\odot}\right)\ \mathrm{yr} \tag{3}$$

Star	$\tau_{\rm dyn}$	$\tau_{\rm KH}$	$\tau_{\rm nucl}$
MS, 1 $\mathrm{M}_\odot$	0.906 h	3.140×10^7 yr	10^{10} yr
MS, 60 $\mathrm{M}_\odot$	6.792 h	9.487×10^3 yr	7.554×10^5 yr
RSG, 15 $\mathrm{M}_\odot$	5.056 yr	4.793 yr	3.358×10^5 yr
WD, 0.6 $\mathrm{M}_\odot$	7.142 s	7.945×10^{10} yr	6×10^{11} yr

For the main sequence stars, these timescales give us an idea of the main-sequence lifetime and post-MS cooling time of the star. There are probably some very interesting consequences of these timescales for the red supergiant, given that its thermal and dynamical timescales are almost the same. For the white dwarf, we know that the nuclear timescale doesn't apply, as it isn't actually fusing anything.

9.2 If nuclear fusion in the sun were to suddenly stop, it would take approximately a thermal timescale to notice; the solar spectrum we observe is largely a product of temperature, and the thermal timescale is the timescale over which a change in temperature would become noticeable.

11.2 (a) For 1 $\mathrm{M}_\odot$, I find $\beta = 0.9996$ and $P_{\rm rad}/P_{\rm gas} = 0.0004$, and for 60 $\mathrm{M}_\odot$, I find $\beta = 0.6867$ and $P_{\rm rad}/P_{\rm gas} = 0.4562$.

(b) For 1 $\mathrm{M}_\odot$ I find a predicted luminosity $L = L_{\rm Edd}(1-\beta) = 14.8\ \mathrm{L}_\odot$, and for 60 $\mathrm{M}_\odot$ I find $L = 7.14 \times 10^5\ \mathrm{L}_\odot$.

(c) My luminosity is off by a factor of $\sim$20 for 1 $\mathrm{M}_\odot$, but is close for 60 $\mathrm{M}_\odot$.

12.2 (a) I used the following equations for radii:

$$\frac{R_{\text{start,H}}}{\text{R}_\odot} = 100\left(\frac{M}{\text{M}_\odot}\right) \tag{4}$$
$$\frac{R_{\text{end,H}}}{\text{R}_\odot} = \frac{R_{\text{start,H}}}{50} \tag{5}$$
$$\frac{R_{\text{end,PMS}}}{\text{R}_\odot} = \left(\frac{M}{\text{M}_\odot}\right)^{0.7} \tag{6}$$

Mass ($\text{M}_\odot$)	$R_{\text{start,H}}$	$R_{\text{end,H}}$	$R_{\text{end,PMS}}$
0.1	10	0.2	0.2
1	100	2	1
10	1000	20	5
100	10^4	200	25

(b) I used the following equations for timescales:

$$\tau_{\text{Hayashi}} = 10^6\left(\frac{M}{\text{M}_\odot}\right)^{-1}\ \text{yr} \tag{7}$$
$$\tau_{\text{PMS}} = 6\times10^7\left(\frac{M}{\text{M}_\odot}\right)^{-2.5}\ \text{yr} \tag{8}$$

Mass ($\text{M}_\odot$)	τ_{Hayashi} (yr)	τ_{PMS} (yr)
0.1	10^7	1.897×10^{10}
1	10^6	6×10^7
10	10^5	1.897×10^5
100	10^4	6×10^2

13.2 I chose to calculate the ratio of final to initial quantities so that I could ignore mass entirely.

$$\mu_0 = (2-1.25Y_0)^{-1} \tag{9}$$
$$\mu_1 = (2-1.25Y_1)^{-1} \tag{10}$$
$$L_1/L_0 = \frac{(2-Y_1)^{-1}\mu_1^4}{(2-Y_0)^{-1}\mu_1^4} \tag{11}$$
$$R_1/R_0 = \frac{(1+X_1)^{0.05}\mu_1^{2/3}}{(1+X_0)^{0.05}\mu_0^{2/3}} \tag{12}$$
$$T_1/T_0 = \frac{(1+X_1)^{-0.5}\mu_1^{0.83}}{(1+X_0)^{-0.5}\mu_0^{0.83}} \tag{13}$$

For $X_1 = Y_1 = 0.49$ and $X_0 = 0.7$, $Y_0 = 0.28$, $L_1/L_0 = 2.28$, $R_1/R_0 = 1.12$, and $T_1/T_0 = 1.23$.