DSA 8020 R Lab 8: Randomized Complete Block Designs and Factorial Designs

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A researcher wishes to determine whether four different tips produce different hardness readings on a Rockwell hardness tester, which operates by pressing a tip into a metal test 'coupon.' Since the coupons are large enough to test four tips, a RCBD (Randomized Complete Block Design) can be used, with one coupon serving as a block. The dataset 'tip_hardness.csv' can be found on Canvas.

1. Load the data into R.

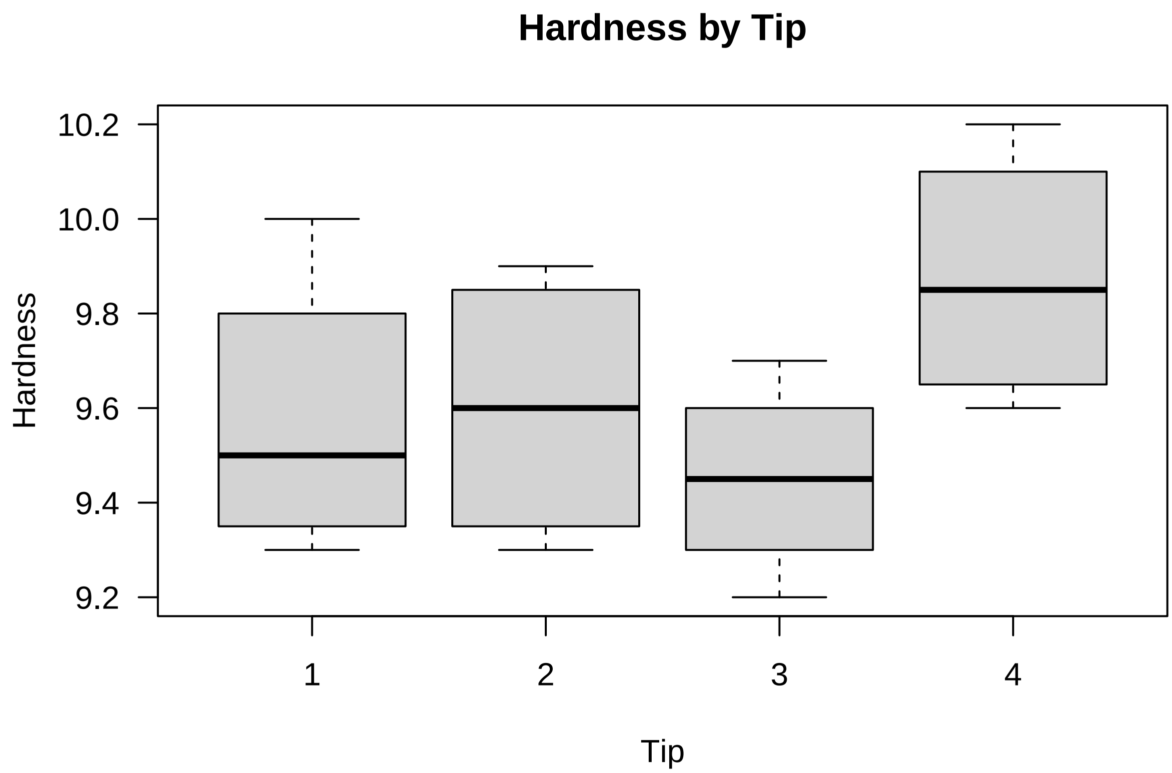
Code:

```r
thardness <- read.csv("tip_hardness.csv", header = TRUE)
thardness$Tip <- as.factor(thardness$Tip)
thardness$Coupon <- as.factor(thardness$Coupon)
```

2. Make side-by-side boxplots by treatment.

Code:

```r
boxplot(Hardness ~ Tip, data = thardness, las = 1,
        main = "Hardness by Tip",
        ylab = "Hardness",
        xlab = "Tip")
```



3. Write down the effects model and explain each term within the model, including the assumptions concerning the random error.

Answer:

The effects model for a Randomized Complete Block Design (RCBD) is:

$$Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}$$

Where:

- Y_{ij}: observed hardness value for tip i on coupon j
- μ: overall mean hardness
- τ_i: effect of tip i (treatment effect)
- β_j: effect of coupon j (block effect)
- ε_{ij}: random error term associated with each observation

Assumptions:

- Errors ε_{ij} are independent and identically distributed.

- Errors have mean 0 and constant variance σ^2.
- Errors follow a normal distribution:

$$\varepsilon_{ij} \sim N(0, \sigma^2)$$

- No interaction between treatments and blocks (additive model).

4. Perform an overall F-test to the treatment effects using ANOVA, state the hypotheses, p-value, and conclusion.

Code:

```
model <- aov(Hardness ~ Tip + Coupon, data = thardness)
summary(model)
```

```
##             Df Sum Sq Mean Sq F value   Pr(>F)
## Tip          3  0.385 0.12833   14.44 0.000871 ***
## Coupon       3  0.825 0.27500   30.94 4.52e-05 ***
## Residuals    9  0.080 0.00889
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Answer:

Hypotheses: - H_0: All tip means are equal - H_a: At least one tip mean differs

The p-value for the Tip factor is 0.000871, which is highly significant (p $<$ 0.001).

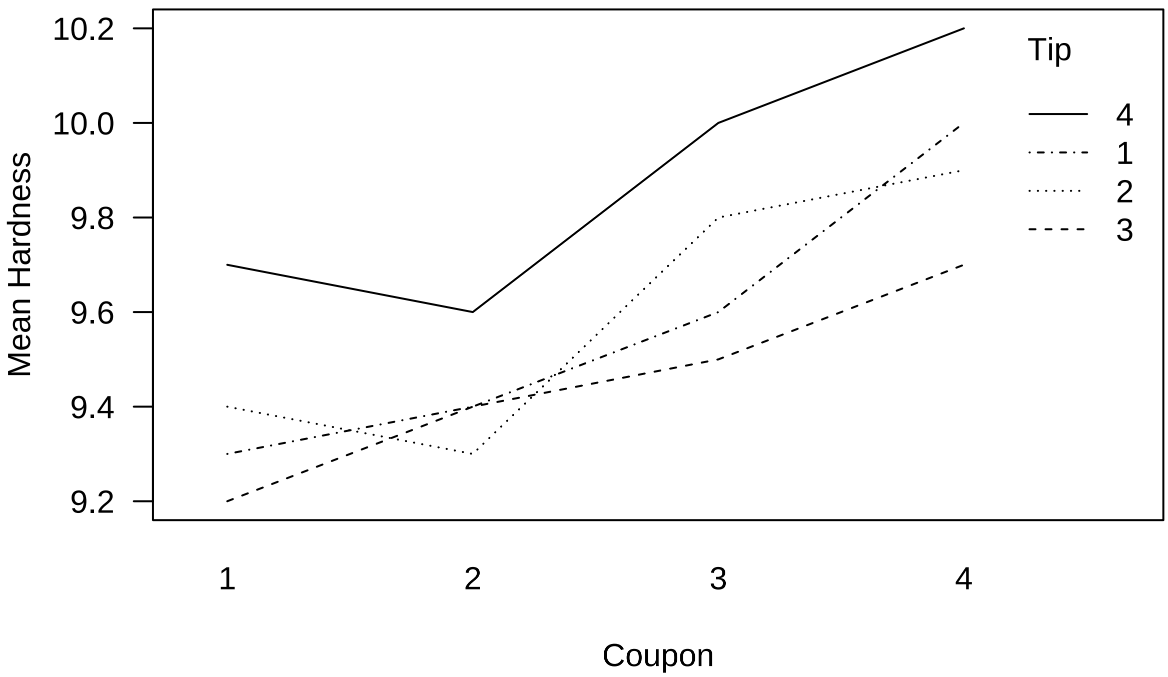
Conclusion: There is strong evidence that the type of tip affects the hardness measurement. We reject the null hypothesis and conclude that at least one tip differs significantly in its effect.

Additionally, the block effect (Coupon) is also highly significant (p $<$ 0.001), confirming that blocking by coupon was useful to control variability.

5. Use an interaction plot to assess the additive assumption.

Code:

```
interaction.plot(thardness$Coupon, thardness$Tip, thardness$Hardness,
                 xlab = "Coupon", ylab = "Mean Hardness",
                 trace.label = "Tip", las = 1)
```



Answer:

The interaction plot (Hardness by Tip across Coupons) shows that the lines for different tips are not parallel and in some cases cross over. This indicates potential interaction between Tip and Coupon.

Conclusion: This suggests the additive model assumption may not hold perfectly — there could be a non-negligible interaction effect between Tip and Coupon. While an RCBD assumes no interaction, this plot suggests caution and a potential violation of that assumption.

In a study to evaluate the impact of a certain bacterial strain (mycoplasma) on the development of birds, birds inoculated with bacteria or a control injection were studied in both cold and warm temperature-controlled rooms. The response variable of interest was the bill length of birds in the rooms. The rooms served as the experimental units, with either cold or warm temperatures, and contained either control or mycoplasma-inoculated birds. Two runs of each combination were performed. The data is given below:

Bact	Warm	Cold
Control	40.37, 41.71	39.77, 40.23
Mycoplasma	40.21, 40.78	39.19, 38.95

6. Enter the data into R.

Code:

```r
bill_data <- data.frame(
  bact = factor(rep(c("Control", "Mycoplasma"), each = 4)),
  temp = factor(rep(c("Warm", "Cold"), times = 4)),
  billLength = c(40.37, 39.77, 41.71, 40.23, 40.21, 39.19, 40.78, 38.95)
)
bill_data
```

```
##         bact temp billLength
## 1    Control Warm      40.37
## 2    Control Cold      39.77
## 3    Control Warm      41.71
## 4    Control Cold      40.23
## 5 Mycoplasma Warm      40.21
## 6 Mycoplasma Cold      39.19
## 7 Mycoplasma Warm      40.78
## 8 Mycoplasma Cold      38.95
```

Answer:

I don't see a question requiring an answer here?

7. Construct an ANOVA table that includes the main effects for bacteria and temperature and their interaction. Test whether the main effects impact bill length.

Code:

```r
anova_model <- aov(billLength ~ bact * temp, data = bill_data)
summary(anova_model)
```

```
##             Df Sum Sq Mean Sq F value Pr(>F)
## bact         1 1.0878  1.0878   3.642 0.1290
## temp         1 3.0381  3.0381  10.171 0.0332 *
## bact:temp    1 0.0741  0.0741   0.248 0.6446
## Residuals    4 1.1949  0.2987
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Answer:

Interpretation:

Bacteria (bact): $p = 0.1290 \rightarrow$ Not statistically significant at the 0.05 level.

Temperature (temp): $p = 0.0332 \rightarrow$ Statistically significant ($p < 0.05$), suggesting temperature does affect bill length.

Interaction (bact:temp): $p = 0.6446 \rightarrow$ Not significant, indicating no evidence of interaction between bacteria and temperature.

Conclusion:

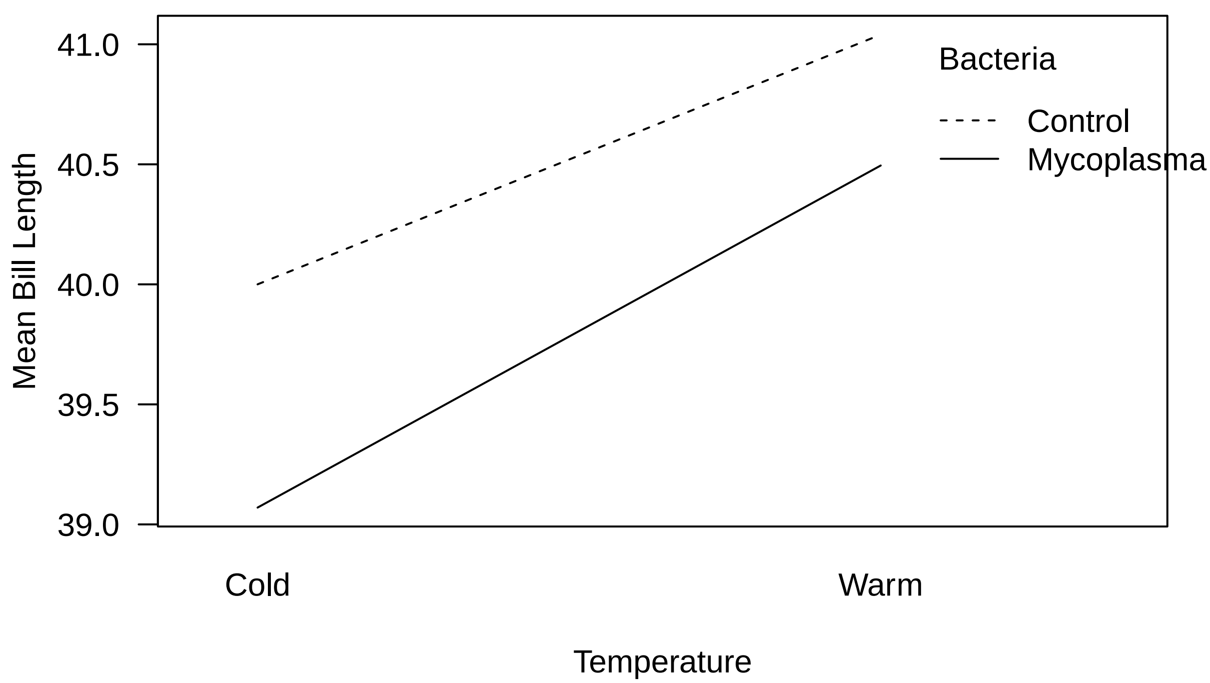
Only temperature has a significant main effect on bird bill length.

There is no significant interaction, and the effect of bacteria is not statistically significant based on this dataset.

8. Make an interaction plot and comment on the interaction effect.

Code:

```
interaction.plot(bill_data$temp, bill_data$bact, bill_data$billLength,
                 xlab = "Temperature", ylab = "Mean Bill Length",
                 trace.label = "Bacteria", las = 1)
```



Answer:

Interpretation:

The interaction plot shows roughly parallel lines for "Control" and "Mycoplasma" across "Cold" and "Warm" temperatures.

Conclusion: This visual evidence aligns with the ANOVA result — no significant interaction exists between bacteria and temperature. The effect of bacteria on bill length does not appear to vary depending on the temperature condition.