

Genetics Algorithm

→ focus on optimization
→ use when population is random
→ best fit will survive.

→ Population

→ Calculate fitness

→ Selection

→ Cross Over

→ Mutation

→ Stopping Criteria.

TSP: salesman needs to visit all the cities only once and return to the starting one while minimizing the total distance travelled.

• Representation: population is represented as a sequence of cities

• Calculate fitness: each route is evaluated to find fitness by calculating total distance travelled for that route

• Selection: solution are selected based on fitness, greater fitness (less distance travelled) is selected.

• Genetic Operators:

→ Crossover: pair of selected routes are combined to create a new route.

→ Mutation: Random changes are introduced.

means swapping or altering cities in routes

• Replacement: new population is replaced by the prev pop

• Iteration: 2-5 steps are repeated until termination condition satisfied

• Termination: Terminate when optimal distance is found.

you have set of items, each with a weight and value Knapsack, and a knapsack with max weight capacity. Goal is to determine the most valuable combination of items that can be placed in the knapsack without exceeding the limit.

• Representation: population represents a combination of items that could be placed in knapsack

→ Represented as binary string

1 (item is included) 0 (item is not included)

• Initialization: Creating initial population (randomly / by using heuristic)

• Evaluation: evaluate every solution.

• Selection: Select parent based on fitness (total value) higher total value higher chance to be selected.

- Genetic Algorithms / Operator:
to create new offspring

→ Crossover: Combined parts of two

selected parents to generate a new solution

eg parent 1 → 1010101011

parent 2 → 1101010010

new parent 1 → 101010101010

new parent 2 → 1101101011

→ Mutation: Involves randomly changing
some bits in the solution.

eg 1010010010

flip the 8 bit from 0 → 1 1010010110

- Replacement: New population replaces prev once
- Iteration: Repeat steps 3-6 (Evaluation - Replace)
- Termination: When optimal solution found (Terminate)

Naive Bayes Classification:

→ Bayes Theorem: m estimate = ?

Person	(Y/N) Covid	(Y/N) Flue	(F/N) Fever				
1	Y	N	Y				$P(\text{fever} \text{Yes}) = 7/10$
2	N	Y	Y				$P(\text{fever} \text{No}) = 3/10$
3	Y	Y	Y				
4	N	N	N				$\frac{\text{Yes}}{\text{Flue}} \quad \frac{\text{No}}{\text{Covid}}$
5	Y	N	Y	Flue	3/7	2/3	
6	N	N	Y	Covid	4/7	2/3	
7	Y	N	Y				
8	Y	N	N				
9	N	Y	Y				
10	N	Y	N				

Given Person (flue, Covid): $3/7 * 4/7 * 7/10 = 0.17$

$$P(\text{Yes}|\text{flue, Covid}) = P(\text{flue}|\text{Yes}) * P(\text{Covid}|\text{Yes}) * P(\text{Yes})$$

$$P(\text{No}|\text{flue, Covid}) = P(\text{flue}|\text{No}) * P(\text{Covid}|\text{No}) * P(\text{No})$$

As the Yes fever class value is greater we predict it to be in Yes data.

Id3

Decision Tree

entropy / info gain \rightarrow high \rightarrow root node.

outcomes \rightarrow on leaf node.

\rightarrow ML algo used for classification / regression task

\rightarrow Decision Tree Structure

\rightarrow Decision nodes

\rightarrow Leaf Nodes

\rightarrow Splitting

\rightarrow Entropy & Information Gain

\rightarrow Pruning

Day	Weather	Temp	Humidity	Wind	Play F/B
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
3	Cloudy	Hot	High	Weak	Yes
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
7	Cloudy	Cool	Normal	Strong	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cold	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Cloudy	Mild	High	Strong	Yes
13	Cloudy	Hot	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

3 2 4

7

8 6

we have to find entropy H info gain to make decision tree.

→ To find root node we have to find ent H info of every data set.

Calculate IG of Weather.

Step 1, entropy of entire dataset \rightarrow play fib.

$$S^H = -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} = 0.94$$

Step 2: entropy of all attributes of weather.

$$\text{entropy of sunny: } S^H = -2 \log_2 \frac{2}{5} - 3 \log_2 \frac{3}{5} = 0.97$$

$$\text{entropy of Cloudy: } S^H = 4 \log_2 \frac{4}{4} - 0 \log_2 0 = 0$$

$$\text{entropy of Rainy: } S^H = -3 \log_2 \frac{3}{5} - 2 \log_2 \frac{2}{5} = 0.97$$

Step 3 Information Gain = entropy of complete data set - $\frac{5}{14} \text{Ent}(S)$ -

$$\frac{4}{14} \text{Ent}(C) - \frac{5}{14} \text{Ent}(R).$$

$$IG = 0.94 - \frac{5}{14}(0.97) - \frac{4}{14}(0) - \frac{5}{14}(0.97) [= 0.247]$$

Calculate Info Gain of Temp.

Step 1, Entropy of entire.

$$S^H = -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} = 0.94.$$

Step 2, Entropy of all attributes:

$$\rightarrow \text{entropy of Hot } S^H = -2 \log_2 \frac{2}{4} - \frac{2}{4} \log_2 \frac{2}{4} = 1.$$

$$\rightarrow \text{entropy of Cold } S^H = -3 \log_2 \frac{3}{4} - \frac{1}{4} \log_2 \frac{1}{4} = 0.81$$

$$\rightarrow \text{entropy of Mild } S^H = -4 \log_2 \frac{4}{4} - \frac{2}{4} \log_2 \frac{2}{4} = 0.91.$$

Step 3, Information Gain:

$$= 0.94 - \frac{4}{14}(1) - \frac{4}{14}(0.81) - \frac{6}{14}(0.91) \boxed{= 0.03}$$

Calculate Information Gain of
Humidity:

Step 1, entropy of entire dataset $\rightarrow 0.94$.

Step 2, entropy of all attributes.

$$\rightarrow \text{entropy of High } \frac{4}{7} + 3, -4 \boxed{= -\frac{3}{7} \log_2 \frac{3}{7} - \frac{4}{7} \log_2 \frac{4}{7} = 0.94}$$

$$\rightarrow \text{entropy of Normal } \frac{4}{7} + 6, -1 \boxed{= -\frac{6}{7} \log_2 \frac{6}{7} - \frac{1}{7} \log_2 \frac{1}{7} = 0.59}$$

Step 3, Information Gain

$$= 0.94 - \frac{7}{14}(0.98) - \frac{7}{14}(0.59) \boxed{= 0.15}$$

Calculate Information Gain of Wind.

Step 1, 0.94 .

Step 2, all attributes

$$\text{entropy of Weak } \frac{6}{8} + 6, -2 \boxed{= -\frac{6}{8} \log_2 \frac{6}{8} - \frac{2}{8} \log_2 \frac{2}{8} = 0.8}$$

$$\text{entropy of Strong } \frac{6}{8} + 3, -3 \boxed{= -\frac{3}{6} \log_2 \frac{3}{6} - \frac{3}{6} \log_2 \frac{3}{6} = 1}$$

$$\text{Step 3}, 0.94 - \frac{8}{14}(0.81) - \frac{6}{14}(1) \boxed{= 0.04}$$

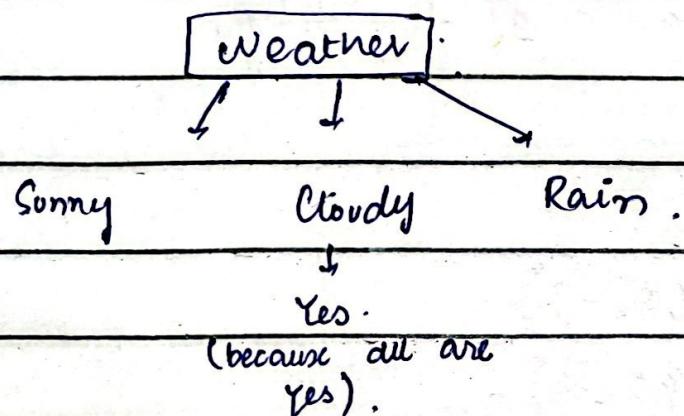
$$\text{Gain (Weather)} = 0.24$$

$$\text{Gain (Temp)} = 0.03$$

$$\text{Gain (Humidity)} = 0.15$$

$$\text{Gain (Wind)} = 0.04$$

This gain will tell us about the root node which is weather (highest) gain.



For Sunny

Day	Weather	Temp	Humidity	Wind	Play P/E	
1	Sunny	No	Hot	High	Weak	No
2	Sunny	No	Hot	High	Strong	No
8	Sunny	No	Mild	High	Weak	No
9	Sunny	Yes	Cold	Normal	Weak	Yes
11	Sunny	Yes	Mild	Normal	Strong	Yes

Step 1 : Entropy of entire data set \rightarrow [Temp]

$$S = -\sum p_i \log_2 p_i = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97$$

Step 2 : all attributes,

$$\rightarrow \text{entropy of hot } \{0, -2\} = -\frac{0}{2} \log_2 \frac{0}{2} - \frac{2}{2} \log_2 \frac{2}{2} = 0$$

$$\rightarrow \text{entropy of mild } \{1, -1\} = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1$$

$$\rightarrow \text{entropy of cold } \{1, 0\} = \frac{1}{1} \log_2 \frac{1}{1} - \frac{0}{1} \log_2 \frac{0}{1} = 0.$$

Step 3 : Info gain

$$= 0.97 - \frac{2}{3}(0) - \frac{2}{3}(1) - \frac{1}{3}(0) \boxed{= 0.57}$$

\hookleftarrow

Info gain of humidity :

Step 1 : 0.97

Step 2 :

$$\text{entropy of high } \{0, -3\} = 0$$

$$\text{entropy of Normal } \{2, 0\} = 0.$$

Step 3 : $\boxed{0.97}$

\hookleftarrow

Info gain of wind.

Step 1 : 0.97

Step 2 :

$$E(S) = \{1, -1\} = 1$$

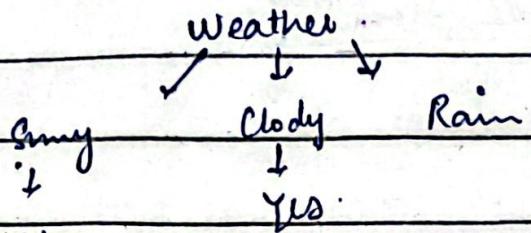
$$E(w) = \{1, -2\} = -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} \boxed{= 0.91}$$

\hookleftarrow

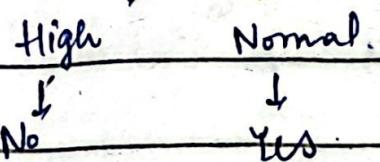
Info gain (Temp) = 0.57 } humidity -
ment

Info gain (Humidity) = 0.97 } root node

Info gain (Wind) = 0.91.



Humidity .



Now Solve for Rain.

Day	Weather	Temp	Humidity	Wind	Play F(B)
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool.	Normal	Weak	Yes
6	Rain	Cool.	Normal	Strong	No.
10	Rain	Mild	Normal	Weak	Yes.
14	Rain	Mild	High	Strong	No.

Info gain of Temp.

$$\text{Step 1: } g + 3, -2 \bar{g} = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.97$$

Step 2:

$$S(\text{Mild}) = g + 2, -1 \bar{g} = -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} = 0.91$$

$$S(\text{Cool}) = g + 1, -1 \bar{g} = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1$$

Step 3:

$$I.g = 0.97 - \frac{3}{5} (0.91) - 2(1) \frac{1}{5} = 0.024$$

Info of Humidity

Step 1, 0.97

Step 2,

$$S(\text{high}) = \frac{1}{3} + 1, -1 \cdot \frac{1}{3} = 1$$

$$S(\text{Normal}) = \frac{1}{3} + 2, -1 \cdot \frac{2}{3} = -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} = 0.91$$

Step 3,

$$I.G = 0.97 - \frac{2}{3}(1) - \frac{3}{3}(0.91) = 0.0245$$

Info of Wind

Step 1, 0.97

Step 2,

$$S(\text{Weak}) = \frac{1}{3} + 3, 0 \cdot \frac{1}{3} = 0$$

$$S(\text{Strong}) = \frac{1}{3} \cdot 0, 3 \cdot \frac{1}{3} = 6$$

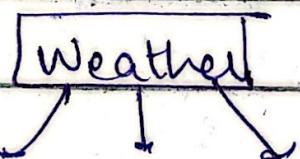
$$\text{Info gain} = 0.97$$

$$S(\text{Temp}) = 0.024$$

$$S(\text{Humidity}) = 0.024$$

$$S(\text{Wind}) = 0.97$$

Wind (highest)



Sunny Cloudy Rain

↓
Thirsty

↓
Yes

↑
Wind

↑
High
A
No

↓
Normal
Yea

↑
Strong
Wt.
Weak
Wt.

- Reduced error pruning Post-prune
 Issues in Decision Tree
1. Overfitting the Data. (it will work good for training data but not for testing data)
 2. Incorporating Continuous valued attributes (convert continuous values to discrete value)
 3. Handling training examples with missing attribute values.
 4. Handling attributes with different costs.
 5. Alternative measures for selecting attributes.
 6. Determining depth of tree for correct classification

Avoid Overfitting in Decision Trees

→ Why overfitting occurs?

Building data that "adapt/rely" to much on the "training data" may lead to overfitting

2. Continuous Values →

eg.

age	bought product	when we have continuous data determine a threshold to split the ages into groups
25	Yes	
30	No	
35	Yes	
40	No	
45	Yes	grp 1 ← if age ≤ 37.5
50	Yes	grp 2 ← if age > 37.5

3. Missing Value :

- 1) Completely ignore (but not preferable)
- 2) fill the missing value with estimated or calculated value.

Study Hrs	Test Score	Grade	
4	85	A	Mean Test Score,
5	90	B	$= \frac{85+90+88}{3} = 85.7$
6	?	C	4
7	80	A	↓ replace it at 3rd Test score.
?	75	B	
3	88	A	

Random Forest

→ extension of decision Tree

→ Decision Tree → 1 tree Random Forest → Multiple Trees

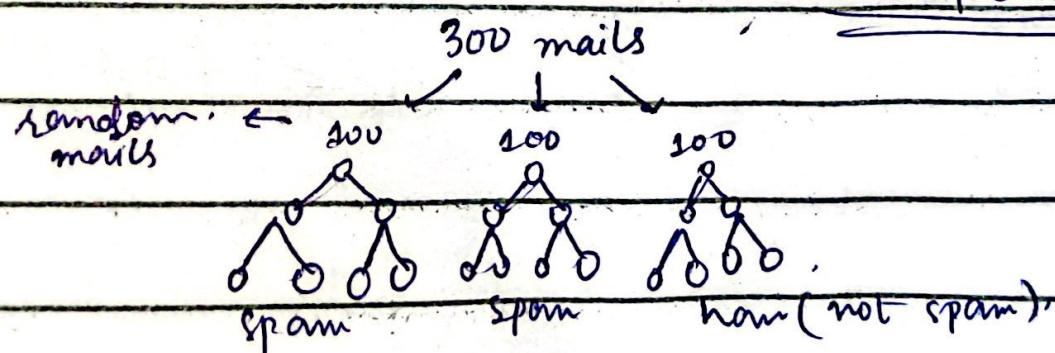
→ ensemble (group) learning.

→ no overfitting (training data works perfect but when we put testing data it didn't work → overfitting).

→ use in classification & regression

more accurate

→ Classification

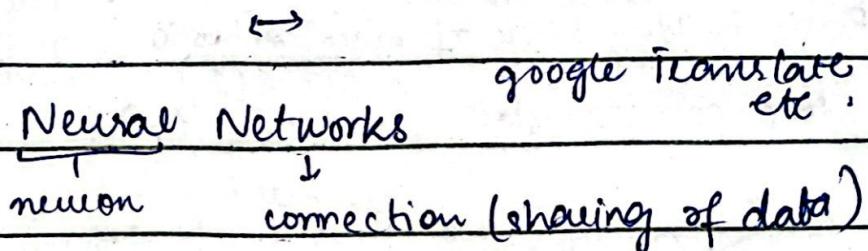


majority wins → spam

→ regression → numeric data.

$$10 \quad \downarrow \quad 20 \quad \downarrow \quad 15$$

mean value is the output $\Rightarrow \frac{10+20+15}{3} = 15$.



Artificial & Brain → Machine

e.g.: human brain → recognize tony stark even if he has beard or shaved bcz our minds are trained but machine might took some time to recognize. On the other hand B⁸ → machine can give output much quicker than human brain.

learning algorithm:

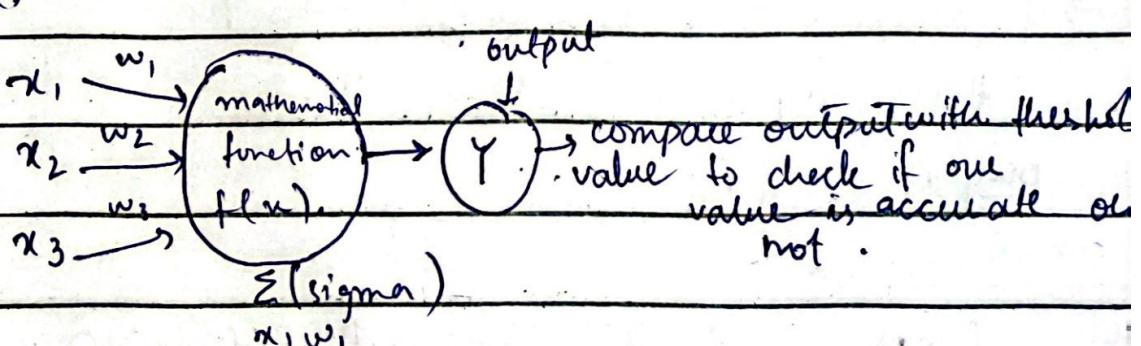
Artificial: Input data → Meaningful info → Learning by Neural Networks

Biological:

Dendrites (take inputs) → Cell body (process the data) → axons (pass data through them)

→ Terminal axons (connected with next dendrites)

Artifical.



MCP.

McCulloch Pitts model.

→ 1st biological neuron model (1943 by Warren McCulloch & Walter Pitts)

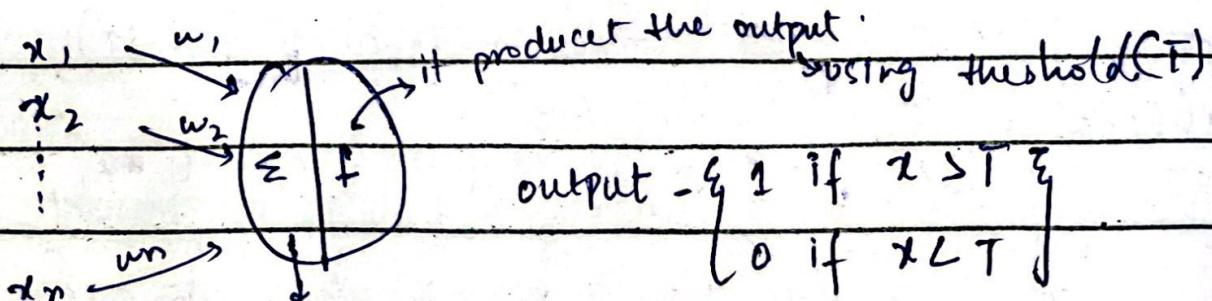
→ aka (linear threshold

Gate model).

→ Basic building block of neural network

→ Directed weight graph is used for connecting neurons.

→ two possible states → Active(1) or Silent(0).



$$\text{output} = \begin{cases} 1 & \text{if } x > T \\ 0 & \text{if } x \leq T \end{cases}$$

aggregates the weighted input into single numeric value.

$$\sum x_1 w_1 + x_2 w_2 + \dots + x_n w_n \cdot f(x)$$

Bias / Threshold → min value of weighted active input for a neuron to fire.

→ if weighted sum is large than T

then $o/p \rightarrow 1$ else 0.

output : $f(a)$

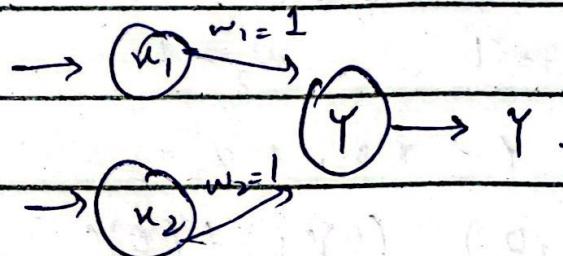
$$f(a) = \sum w_i x_i - T$$

this is a function that $\theta(x) = 1$ if $x > 0$.
 $\theta(x) = 0$ if $x \leq 0$

Implementation of MCP. (in AND gate).

x_1	x_2	y	Threshold \rightarrow neuron will fire if both inputs are high in this case.
1	1	1	
1	0	0	
0	1	0	
0	0	0	

$$w_1 = 1 \quad w_2 = 1$$



$$(1,1); y_{in} = x_1 w_1 + x_2 w_2 = (1)(1) + (1)(1) = 1+1 = 2$$

$$(1,0); y_{in} = x_1 w_1 + x_2 w_2 = (1)(1) + (0)(1) = 1+0 = 1$$

$$(0,1); y_{in} = (0)(1) + (1)(1) = 0+1 = 1$$

$$(0,0) = (0)(1) + (0)(1) = 0+0 = 0$$

if θ (threshold) value is ≥ 2 , (neuron will fire).

can also be obtained by

$$\theta \geq nw - p$$

$$n = 2, \quad \theta = 2, \quad p = 0$$

$n \rightarrow$ no of neurons.

$$w = 1$$

$w \rightarrow$ +ve ~~and~~ weight

$$p = 0 \text{ (no inhibitory weight)}$$

$p \rightarrow$ neg. weight

$$\theta \geq (2)(1) - 0$$

$$\theta \geq 2$$

outputs

$$\begin{cases} 1 & \text{if } \theta \geq 2 \\ 0 & \text{if } \theta < 2 \end{cases}$$

Implementation of MCP (ANDNOT).

x_1	x_2	y
0	0	0
0	1	0
1	0	1
1	1	0

$$w_1 = 1 \quad w_2 = 1$$

$$y = x_1 w_1 + x_2 w_2$$

$$(0, 0) = (0)(1) + (0)(1) = 0$$

$$(0, 1) = (0)(1) + (1)(1) = 1$$

$$(1, 0) = (1)(1) + (0)(1) = 1$$

$$(1, 1) = (1)(1) + (1)(1) = 2$$

change weight values :

$$w_1 = 1 \quad w_2 = -1$$

$$(0, 0) = (0)(1) + (0)(-1) = 0$$

$$(0, 1) = (0)(1) + (1)(-1) = -1$$

$$(1, 0) = (1)(1) + (0)(-1) = 1$$

$$(1, 1) = (1)(1) + (1)(-1) = 0$$

if $\theta = 1$ then neuron will fire.

$$I = x_1 x_2 + \bar{x}_1 + \bar{x}_2$$



Artificial Neural Network.

" An ANN is an information-based model that is inspired by the way a biological nervous system works, such as brain ".

→ This model tries to replicate the most basic function of the brain.

→ ANN is composed of highly interconnected processing units (neurons) to solve specific problem.

→ Like human being, ANN learn by example.

→ configured for a specific application.

→ each neuron is connect by a "link"

→ each link is associated with weight ~~to~~ which contain ~~of~~ info about input signal

→ This info is used by neural network to solve a prob

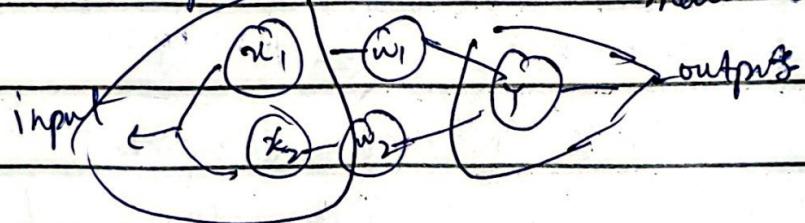
→ They have the ability to learn, recall a train pattern or data

Neuron

→ each neuron has an internal state of its own

→ This internal state is called activation level of neuron.

→ The activation signal is transmitted to other neurons



3 Basic Components of ANN:

→ model synaptic interconnections

single layer feed forward
multilayer "

→ training or learning rules adopting for

single node with its own feedback
single layer recurrent network

updating by adjusting the connection weights

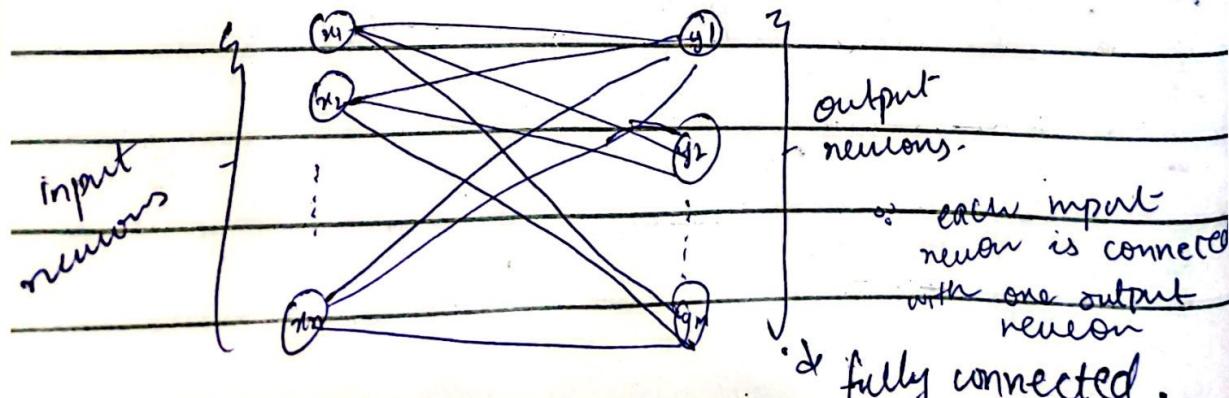
multilayer "

→ the activation function.

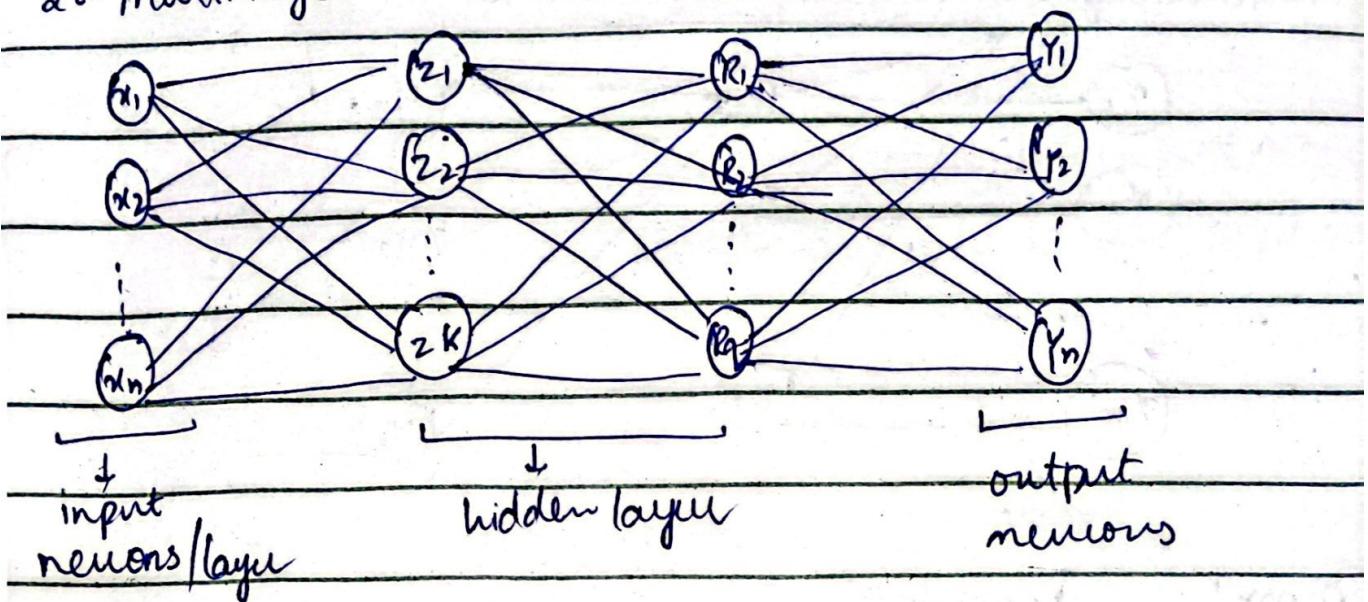
3 point

2. Connection

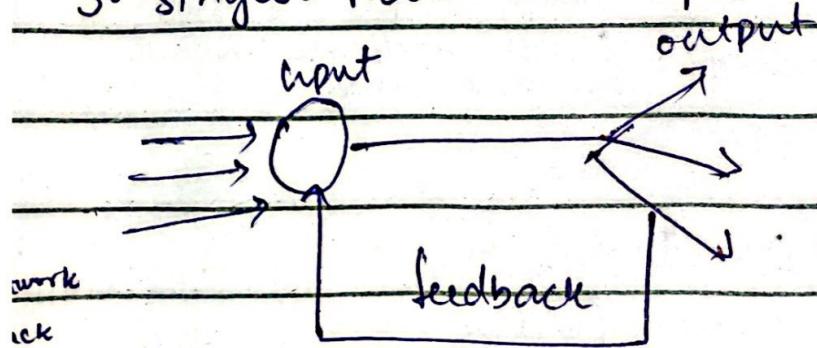
① Single layer feed forward network.



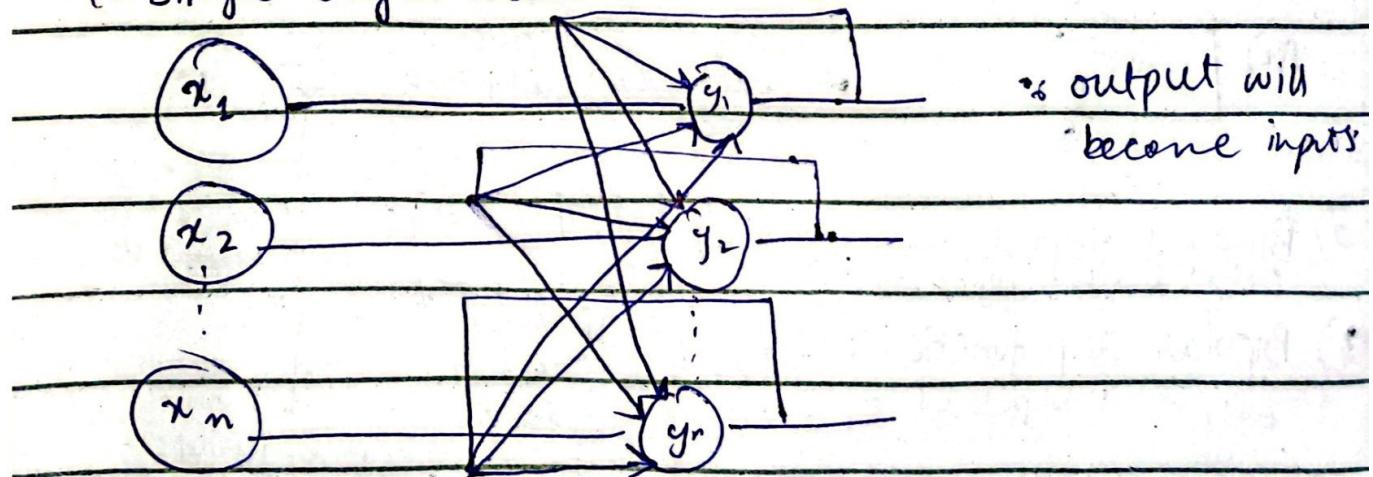
2. Multilayer



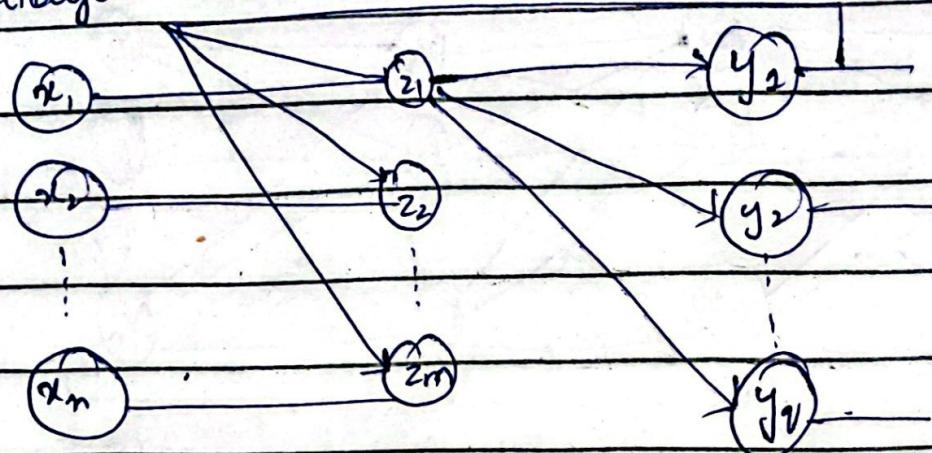
3. Single-Node with feedback



4. Single layer recurrent network



6. Multilayer Recurrent Network



② point learning

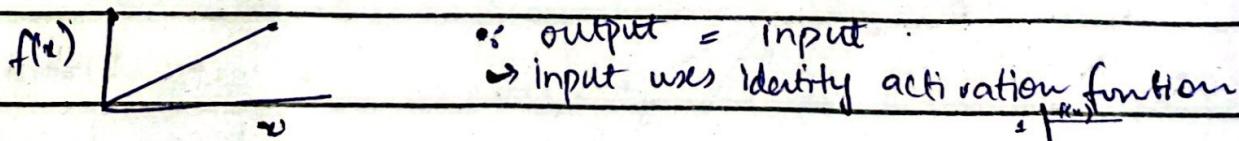
- ① Parameter learning: updates the connecting weight.
- ② Structural learning: focuses on change in network structure.

3rd point

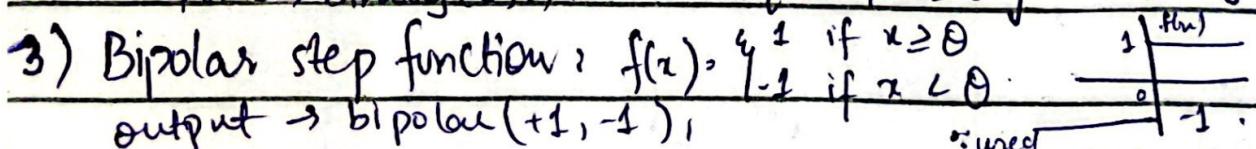
Activation Function.

There are several activation functions.

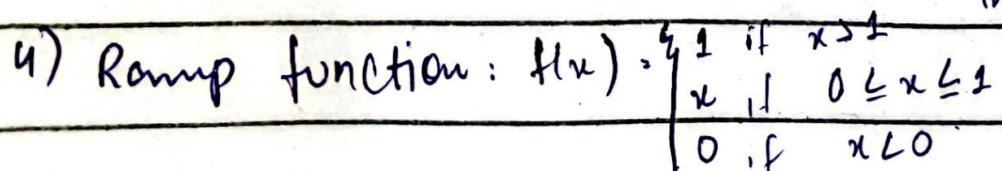
- 1) Identity function: linear $\rightarrow f(x) = x$ for all x .



- 2) Binary step function: $f(x) \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$ ↳ used in single layer nets



- 3) Bipolar step function: $f(x) \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$ ↳ used in single layer nets



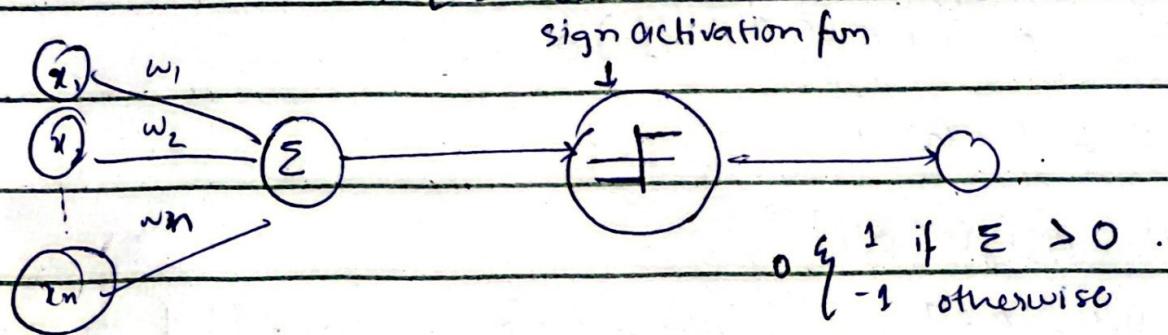
- 4) Ramp function: $f(x) \begin{cases} 1 & \text{if } x > 1 \\ x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x < 0 \end{cases}$

Perception

→ used to build ANN

→ takes real valued inputs, calculates a linear combination, then output 1 if result > threshold and -1 if < threshold

$$\theta(x_1, \dots, x_n) = \begin{cases} 1 & \text{if } w_0 + w_1 x_1 + w_2 x_2 + \dots + w_n x_n > 0 \\ -1 & \text{otherwise} \end{cases}$$



AND GATE PERCEPTRON TRAINING RULE

→ ANN

$$\rightarrow w_1 = 1.2$$

$$A \quad B \quad A \wedge B$$

$$w_2 = 0.6$$

$$0 \quad 0 \quad 0$$

$$\text{Threshold} = 1$$

$$0 \quad 1 \quad 0$$

$$\text{Learning Rate} = 0.5$$

$$1 \quad 0 \quad 0$$

$$(i) w_i * x_i$$

$$1 \quad 1 \quad 1$$

$$Y = w_1 w_1 + x_2 w_2$$

$$\bullet (0, 0) = (0)(1.2) + (0)(0.6) = 0.$$

$Y <$ threshold As, targeted value ($A \wedge B$) and output is same so no weight updation.

$$(0, 1) = (0)(1.2) + (1)(0.6) = 0.6 \quad " \quad "$$

$$(1, 0) = (1)(1.2) + (0)(0.6) = 1.2$$

As, $Y >$ threshold value. Output = 1, As target ($A \wedge B$) by output are diff there would be some weight updation.

→ Through Perceptron Training Rule's

$$w_i^t = w_i + n(t - o) \xrightarrow{\text{actual output}} x_i$$

for 1st case : targeted value $t = 1, B = 0$ $\xi = 0$.
 $w_1 = w_1 + n(t - o)x_1$ $o = 1$.
 $w_1 = 1 \cdot 2 + 0.5(0 - 1)1$ $n = 0.5$.
 $= 0.7$

$$w_2 = w_2 + n(t - o)x_2$$

 $w_2 = 0.6 + 0.5(0 - 1)0$
 $= 0.6$

new weights :

$$w_1 = 0.7 \quad w_2 = 0.6 \quad \text{threshold} = 1 \quad n = 0.5$$

(i) $y = x_1 w_1 + x_2 w_2$.

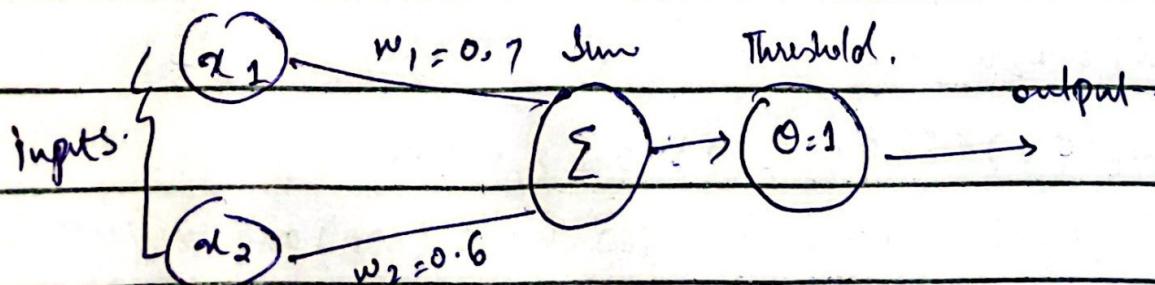
$$(0,0) = (0)(0.7) + (0)(0.6) = 0$$

$$(0,1) = (0)(0.7) + (1)(0.6) = 0.6 = 0$$

$$(1,0) = (\frac{1}{0})(0.7) + (0)(0.6) = 0.7 = 0$$

$$(1,1) = (1)(0.7) + (\frac{1}{0})(0.6) = 1.3$$

$y > \text{threshold}$ output = 1 $\because \text{output} = (A \wedge B)$ no weight update



OR GATE PERCEPTRON TRAINING RULE

→ ANN

$$w_1 = 0.6$$

$$w_2 = 0.6$$

$$A \quad B \quad Y = A+B$$

$$0 \quad 0 \quad 0$$

$$\text{Threshold} = 1$$

$$0 \quad 1 \quad 1$$

$$\text{Learning rate} = n = 0.5 \quad 1 \quad 0 \quad 1$$

$$1 \quad 1 \quad 1$$

(i) $y = x_1 w_1 + x_2 w_2$

$$(0, 0) = (0)(0.6) + (0)(0.6) = 0$$

$$(0, 1) = (0)(0.6) + (0+1)(0.6) = 0.6 - 0 = 0$$

$$(1, 0) = (1)(0.6) + (0)(0.6) = 0.6 - 0 = 0$$

targeted value = 1. Output = 0

~~weight~~ weight updation:

$$w_i = w_i + n(t-o)x_i \quad \therefore w_1 = 0.6 \quad w_2 = 0.6$$

$$w_1 = w_1 + n(t-o)x_1 \quad x_1 = 0 \quad x_2 = 1 \\ n = 0.5 \quad t = 1$$

$$w_1 = 0.6 + 0.5(1-0)0 \quad o = 0$$

$$w_1 = 0.6$$

$$w_2 = 0.6 + 0.5(1-0)1$$

$$w_2 = 1.1$$

new weights =

$$w_1 = 0.6 \quad w_2 = 1.1$$

$$Y = x_1 w_1 + x_2 w_2$$

$$w_1 = 0.6$$

$$w_2 = 1.1$$

$$(0,0) = (0)(0.6) + (0)(1.1) = 0$$

$$(0,1) = (0)(0.6) + (1)(1.1) = 1.1$$

$y > \text{threshold}$ $o = 1$ $o \neq t$ (no weight update)

$$(1,0) = (1)(0.6) + (0)(1.1) = 0.6$$

$y < \text{threshold}$ $o = 0$ $o \neq t$ (weight update)

$$\text{or } w_1 = 0.6, w_2 = 1.1, x_1 = 1, x_2 = 0, n = 0.5, o = 0, t = 1$$

$$w_i = w_i + n(t - o)x_i$$

$$w_1 = w_1 + n(t - o)x_1$$

$$w_1 = 0.6 + 0.5(1 - 0)1 =$$

$$= 1.1$$

$$w_2 = 1.1 + 0.5(1 - 0)0$$

$$= 1.1$$

new weights

$$w_1 = 1.1 \quad w_2 = 1.1$$

$$Y = x_1 w_1 + x_2 w_2$$

$$(0,0) = (0)(1.1) + (0)(1.1) = 0$$

$o < \text{threshold}$ output = 0 $o \neq t$ (no weight update)

$$(0,1) = (0)(1.1) + (1)(1.1) = 1.1$$

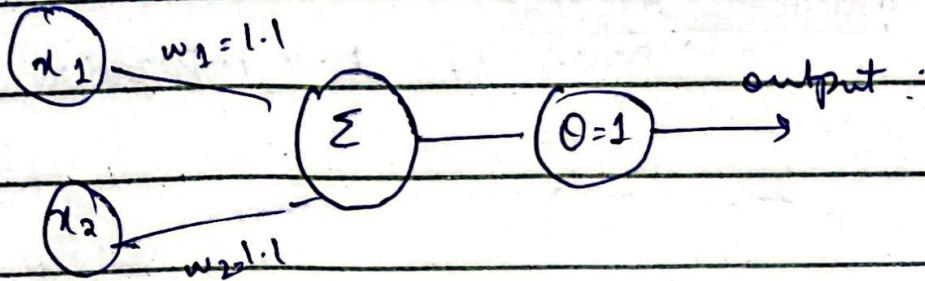
$y > \text{threshold}$ output = 1 $o \neq t$ (n.w.u)

$$(1,0) = (1)(1.1) + (0)(1.1) = 1.1$$

$y > \text{threshold}$ output = 1 $o \neq t$ (n.w.u)

$$(1, 1) = (1)(1 \cdot 1) + (1)(1 \cdot 1) = 2 \cdot 2.$$

$y > \text{threshold}$ output = 1 $o = \tau(n \cdot w \cdot v)$



XOR PERCEPTRON TRAINING RULE:

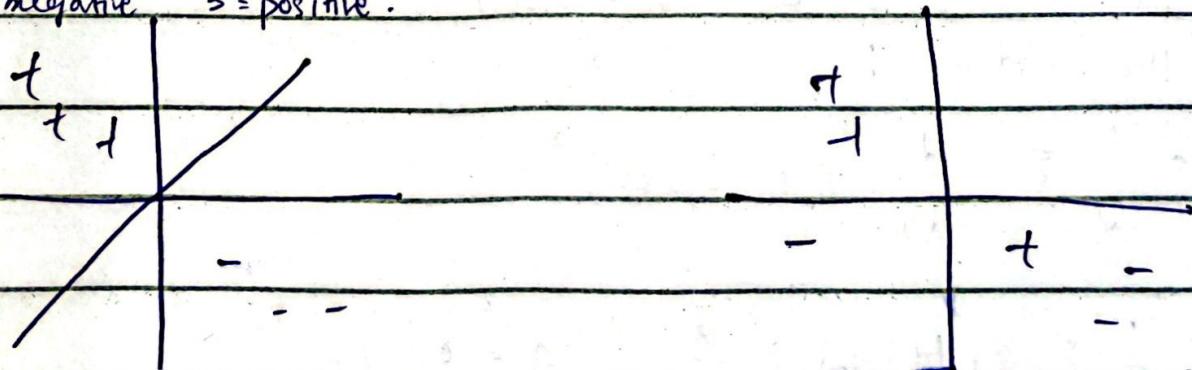
x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

GRADIENT DESCENT BY DELTA RULE.

linearly separable

non linearly separable

total = 6 data points.
3 = negative 3 = positive.



given data is separable
with a straight line

or no straight line
to separate data.

→ Perceptron Rule search successful weight if (linearly separable)

→ Delta Rule → If not linearly separable

↳ use gradient descent

↳ basis of back propagation.

$$\Delta w = -n \nabla E(\vec{w})$$

n = learning rate (determines step size in gradient desc)

-ve sign → shows we want to move the weight vector in direction that "decreases" E

Component form: A.k.A

$$w_i \rightarrow w_i + \Delta w_i$$

$$\therefore \Delta w_i = -n \frac{\partial E}{\partial w_i}$$

$$\Delta E(\vec{w}) = \frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \dots, \frac{\partial E}{\partial w_n}$$

for all training eg

$$\text{value of } E(\vec{w}) = \frac{1}{2} \sum_{d=0}^D (t_d - o_d)^2$$

$$\frac{dE}{dw_i} = \frac{d}{dw_i} \frac{1}{2} \sum (t_d - o_d)^2$$

$$= \frac{1}{2} \sum \frac{d}{dw_i} (t_d - o_d)^2$$

$$= \frac{1}{2} \sum (t_d - o_d) \frac{d}{dw_i} (t_d - o_d)$$

constant (as it is)

$$= \sum_i (t_d - o_d) \frac{d}{d w_i} (t_d - \vec{w} \cdot \vec{x}_i) \cdot$$

derivation is 0 in all cases except when vector w become w_i , the derivation would be 1 in that case.

$$\therefore o(\vec{x}) = \vec{w} \cdot \vec{x}$$

$$\frac{dE}{d w_i} = \sum_{d \in D} (t_d - o_d) (-x_{id}).$$

$$\Delta w_i = -n \frac{dE}{d w_i}$$

$$= -n \sum_{d \in D} (t_d - o_d) x_{id}$$

$$\Delta w_i = n \sum_{d \in D} (t_d - o_d) x_{id}$$

$$w_i \leftarrow w_i + \Delta w_i$$

GRADIENT DESCENT ALGO.

- used to separate the non linearly separable data
- weights are updated by:

$$w_i \leftarrow w_i + \Delta w_i$$

$$\text{where, } \Delta w_i = n \sum_{d \in D} (t_d - o_d) x_{id}.$$

Backpropagation In ANN

→ Neural network based technique

disadvantage of neural network

→ it takes long time for training

adv:

→ tolerate noisy data

→ good for unrecognize (untrained) patterns

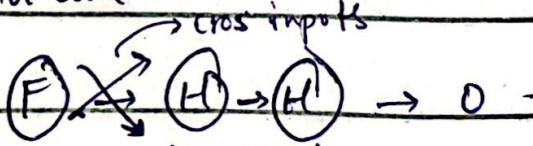
∴ Backpropagation is an iterative algo.

[First layer] → [Hidden layer] → [Output layer]

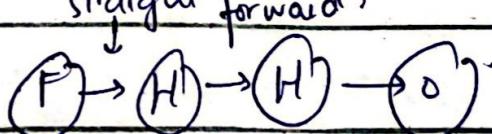
dataset, target attribute, weight, bias → already known

Two layer → 

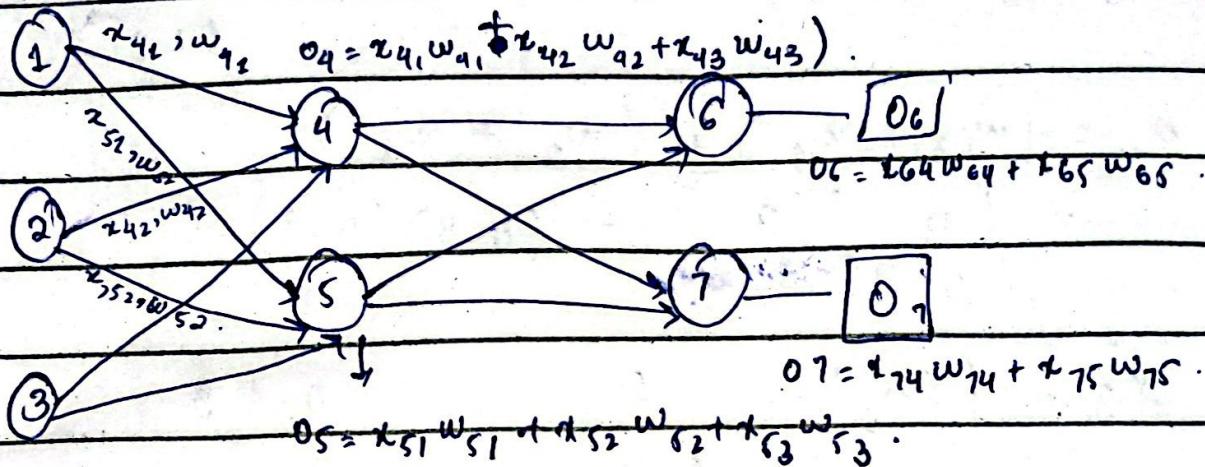
do not cont.

Feed Forward → 

multiple hidden layers.

Fully - connected → 

→ compute output o_m of every unit.



→ each network k , calculate error δ_k for output layer. o_k , t_k .

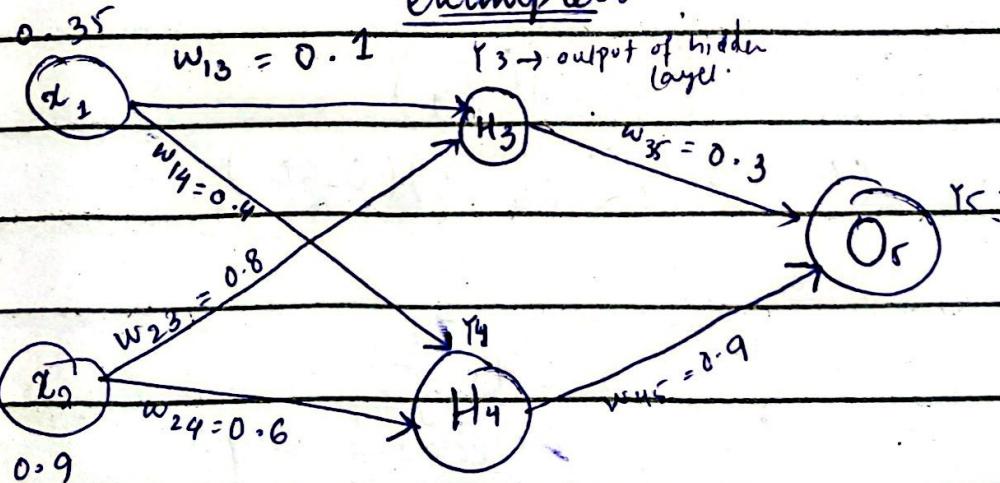
$$\delta_k \leftarrow o_k (1 - o_k) (t_k - o_k)$$

output target

→ each network h , calculate its error term δ_h for hidden layers δ_4, δ_5

$$\delta_h \leftarrow o_h (1 - o_h) \sum_{k \in \text{outputs}} w_{hk} \delta_k$$

examples:



Forward pass = Compute output for y_3, y_4 and y_5 .

$$a_j = \sum_i (w_{ij} * x_i) \quad Y_j = F(a_j) = \frac{1}{1 + e^{-a_j}}$$

$$a_1 = (w_{13} * x_1) + (w_{23} * x_2)$$

$$= (0.1)(0.35) + (0.8)(0.9) \\ = 0.75$$

$$Y_3 = f(a_1) = \frac{1}{1 + e^{-0.75}} = 0.68$$

$$a_2 = (w_{14})(x_1) + (w_{24})(x_2)$$

$$= (0.4)(0.35) + (0.6)(0.9) \\ = 0.68$$

$$Y_4 = \frac{1}{1 + e^{-0.68}} = 0.66$$

$$a_3 = (w_{35})(y_3) + (w_{45})(y_4)$$

$$= (0.3)(0.68) + (0.9)(0.66)$$

$$= 0.801$$

$$Y_5 = \frac{1}{1 + e^{-0.801}} = 0.69 \rightarrow$$

Network output

$$\text{error} = Y_{\text{target}} - Y_5$$

$$= 0.5 - 0.69 = -0.19$$

learning rate
constant

weight updation: $w_{ji} = \eta \delta_{ji}$

if j is output unit,

$$\delta_j = o_j(1-o_j)(t_j - o_j)$$

$$\delta_j = o_j(1-o_j) \sum_k \delta_k w_{kj} \rightarrow \text{if } j \text{ is a hidden unit}$$

Backward pass:

$$\begin{aligned}\delta_5 &= Y(1-Y)(y_{\text{target}} - Y) \\ &= (0.69)(1-0.69)(0.5 - 0.69) \\ &= -0.0406\end{aligned}$$

for hidden:

$$\begin{aligned}\delta_3 &= Y_3(1-Y_3)w_{35} * \delta_5 \\ &= (0.68)(1-0.68)(0.3)(-0.0406) \\ &= -0.0026\end{aligned}$$

$$\begin{aligned}\delta_4 &= Y_4(1-Y_4)(w_{45})(\delta_5) \\ &= (0.66)(1-0.66)(0.9)(-0.0406) \\ &= -0.0082\end{aligned}$$

$$\Delta w_{ji} = n \delta_j o_i$$

$$\Delta w_{45} = n \delta_5 Y_4 = (1)(-0.0406)(0.66) = -0.02$$

now we will add this to the old weight value to find the new weight updation.

$$\begin{aligned}w_{45}(\text{new}) &= \Delta w_{45} + w_{45}(\text{old}) = -0.0269 + 0.9 \\ \Delta w_{45}(\text{new}) &= 0.03971\end{aligned}$$

$$\Delta w_{14} = n \delta_4 x_1 = (1)(-0.0082)(0.35) \\ = -0.00287.$$

$\Delta w_{14}(\text{new}) = \Delta w_{14} + w_{14}(\text{old}).$

$$w_{14} = -0.00287 + 0.4 \boxed{= 0.3971}$$

we will find updated weight for all weights

again using forward pass

now find y_3, y_4, y_5 values by using updated weights

$$a_1 = (w_{13} * x_1) + (w_{23} * x_2) \\ = (0.0991)(0.35) + (0.79)(0.9)$$

$$a_1 = 0.7525$$

$$y_3 = \frac{1}{1 + e^{-0.7525}} = 0.6797.$$

:

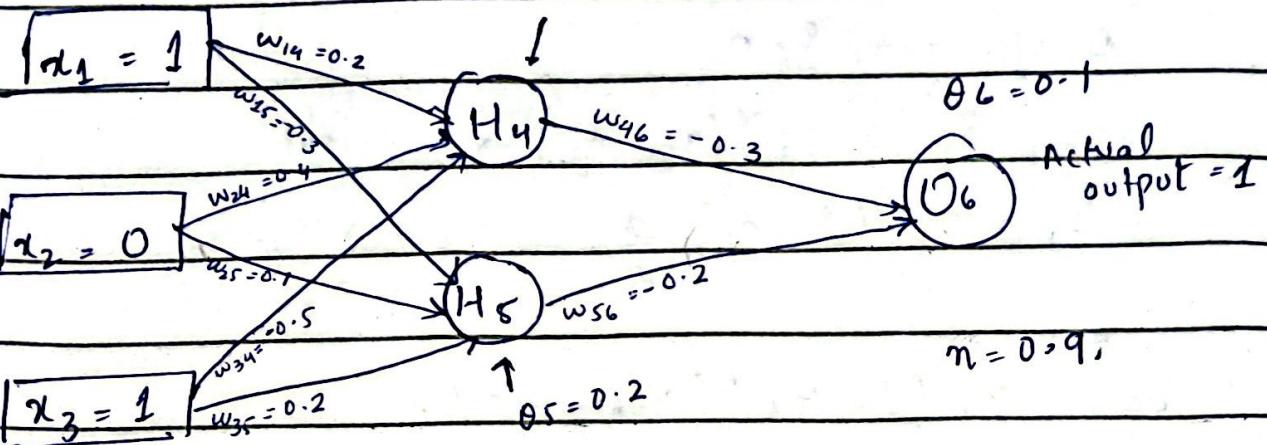
$$y_5 = 0.68 \rightarrow \text{Network output}$$

$$\text{Error} = Y_{\text{target}} - Y_5$$

$$0.5 - 0.68 = -0.182$$

MLP (backpropagation)

$$\theta_4 = -0.4$$



Forward pass = y_4, y_5, y_6 .

$$a_j = \sum_i (w_{ij} * x_i) \quad Y_j = F(a_j) = 1 / (1 + e^{-a_j})$$

$$a_4 = (w_{14})(x_1) + (w_{24})(x_2) + (w_{34})(x_3) + \theta_4$$

$$a_4 = -0.7$$

$$y_4 = 1 / (1 + e^{-0.7}) = 0.332.$$

$$a_5 = (w_{15})(x_1) + (w_{25})(x_2) + (w_{35})(x_3) + \theta_5$$

$$a_5 = 0.1$$

$$y_5 = 1 / (1 + e^{-0.1}) = 0.525.$$

$$a_6 = (w_{46})(y_4) + (w_{56})(y_5) + \theta_6$$

$$a_6 = -0.105$$

$$y_6 = 1 / (1 + e^{0.105}) = 0.474$$

$$\text{Error} = Y_{\text{target}} - Y_6 = 0.526$$

↑ update weight
to get rid of Ans error

Backward pass = $\delta_4, \delta_5, \delta_6$.

$$\Delta w_{ji} = n \delta_j o_i$$

$$\delta_j = o_j (1 - o_j) (t_j - o_j) \rightarrow \text{output}$$

$$\delta_j = o_j (1 - o_j) \sum_k \delta_k w_{kj} \rightarrow \text{hidden}$$

$$\delta_6 = \gamma_6 (1 - \gamma_6) (\gamma_{\text{target}} - \gamma_6)$$

$$= (0.474) (1 - 0.474) (1 - 0.474)$$

$$= 0.1311$$

for hidden:

$$\delta_5 = \gamma_5 (1 - \gamma_5) (\delta_6) (w_{56}) \\ = -0.0065$$

$$\delta_4 = \gamma_4 (1 - \gamma_4) (\delta_5) (w_{46}) \\ = -0.0087$$

$$\Delta w_{ij} = n \delta_j o_i$$

$$\Delta w_{46} = n \delta_6 \gamma_4 = 0.039$$

$$\Delta w_{46}(\text{new}) = \Delta w_{46} + \Delta w_{46}(\text{old}) = -0.261$$

in d. all.

again use forward pass.

to find $\gamma_4, \gamma_5, \gamma_6$. with
updated weights

$$y_4 = 0.32$$

$$y_5 = 0.52$$

$y_6 = 0.51 \rightarrow$ network output

$$\text{Error} = Y_{\text{target}} - Y_6.$$

$$= 1 - 0.51 = 0.48.$$