(DE)

2nd Order D.E

$$ay'' + by' + cy = 0$$

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

Is $y = t^3$ is sol of $ay'' + by' + cy = 0$

$y' = 3t^2$

$y'' = 6t$

$a6t + 3bt^2 + ct^3 = 0$

~~is~~

so $y = t^3$ is not sol of this

& It must satisfy

$e^{\overset{\text{constant}}{\lambda} t}$

If $y = e^{\lambda t}$

$y' = \lambda e^{\lambda t}$

$y'' = \lambda^2 e^{\lambda t}$

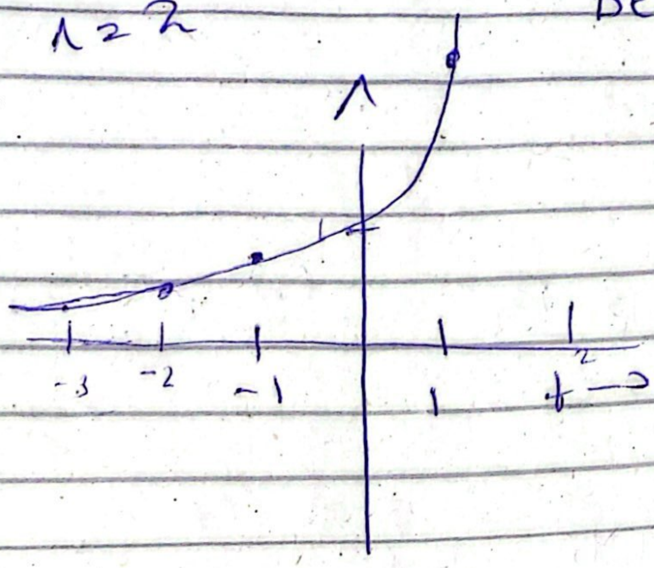
$a\lambda^2 e^{\lambda t} + b\lambda e^{\lambda t} + ce^{\lambda t} = 0$

$\underbrace{e^{\lambda t}}(a\lambda^2 + b\lambda + c) = 0$

& can never be 0

It can never be 0

$r = 2$



$e^{rt}(ar^2 + br + c) = 0$

→ Now this must be 0

characteristic eq.

$ar^2 + br + c = 0$ (Quad eq)

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Based on value of λ three cases

Real & unequal | Real, equal | Imaginary

EXP

$$4y'' - 5y' - 6y = 0$$

characteristic eq is

$$4\lambda^2 - 5\lambda - 6 = 0 \quad \text{—} \ (1)$$

$$\lambda = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-6)}}{2(4)}$$

$$= \frac{5 \pm \sqrt{25 + 96}}{8}$$

$$\boxed{\lambda = 2, -\frac{3}{4}}$$

$$= \frac{5 \pm \sqrt{121}}{8} = \frac{5 \pm 11}{8}$$

$\lambda = 2, \frac{-3}{4}$

$y = e^{\lambda t}$

$y_1 = e^{2t}$

$y_2 = e^{-3/4 t}$

As $y_1 = e^{2t}$ is sol of ①

So,

$y_1 = e^{2t}$

$y_1' = 2e^{2t}$

$y_1'' = 4e^{2t}$

$4(4e^{2t}) - 5(2e^{2t}) - 6(e^{2t}) = 0$

$16e^{2t} - 10e^{2t} - 6e^{2t} = 0$

$16e^{2t} - 16e^{2t} = 0$

$\boxed{0 = 0}$

So, $y_1 = e^{2t}$ is sol of eq (1)

Now, assume $y_2 = e^{-\frac{3}{4}t}$ is sol of (1)

$$y_2' = \left(\frac{-3}{4}\right) e^{-\frac{3}{4}t}$$

$$y_2'' = \left(\frac{-3}{4}\right)\left(\frac{-3}{4}\right) e^{-\frac{3}{4}t}$$

$$= \frac{9}{16} e^{-\frac{3}{4}t}$$ Put in (1)

$$\cancel{4}\left(\frac{9}{\cancel{16}_{4}}\right) e^{-\frac{3}{4}t} - 5\left(\frac{-3}{4}\right) e^{-\frac{3}{4}t} - 6\left(e^{-\frac{3}{4}t}\right) = 0$$

$$\frac{9}{4} e^{-\frac{3}{4}t} + \frac{15}{4} e^{-\frac{3}{4}t} - 6 e^{-\frac{3}{4}t} = 0$$

$$\frac{9e^{-\frac{3}{4}t} + 15e^{-\frac{3}{4}t} - 24e^{-\frac{3}{4}t}}{4} = 0$$

$\boxed{0 = 0}$ $\qquad 9e^{-\frac{3}{4}t} - 9e^{-\frac{3}{4}t} = 0$

some constant mult of that

$y_1 = ce^{2t}$ & $y_2 = ce^{-\frac{3}{4}t}$

Also sol of (1)

$$\boxed{y = c_1 e^{2t} + c_2 e^{-\frac{3}{4}t}}$$

overall sol of that

Diffrential eq

Put also it in (1).

→ From roots direct
make this eq
as far as you asked
to do some other
thing

*9.1 – 9.6

D.E

Exp:

$$4y'' - 5y' - 6y = 0$$

Characteristic eq is

$$4\lambda^2 - 5\lambda - 6 = 0$$

$$\lambda = \frac{-(-5) \pm \sqrt{25 - 4(4)(-6)}}{8}$$

$$= \frac{5 \pm \sqrt{25 + 96}}{8}$$

$$\frac{5 - 11}{8}$$

$$= \frac{5 \pm 11}{8}$$

$$= \frac{16}{8}, \quad \frac{-\cancel{6}3}{\cancel{8}4}$$

Exp

$$y'' - 7y' = 0$$

characteristic eq is

$$\lambda^2 - 7\lambda = 0$$

$$\lambda(\lambda - 7) = 0$$

$$\left.\lambda = 0 \quad \lambda = 7\right]$$ Real & clear.

$$y(t) = c_1 e^{0t} - c_2 e^{7t}$$

$$y(t) = c_1 - c_2 e^{7t}$$

Case (when roots are imaginary)

$$y'' + 4y' + 5y = 0 \quad — \quad (1)$$

characteristic eq is

$$\lambda^2 + 4\lambda + 5 = 0$$

$$= \frac{-4 \pm \sqrt{16 - 4(1)(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{16-20}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$\boxed{\begin{array}{l} \lambda = -2 + i \\ \lambda = -2 - i \end{array}}$$

$$= \frac{-4 \pm 2i}{2}$$

$$\lambda = \alpha \pm i\beta$$

$$\lambda = \frac{-2 \pm i}{\cancel{2}}$$

$\alpha \pm ib$ → Realpat

↳ Imaginy pat

$$y = e^{\alpha t}\left[c_1 \cos\beta t + c_2 \sin\beta t\right]$$

$$y = e^{-2t}\left[c_1 \cos\beta(1) + c_2 \sin t(1)\right]$$

$$y = e^{-2}\left[c_1 \cos t + c_2 \sin t\right]$$

is a sol of eq 1

How formula is drived ?

$\lambda = \alpha \pm i\beta$

$\lambda_1 = \alpha + i\beta$ $\quad\quad$ $\lambda_2 = \alpha - i\beta$

$$y = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

$$y = c_1 e^{\alpha t + i\beta} + c_2 e^{\alpha - i\beta}$$

$$= c_1 e^{\alpha t} \cdot e^{i\beta t} + c_2 e^{\alpha t} e^{-i\beta t}$$

$$= e^{\alpha t}\left[c_1 e^{i\beta t} + c_2 e^{-i\beta t}\right]$$

Eulers identity $e^{i\theta} = \cos\theta + i\sin\theta$

put value

$$= e^{\alpha t}\left[c_1\left(\cos\beta t + i\,\overset{\sin}{\cancel{\cos}}\,\beta t\right) + c_2\left(\cos(-\beta t) + i\sin(-\beta t)\right)\right]$$

$$= e^{\alpha t}\left[c_1\cos\beta t + c_1 i\,\overset{\sin}{\cancel{\cos}}\,\beta t + c_2\cos\beta t - c_2 i\sin\beta t\right]$$

Another word

$$= e^{\alpha t}\left[\cos\beta t\,(c_1 + c_2) + \sin\beta t\,(c_1 i - c_2 i)\right]$$

can also call the c_1 & c_2

$$= e^{\alpha t}\left[K_1 \cos\beta t + K_2 \sin\beta t\right]$$

Numerical.

9.10
$$\boxed{y'' - 6y' + 25y = 0}$$

$$r^2 - 6r + 25 = 0$$

$$r = \frac{-(-6) \pm \sqrt{36 - 4(1)(25)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 - 100}}{2(1)}$$

$$y = c_1 e^{\alpha t + i\beta} + c_2 e^{\alpha \cancel{+} - i\beta}$$

$$= c_1 e^{\alpha t} \cdot e^{i\beta t} + c_2 e^{\alpha t} \cdot e^{-i\beta t}$$

$$= e^{\alpha t}\left[c_1 e^{i\,\overset{\theta}{\beta t}} + c_2 e^{-i\,\overset{\theta}{\beta t}}\right]$$

Eulers identity $e^{i\theta} = \cos\theta + i\sin\theta$

put vale

$$= e^{\alpha t}\left[c_1\left(\cos\beta t + i\overset{\sin}{\cancel{\cos}}\beta t\right) + c_2\left(\cos(-\beta t) + i\sin(-\beta t)\right)\right]$$

$$= e^{\alpha t}\left[c_1\cos\beta t + c_1 i\overset{\sin}{\cancel{\cos}}\beta t + c_2\cos\beta t - c_2 i\sin\beta t\right]$$

Another word

$$= e^{\alpha t}\left[\cos\beta t\,(c_1+c_2) + \sin\beta t\,(c_1 i - c_2 i)\right]$$

can also call the c_1 & c_2

$$= e^{\alpha t}\left[K_1\cos\beta t + K_2\sin\beta t\right]$$

Numerical.

$\boxed{9.10 \quad y'' - 6y + 25y = 0}$

$r^2 - 6r + 25 = 0$

$$r = \frac{-(-6) \pm \sqrt{36 - 4(1)(25)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36-100}}{2(1)}$$

$$= \frac{6 \pm \sqrt{-64}}{2\cancel{1}}$$

$$= \frac{6 \pm 8i}{2\cancel{1}}$$

$$= \frac{\cancel{2}(3 \pm 4i)}{\cancel{2}}$$

$$= 3 \pm 4i$$

$\alpha = 3$

$\beta = 4$

$$y = e^{3t}\left[c_1 \cos 4t + c_2 \sin 4t\right]$$

Case: Real & Equal

9/4/22

$9.12 = y'' - 8y' + 16y = 0$

characteristic eq is

$r^2 - 8r + 16 = 0$

$$r = \frac{8 \pm \sqrt{64 - 4(1)(16)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{0}}{2} \quad \frac{8+0}{2} \quad \frac{8-0}{2}$$

$$\boxed{r_1 = 4 \qquad r_2 = 4}$$

L Real & Equal

$$y = c_1 e^{4t} + c_2 e^{4t}$$

$$= e^{4t}(c_1 + c_2)$$

$\hookrightarrow$ Also wrong

$$y = K e^{4t} \quad -(1)$$ ← as some component is missing

$$K \rightarrow K(t)$$

$$y = K(t) e^{4t}$$

Take Derivative Two times

$$y' = K'(t) e^{4t} + 4 e^{4t} K(t)$$

$$y'' = K''(t) e^{4t} + 4 e^{4t} K'(t) + 4\left[K'(t) e^{4t} + 4 e^{4t} K(t)\right]$$

$$y'' = 4K'(t) e^{4t} + 16 e^{4t} K(t) + K''(t) e^{4t} + 4 e^{4t} K'(t)$$

$$y'' = 16Ke^{4t} + 8\bar{K}(t)e^{4K} + K''(t)e^{4t}$$

Put in original eq

$$16Ke^{4t} + 8\bar{K}(t)e^{4K} + \bar{\bar{K}}(t)e^{4t}$$

$$-8\left(K'(t)e^{4t} + 4e^{4t}K(t)\right)$$

$$+16 = 0$$

$$\cancel{16Ke^{4t}} + \cancel{8\bar{K}(t)e^{4t}} + \bar{\bar{K}}(t)e^{4t}$$

$$-\cancel{8\bar{K}(t)e^{4t}} - 32e^{4t}K(t)$$

$$+16K(t) = 0$$

e^{4t}

Can never be zero

$$\ddot{K}(t)e^{4t} = 0$$

Integrating two times

$$\ddot{K}(t) = 0$$

$$\dot{K}(t) = c_1$$

$$K(t) = c_1 t + c_2$$

Put in ①

$$y = (c_1 t + c_2)e^{4t}$$

$$\boxed{y = c_1 t e^{4t} + c_2 e^{4t}}$$

↳ General sol

$y = c_1 t e^{4t} + c_2 e^{4t}$

K_2 $\quad$ K_1

$y = K_1 e^{4t} + K_2 t e^{4t}$

Just multiplied by t

Indepet variable

Don't do whole procs untill you are do do so

$y = K_1 e^{4t}$ → fond ca-4q

→ $y = K_1 e^{4t} + K_2 t e^{4t}$

(D.E) $y = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$

λ_1, λ_2

Real, unequal

Characteristic eq, roots → Real, equal

→ Imaginary

$c_1 e^{\lambda t} + c_2 t \cdot e^{\lambda t} + c_3 t^2 e^{\lambda t} \ldots$

$\lambda = \alpha \pm iB.$

Real

Imaginary

$$y = e^{\alpha t}\left[c_1 \cos \beta t + c_2 \sin \beta t\right]$$

10.1

$$y''' - 6y'' + 11y' - 6y = 0$$

characteristic eq is

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

root 1

$$\begin{array}{c|cccc}
1 & 1 & -6 & 11 & -6 \\
 & \downarrow & 1 & -5 & 6 \\
\hline
 & 1 & -5 & 6 & \boxed{0}
\end{array}$$

$= \lambda^2 - 2\lambda - 3\lambda + 6 = 0$

$\lambda(\lambda - 2) - 3(\lambda - 2)$

$\lambda^2 - 5\lambda + 6 = 0$

$= \dfrac{5 \pm \sqrt{25 - 4(1)(6)}}{2}$

$= \dfrac{5 \pm \sqrt{25 - 24}}{2}$

$= 3, 2$

$\lambda_1 = 1 \quad , \lambda_2 = 2 \quad , \lambda_3 = 3$

$y(x) = c_1 e^{x} + c_2 e^{2x} + c_3 e^{2x}$

10.2 $\quad y^4 - 9y^2 + 20y = 0$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -9 & 20 \\ & \downarrow & 1 & 1 & -8 \\ \hline & 1 & 1 & -8 & \end{array}$$

$\cancel{\lambda^4 - 9\lambda^2 +}$

$\lambda^4 + 0\lambda^3 - 9\lambda^2 + 0\lambda + 20 = 0$

$\lambda_1 = 2$

$$\begin{array}{c|ccccc} 2 & 1 & 0 & -9 & 0 & 20 \\ & \downarrow & 2 & 4 & -10 & -20 \\ \hline & 1 & 2 & -5 & -10 & \boxed{0} \end{array}$$

$\lambda_2 = -2$

$$\begin{array}{c|cccc} -2 & 1 & 2 & -5 & -10 \\ & \downarrow & -2 & 0 & 10 \\ \hline & 1 & 0 & -5 & \boxed{0} \end{array}$$

$r^2 - 5r = 0$

$r(r-5) = 0$

$r_3 = 0 \quad r_4 = 5$

$r^2 = 5$

$r = \pm\sqrt{5}$

$r_3 = +\sqrt{5}$

$r_4 = -\sqrt{5}$

$$y(x) = c_1 e^{2x} + c_2 e^{-2x} + c_3 e^{\sqrt{5}x} + c_4{}^{-\sqrt{5}x}$$

Q.4

$$y''' - 6y'' + 2y' + 36y = 0$$

characteristic eq is

$$r^3 - 6r^2 + 2r + 36 = 0$$

$$\begin{array}{r|rrrr} & 1 & -6 & 2 & 36 \\ 2 & \downarrow & 2 & -8 & -12 \\ \hline & 1 & -4 & -6 & \end{array}$$

$$\begin{array}{r|rrrr} & 1 & -6 & 2 & 36 \\ 5 & \downarrow & 5 & -5 & \\ \hline & 1 & -1 & -3 & \end{array}$$

$$\begin{array}{r|rrrr} & 1 & -6 & 2 & 36 \\ 3 & \downarrow & 3 & -9 & \\ \hline & 1 & -3 & -7 & \end{array}$$

$$\begin{array}{r|rrrr} & 1 & -6 & 2 & 36 \\ 4 & \downarrow & 4 & -8 & \\ \hline & 1 & -2 & -6 & \end{array}$$

$$\begin{array}{r|rrrr} & 1 & -6 & 2 & 36 \\ -6 & \downarrow & -6 & 0 & -12 \\ \hline & 1 & 0 & 2 & \end{array}$$

$$\begin{array}{r|rrrr} & 1 & -6 & 2 & 36 \\ 8 & \downarrow & 8 & -16 & \\ \hline & 1 & -2 & -14 & \end{array}$$

$$\begin{array}{r|rrrr} & 1 & -6 & 2 & 36 \\ -2 & \downarrow & -2 & 16 & -36 \\ \hline & 1 & -8 & 18 & \boxed{0} \end{array}$$

$\begin{array}{r} 18 \\ 4 \\ \hline 72 \end{array}$

$$\lambda^2 - 8\lambda + 18 = 0$$

$$= \frac{8 \pm \sqrt{64 - 4(1)(18)}}{2}$$

$$= \frac{8 \mp \sqrt{64 - 72}}{2}$$

$$= \frac{8 \pm \sqrt{-8}}{2}$$

-2×4

$$= \frac{8 \pm 2\sqrt{-2}}{2}$$

$= 4 \pm 2i$

$4+\sqrt{2}i \quad , \quad 4-\sqrt{2}i$

$= e^{-2x}$

$\sqrt{-1 \times 2}$

$\sqrt{2}$

$e^{4x}\left[c_1 \cos\sqrt{2}\,x + c_2 \sin\sqrt{2}x\right]$

$= c_1 e^{-2x} + c_2 e^{4x}\left[c_3 \cos\sqrt{2}x + c_4 \sin\sqrt{2}x\right]$

$$y^{4} + 8y^{3} + 24y^{2} + 32y'$$
$$+ 16y = 0$$

Charactaristic eq is

$$r^{4} + 8r^{3} + 24r^{2} + 32r$$
$$+ 16 = 0$$

$$\begin{array}{r|rrrrr}
 & 1 & 8 & 24 & 32 & 16 \\
-2 & 1 & -2 & -12 & -24 & -16 \\
\hline
 & 1 & 6 & 12 & 8 & \boxed{0}
\end{array}$$

$r_1 = -2$

$r_2 = -2$

$$\begin{array}{r|rrrr}
 & 1 & 6 & 12 & 8 \\
-2 & 1 & -1 & -5 & -7 \\
\hline
 & 1 & 5 & 7 &
\end{array}$$

$$\boxed{y(x) = c_1 e^{-2x} + c_2 x e^{-2x}}$$

$$\begin{array}{c|cccc} & 1 & 6 & 12 & 8 \\ -2 & \downarrow & -2 & -8 & -8 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$+ c_3 x^2 e^{-2x}$

$+ c_4 x^3 e^{-2x}$

$\lambda^2 + 4\lambda + 4 = 0$

$$= \frac{-4 \pm \sqrt{16 - 4(1)(4)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16-16}}{2}$$

$\lambda_3 = -2$

$$= \frac{-4 + \sqrt{0}}{2}$$

$\lambda_4 = -2$

$$= \frac{-4+}{2}$$

{DE}

$y'' - y' - 2y = 4x^2$ Non-Homogeneous.

$y'' - y' - 2y = 0$ (Homogeneous version)

characteristic eq is

$r^2 - r - 2 = 0$

$r^2 - 2r + r - 2 = 0$

$r(r-2) + 1(r-2) = 0$

$r_1 = -1$ | $r_2 = 2$

Roots are real & unequal

$$\boxed{y_h(x) = c_1 e^{2x} + c_2 e^{-x}}$$

So,

$y_p = ?$

Becau highest po on right side is 2

$y_p = \underbrace{Ax^2 + Bx + C}$

Have to find valu of A, B, C.

$y_p' = 2Ax + B$

$y_p'' = 2A$ Put in original eq

~~by~~ A

$2A - (2Ax + B) - 2(Ax^2 + Bx + C) = 4x^2$

$2A - 2Ax - B - 2Ax^2 - 2Bx - 2C = 4x^2$

$-2Ax^2 + (-2A - 2B)x + (2A - B - 2C) = 4x^2$

$-2A = 4$ $\quad -2A - 2B = 0$

$A = -2$ $\quad B = 2$ * By comparing coefficient

$2A - B - 2C = 0$

$y_p = -2x^2 + 2x - 3$

$\boxed{C = -3}$

$\boxed{y = y_p + y_h}$

$$\boxed{y = c_1 e^{2x} + c_2 e^{-x} + -2x^2 + 2x - 3}$$

$\hookrightarrow$ Sol.

$y'' - y' - 2y = e^{3x}$

characteristic eq is. $r_1 = -1$

$r^2 - r - 2 = 0$ $r_2 = 2.$

$r^2 - 2r + r - 2 = 0$

$r(r-2) + 1(r-2)$

$$y_h = c_1 e^{2x} + c_2 e^{-x}$$

$$y_p = Ax^2 + Bx + C$$

$$\bar{y}_p = 2Ax + B$$

$$\bar{\bar{y}}_p = 2A$$

$$2A - (2Ax + B) - 2(Ax^2 + Bx + C) = e^{3x}$$

$$2A - 2Ax - B - 2Ax^2 - 2Bx - 2C = e^{3x}$$

$$-2Ax^2 + (-2A - 2B)x + (2A - B - 2C) = e^{3x}$$

will be exp nature in the ce.

$y_p = Ae^{3x}$

$\bar{y}_p = 3Ae^{3x}$

$\bar{\bar{y}}_p = 9Ae^{3x}$

$9Ae^{3x} - (3Ae^{3x})$
$-2(Ae^{3x}) = e^{3x}$

$9Ae^{3x} - 3Ae^{3x}$
$-2Ae^{3x} = e^{3x}$

$4A\cancel{e^{3x}} = \cancel{e^{3x}}$

$$\boxed{A = \frac{1}{4}}$$

$y = y_h + y_p$

$y_p = \frac{1}{4} e^{3x}$

$$\boxed{y = c_1 e^{2x} + c_2 e^{-x} + \frac{1}{4} e^{3x}}$$

$y'' - y' - 2y = \sin 2x \rightarrow \theta$.

sinosidal funct

$y_p = A \sin\theta + B \cos\theta$

characteristic eq

$r^2 - r - 2 = 0$

r $\boxed{y_h = c_1 e^{2x} + c_2 e^{-x}}$

$$\bar{y}_P = 2A\cos 2x - 2B\sin 2x$$

$$\bar{\bar{y}}_P = -4A\sin 2x - 4B\cos 2x$$

$$-4A\sin 2x - 4B\cos 2x$$
$$-(2A\cos 2x - 2B\sin 2x)$$
$$-2(A\sin 2x + B\cos 2x)$$
$$= \sin 2x$$

$$-4A\sin 2x - 4B\cos 2x - 2A\cos 2x + 2B\sin 2x - 2A\sin 2x - 2B\cos 2x$$

$$= \sin 2x$$

A

$$(-4A+2B-2A)\sin 2x$$

$$+(-4B-2A-2B)\cos 2x = \sin 2x$$

$$(-6A+2B)\sin 2x + (-6B-2A)\cos 2x = \sin 2x$$

$$-6A+2B=1 \quad \Big| \quad -6B-2A=0$$

$$\cancel{-6A}-2B=1$$
$$\cancel{-6A}-18B=0$$
$$\cancel{+}\quad +$$
$$20B=1 \Rightarrow \boxed{B=\frac{1}{20}}$$

put in

$$-6A+\cancel{2}\left(\frac{1}{\cancel{20}_{10}}\right)=1 \qquad -60A+1=10$$

$$=9/3$$

$$\frac{10(-6A)+1}{10}=1 \qquad \boxed{A=-3/20}$$

$$y_p = -\frac{3}{20}\sin 2x + \frac{1}{20}\cos 2x$$

$$y = y_h + y_p.$$

$$\boxed{y = -\frac{3}{20}\sin 2x + \frac{1}{20}\cos 2x + c_1 e^{2x} + c_2 e^{-x}}$$

$$y''' - 6y'' + 11y' - 6y = 0 \quad 2xe^{-x}$$

Multiplication
of polynomial
& exp

2 will
not matter

$$2x \times e^{-x}$$

$$y_p = (Ax+B)e^{-x}$$

(D.E)

$y'' + 4y' + 8y = \sin x$

y_h

characteristic eq is

$r^2 + 4r + 8 = \sin x$

$r^2 + \cancel{4r + 4r} + 8 = \sin x$

$= \frac{-4 \pm \sqrt{16 - 4(1)(8)}}{2}$

$= \frac{-4 \pm \sqrt{16 - 32}}{2}$ $\boxed{r = -2 \pm 2i}$

$= \frac{-4 \pm \sqrt{-16}}{2}$

$= \frac{-4 \pm 2\sqrt{-4}}{2}$

$= \cancel{2}\left(\frac{-2 \pm \sqrt{-4}}{\cancel{2}}\right)$

$$y_h = e^{-2x}(c_1\cos 2x + c_2 \sin 2x)$$

$\cancel{Ax^2+Bx+1}$

$A = \frac{7}{65}$

$$y_p = A\sin x + B\cos x$$

$$y_p' = A\cos x - B\sin x$$

$$y_p'' = -A\sin x - B\cos x$$

Put

$$(-A\sin x - B\cos x) + 4(A\cos x - B\sin x)$$

$$+ 8(A\sin x + B\cos x) = \sin x$$

$$-A\sin x - B\cos x + 4A\cos x - 4B\sin x$$

$$+ 8A\sin x + 8B\cos x = \sin x$$

$$7A\sin x - 4B\sin x + 7B\cos x + 4A\cos x$$

$$= \sin x$$

$$\sin x(7A - 4B) + (7B + 4A)\cos x = \sin x$$

$$7A - 4B = 1$$

$$\cancel{7B} + 4A = 0$$

$$4A + 7B = 0$$

$$28A - 16B = 4$$

$$28A + 49B = 0$$

$$-65B = 4$$

$$\frac{4}{65}$$

$$\begin{array}{r} 49 \\ 16 \\ \hline 33 \end{array} \qquad \begin{array}{r} 49 \\ 16 \\ \hline 65 \end{array}$$

$$B = \frac{-4}{65} \qquad A = \frac{7}{65}$$

$$y = y_h + y_p$$

$$y(0)=1$$
$$\bar{y}(0)=0$$ } → Initial condition

$$y(x)=e^{-2x}(c_1\cos 2x+c_2\sin 2x)$$
$$+\frac{7}{65}\sin x-\frac{4}{65}\cos x$$

Put $x=0$ in $y(x)$

$$y(0)=c_1-\frac{4}{65}$$
$$1=c_1-\frac{4}{65}$$
$$1+\frac{4}{65}=c_1$$
$$\frac{65+4}{65}=c_1$$
$$\boxed{c_1=\frac{69}{65}}$$

$$y'(x)=e^{-2x}(-2c_1\sin 2x+2c_2\cos 2x)$$
$$-2e^{-2x}(c_1\cos 2x+c_2\sin 2x)$$
$$+\frac{7}{65}\cos x$$
$$+\frac{4}{65}\sin x$$

Put $x=0$ in it

$$y'(0)=2c_2-2c_1+\frac{7}{65}$$
$$0=2c_2-2c_1+\frac{7}{65}$$
$$0=2c_2-2\left(\frac{69}{65}\right)+\frac{7}{65}$$
$$2c_2=2\left(\frac{69}{65}\right)+\frac{7}{65}$$

$$2c_2 = \frac{138}{65} + \frac{\cancel{7}}{65}$$

$$c_2 = \frac{138}{65 \times 2}$$

$$\boxed{c_2 = \frac{138}{130}}$$

$$\begin{array}{r} 69 \\ \times 2 \\ \hline 138 \end{array}$$

13.4

$$y''' - 6y'' + 11y' - 6y = 0$$

As it is already homogen so no need for $y = y_p + y_h$

$y'(\pi) = 0$

$y(\pi) = 0$

$y''(\pi) = 1$

characteristic eq is.

Ax^0

A

	1	-6	11	-6
1	1	1	5	6
	1	-5	6	$\boxed{0}$

$\lambda_1 = 1$

$2x^2$

$\cancel{y^2 - 5y + 6}$

$\lambda^2 - 5\lambda + 6$

$= \frac{5 \pm \sqrt{25 - 4(1)(6)}}{2}$

$= \frac{5 \pm \sqrt{1}}{2}$

$\lambda_2 = 3 \quad \lambda_3 = 2.$

$$\boxed{y_h = c_1 e^{1x} + c_2 e^{2x} + c_3 e^{3x}}$$

$\cancel{y_p = A}$

y

$$y'(x) = c_1 e^{x} + 2c_2 e^{2x} + 3c_3 e^{3x}$$

$$y''(x) = c_1 e^{x} + 4c_2 e^{2x} + 9c_3 e^{3x}$$

put $x = \pi$ in $y(x)$

$$y(\pi) = c_1 e^{\pi} + c_2 e^{2\pi} + c_3 e^{3\pi}$$

$y(\pi) = \cancel{c_1 e^{\pi}} + c$

$$\boxed{c_1 e^{\pi} + c_2 e^{2\pi} + c_3 e^{3\pi} = 0} \quad - (1)$$

Put $x = \pi$ in

$$y'(\pi) = c_1 e^{\pi} + 2c_2 e^{2\pi} + 3c_3 e^{3\pi}$$

$$0 = c_1 e^{\pi} + 2c_2 e^{2\pi} + 3c_3 e^{3\pi} \quad - (2)$$

Put $x = \pi$ in $\bar{y}(\pi)$ $\quad c_3 = \frac{1}{2e^{3\pi}}$

$$c_1 e^{\pi} + 4c_2 e^{2\pi} + 9c_3 e^{3\pi} = 1 \quad (3)$$

solving (2) & (3)

$$\cancel{c_1 e^{\pi}} + 4c_2 e^{2\pi} + 9c_3 e^{3\pi} = 1$$
$$\cancel{c_1 e^{\pi}} + 2c_2 e^{2\pi} + 3c_3 e^{3\pi} = 0$$

$$2c_2 e^{2\pi} + 6c_3 e^{3\pi} = 1 \quad (4)$$

Now solving

$$\cancel{c_1 e^{\pi}} + 4c_2 e^{2\pi} + 9c_3 e^{3\pi} = 1$$
$$\cancel{c_1 e^{\pi}} + c_2 e^{2\pi} + c_3 e^{3\pi} = 0$$

$$3c_2 e^{2\pi} + 8c_3 e^{3\pi} = 1 \quad (5)$$

$$\cancel{6c_2 e^{2\pi}} + 18c_3 e^{3\pi} = 3 \qquad 2c_3 e^{3\pi} = 1$$
$$\cancel{-6c_2 e^{2\pi}} \pm 16c_3 e^{3\pi} = 2 \qquad c_3 e^{3\pi} = \frac{1}{2}$$

$c_3 = \frac{1}{2}e^{-3\pi}$

Put in 5

$$3c_2e^{2\pi} + 8c_3e^{3\pi} = 1$$

$$3c_2e^{2\pi} + \cancel{8}^{4}\left(\frac{1}{\cancel{2}}e^{-3\pi}\right)e^{3\pi} = 1$$

$$3c_2e^{2\pi} + 4e^{-3\pi+3\pi} = 1$$

$$3c_2e^{2\pi} + 4 = 1$$

$$= \frac{-\cancel{3}}{\cancel{3}e^{2\pi}}$$

$$\boxed{c_2 = -e^{-2\pi}}$$

Put both c_3 & c_2 in 1

$$c_1e^{\pi} + c_2e^{2\pi} + c_3e^{3\pi} = 0$$

$$c_1e^{\pi} + (-e^{-2\pi})e^{2\pi} + \left(\frac{1}{2}e^{-3\pi}\right)e^{3\pi} = 0$$

$-1 + \frac{1}{2}$.

$c_1 e^{\pi} - 1 + \frac{1}{2} = 0$

$+ \quad -1$

$c_1 e^{\pi} = \frac{1}{2}$

$c_1 = \frac{1}{2e^{\pi}}$

$\boxed{c_1 = \frac{1}{2} e^{-\pi}}$

$y'' - y' - 2y = e^{3x}$ $\qquad y(0) = 1$

$y'(0) = 2$

$r^2 - r - 2\cancel{y} = 0$

$r = 2$

$r = -1$

$r^2 - 2r + r - 2 = 0$

$r(r-2) + 1(r-2) = 0$ $\qquad y_h = c_1 e^{2x} + c_2 e^{-x}$

$y_p = Ae^{3x}$ $\qquad y_p'' = 9Ae^{3x}$

$y_p' = 3Ae^{3x}$ $\qquad \cancel{9A}$ $\qquad A = \frac{1}{4}$

$$\boxed{y = c_1 e^{2x} + c_2 e^{-x} + \frac{1}{4} e^{3x}}$$

Put $x = 0$

$$y(0) = c_2 e^{0} + c_1 e^{2(0)} + \frac{1}{4} e^{0}$$

$$1 = c_1 + c_2 + \frac{1}{4}$$

$$c_1 + c_2 = 1 - \frac{1}{4}$$

$$c_1 + c_2 = \frac{3}{4}$$

$$y(x) = \frac{1}{12} e^{2x} + \frac{2}{3} e^{-x} + \frac{1}{4} e^{3x}$$

$$\bar{y}(x) = -c_1 e^{-x} + 2c_2 e^{2x} + \frac{3}{4} e^{3x}$$

$$\bar{y}(0) = -c_1 + 2c_2 + \frac{3}{4}$$

بس عكس نقول

$$2 - \frac{3}{4} = 2c_2 - c_1$$

$$\frac{8-3}{4} = 2c_2 - c_1$$

$$2c_2 - c_1 = \frac{5}{4}$$

Put in

$$c_1 + c_2 = \frac{3}{4}$$

$$c_1 + \frac{2}{3} = \frac{3}{4}$$

$$\cancel{c_1} + c_2 = \frac{3}{4}$$

$$-\cancel{c_1} + 2c_2 = \frac{5}{4}$$

$$c_1 = \frac{3}{4} - \frac{2}{3}$$

$$3c_2 = \frac{8}{4}$$

$$= \frac{9-8}{12}$$

$$\boxed{c_2 = \frac{2}{3}}$$

$$\boxed{c_1 = \frac{1}{12}}$$

(DE)

$$3\frac{dx}{dt} + 2x = 1$$

$$\frac{dx}{dt} + \frac{2}{3}x = \frac{1}{3} \quad \text{— (1)}$$

x_h

$$\frac{dx}{dt} + \frac{2}{3}x = 0$$ (Homogenous sol).

$$\lambda + \frac{2}{3} = 0 \Rightarrow \boxed{\lambda = -\frac{2}{3}}$$

$$x_h(t) = c_1 e^{-2/3\, t}$$

variation of parameter

$$x(t) = c_1(t) e^{-2/3\, t}$$

$$\frac{dx}{dt} = c_1(t) \cdot e^{-\frac{2}{3}t}\left(-\frac{2}{3}\right) + e^{-2/3\, t} c_1'(t)$$

$$\frac{dx}{dt} = -\frac{2}{3} c_1(t) e^{-\frac{2}{3}t} + e^{-\frac{2}{3}t} \bar{c}_1(t)$$

by (1)

$$\cancel{-\frac{2}{3} c_1(t) e^{-\frac{2}{3}t}} + e^{-\frac{2}{3}t} \bar{c}_1(t) \cancel{+ \frac{2}{3} c_1(t) e^{-\frac{2}{3}t}} = \frac{1}{3}$$

$$\bar{c}_1(t) e^{-\frac{2}{3}t} = \frac{1}{3}$$

$$\bar{c}_1(t) = \frac{1}{3} e^{\frac{2}{3}t}$$

$$c_1(t) = \frac{1}{3} \frac{e^{\frac{2}{3}t}}{\frac{2}{3}} + \cancel{c}$$

substitute vale of $c_1(t)$ in

$$x(t) = c_1(t) e^{-\frac{2}{3}t}$$

$$x(t) = \left(\frac{1}{\frac{3}{2}} e^{\frac{3}{2}t} + K_1 \right) e^{-\frac{2}{3}t}$$

$$\boxed{x(t) = \underbrace{\frac{1}{2}}_{x_p} + \underbrace{K_1 e^{-\frac{2}{3}t}}_{x_h}}$$

$x(t) = x_p + x_h$

If I have just to find x_p?

ignore integration constant

$$2x'' - 6x' + 4x = 6e^{2t}$$

$$x'' - 3x' + 2x = 3e^{2t}$$

$$r^2 - 3r + 2 = 0$$

$$r = \frac{3 \pm \sqrt{9-8}}{2}$$

$$r = \frac{3 \pm 1}{2}$$

$r = 2$ $\quad\quad$ $r = 1$

$$x_h(t) = c_1 e^{2t} + c_2 e^{t}$$

→ find x_p?

$$x_p(t) = c_1(t)e^t + c_2(t)e^{2t}$$

$$\bar{c}_1(t)e^t + \bar{c}_2(t)e^{2t} = 0$$

$$\bar{c}_1(t)e^t + 2\bar{c}_2(t)e^{2t} = 3e^{2t}$$

(2) − (1)

$$\bar{c}_2(t)e^{2t} = 3e^{2t}$$

$$\boxed{\bar{c}_2(t) = 3} \quad (3)$$

by (1)

$$\bar{c}_1(t)e^t + 3e^{2t} = 0$$

$$\boxed{\bar{c}_1(t) = -3e^t} - (4)$$

$$c_1(t) = -3e^{t}$$

$$c_2(t) = 3t$$

$$x_p(t) = -3e^{t} \cdot e^{t} - 3e^{t} \cdot e^{4t}$$

$$x_p(t) = -3e^{2t} + 3te^{2t}$$