LINEAR ALGEBRA and its Applications

linear Algebra by David. C. Lay, Steven R. Lay

5th Edition.

2) Elementry Linear Algebra Howard Anton

Ass 05
Quizes 15
H/ class Part 05
Mid. 35
Finals 40

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L#1 9th Jan 2022

INTRODUCTION

linear.

$a_1x_1 + a_2x_2 + a_3x_3 \ldots a_nx_n = b$

$xy + 1 = 0$ (non linear)

2 unknows multiplying

Two diff chezon ka akhtha behaviour

Trignometry (non-linear)

$x+y = 2$ (linear)

Matrix and system of leq

★ cross terms not allowed

↳ sin, cosine not allowed → sinosoidal

coefficients are expressed as matrixes

Face, images are matrices.

Calculus = Algebra + Geometry.

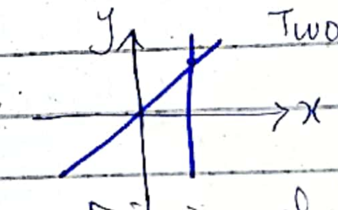
Linear = one constant + one variable, unknowns power 1

linear systems are trackable. (linear approx)

Consistent

Linear system can be consistent or non consisten.

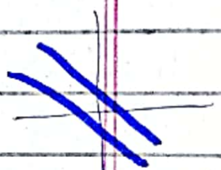
→ When at least one solution exists it is consistent system wrna 0 solution Tu non consistent system.



Two linear equat at one point of intersection

⇒ it is solution

1. consistent

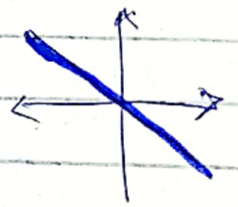


No solution

— Two lines superimposed on each other

It has many solutions (consistent)

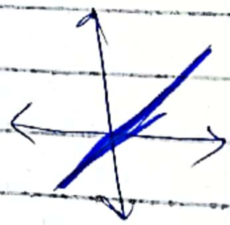
$y = -x$



a = cofficient constan

x = unknown

$y = +x$



$$1x_1 + 1x_2 + 2x_3 = 6$$
$$2x_1 + 4x_2 + 3x_3 = 2$$
$$3x_1 + 5x_2 + 4x_3 = 9$$

cefficient matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & 3 \\ 3 & 5 & 4 \end{bmatrix}$$

Augmented matrix

$$\begin{bmatrix} 1 & 1 & 2 & 6 \\ 2 & 4 & 3 & 2 \\ 3 & 5 & 4 & 9 \end{bmatrix}$$

Lecture-2

1- Multiply an eq by non zero constant
2- Interchange two equation
3- Add a multiple of one equation to another.

* Echelon form can be different for diff one augmented matrix.

$x_1 - 2x_2 + x_3 = 0$
$2x_2 - 8x_3 = 8$
$5x_1 - 5x_3 = 10$ ① Write augmented matrix

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & 8 & 8 \\ 5 & 0 & -5 & 10 \end{bmatrix}$$

Multi ~~eq~~ row ① × 5 − with ~~eq~~ row ③

5 −10 5 0

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & 8 & 8 \\ 0 & -10 & -15 & 10 \end{bmatrix}$$ row ② × $\frac{1}{2}$,

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & 4 & 4 \\ 0 & -10 & -10 & 10 \end{bmatrix}$$ row ① × ⑤ and add in row ③.

* 5 −10 15 0

$$\begin{bmatrix} 1 & -2 & & \\ 0 & 1 & & \\ 0 & & & \end{bmatrix}$$

$x_3 = -1$

* 0 2 8 8

$x_2 = 0$

$$\begin{bmatrix} 1 & 0 & 4 & 8 \\ 0 & 1 & 4 & 4 \\ 0 & -10 & -10 & 10 \end{bmatrix}$$

$x_1 = 1$

System is consistent, unique it has one solution only.

$$x_2 - 4x_3 = 8$$
$$2x_1 - 3x_2 + 2x_3 = 1$$
$$4x_1 - 8x_2 + 12x_3 = 1$$

$$\begin{bmatrix} 0 & 1 & -4 & 8 \\ 2 & -3 & +2 & 1 \\ 4 & -8 & 12 & 1 \end{bmatrix}$$ interchanging rows 1 & 2

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 4 & -8 & 12 & 1 \end{bmatrix}$$ row 1 × 2

row 1 × -2 add in row 3

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & -2 & 8 & -1 \end{bmatrix}$$

row 2 × 2 + row 3.

2-2

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ \boxed{0 \quad 0 \quad 0 \quad 15} \end{bmatrix}$$ inconsistent system

typical inconsistent system

$[0 \quad 0 \quad 0 \quad 0]$ it can be

If four zeros ↘ consistent

constant

So, some free variable is there

For what values of h and k ~~this~~ ~~consist~~ system will be consistent?

$$2x_1 - x_2 = h$$
$$-6x_1 + 3x_2 = k$$

$$\begin{bmatrix} 2 & -1 & h \\ -6 & +3 & k \end{bmatrix} \sim$$

$+6 \quad -3 \quad 3h$

$$\begin{bmatrix} 2 & -1 & h \\ 0 & 0 & 3h+k \end{bmatrix} \checkmark$$

leading entry of 2

It can be inconsistent but it can't be.

$3h + k \neq 0$ It will be consistent.

$-3 \quad +9$

$h = 3$

$k = 9$

| multiples of 3 |

Echelon Form

Different levels.

Step like pattern.

A rectangular matrix is in echelon form if it has following 3 properties

①
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

① All non zero rows are above of any rows of zeros.

② Each leading entry of the row is in a column to the right of leading entry of the row above it.

③ Rest of the elements below the leading entry are zero

in a ~~column~~ row

Then matrix is in echelon form

★ Echelon form is for convenience

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 6 & 5 \\ 0 & 0 & 4 \end{bmatrix}$$

★ reduced row echelon form
↳ Gaussian Eliminatio

Reduced row echelon form
Example identity matrix

Row reduction algorithm
don't need to think
Gauss Jordan elimination method.

Pivot column → that has non zero entities
Pivot pos → left coloum non zero's ع اوپر

↓ Forward pass | ↑ backward pass | ★ leading entries zeros کے اوپر ہونے چاہیے

= from bottom to top
① left to righ

reduced echelon form — (unique) solution
★ You can't change it

leading variables: → basic variable
→ exists in pivot position

[nikal ker jo variables bchay gay
[those will be free variables
↳ 1 free variable available → it has many solution

Aug

$$\left[\begin{array}{cccc|c|c} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{array}\right]$$

You can get diff echelon forms but only 1 reduced echelon form.

① left most non zero column

$$\begin{bmatrix} 3 & -7 & 8 & -5 & 8 & 9 \\ 0 & \boxed{3} & -6 & 6 & 4 & -5 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

$-9-(-7)$

row 3 − row 1

$$\begin{bmatrix} 3 & -7 & 8 & -5 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \\ 0 & -2 & 4 & -4 & -2 & 6 \end{bmatrix}$$

ignore row ①

Mathematics library languages

row echelon form + reduced echelon form.

0	6	−36	36	24	−30
0	6	24	24	−12	18

$24-(-36)$

$$\begin{bmatrix} 0 & 0 & 60 & -12 & -36 & 48 \end{bmatrix}$$

36
+24
60

24
+12
36

18
+30
48

Tip:- reduced echelon form hi practice krein

leading pivot position = zero. now

Linear Algebra

Lecture #3:-

$$\begin{bmatrix} 1 & 6 & 2^{\neq 0} & -5 & -2 & -4 \\ 0 & 0 & 2^{\neq 1} & -8 & -1 & 3 \\ 0 & 0 & 0^{\neq 0} & 0 & 1 & 7 \end{bmatrix}$$

Echelon form

pivot column

$$= \begin{bmatrix} 1 & 6 & 2 & -5 & -2 & -4 \\ 0 & 0 & 1 & -4 & -1/2 & 3/2 \end{bmatrix}$$

row $2/2$

Start from right most

right most but not pivot

Backward pass algorithm

☆ up and left

$$\begin{bmatrix} 1 & 6 & 2 & -5 & -2 & -4 \\ 1 & 1 & 3 & -7 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 8 & 4 & -3 & 0 & -2 \\ 1 & 1 & 3 & -7 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 8 & 4 & -3 & 0 & -2 \\ 1/3 & 1/3 & 1 & -7 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \text{leading} \end{array}$$

$$\begin{bmatrix} 1 & 6 & 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & -4 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

reduce echelon form

x_2 & x_4 = free variables

Expression of Solution set is called parametric form.

$x_1 + 6x_2 + 5x_4 = 0$

$x_1 = -6x_2 - 5x_4$ — ①

x_2 is free

$x_3 = 4x_4 + 5$ x_4 is free

$x_5 = 7$

① Forward pass
② Backward pass
③
Mechanical Procedure

- not implemented in the computers.
- restricted to forward pass
- backward pass mai computations بچ ajati hain.

$A_{m \times n}$ $m \geq 30$ ∵ $n = m+1$

$O(m^3) \rightarrow$ Forward Pass. Big-O (how complex is your

$O(m^2)$ Backward pass

- Computers compute echelon form then use back substitution $\boxed{12:45}$

Matrix A

pivot pivot

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$$

$r_3 - 3r_1$

$$\begin{pmatrix} 1 & 3 & 5 & 7 \\ 0 & 2 & 4 & 6 \\ 0 & 2 & 4 & -4 \end{pmatrix}$$

$(-2)(-7)$

$$\begin{pmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 4 \end{pmatrix}$$

row2 × 2

0 2 4 3

1 3 5 7

$$\begin{pmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0-0 & 2-2 & 4-4 & 3+4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 7 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 0 & \boxed{-10} \end{bmatrix}$$

→ scaling kr k kina hai $\boxed{\text{row 1} \times}$ add(subt)

$$\begin{pmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

row3 × 12 ⊕ row2

$$\begin{pmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \sim \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

left up (Backward)

$$\begin{pmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$x_1 + h x_2 = 2$ $\quad h = ?$

$4x_1 + 8x_2 = k$ $\quad k = ?$

① system is inconsistent → no solution exist last row = 0's or end ≠ non zero

② system has unique solution.

③ system has many solutions

$$\begin{bmatrix} 1 & h & 2 \\ 4 & 8 & k \end{bmatrix}$$

$\{0 \ 0 \ 0 \ 0 \ b\}$ inconsistent

$4 \quad 4h \quad 8$ make echelon form.

$0 \quad 8-4h \quad k-8$

$$\begin{bmatrix} \boxed{1} & h & 2 \\ 0 & \underbrace{8-4h}_{\text{leading}} & k-8 \end{bmatrix}$$ It is in echelon form.

zero

~~$4 \quad 4h$~~ $\quad 8$ add

$8-4h = 0$ ① system inconsistent

$k-8 \neq 0$

$\boxed{\begin{matrix} h = 2 \\ k \neq 8 \end{matrix}}$

$\{0 \ 0 \ 0 \ 0 \ 0\}$ → consistent

$\{0 \ 0 \ 0 \ 0 \ b\}$ → inconsistent

$h = 2 \quad k = 8$

③ system is consistent

② make reduced echelon form for unique solution

$$\begin{bmatrix} 1 & h & 2 \\ 0 & \underset{①}{8-4h} & k-8 \end{bmatrix}$$

$0 \cdot \dfrac{8-4h}{8-4h} \quad \dfrac{k-8}{8-4h}$

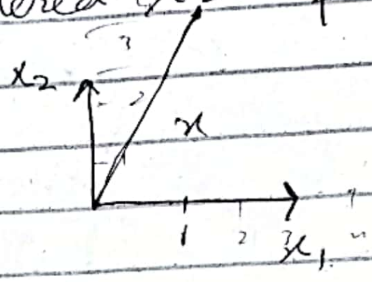
$$\begin{bmatrix} 1 & h & 2 \\ 0 & 1 & \cfrac{\cancel{8-4h}}{\cfrac{k-8}{8-4h}} \end{bmatrix}$$

$\dfrac{k-8}{8-4h}$ $\quad h \neq 2$ denominator $\neq 0$

$k =$ any value.

vector ordered list of numbers.

$x = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$



direction + magnitude

$\mathbb{R}^2$

$v \in \mathbb{R}^3$ vector belongs to $\mathbb{R}^3$

$v = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $\mathbb{R}^n$

rows in table are vectors

$\mathbb{R}^2$ vector can't be compared with $\mathbb{R}^3$

vectors addition / subtraction

$\{v_1, v_2, \dots, v_p\}$ $\mathbb{R}^n$ (linear combination)
$\{c_1, c_2, \dots, c_p\}$ scalars / weights

linear combind in vector :–

$c_1v_1 + c_2v_2 + \dots + c_pv_p = y \in \mathbb{R}^n$ (lies in n dimension space)

↳ **vector addition equation**

Given vectors: v_1, v_2, b

Is 'b' a linear combination of v_1 and v_2 = ?

$x_1 v_1 + x_2 v_2 = b$

↓ weights unknown.

$$a_1 = \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} ; \quad a_2 = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} ; \quad b = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}$$

$$x_1 a_1 + x_2 a_2 = b$$

$$x_1 \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ -2x_1 \\ -5x_1 \end{bmatrix} + \begin{bmatrix} 2x_2 \\ 5x_2 \\ 6x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}$$

same size so can be combined

$$\begin{bmatrix} x_1 + 2x_2 \\ -2x_1 + 5x_2 \\ -5x_1 + 6x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}$$

(linear system)
it is a

$$\begin{bmatrix} 1 & 2 & 7 \\ -2^{\times 2} & 5 & 4 \\ -5 & 6 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 7 \\ 0 & 7 & 6 \\ -5 & 6 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 7 \\ 0 & 7 & 6 \\ 0 & 11 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 7 \\ 0 & 1^{\times 11} & 6/7^{\times 11} \\ 0 & 11 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 6/7 \\ 0 & 0 & 2 - \frac{66}{7} \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 6/7 \\ 0 & 0 & -\frac{52}{7} \end{bmatrix}$$

$0 \quad 11 \quad 66/7$

$\frac{14-66}{7} = \frac{-52}{7}$

$$\begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 2 \end{bmatrix}$$

reduced echelon form.

$$\underset{\text{Final}}{\sim}\begin{bmatrix}1 & 0 & 3\\0 & 1 & 2\\0 & 0 & 0\end{bmatrix} = \text{unique solution}$$

3x3 can't be identity matrix = ~~consistent~~

$$\begin{bmatrix}x_1 = 3\\x_2 = 2\end{bmatrix}$$

$$3\begin{bmatrix}1\\-2\\-5\end{bmatrix}+2\begin{bmatrix}2\\5\\6\end{bmatrix}=\begin{bmatrix}7\\4\\-3\end{bmatrix}$$

Span

3-22

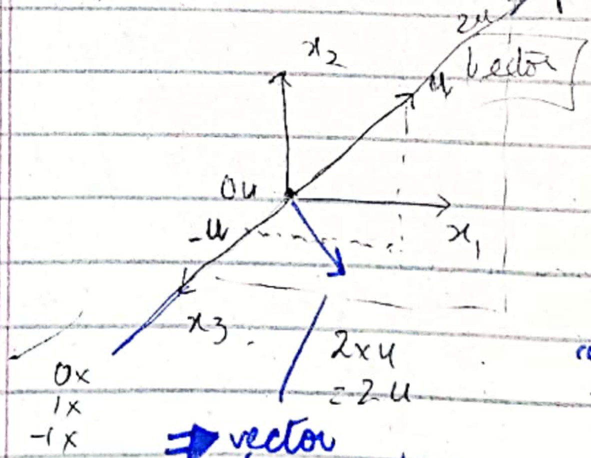
The set of all linear combinations of $v_1 \ldots v_p$ is known as span.

$$\text{span}\{v_1, \ldots v_p\} \subseteq \mathbb{R}^n$$

Subset of $\mathbb{R}^n$

$$cv_1 = cv_1 + 0v_2 + \ldots + 0v_p$$

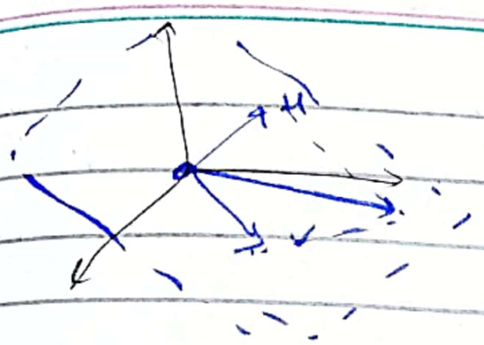
↳ Generated by this formula is called "Span"



⇒ vector here isn't in its span

سارے vectors generate کر سکتا ہے۔ بس۔

So it is "span u"



span$\{u, v\}$

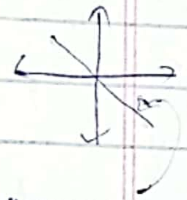
plane is open
all are part of
span.

Draw a perpendicular
(least square solution)

Lect #4

This slo

Th



span$\{u, v\} = ?$

$u = \begin{bmatrix} 6 \\ -9 \end{bmatrix}$ $\quad v = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$

Quiz on Friday

$x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

span$\{x, y\} = ?$

$$x_1 \begin{bmatrix} 6 \\ -9 \end{bmatrix} + x_2 \begin{bmatrix} -4 \\ 6 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$Ax = b$$

$\begin{bmatrix} 6 & -4 & a_1 \\ -9 & 6 & a_2 \end{bmatrix}$ linear form corresponding to this

$r_2 = \frac{3}{2} r_1 + r_2$

$$\left[\begin{array}{cc|c} 6 & -4 & a_1 \\ 0 & 0 & \frac{3}{2}a_1 + a_2 \end{array}\right]$$

$\frac{3}{2} a_1 + a_2 = 0$ should b to make system consistent

$a_1 = -\frac{2}{3} a_2 \quad \therefore y = mx + c \rightarrow$ slope $= 0$

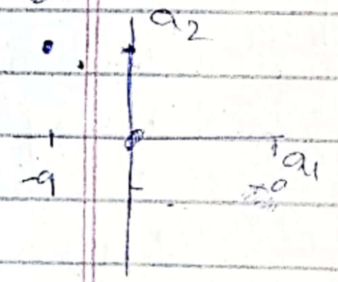
span $\{x, y\} = ?$ $\quad x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \; y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

2D

cartesian

$a_1 = 0$ $\quad a_1 = -9$

$a_2 = 0$ $\quad a_2 = 6$



$v_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}; \; v_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}, \; v_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$

$y = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix} \rightarrow$ augmented (ارسم)

لو ال vector = span من ...

إيه value ال $h = ?$

$$\begin{bmatrix} 1 & 5 & -3 \\ -1 & -4 & 1 \\ 2 & -7 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & -3 & -4 \\ -1 & -4 & 1 & 3 \\ 2 & -7 & 0 & h \end{bmatrix}$$

→ coefficient some value.

last of #

pivot of column
Ehi is column

$\boxed{\text{echelon form convert}}$

$R_2 + R_1$

$$\begin{bmatrix} 1 & 5 & -3 & -4 \\ 0 & -1 & -2 & -1 \\ 2 & -7 & 0 & h \end{bmatrix}$$

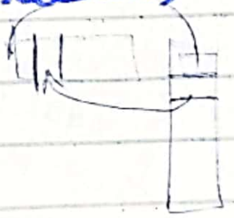
$h - 5 = 0$

$h = 5$ In this vector relies the span.

Matrix multiplication

$$\begin{bmatrix} a_{11} & & \\ a_{21} & & \\ & & \end{bmatrix} \quad \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$A_{m \times n}$ $\qquad$ X



Dot product
Inner-Product

Vector Equation

Scalar$_1$ * vector$_1$ + Scalar$_2$ * vector$_2$

+ ... = [ ]

$c_1 v_1 + c_2 v_2 + \dots + c_n v_n$

$= x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{21} \\ \vdots \\ a_{m2} \end{bmatrix} + \dots$

$Ax = b$ is same as vector eq

$\boxed{x_1 a_1}$

$Ax = b$

$\Updownarrow$

$x_1 a_1 + x_2 a_2 + \dots + x_n a_n =$

$\Updownarrow$

$[a_1 \; a_2 \dots a_n \; b] = [A \;\; b]$

$$A = \begin{bmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

You have to tell all b's jinkai liyai system consistent hoga b ya nhi

possible questions
Gausian
Qs are for your geometric intuition

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$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & b_1 \\ -4 & 2 & -6 & b_2 \\ -3 & -2 & -7 & b_3 \end{array}\right]$$

$$\begin{matrix} 1 & 3 & 4 & b_1 \\ 0 & 10 & 10 & 4b_1 - b_2 \\ 0 & -11 & -19 & b_3 - 3b_1 \end{matrix}$$

$R_1 + R_2$

$$\begin{matrix} -3 & 9 & 12 & -3b_1 \end{matrix}$$

$-3R_1 + R_2$

$R_2 - 3R_1$

$$\begin{bmatrix} 1 & 3 & 4 & b_1 \\ 0 & 10 & 10 & 4b_1 - b_2 \\ 0 & -11 & -19 & b_3 - 3b_1 \end{bmatrix}$$

Span can be 2D - 3D space

Are its span

All b's that satisfy this equation

$b_1 - \frac{1}{2} b_2 + b_3 = 0$

system will be consistent

b_1, b_2 value put

Hyperplane:-

$b=0$

$AX=b$

special case when $AX=0$

special system → homogenious system of linear equations

↳ It is always consistent, ek jawab zaroor dega,

① infinitely many ② unique.

$A \quad X = 0$

A → data matrix (known)

X → unknown weight

$x_1 a_1 + x_2 a_2 + \dots + x_n a_n = 0$

(My) $X=0$ $\mathbb{R}^n$

↳ trivial solution

ordinary unimeaning [illegible], common

میرا مطلب نظر آ رہا ہے

can take any value.

[A 0] → non trivial

free variable لیا کو

موجود ہے

↳ solution

کیا ہوگا

Homogenous system

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 2x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 8x_3 = 0$$

does non trivial solution exists??

[A 0] اسے کیسے لکھنا ہے؟

$$\begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & +4 \\ 6 & 1 & -8 \end{bmatrix}$$

$R_2 + R_1$ $\quad$ $2R_1$

$$\begin{bmatrix} 3 & 5 & -4 \\ 0 & -3 & 0 \\ 6 & 1 & -8 \end{bmatrix}$$

$R_3 - 2R_1$

~~6 10 -8~~

Convert it into echelon form = Determines system is consistent.

$$\begin{bmatrix} 3 & 5 & -4 \\ 0 & -3^{\times 3} & 0 \\ 0 & -9 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 5 & -4 \\ 0 & -3/_{-3} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

⇒ system is consistent with free variable.

⇒ A homogenous system has a non-trivial solution if and only if it has atleast one free variable

$$\begin{bmatrix} 3 & 5/_3 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\begin{bmatrix} 1 & 5/3 & -4/3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

x_3 free variable

x_1 & x_2 bound variable

Solution Set

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4/3\,x_3 \\ 0 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 4/3 \\ 0 \\ 1 \end{bmatrix}$$

it is free $\quad$ t is free $\quad$ $t \in \mathbb{R}$

$\boxed{x = t(v)}$ → equation of line in explicit form

⇒ Parametric vector form solution set

Equation of a line in implicit form

$\boxed{\text{span}}$ will pass from origin ik plain hai

$10x_1 - 3x_2 - 2x_3 = 0$ Equation of a plane

LINEAR h #5 25th Jan 2023

r_3 space mai zero vector

3 components hon gay

$r_n \to$ main n compo

$r_2 \to$ 2 components

$x_1 a_1 + \dots + x_n a_n = 0$

$[A \; b]$ $[A \; 0]$

Homogenous → consistent

↳ (at least one solution will exist)

2 types of solutions exist

↳ Trivial = vector contains where all weights are zero

↳ Non-trivial = {atleast one free variable exists.} (not all zeros)

(m, c) pa

$y = mx + c$

$p = x\cos\theta + y\sin\theta$

(p, θ) parameter {implicit equations in the form of variables}

row

$$X = \begin{cases} x_1 = 4/3 \; x_3 \\ x_2 = 0 \\ x_3 \text{ is free} \end{cases} \qquad X = x_3 \begin{bmatrix} 4/3 \\ 0 \\ t \end{bmatrix}$$

$\boxed{\text{vector } \vec{v}}$

$\uparrow \vec{v}$

$= t\vec{v}$

$$10x_1 - 3x_2 - 2x_3 = 0$$

linear system ↘ only one row.

$x = t\vec{v}$

$AX = 0$ (row vector)

$$\frac{10x_1}{10} = \frac{3x_2}{10} + \frac{2x_3}{10}$$

pivot column.

$[\,10 \quad -3 \quad 2\,]$

Pivot positive zero.

reduced echelon form

$\therefore [x_1 - 0.3x_2 - 0.2x_3 = 0$

Solution set:- $X = \boxed{x_1 = 0.3x_2 + 0.2x_3}$

x_1 ___

x_2 ___

x_3 ___

x_2, x_3 free

$$X = \begin{bmatrix} 0.3x_2 + 0.2x_3 \\ x_2 \\ x_3 \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{cases} x_1 = 0.3x_2 + 0.2x_3 \\ x_2 \\ x_3 \end{cases}$$

we can change it differently b/c x_2 & x_3 both are free variables

$$\underbrace{\begin{bmatrix} 0.3x_2 + 0.2x_3 \\ x_2 \\ x_3 \end{bmatrix}}_{\text{splitting}} = \begin{bmatrix} 0.3x_2 \\ x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.2x_3 \\ 0 \\ x_3 \end{bmatrix}$$

free variable ki jga 1 likh liya

$$\begin{bmatrix} 0.3x_2 \\ x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.2x_3 \\ 0 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 0.3 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0.2 \\ 0 \\ 1 \end{bmatrix}$$

s t

$x = s\vec{u} + t\vec{v}$

{parametric} $s, t \in \mathbb{R}$

* Jitnai free variables hain utnai parametric mai constants hon gay
* names arranged (customery)

$3x_1 + 5x_2 - 4x_3 = 7$
$-3x_1 - 2x_2 + 4x_3 = -1$
$6x_1 + x_2 - 8x_3 = -4$

non homogenous system

$$\begin{bmatrix} 3 & 5 & -4 & 7 \\ -3 & -2 & 4 & -1 \\ 6 & 1 & -8 & -4 \end{bmatrix}$$

reduced row echelon form

$$\begin{bmatrix} 1 & 5/3 & -4/3 & 7/3 \\ 0 & 3 & 0 & 6 \\ 0 & -9 & 0 & 10 \end{bmatrix}$$

$R_2 + R_1$

$-3+3$ $-2+5$ $4-4$ $-1+7$

$6-6$ $-8-(-8)$ $-4+14$

$6R_1$

$R_3 - 6R_1$

$$\begin{bmatrix} 1 & 5/3 & -4/3 & 7/3 \\ 0 & 3/3 & 0 & 6/3 \\ 0 & -9+9 & 0 & -18+ \end{bmatrix}$$

$7-9=-2$

$$\begin{bmatrix} 1 & 5/3 & -4/3 & 7/3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & -9 \end{bmatrix}$$

$9R_2$

$R_1 - 9R_2$

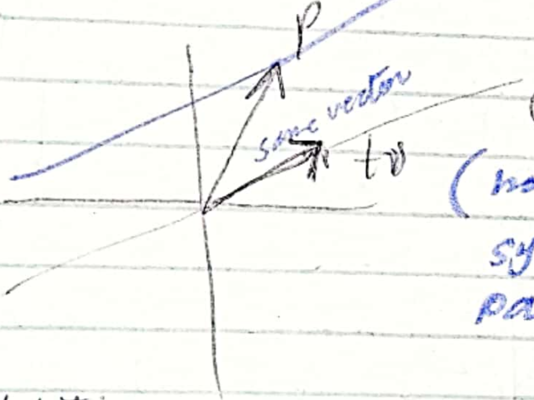
?? $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 + \frac{4}{3} x_3 \\ 2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{4}{3} x_3 \\ 0 \\ x_3 \end{bmatrix}$

$$X = \underset{\uparrow \; p}{\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}} + \underset{\downarrow \; (t)}{x_3} \underset{\uparrow \; v}{\begin{bmatrix} 4/3 \\ 0 \\ 1 \end{bmatrix}}$$

$x = p + x_3 v$

$\boxed{x = p + t v}$

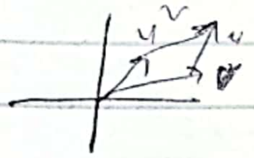
Idea



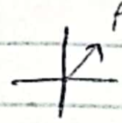
(non homogeneous solutions lies here)
$(Ax = b)$

$(Ax = 0)$
(homogeneous system k parallel)

2 vector addition



If $t = 0$ If
$x = p + 0$ $t = 1$
$px = p + v$



$Ax = b$ t can be anything
for some "b"
and a solution 'p'

$x = p + v$

$w = p + v_h$
this value is always true

vector ko move kiya hai isma
" vector translation "

$10x_1 - 3x - 2x_3 = 7$ non homogeneous

$W = P + \text{span}\{u, v\}$

$W = P + (s\bar{u} + t\bar{v})$

⇒ Chemical Equations Balance Using

$x_1 \, C_3H_8 + x_2 \, O_2 \rightarrow x_3 \, CO_2 + x_4 \, H_2O$

In vector form $\begin{bmatrix} C \\ H \\ O \end{bmatrix} =$

$$x_1 \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix} = 0$$

Homogeneous system of equations

+ Signs are accounted for

(left to right 1st non zero is pivot position)

① nonzero

$$\begin{bmatrix} 3 & 0 & -1 & 0 & 0 \\ 8 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}$$

pivot posit

Rows interchange

Solution set is define reduced echelon form.

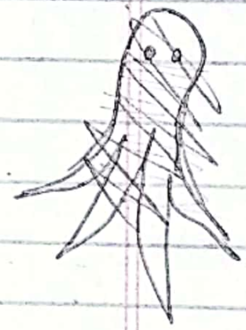
$$\begin{bmatrix} 1 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 8/3 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{bmatrix}$$

ختم

$$\begin{bmatrix} 1 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & -8/3 & -2 & 0 \\ 0 & 1 & -1 & -1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & -16 & 1 & 0 \\ 0 & 1 & -1 & -1/2 & 0 \end{bmatrix}$$

$$\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \begin{bmatrix} 1 & 0 & 0 & -1/4 & 0 \\ 0 & 1 & 0 & -5/4 & 0 \\ 0 & 0 & 1 & -3/4 & 0 \end{bmatrix}$$



x_4 = free variable   reduced echelons.

$$X = \begin{cases} x_1 = x_4/4 \\ x_2 = 5/4\, x_4 \\ x_3 = 3/4\, x_4 \\ x_4 \text{ is free} \end{cases}$$

lets suppose $x_4 = 4$ = fraction khtm hoti thi isliyai 4 lai liya

$x_1 = 1$

$x_2 = 5$

$x_3 = 3$

x_4 is free 4

$1C_3H_8 + 5O_2 \longrightarrow 3O_2 + 4H_2O$

Is system consistent or x ? ~Echelon form

pivot entries

★ کے اوپر اور نیچے zeros ہونے چاہیئے