Exercise 5.2

2. $\int x\sin x\,dx = \sin x - x\cos x + C$

$= \frac{d}{dx}(\sin x - x\cos x + c)$

$= \frac{d}{dx}(\sin x) - \frac{d}{dx}(x\cos x) + \frac{d}{dx}(c) = \frac{d}{dx}(\sin x) - x\frac{d}{dx}(\cos x) - \cos x\frac{d}{dx} + 0.$

$= \cancel{\cos x} - x(-\sin x) + \cancel{\cos x}(1) + 0$

$= + x\sin x$

b) $\int \frac{dx}{(1-x^2)^{3/2}} = \frac{x}{\sqrt{1-x^2}} + c.$

$= \frac{d}{dx}\left(\frac{x}{\sqrt{1-x^2}} + c\right)$

$= \frac{d}{dx}\left(\frac{x}{\sqrt{1-x^2}}\right) + \frac{d}{dx}(c).$

$= \frac{(\sqrt{1-x^2})\,d/dx\,(x) - (x)\,d/dx\,(\sqrt{1-x^2})}{(\sqrt{1-x^2})^2} + \frac{d}{dx}(c).$

$= \frac{(\sqrt{1-x^2}) - x\left(\frac{-2x}{2\sqrt{1-x^2}}\right)}{1-x^2} + 0.$

$= \frac{\frac{(\sqrt{1-x^2})^2 + x^2}{\sqrt{1-x^2}}}{1-x^2} = \frac{1-x^2+x^2}{(1-x^2)^{3/2}} = \frac{1}{(1-x^2)^{3/2}}$

5. $\frac{d}{dx}(\sqrt{x^3+5})$

$= \frac{d}{dx}\left[(x^3+5)^{1/2}\right] = \frac{1}{2}(x^3+5)^{1/2-1}\frac{d}{dx}(x^3+5)$

$= \frac{1}{2}(x^3+5)^{-1/2}(3x^2)$

$= \frac{3x^2}{2\sqrt{(x^3+5)}}$

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6. $\frac{d}{dx}\left(\frac{x}{x^2+3}\right) = \frac{(x^2+3)\, d/dx\,(x) - (x)\, d/dx\,(x^2+3)}{(x^2+3)^2}$

$= \frac{(x^2+3) - x(2x)}{(x^2+3)^2}$

$= \frac{x^2+3-2x^2}{(x^2+3)^2} = \frac{3-x^2}{(x^2+3)^2}$

$\int \frac{3-x^2}{(x^2+3)^2} = \frac{x}{x^2+3} + c.$

7. $\frac{d}{dx}(\sin(2\sqrt{x})) = \cos(2\sqrt{x}) \cdot \frac{d}{dx}(2\sqrt{x})$

$= \cos(2\sqrt{x}) \cdot \frac{2}{2\sqrt{x}}$

$= \frac{\cos 2\sqrt{x}}{\sqrt{x}}$

$\int \frac{\cos 2\sqrt{x}}{\sqrt{x}} = \sin 2\sqrt{x} + c.$

8. $\frac{d}{dx}(\sin x - x\cos x) = \cos x - (\cos x - x\sin x) = -x\sin x$

Integral will be $\int(-x\sin x\, dx) = -\int x\sin x\, dx$

11. $\int\left(5x + \frac{2}{3x^5}\right)dx = \int 5x\, dx + \int \frac{2}{3x^5}\, dx.$

$= 5\int x\, dx + \frac{2}{3}\int \frac{1}{x^5}\, dx.$

$= \frac{5x^2}{2} + \frac{2}{3}\left(\frac{x^{-4}}{-4}\right) + c.$

$= \frac{5x^2}{2} - \frac{1}{6x^4} + c$

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15. $\int x(1+x^3)\,dx.$

$= \frac{1}{2}x^2 + \frac{1}{5}x^5 + C.$

Now, we will differentiate the answer to check.

$\frac{d}{dx}\left(\frac{x^2}{2} + \frac{x^5}{5} + c\right) = \frac{1}{2} \cdot \frac{d}{dx}(x^2) + \frac{1}{5} \cdot \frac{d}{dx}(x^5) + \frac{d}{dx}(c).$

$= \frac{1}{2}(2x) + \frac{1}{5}(5x^4) + 0.$

$= x + x^4$

$= x(1+x^3).$

11- $\int\left[5x + \frac{2}{3x^5}\right]dx$

$= \int 5x\,dx + \int \frac{2}{3x^5}\,dx$

$= \frac{5x^2}{2} + \frac{2}{3}\left(\frac{x^{-4}}{-4}\right) + c$

$= \frac{5x^2}{2} + \frac{2}{3}\left(\frac{-1}{4}\right)\left(\frac{1}{x^4}\right) + c.$

$= \frac{5x^2}{2} - \frac{2}{12}\left(\frac{1}{x^4}\right) + c$

$= \frac{5x^2}{2} - \frac{1}{6x^4} + c$

12- $\int\left[x^{-1/2} - 3x^{7/5} + \frac{1}{9}\right]dx$

$= \int x^{-1/2}\,dx - 3\int x^{7/5} + \int \frac{1}{9}\,dx$

$= 2x^{1/2} - 3\left(\frac{x^{7/5+1}}{7/5+1}\right) + \frac{1}{9}x + c$

$= 2x^{1/2} - \frac{5}{4}x^{12/5} + \frac{1}{9}x + c$

13. $\int\left[x^{-3} - 3x^{1/4} + 8x^2\right]dx$

$= \int x^{-3}\,dx - 3\int x^{1/4}\,dx + 8\int x^2\,dx$

$= \frac{x^{-2}}{-2} - \frac{3x^{5/4}}{5/4} + 8\frac{x^3}{3} + c$

$= -\frac{1}{2}x^{-2} - \frac{12x^{5/4}}{5} + \frac{8x^3}{3} + c$

14. $\int\left[\frac{10}{y^{3/4}} - \sqrt[3]{y} + \frac{4}{\sqrt{y}}\right]dy.$

$= 10\int y^{-3/4}\,dy - \int \sqrt[3]{y}\,dy + 4\int y^{-1/2}\,dy$

$= (4)(10)y^{1/4} - \frac{3}{4}y^{4/3} + 4y^{1/2}(2)$

$= 40y^{1/4} - \frac{3}{4}y^{4/3} + 8y^{1/2} + c$

$\frac{1}{3} + \frac{3}{3} = \frac{4}{3}$ $-\frac{3}{4} + \frac{4}{4} = \frac{1}{4}$ $3 \div \frac{5}{4}$ $\frac{3 \times 4}{5}$ $\frac{1}{4} + \frac{4}{4}$ $\frac{5}{4}$

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16. $\int (2+y^2)^2 dy$

$= \int (4 + 4y^2 + y^4) dy$

$= \int 4dy + \int 4y^2 dy + \int y^4 dy$

$= 4y + \frac{4y^3}{3} + \frac{y^5}{5} + c.$

$= 4y + \frac{4}{3}y^3 + \frac{1}{5}y^5 + c.$

17. $\int x^{1/3}(2-x)^2 dx.$

$= \int x^{1/3}(4 - 4x + x^2) dx.$

$= \int (4x^{1/3} - 4x^{4/3} + x^{7/3}) dx$

$= 4\int x^{1/3}dx - 4\int x^{4/3}dx + \int x^{7/3}dx$

$= \frac{4(3)x^{4/3}}{4} - \frac{4x^{7/3}}{7}(3) + \frac{3x^{10/3}}{10} + c$

$= 3x^{4/3} - \frac{12}{7}x^{7/3} + \frac{3}{10}x^{10/3} + c.$

18. $\int (1+x^2)(2-x) dx$

$= \int (2 - x + 2x^2 - x^3) dx.$

$= \int 2dx - \int x dx + \int 2x^2 dx - \int x^3 dx$

$= 2x - \frac{x^2}{2} + \frac{2x^3}{3} - \frac{x^4}{4} + c.$

19. $\int \frac{x^5 + 2x^2 - 1}{x^4} dx.$

$= \int \left(\frac{x^5}{x^4} + \frac{2x^2}{x^4} - \frac{1}{x^4}\right) dx$

$= \int x dx + 2\int x^{-2} dx - \int x^{-4} dx$

$= \frac{x^2}{2} + \frac{2}{x} + \frac{1}{3x^3} + c$

20. $\int \frac{1 - 2t^3}{t^3} dt.$

$= \int \left(\frac{1}{t^3} - \frac{2\cancel{t^3}}{\cancel{t^3}}\right) dt$

$= \int t^{-3} dt - 2\int dt$

$= \frac{t^{-2}}{-2} - 2t + c$

21. $\int \left[\frac{2}{x} + 3e^x\right] dx.$

$= 2\int \frac{1}{x} dx + 3\int e^x dx$

$= 2\ln x + 3e^x + c$

22. $\int \left[\frac{1}{2t} - \sqrt{2}e^t\right] dt.$

$= \frac{1}{2}\int \frac{1}{t} dt - \sqrt{2}\int e^t dt.$

$= \frac{1}{2}\ln t - \sqrt{2}e^t + c.$

23. $\int [3\sin x - 2\sec^2 x] dx$

$= 3\int \sin x - 2\int \sec^2 x dx$

$= 3(-\cos x) - 2(\tan x) + c$

$= -3\cos x - 2\tan x + c$

24. $\int [\csc^2 t - \sec t \tan t] dt$

$= \int \csc^2 t dt - \int \sec t \tan t dt$

$= -\cot t - \sec t + c$

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Check Your Answers by Differentiating

16. $\frac{d}{dy}\left[\frac{y^5}{5}+\frac{4y^3}{3}+4y+c\right]$

$= \frac{1}{5}\frac{d}{dy}(y^5)+\frac{4}{3}\frac{d}{dy}(y^3)+4\frac{d}{dy}(y)+\frac{d}{dy}(c).$

$= \frac{1}{5}(5y^4)+\frac{4}{3}(3y^2)+4+0$

$= y^4+4y^2+4 = (y^2+2)^2$

17. $\frac{d}{dx}\left[3x^{4/3}-\frac{12}{7}x^{7/3}+\frac{3}{10}x^{10/3}\right]$

$= 3\cdot\frac{4}{3}x^{1/3}-\frac{12}{7}\cdot\frac{7}{3}x^{4/3}+\frac{3}{10}\cdot\frac{10}{3}x^{7/3}$

$= 4x^{1/3}-4x^{4/3}+x^{7/3}$

$= x^{1/3}(4-4x+x^2).$

$= x^{1/3}(2-x)^2$

18. $\frac{d}{dx}\left[\frac{-x^4}{4}+\frac{2x^3}{3}-\frac{x^2}{2}+2x+C\right].$

$= -\frac{1}{4}\frac{d}{dx}x^4+\frac{2}{3}\frac{d}{dx}(x^3)-\frac{1}{2}\frac{d}{dx}(x^2)+2\frac{d}{dx}(x)+\frac{d}{dx}(c)$

$= \frac{-1}{4}(4x^3)+\frac{2}{3}(3x^2)-\frac{1}{2}(2x)+2(1)+0$

$= -x^3+2x^2-x+2$

$= x^2(-x+2)+1(-x+2)$

$= (x^2+1)(-x+2)$

19. $\frac{d}{dx}\left[\frac{1}{2}x^2-\frac{2}{x}+\frac{1}{3x^3}\right]$

$= \frac{1}{2}\frac{d}{dx}(x^2)-2\frac{d}{dx}(x^{-1})+\frac{1}{3}\frac{d}{dx}(x^{-3})$

$= \frac{1}{2}(2x)-2(-x^{-2})+\frac{1}{3}(-3x^{-4})$

$= x+\frac{2}{x^2}-\frac{1}{x^4}$

$= \frac{x^7+2x^4-x^2}{x^6}$

$= \frac{x^2(x^5+2x^2-1)}{x^6}$

$= \frac{x^5+2x^2-1}{x^4}$

20. $\frac{d}{dx}\left[-\frac{1}{2}t^{-2}-2t\right]$

$= -(-2)\cdot\frac{1}{2}t^{-2-1}-2$

$= t^{-3}-2$

$= \frac{1}{t^3}-2 = \frac{1-2t^3}{t^3}$

21. $\frac{d}{dx}(2\ln x+3e^x+C)$

$= 2\frac{d}{dx}(\ln x)+3\frac{d}{dx}(e^x)+\frac{d}{dx}(c).$

$= \frac{2}{x}+3e^x+0$

$= \frac{2}{x}+3e^x$

22. $\frac{d}{dx}\left(\frac{\ln t}{2}-\sqrt{2}e^t+c\right).$

$= \frac{1}{2}\frac{d}{dt}(\ln t)-\sqrt{2}\frac{d}{dt}(e^t)+\frac{d}{dt}(c).$

$= \frac{1}{2t}-\sqrt{2}e^t+0 \Rightarrow \frac{1}{2t}-\sqrt{2}e^t$

$+ (-2)\left(-\frac{1}{2}\right)t^{-3}$

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23- $\frac{d}{dx}\left[-3\cos x - 2\tan x + c\right]$

$= -3\frac{d}{dx}(\cos x) - 2\frac{d}{dx}(\tan x) + \frac{d}{dx}(c).$

$= -3(-\sin x) - 2(\sec^2 x) + 0$

$= 3\sin x - 2\sec^2 x$

24- $\frac{d}{dx}\left[-\cot t - \sec t + c\right].$

$= -\frac{d}{dt}(\cot t) - \frac{d}{dt}(\sec t) + \frac{d}{dt}(c).$

$= -(-\csc^2 t) - (\sec t \tan t) + 0$

$= \csc^2 t - \sec t \tan t$.

25- $\int \sec x(\sec x + \tan x)dx.$

$= \int \sec^2 x\,dx + \int \tan x \sec x\,dx.$

$= \tan x + \sec x + c.$

26- $\int \csc x(\sin x + \cot x)dx.$

$= \int \frac{1}{\sin x}(\sin x)dx + \int \frac{1}{\sin x}\cot x\,dx.$

$= \int 1\,dx + \int \cot x \csc x\,dx$

$= x + (-\csc x) + c$

$= x - \csc x + c.$

$\frac{d}{dx}(x - \csc x + c)$

$1 - \frac{d}{dx}(\csc x) + \frac{d}{dx}(c)$

$= 1 - (-\cot x \csc x) + 0 = 1 + \csc x \cot x$

$= \csc x(\sin x + \cot x)$

27- $\int \frac{\sec\theta}{\cos\theta}d\theta.$

$= \int \sec\theta \cdot \frac{1}{\cos\theta}d\theta.$

$= \int \sec\theta\,\sec\theta\,d\theta$

$= \int \sec^2\theta\,d\theta.$

$= \tan\theta + c.$

$\frac{d}{d\theta}\left[\tan\theta + c\right].$

$= \sec^2\theta + 0.$

$= \sec\theta \cdot \sec\theta.$

$= \sec\theta \cdot \frac{1}{\cos\theta} = \frac{\sec\theta}{\cos\theta}.$

28- $\int \frac{dy}{\csc y}$

$= \int \frac{1}{\csc y}dy.$

$= \int \sin y\,dy$

$= -\cos y + c$

$\frac{d}{dy}\left[-\cos y + c\right]$

$= -(-\sin y) + 0.$

$= \sin y = \frac{1}{\csc y}$

29- $\int \frac{\sin x}{\cos^2 x}dx$

$= \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}dx$

$= \int \tan x \sec x\,dx = \sec x + c$

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30. $\int \left[\phi + \frac{2}{\sin^2\phi} \right] d\phi$

$= \int \phi \, d\phi + 2\int \frac{1}{\sin^2\phi} d\phi$

$= \frac{1}{2}\phi^2 - 2\cot\phi + c$

31. $\int \left[1 + \sin^2\theta \csc\theta \right] d\theta.$

$= \int 1 \, d\theta + \int \sin^2\theta \frac{1}{\sin\theta} d\theta$

$= \theta + (-\cos\theta) + c$

$= \theta - \cos\theta + c$

32. $\int \frac{\sec x + \cos x}{2\cos x} dx.$

$= \int \frac{\sec x}{2\cos x} dx + \int \frac{\cos x}{2\cos x} dx$

$= \frac{1}{2}\int \sec^2 x \, dx + \frac{1}{2}\int dx$

$= \frac{1}{2}\tan x + \frac{1}{2}x + C.$

33. $\int \left[\frac{1}{2\sqrt{1-x^2}} - \frac{3}{1+x^2} \right] dx.$

$= \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}} dx - 3\int \frac{1}{1+x^2} dx.$

$= \frac{1}{2}\sin^{-1}x - 3\tan^{-1}x + c$

34. $\int \left[\frac{4}{x\sqrt{x^2-1}} + \frac{1+x+x^3}{1+x^2} \right] dx.$

$= 4\int \frac{1}{x\sqrt{x^2-1}} dx + \int \frac{1}{1+x^2} dx + \int \frac{x\cancel{(1+x^2)}}{\cancel{1+x^2}}$

$= 4\sec^{-1}x + \tan^{-1}x + \frac{x^2}{2} + C$

35. $\int \frac{1}{1+\sin x} dx.$

$= \int \frac{1}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x} dx$

$= \int \frac{1-\sin x}{1-\sin^2 x} dx.$

$= \int \frac{1-\sin x}{\cos^2 x} dx.$

$= \int \frac{1}{\cos^2 x} dx - \int \frac{\sin x}{\cos^2 x} dx.$

$= \int \sec^2 x \, dx - \int \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} dx.$

$= \tan x - \int \sec x \tan x \, dx$

$= \tan x - \sec x + c$

36. $\int \frac{1}{1+\cos 2x} dx \; ; \cos 2x = 2\cos^2 x -$

$= \int \frac{1}{1+2\cos^2 x - 1} dx = \int \frac{1}{2\cos^2 x} dx$

$= \frac{1}{2}\int \frac{1}{\cos^2 x} dx = \frac{1}{2}\int \sec^2 x \, dx$

$= \frac{1}{2}\tan x + c.$

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43. a) $\frac{dy}{dx} = 3\sqrt{x}$, $y(1) = 2$

$y(x) = \int x^{1/3} dx$

$= \frac{3}{4} x^{4/3} + C$.

$y(1) = \frac{3}{4} + C = 2$

$C = 2 - \frac{3}{4} = \frac{8-3}{4} = \frac{5}{4}$

$y(x) = \frac{3}{4} x^{4/3} + \frac{5}{4}$

b) $\frac{dy}{dt} = \sin t + 1$, $y\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$y(t) = \int (\sin t + 1) dt$

$= -\cos t + t + C$

$y\left(\frac{\pi}{3}\right) = -\frac{1}{2} + \frac{\pi}{3} + C = \frac{1}{2}$

$C = 1 - \frac{\pi}{3}$

$y\left(\frac{\pi t}{3}\right) = -\cos t + t + 1 - \frac{\pi}{3}$

c) $\frac{dy}{dx} = \frac{x+1}{\sqrt{x}}$, $y(1) = 0$

$y(x) = \int \frac{x+1}{\sqrt{x}} dx$.

$= \int (x^{1/2} + x^{-1/2}) dx$

$= \frac{2}{3} x^{3/2} + 2x^{1/2} + C$

$y(1) = 0 = \frac{8}{3} + C$, $C = -\frac{8}{3}$

$y(x) = \frac{2}{3} x^{3/2} + 2x^{1/2} - \frac{8}{3}$

44. a) $\frac{dy}{dx} = \frac{1}{(2x)^3}$, $y(1) = 0$

$y(x) = \int \left(\frac{1}{8} x^{-3}\right) dx$.

$= -\frac{1}{16} x^{-2} + C$.

$y(1) = 0 = -\frac{1}{16} + C \Rightarrow C = \frac{1}{16}$.

$y(x) = -\frac{1}{16} x^{-2} + \frac{1}{16}$.

b) $\frac{dy}{dt} = \sec^2 t - \sin t$, $y\left(\frac{\pi}{4}\right) = 1$

$y(x) = \tan t + \cos t + C$

$y\left(\frac{\pi}{4}\right) = 1 = 1 + \frac{\sqrt{2}}{2} + C$

$C = -\frac{\sqrt{2}}{2}$, $y(t) = \tan t + \cos t - \frac{\sqrt{2}}{2}$

c) $\frac{dy}{dx} = x^2\sqrt{x^3}$, $y(0) = 0$

$y(x) = \int x^{7/2} dx = \frac{2}{9} x^{9/2} + C$

$y(0) = 0$, $C = 0$

$y(x) = \frac{2}{9} x^{9/2}$.

Q45. a) $\frac{dy}{dx} = 4e^x$, $y(0) = 1$

$= 4e^x + C$

$y(0) = 1 = 4 + C \Rightarrow C = -4 + 1$

$C = -3$

$= 4e^x - 3$

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b. $\frac{dy}{dt} = \frac{1}{t}$, $y(-1) = 5$

$y(t) = \int t^{-1} dt = \ln t + C$

$y(-1) = 5 = \cancel{\ln t + C}$

$\Rightarrow C = 5$

$y(t) = \ln t + 5$

46-a) $\frac{dy}{dt} = \frac{3}{\sqrt{1-t^2}}$, $y\left(\frac{\sqrt{3}}{2}\right) = 0$

$y = \int \frac{3}{\sqrt{1-t^2}} dt = 3\sin^{-1} t + C$

$y\left(\frac{\sqrt{3}}{2}\right) = 0 = \pi + C \Rightarrow C = -\pi.$

$y = 3\sin^{-1} t - \pi$

b) $\frac{dy}{dx} = \frac{x^2-1}{x^2+1}$, $y(1) = \frac{\pi}{2}$

$y = \int \left(1 - \frac{2}{x^2+1}\right) dx$

$= x - 2\tan^{-1} x + C$

$y(1) = \frac{\pi}{2} = 1 - 2\frac{\pi}{4} + C$

$C = \pi - 1$

$y = x - 2\tan^{-1} x + \pi - 1$

47- $v(t) = 32t$, $s(0) = 20$

$v(t) = s'(t)$.

$= 16t^2 + c$

$s(0) = 16(0)^2 + c = 20 \Rightarrow c = 20$

$s(t) = 16t^2 + 20$

48- $v(t) = \cos t$; $s(0) = 2$

$= \sin t + c$

$s(0) = \sin(0) + c = 2$

$\Rightarrow c = 2$.

$s(t) = \sin t + 2$

49- $v(t) = 3\sqrt{t}$; $s(4) = 1$

$\int 3t^{1/2} dt = 3 \cdot \frac{2}{3} \cdot t^{3/2} = 2t\sqrt{t} + c$

$s(4) = 2(4)\sqrt{4} + c = 1$

$8 \cdot 2 + c = 1 \Rightarrow c = 1 - 16$

$c = -15$

$s(t) = 2t\sqrt{t} - 15$

50- $v(t) = 3e^t$; $s(1) = 0$.

$= 3e^t + c$

$s(1) = 3e^1 + c = 0$

$c = -3e$

$s(t) = 3e^t - 3e$

Exercise 5.3

1- a) $\int 2x(x^2+1)^{23} dx$; $u = x^2 + 1$

$du = 2x\,dx$

$\int u^{23} du = \frac{u^{24}}{24} + c$

$= \frac{(x^2+1)^{24}}{24} + c$.

b) $\int \cos^3 x \sin x\, dx$; $u = \cos x$

$du = -\sin x\, dx$

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$= -\int u^3 du$

$= -\frac{u^4}{4} + c.$

$= -\frac{\cos^4 x}{4} + c.$

2-a) $\int \frac{1}{\sqrt{x}} \sin\sqrt{x}\, dx$; $u = \sqrt{x}$

$du = \frac{1}{2} x^{-1/2} \Rightarrow \frac{1}{2\sqrt{x}} dx$

$2 du = \frac{1}{\sqrt{x}} dx.$

$= \int 2 \sin u du$

$= -2\cos u + c$

$= -2\cos\sqrt{x} + c$

b) $\int \frac{3x dx}{\sqrt{4x^2+5}}$; $u = 4x^2 + 5$

$du = 8x dx$. To get,

$\frac{3 du}{8} = \frac{3 \times 8x dx}{8}$

$\frac{3}{8} du = 3x dx$

$\int \frac{3 du}{8\sqrt{u}}$

$= \frac{3}{8} \int u^{-1/2} du \Rightarrow \frac{3}{8} \cdot 2\sqrt{u} + c$

$= \frac{6}{8} \sqrt{u} + c$

$= \frac{6\sqrt{4x^2+5}}{8} + c$

3- a) $\int \sec^2(4x+1) dx$; $u = 4x+1$

$du = 4dx$

$\frac{du}{4} = dx$

$= \int \frac{\sec^2 u \, du}{4}$

$= \frac{\tan u}{4} + c$

$= \frac{\tan(4x+1)}{4} + c$

b) $\int y\sqrt{1+2y^2} dy$; $u = 1+2y^2$

$du = 4y dy.$

$\frac{du}{4} = y dy.$

$= \int \frac{\sqrt{u}}{4} du.$

$= \frac{1}{4} \cdot \frac{2(u^{3/2})}{3} + c.$

$= \frac{u^{3/2}}{6} + c.$

$= \frac{(1+2y^2)^{3/2}}{6} + c,$

4- a) $\int \sqrt{\sin\pi\theta} \cos\pi\theta \, d\theta$; $u = \sin\pi\theta$

$du = \pi\cos\pi\theta d\theta.$

$\int \frac{\sqrt{u}}{\pi} du = \frac{1}{\pi} \cdot \frac{2(u^{3/2})}{3} + c$

$= \frac{2}{3\pi} (\sin\pi\theta)^{3/2} + c.$

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b) $\int (2x+7)(x^2+7x+3)^{4/5} dx$

$u = x^2+7x+3$

$du = 2x+7$

$= \int u^{4/5} du$

$= \frac{5}{9} u^{9/5} + c$

$= \frac{5}{9}(x^2+7x+3)^{9/5} + c$

5-a) $\int \cot x \csc^2 x dx$; $u = \cot x$

$du = -\csc^2 x$.

$= -\int u du$

$= -\frac{u^2}{2} + c$

$= -\frac{\cot^2 x}{2} + c$

b) $\int (1+\sin t)^9 \cos t dt$; $u = 1+\sin t$

$du = \cos t$.

$= \int u^9 du$

$= \frac{u^{10}}{10} + c = \frac{(1+\sin t)^{10}}{10} + c$

6-a) $\int \cos 2x dx$; $u = 2x$

$du = 2 dx$ $\frac{du}{2} = dx$.

$= \int \frac{\cos \cancel{2x} u du}{2}$ 2

$= \frac{\sin u}{2} + c$.

$= \frac{\sin(2x) + c}{2}$

b) $\int x \sec^2 x^2 dx$; $u = x^2$

$du = 2x dx$ $\frac{du}{2} = x dx$.

$= \int \frac{\sec^2 u}{2} du$

$= \frac{\tan u}{2} + c$.

$= \frac{\tan x^2}{2} + c$.

7- a) $\int x^2 \sqrt{1+x} dx$; $u = 1+x$ $\Rightarrow x = u-1$

$du = 1 dx$

$= \int (u-1)^2 \sqrt{u} du$

$= \int (u^2 - 2u + 1) \cdot u^{1/2} du$.

$= \int (u^{5/2} - 2u^{3/2} + u^{1/2}) du$

$= \frac{2u^{7/2}}{7} - \frac{4u^{5/2}}{5} + \frac{2u^{3/2}}{3} + c$.

$= \frac{2(x+1)^{7/2}}{7} - \frac{4(x+1)^{5/2}}{5} + \frac{2(x+1)^{3/2}}{3} + c$

b) $\int [\csc(\sin x)]^2 \cos x dx$; $u = \sin x$

$du = \cos x dx$

$= \int [\csc(u)]^2 du$

$= -\cot u + c$

$= -\cot(\sin x) + c$

8-a) $\int \sin(x - \pi) dx$; $u = x - \pi$

$du = dx$

$= \int \sin(u) du$

$= -\cos(u) + c = -\cos(x - \pi) + c$

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b) $\int \frac{5x^4}{(x^5+1)^2}dx$; $u = x^5+1$

$du = 5x^4\,dx$.

$= \int \frac{du}{(u)^2} = \frac{-1}{u} + c = \frac{-1}{(x^5+1)} + c$.

9-a) $\int \frac{dx}{x\ln x}$; $u = \ln x$.

$du = \frac{1}{x}dx$

$= \int \frac{du}{u}$

$= \ln|u| + c$

$= \ln|\ln(x)| + c$

b) $\int e^{-5x}dx$; $u = -5x$

$du = -5dx \Rightarrow \frac{du}{-5} = dx$

$= \int \frac{e^u du}{-5} = -\int \frac{e^u}{5} du$

$= \frac{-e^u}{5} + c$

$= \frac{-e^{-5x}}{5} + c$

10-a) $\int \frac{\sin 3\theta}{1+\cos 3\theta} d\theta$; $u = 1+\cos 3\theta$.

$du = -3\sin(3\theta)d\theta \Rightarrow \frac{-du}{3} = \sin 3\theta d\theta$.

$= \int \frac{-du}{3u} = \frac{-1}{3}\ln|u| + c$

$= -\frac{1}{3}\ln(1+\cos 3\theta) + c$

b) $\int \frac{e^x}{1+e^x}dx$; $u = 1+e^x$

$du = e^x\,dx$.

$= \int \frac{du}{u} = \ln|u| + c$

$= \ln(1+e^x) + c$

11-a) $\int \frac{x^2dx}{1+x^6}$; $u = x^3$

$du = 3x^2dx \Rightarrow \frac{du}{3} = x^2dx$.

$= \int \frac{du}{3(1+u^2)}$

$= \frac{1}{3}\int \frac{1}{1+u^2}du$

$= \frac{1}{3}\tan^{-1}u + c$

$= \frac{1}{3}\tan^{-1}(x^3) + c$

b) $\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$; $u = \ln x$.

$du = \frac{1}{x}dx$.

$= \int \frac{1}{\sqrt{1-u^2}}du$

$= \sin^{-1}u + c$

$= \sin^{-1}(\ln x) + c$.

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12- a) $\int \frac{dx}{x\sqrt{9x^2-1}}$; $u = 3x$

$du = 3dx \Rightarrow \frac{du}{3} = dx$.

$= \int \frac{du/3}{\frac{u}{3}\sqrt{9(\frac{u}{3})^2-1}}$

$= \int \frac{du}{u\sqrt{u^2-1}}$

= [illegible]

$= \sec^{-1}|u| + c$

$= \sec^{-1}|3x| + c$

b) $\int \frac{dx}{\sqrt{x}(1+x)}$; $u = \sqrt{x} \Rightarrow u^2 = x$

$du = \frac{dx}{2\sqrt{x}} \Rightarrow 2du = \frac{dx}{\sqrt{x}}$

$= \int \frac{2du}{u^2+1} = 2\int \frac{1}{u^2+1} du$

$= 2\tan^{-1} u + c$

$= 2\tan^{-1}\sqrt{x} + c$

15- $\int (4x-3)^9 dx$

$u = 4x-3$

$du = 4dx$

$\frac{du}{4} = dx$

$\int \frac{u^9 du}{4}$

$= \frac{1}{4} \cdot \frac{1}{10} u^{10} + c$

$= \frac{1}{40}(4x+3)^{10} + c$

16- $\int x^3\sqrt{5+x^4}\, dx$

$u = 5+x^4$

$du = 4x^3 dx$.

$\frac{du}{4} = x^3 dx$.

$= \int \frac{\sqrt{u}\, du}{4}$.

$= \frac{1}{4} \cdot \frac{2}{3} u^{3/2} + c$

$= \frac{1}{6} u^{3/2} + c = \frac{1}{6}(5+x^4)^{3/2} + c$

17- $\int \sin 7x\, dx$.

$u = 7x$

$du = 7dx. \Rightarrow \frac{du}{7} = dx$.

$= \int \frac{\sin u\, du}{7}$

$= \frac{1}{7}(-\cos u) + c$

$= -\frac{1}{7}\cos(7x) + c$

18- $\int \cos\frac{x}{3}\, dx$.

$u = \frac{x}{3}$, $du = \frac{1}{3}dx$

$= \int 3\cos u\, du \Rightarrow 3du = dx$

$= 3\int \cos u\, du$

$= 3\sin u + c$

$= 3\sin\frac{x}{3} + c$.

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19 - $\int \sec 4x \tan 4x \, dx$.

$u = 4x$

$du = 4\,dx \Rightarrow \frac{du}{4} = dx$

$= \int \frac{\sec u \tan u \, du}{4}$

$= \frac{1}{4} \sec u + c$

$= \frac{1}{4} \sec 4x + c$

20 - $\int \sec^2 5x \, dx$

$u = 5x$

$du = 5\,dx \Rightarrow \frac{du}{5} = dx$

$= \int \frac{\sec^2 u \, du}{5}$

$= \frac{1}{5} \tan u + c$

$= \frac{1}{5} \tan 5x + c$

21 - $\int e^{2x} dx$.

$u = 2x$

$du = 2\,dx \Rightarrow \frac{du}{2} = dx$,

$= \int \frac{e^{u} du}{2}$

$= \frac{1}{2} e^{u} + c$

$= \frac{1}{2} e^{2x} + c$

22 - $\int \frac{dx}{2x}$

$= \frac{1}{2} \int \frac{dx}{x}$

$= \frac{1}{2} \ln |x| + c$.

23 - $\int \frac{dx}{\sqrt{1 - 4x^2}}$

$u = 2x$

$du = 2\,dx \Rightarrow \frac{du}{2} = dx$.

$= \int \frac{du}{2\sqrt{1-u^2}}$

$= \frac{1}{2} \int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2} \sin^{-1} u + c$

$= \frac{1}{2} \sin^{-1} 2x + c$.

24 - $\int \frac{dx}{1+16x^2}$

$u = 4x$.

$du = 4\,dx \Rightarrow \frac{du}{4} = dx$

$= \int \frac{du}{4(1+u^2)}$

$= \frac{1}{4} \tan^{-1} u + c$

$= \frac{1}{4} \tan^{-1} 4x + c$

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25- $\int t\sqrt{7t^2+12}\,dt$.

Let $u = 7t^2+12$

$du = 14t\,dt \Rightarrow \frac{du}{14} = t\,dt$

$= \int \frac{du}{14}\sqrt{u}$

$= \frac{1}{14} \cdot \frac{2}{3} u^{3/2} + c$

$= \frac{1}{7} u^{3/2} + c$

$= \frac{1}{7} (7t^2+12)^{3/2} + c$

26- $\int \frac{x}{\sqrt{4-5x^2}}\,dx$

$u = 4-5x^2$

$du = -10x\,dx \Rightarrow \frac{du}{-10} = x\,dx$.

$= -\int \frac{du}{10\sqrt{u}}$

$= -\frac{1}{10} \int \frac{du}{\sqrt{u}}$

$= -\frac{1}{10} \cdot 2u^{1/2} + c$

$= \frac{-1}{5} \sqrt{u} + c$

$= \frac{-1}{5} \sqrt{(4-5x^2)} + c$

27- $\int \frac{6}{(1-2x)^3}\,dx$.

$1-2x = u$

$du = -2\,dx \Rightarrow \frac{du}{-2} = dx$.

$= \int \frac{6}{(-2)u^3}\,du$

$= \int \frac{-3}{u^3}\,du$

$= -3\,\frac{u^{-2}}{-2} + c$

$= \frac{+3}{2} u^{-2} + c$

$= \frac{3}{2} (1-2x)^{-2} + c \Rightarrow \frac{3(1)}{2(1-2x)^2} +$

28- $\int \frac{x^2+1}{\sqrt{x^3+3x}}\,dx$

$u = x^3+3x$

$du = 3x^2+3\,dx$.

$du = 3(x^2+\,dx)$

$\frac{du}{3} = x^2+1\,dx$.

$= \int \frac{du}{3\sqrt{u}}$

$= \frac{1}{3} \cdot 2u^{1/2} + c$

$= \frac{2}{3}\sqrt{u} + c$

$= \frac{2}{3}\sqrt{x^3+3x} + c$

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29. $\int \frac{x^3}{(5x^4+2)^3} dx$.

$u = 5x^4 + 2$

$du = 20x^3 dx$.

$\frac{du}{20} = x^3 dx$.

$= \int \frac{du}{20u^3}$

$= \frac{1}{20} \int u^{-3} du$

$= \frac{1}{20} \cdot \frac{u^{-2}}{-2} + c$.

$= \frac{1}{-40} u^{-2} + c$

$= \frac{1}{-40} (5x^4+2)^{-2} + c$

30. $\int \frac{\sin(1/x)}{3x^2} dx$.

$u = 1/x = x^{-1}$

$du = \frac{-1}{x^2} dx \Rightarrow$

$= \int \frac{\sin u}{3} du$

$= \frac{-1}{3}(-\cos u) + c$

$= \frac{1}{3} \cos\left(\frac{1}{x}\right) + c$

31. $\int e^{\sin x} \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$.

$= \int e^u du$.

$= e^u + c$

$= e^{\sin x} + c$.

32. $\int x^3 e^{x^4} dx$.

$u = x^4$

$du = 4x^3 dx \Rightarrow \frac{du}{4} = x^3 dx$.

$= \int \frac{e^u du}{4}$

$= \frac{1}{4} e^u + c$

$= \frac{1}{4} e^{x^4} + c$.

33. $\int x^2 e^{-2x^3} dx$

$u = -2x^3$

$du = -6x^2 dx$.

$\frac{du}{-6} = x^2 dx$

$= \int \frac{e^u du}{-6}$

$= -\frac{1}{6} \int e^u du = \frac{-1}{6} e^u + c$

$= \frac{-1}{6} e^{-2x^3} + c$

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34- $\int \frac{e^x+e^{-x}}{e^x-e^{-x}} dx$.

$u = e^x - e^{-x}$

$du = e^x + e^x dx$

$= \int \frac{du}{u}$

$= \ln u + c$.

$= \ln|e^x - e^{-x}| + c$

35- $\int \frac{e^x}{1+e^{2x}} dx$.

$u = \cancel{1+} e^{\cancel{2}x}$

$du = e^x du$

$= \int \frac{du}{1+u^2}$

$= \tan^{-1} u + c$

$= \tan^{-1} e^x + c$

36- $\int \frac{t}{t^4+1} dt$

$u = t^2$

$du = 2t dt \Rightarrow \frac{du}{2} = t\, dt$

$= \int \frac{du}{2(1+u^2)} = \frac{1}{2} \int \frac{du}{1+u^2}$

$= \frac{1}{2} \tan^{-1} u + c$

$= \frac{1}{2} \tan^{-1} t^2 + c$

37- $\int \frac{\sin(5/x)}{x^2} dx$.

$u = 5/x = 5x^{-1}$

$du = -5x^{-2} dx$

$\frac{du}{-5} = \frac{1}{x^2} dx$.

$= -\frac{1}{5} \int \sin u\, du$

$= -\frac{1}{5}(-\cos u) + c$

$= \frac{+1}{5} \cos \frac{5}{x} + c$.

38- $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx$.

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx. \Rightarrow 2du = \frac{1}{\sqrt{x}} dx$

$= 2\int \sec^2 u\, du$

$= 2 \tan u + c$

$= 2 \tan \sqrt{x} + c$

39- $\int \cos^4 3t \sin 3t\, dt$.

$u = \cos 3t$

$du = -3 \sin 3t\, dt$.

$\frac{du}{-3} = \sin 3t\, dt$

$= \int \frac{u^4}{-3} du = \frac{-1}{3} \int u^4 du$

$= \frac{-1}{3} \frac{u^5}{5} + c = \frac{-1}{15} (\cos 3t)^5 + c$

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40- $\int \cos 2t \sin^5 2t\,dt$.

$u = \sin 2t$

$du = 2\cos 2t\,dt$

$\frac{du}{2} = \cos 2t\,dt$.

$= \int \frac{u^5\,du}{2} = \frac{1}{2}\int u^5\,du$

$= \frac{1}{2}\cdot\frac{u^6}{6} + c$.

$= \frac{1}{12}\sin^6 2t + c$.

41- $\int x\sec^2(x^2)\,dx$.

$u = x^2$

$du = 2x\,dx \Rightarrow du/2 = x\,dx$.

$= \int \frac{\sec^2 u\,du}{2}$

$= \frac{1}{2}\int \sec^2 u\,du = \frac{1}{2}\tan u + c$

$= \frac{1}{2}\tan x^2 + c$

42- $\int \frac{\cos 4\theta}{(1+2\sin 4\theta)^4}\,d\theta$

$u = 1+2\sin 4\theta$

$du = 8\cos 4\theta\,d\theta \Rightarrow \frac{du}{8} = \cos 4\theta\,d\theta$.

$= \int \frac{du}{8(u)^4} = \frac{1}{8}\int \frac{du}{u^4} = \frac{1}{8}\int u^{-4}\,du$

$= \frac{1}{8}\cdot\frac{u^{-3}}{-3} + c = -\frac{1}{24}(1+2\sin 4\theta)^{-3} + c$

43- $\int \cos 4\theta\sqrt{2-\sin 4\theta}\,d\theta$.

$u = 2-\sin 4\theta$

$du = -4\cos 4\theta\,d\theta$.

$\Rightarrow du/-4 = \cos 4\theta\,d\theta$

$= \int \frac{\sqrt{u}\,du}{-4}$

$= -\frac{1}{4}\int \sqrt{u}\,du = -\frac{1}{4}\int u^{1/2}\,du$

$= -\frac{1}{4}\cdot\frac{2}{3}u^{3/2} + c$

$= -\frac{1}{6}u^{3/2} + c = -\frac{1}{6}(2-\sin 4\theta)^{3/2} + c$

44- $\int \tan^3 5x\sec^2 5x\,dx$

$u = \tan 5x$

$du = 5\sec^2 5x\,dx$.

$\frac{du}{5} = \sec^2 5x\,dx$.

$= \int \frac{u^3\,du}{5} = \frac{1}{5}\int u^3\,du$

$= \frac{1}{5}\cdot\frac{u^4}{4} + c$

$= \frac{1}{20}u^4 + c$

$= \frac{1}{20}\tan^4 5x + c$

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45- $\displaystyle\int \frac{\sec^2 x\,dx}{\sqrt{1-\tan^2 x}}$

$u = \tan x$

$du = \sec^2 x\,dx$,

$= \displaystyle\int \frac{du}{\sqrt{1-u^2}}$

$= \sin^{-1} u + c$

$= \sin^{-1}(\tan x) + c$

46- $\displaystyle\int \frac{\sin\theta}{\cos^2\theta + 1}\,d\theta$.

$u = \cos\theta$

$du = -\sin\theta\,d\theta$.

$= \displaystyle\int \frac{-du}{1+u^2} = -\int \frac{du}{1+u^2}$

$= -\tan^{-1} u + c$

$= -\tan^{-1}(\cos\theta) + c$

47- $\displaystyle\int \sec^3 2x \tan 2x\,dx$

$u = \sec 2x$

$du = 2\sec 2x \tan 2x\,dx$

$\dfrac{du}{2} = 2\sec 2x \tan 2x\,dx$.

$= \dfrac{1}{2}\displaystyle\int u^2\,du$

$= \dfrac{1}{2} \cdot \dfrac{u^3}{3} + c = \dfrac{1}{6} u^3 + c$

$= \dfrac{1}{6} \sec^3 2x + c$

48. $\displaystyle\int [\sin(\sin\theta)] \cos\theta\,d\theta$.

$u = \sin\theta$

$du = \cos\theta\,d\theta$.

$= \displaystyle\int [\sin u]\,du$.

$= -\cos u + c$.

$= -\cos(\sin\theta) + c$

49- $\displaystyle\int \frac{dx}{e^x} = \int e^{-x}\,dx$.

$u = -x$

$du = -dx \Rightarrow -du = dx$.

$= -\displaystyle\int e^u\,du$

$= -e^u + c$

$= -e^{-x} + c$

$= -1/e^x + c$

50- $\displaystyle\int \sqrt{e^x}\,dx = \int e^{x/2}\,dx$.

$u = x/2$

$du = dx/2 \Rightarrow 2du = dx$

$= 2\displaystyle\int e^u\,du$

$= 2e^u + c$

$= 2e^{x/2} + c$

$= 2\sqrt{e^x} + c$

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51- $\int \frac{dx}{\sqrt{x}\, e^{(2\sqrt{x})}}$

$u = 2\sqrt{x}$

$du = \frac{1}{\sqrt{x}} dx.$

$= \int \frac{du}{e^u}$

$= \int e^{-u} du$

$= -e^{-u} + c$

$= -\frac{1}{e^u} + c = \frac{-1}{e^{2\sqrt{x}}} + c$

52- $\int \frac{e^{\sqrt{2y+1}}}{\sqrt{2y+1}} dy$

$u = \sqrt{2y+1}$

$du = \frac{1}{\sqrt{2y+1}} dy.$

$= \int e^{u} du$

$= e^u + c$

$= e^{\sqrt{2y+1}} + c$

53- $\int \frac{y}{\sqrt{2y+1}} dy$; $u = \sqrt{2y+1} \Rightarrow y = \frac{u^2-1}{2}$

$du = \frac{1}{\sqrt{2y+1}} dy$

$= \int \left(\frac{1}{2} u^2 - 1\right) du$

$= \frac{1}{2} \int u^2 - 1\, du = \frac{1}{2}\left(\frac{u^3}{3} - u\right) + c$

$= \frac{(2y+1)^{3/2}}{6} - \frac{(2y+1)^{1/2}}{2} + c$

54- $\int x\sqrt{4-x}\, dx.$

$4 - x = u \Rightarrow x = 4 - u.$

$du = -dx \Rightarrow -du = dx.$

$= \int -(4-u)\sqrt{u}\, du$

$= \int u^{3/2} - 4u^{1/2} du$

$= \frac{2}{5} u^{5/2} - \frac{4u^{3/2}}{3}(2) + c$

$= \frac{2}{5}(4-x)^{5/2} - \frac{8}{3}(4-x)^{3/2} + c$

55- $\int \sin^3 2\theta\, d\theta$

$= \int \sin 2\theta \sin^2 2\theta\, d\theta$

$= \int \sin 2\theta (1 - \cos^2 2\theta)\, d\theta.$

$u = \cos 2\theta.$

$du = -2\sin 2\theta$, $-\frac{du}{2} = \sin 2\theta.$

$= -\int \frac{(1-u^2)\, du}{2}$

$= \frac{-1}{2}\int du + \frac{1}{2}\int u^2 du$

$= \frac{-1}{2} u + \frac{1}{2} \frac{u^3}{3} + c$

$= -\frac{\cos 2\theta}{2} + \frac{(\cos 2\theta)^3}{6} + c$

56- $\int \sec^4 3\theta\, d\theta.$

$u = \tan 3\theta$, $du = 3\sec^2 3\theta\, d\theta$

$= \int \sec^2 3\theta . \sec^2 3\theta\, d\theta.$

$= \int (1 + \tan^2 3\theta) \sec^2 3\theta\, d\theta.$

$= \frac{1}{3}\int (1 + u^2)\, du = \frac{1}{3} u + \frac{u^3}{9} + c$

$= \frac{1}{3}\tan 3\theta + \frac{\tan^3 3\theta}{9} + c$

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Exercise 6.1

1- $A = \int_a^b [f(x) - g(x)]dx$.

$A = \int_{-1}^{2} [(x^2+1) - (x)]dx$

$= \int_{-1}^{2} (x^2+1-x)dx$

$= \int_{-1}^{2} x^2dx + \int_{-1}^{2} 1dx - \int_{-1}^{2} xdx$.

$= \frac{x^3}{3}\Big|_{-1}^{2} + x\Big|_{-1}^{2} - \frac{x^2}{2}\Big|_{-1}^{2}$

$= \left[\frac{2^3-(-1)^3}{3}\right] + \left[\frac{2+1}{1}\right] - \left[\frac{2^2-(-1)^2}{2}\right]$.

$= \left[\frac{8+1}{3}\right] + \left[3\right] - \left[\frac{4-1}{2}\right]$.

$= \cancel{\frac{9}{3}}3 + 3 - \frac{3}{2}$

$= 6 - \frac{3}{2} = \frac{12-3}{2} = \frac{9}{2}$.

2- $A = \int_0^4 \left[\sqrt{x} - \left(-\frac{1}{4}x\right)\right]dx$.

$= \int_0^4 x^{1/2}dx + \int_0^4 \frac{1}{4}xdx$.

$= \frac{x^{3/2}}{3/2}\Big|_0^4 + \frac{1}{4}\cdot\frac{x^2}{2}\Big|_0^4$

$= \frac{2x^{3/2}}{3}\Big|_0^4 + \frac{x^2}{8}\Big|_0^4$

$= \frac{2(4)^{3/2} - 2(0)^{3/2}}{3} + \frac{(4)^2-(0)^2}{8}$

$= \frac{16}{3} + \cancel{\frac{16}{8}}2 = \frac{16+2}{3}$

$= \frac{16+6}{3} = \frac{22}{3}$.

3- $A = \int_1^2 \left[y - \frac{1}{y^2}\right]dy$.

$= \int_1^2 ydy - \int_1^2 y^{-2}dy$

$= \frac{y^2}{2}\Big|_1^2 - \frac{y^{-1}}{-1}\Big|_1^2$

$= \frac{(2)^2-(1)^2}{2} + \left[(2)^{-1}-(1)^{-1}\right]$

$= \frac{4-1}{2} + \frac{1-2}{2} = \frac{3-1}{2} = \frac{2}{2}$

4- $A = \int_0^2 [(2-y^2)-(-y)]dy$

$= \int_0^2 [2-y^2+y]dy$.

$= \int_0^2 2dy - \int_0^2 y^2dy + \int_0^2 ydy$

$= 2y\Big|_0^2 - \frac{y^3}{3}\Big|_0^2 + \frac{y^2}{2}\Big|_0^2$

$= (2\cdot2-2\cdot0) - \frac{(2)^3-(0)^3}{3} + \frac{(2)^2-(0)^2}{2}$

$= (4-0) - \frac{8-0}{3} + \frac{4-0}{2} = \frac{10}{3}$

5-a) $A = \int_0^2 [2x-x^2]dx$

$= \int_0^2 2xdx - \int_0^2 x^2dx$

$= \frac{2x^2}{2}\Big|_0^2 - \frac{x^3}{3}\Big|_0^2$

$= ((2)^2-(0)^2) - \frac{((2)^3-(0)^3)}{3}$

$= 4 - \frac{8}{3} = \frac{4}{3}u^2$

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b) $\int_0^4 \left[\sqrt{y} - \frac{y}{2}\right] dy$.

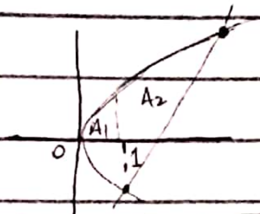
$= \int_0^4 \sqrt{y}\,dy - \int_0^4 \frac{y}{2}\,dy$.

$= \frac{2\sqrt{y^3}}{3}\Big|_0^4 - \frac{1}{2}\cdot\frac{y^2}{2}\Big|_0^4$

$= \frac{2\sqrt{(4)^3 - 0}}{3} - \frac{1}{4}\left[4^2 - 0^2\right]$

$= \frac{2\sqrt{64}}{3} - \frac{1}{4}(16) = \frac{16}{3} - 4 = \frac{4}{3}u^2$

6 - a)



$A_1 = \int_0^1 \left[\sqrt{4x} - (-\sqrt{4x})\right] dx$.

$= \int_0^1 \left[\sqrt{4x} + \sqrt{4x}\right] dx = \int_0^1 \left(2\sqrt{4x}\right) dx = \int_0^1 (2.2\sqrt{x})\,dx$

$= \int_0^1 4\sqrt{x}\,dx = 4\int_0^1 \sqrt{x}\,dx = 4\left[\frac{2\sqrt{x^3}}{3}\right]\Big|_0^1$

$A_1 = \frac{8}{3}u^2$

$A_2 = \int_1^4 \left[\sqrt{4x} - (2x-4)\right] dx$.

$= \int_1^4 \left(\sqrt{4x} - 2x + 4\right) dx$

$= \int_1^4 \sqrt{4x}\,dx - \int_1^4 2x + \int_1^4 4\,dx$

$= \frac{4\sqrt{x^3}}{3}\Big|_1^4 - x^2\Big|_1^4 + 4x\Big|_1^4$

$= \frac{4\sqrt{(4)^3 - (1)^3}}{3} - \left((4)^2 - 1^2\right) + \left[4(4) - 4(1)\right]$

$= \left(\frac{32-4}{3}\right) - 15 + 12 = \frac{19}{3}$

$A = A_1 + A_2 = \frac{8}{3} + \frac{19}{3} = \frac{27}{3}u^2 = 9u^2$

b) $\int_{-2}^4 \left[\frac{y+4}{2} - \frac{y^2}{4}\right] dy$.

$= \frac{y^2}{4} + 2y - \frac{y^3}{12}\Big|_{-2}^4$

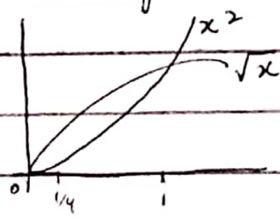
$= \frac{(4)^2 - (-2)^2}{4} + 2\left[4+2\right] - \left[\frac{(4)^3}{12} - \frac{(-2)^3}{12}\right]$

$= \frac{16-4}{4} + 2[6] - \left[\frac{64+8}{12}\right]$

$= \frac{12}{4} + \frac{12-72}{12} = \frac{36+144-72}{12}$

$= \frac{108}{12} = 9$

7 - $y = x^2$, $y = \sqrt{x}$, $x = \frac{1}{4}$, $x = 1$.



$= \int_{1/4}^1 \left(\sqrt{x} - x^2\right) dx$

$= \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_{1/4}^1 = \left[\frac{2}{3}(1)^{3/2} - \frac{1^3}{3}\right] - \left[\frac{2}{3}\left(\frac{1}{4}\right)^{3/2} - \frac{\left(\frac{1}{4}\right)^3}{3}\right]$

$= \left[\frac{2}{3} - \frac{1}{3}\right] - \left[\frac{2}{3}\left(\frac{1}{8}\right) - \frac{1}{192}\right]$

$= \frac{1}{3} - \frac{15}{192} = \frac{64}{192} - \frac{15}{192} = \frac{49}{192}$

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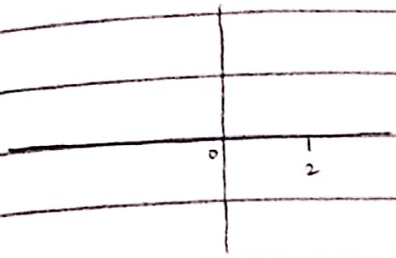
8- $y = x^3 - 4x$, $y = 0$, $x = 0$, $x = 2$.



x	0	2
y	0	0

$$= \int_0^2 [0 - x^3 + 4x]\,dx.$$

$$= -\int_0^2 x^3\,dx + \int_0^2 4x\,dx = -\frac{x^4}{4}\Big|_0^2 + \frac{\cancel{4}^{\,2}x^2}{\cancel{2}}\Big|_0^2$$

$$= -\frac{16}{4} + 2 \cdot 2^2 + 0 - 0 = 4$$

9- $y = \cos 2x$, $y = 0$, $x = \pi/4$, $x = \pi/2$.

$$A = \int_{\pi/4}^{\pi/2} [0 - \cos 2x]\,dx.$$

$$= \int 0\,dx - \frac{\sin 2x}{2}\Big|_{\pi/4}^{\pi/2}$$

$$= \frac{-\sin \pi}{2} + \frac{\sin(\pi/2)}{2} = 0 + \frac{1}{2} = \frac{1}{2}$$

10- $y = \sec^2 x$, $y = 2$, $x = -\pi/4$, $x = \pi/4$.

$$= \int_{-\pi/4}^{\pi/4} [2 - \sec^2 x]\,dx = \int_{-\pi/4}^{\pi/4} 2\,dx - \int_{-\pi/4}^{\pi/4} \sec^2 x\,dx$$

$$= 2x\Big|_{-\pi/4}^{\pi/4} - \tan x\Big|_{-\pi/4}^{\pi/4}$$

$$= \left[\frac{\pi}{2} + \frac{\pi}{2}\right] - \left[\tan\frac{\pi}{4} - \tan\frac{-\pi}{4}\right]$$

$$= \pi - [1 + 1] = \pi - 2$$

11- $x = \sin y$, $x = 0$, $y = \pi/4$, $y = 3\pi/4$

$$= \int_{\pi/4}^{3\pi/4} [\sin y - 0]\,dx.$$

$$= \int \sin y\,dy = -\cos y\Big|_{\pi/4}^{3\pi/4}$$

$$= -\cos 3\pi/4 + \pi/4 = -\left(\frac{-\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2}$$

$$= \sqrt{2}.$$

12- $x^2 = y$, $x = y - 2$

$$A = \int_{-1}^{2} [x + 2 - x^2]\,dx.$$

$$= \frac{x^2}{2} + 2x - \frac{x^3}{3}\Big|_{-1}^{2}$$

$$= \frac{4}{2} + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}$$

$$= \frac{3}{2} + 6 - 3 = \frac{9}{2}$$

13- $y = e^x$, $y = e^{2x}$, $x = 0$, $x = \ln 2$

$$A = \int_0^{\ln 2} [e^{2x} - e^x]\,dx.$$

$$= \left(\frac{1}{2} e^{2x} - e^x\right)\Big|_0^{\ln 2}$$

$$= \left(\frac{1}{2} e^{2\ln 2} - e^{\ln 2}\right) - \left(\frac{1}{2} e^0 - e^0\right)$$

$$= \left(\frac{1}{2} e^{\ln 2^2} - e^{\ln 2}\right) - \left(\frac{1}{2} - 1\right) = \left(\frac{4}{2} - 2\right) - \left(\frac{-1}{2}\right)$$

$$= 1/2$$

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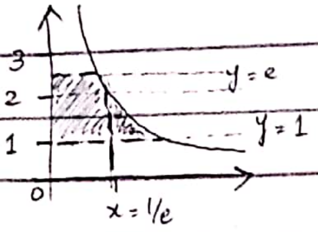
14.



$$A_1 = \int_0^{1/e} (e-1)\,dx.$$

$$= \int_0^{1/e} (e-1) = \int_0^{1/e} (e-1)x = 0.367x(e-1) - 0.(e-1).$$

$$= 0.3679(e-1)$$

$$= 0.6322$$

$$A_2 = \int_{1/e}^{1} \left(\frac{1}{x} - 1\right)dx.$$

$$= \ln|x| \Big|_{1/e}^{1} - x\Big|_{1/e}^{1}$$

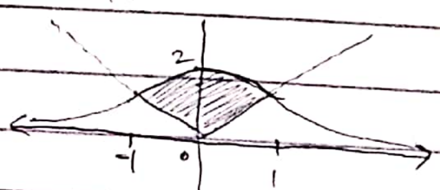
$$= (\ln 1 - \ln 0.367) - (1 - 0.367)$$

$$= 0.99994 - 0.6321$$

$$= 0.3678$$

$$A = A_1 + A_2 = 0.6322 + 0.3678 = 1$$

15. $y = \dfrac{2}{1+x^2}$, $y = |x|$



$$A = \int_{-1}^{1} \left(\frac{2}{1+x^2} - |x|\right)dx$$

$$= \int_{-1}^{0} \left(\frac{2}{1+x^2} - (-x)\right)dx + \int_0^1 \left(\frac{2}{1+x^2} + (-x)\right)dx$$

$$= \int_{-1}^{0} \left(\frac{2}{1+x^2} + x\right) + \int_0^1 \left(\frac{2}{1+x^2} - x\right)dx$$

$$= \left(2\tan^{-1}x\Big|_{-1}^{0} + \frac{x^2}{2}\Big|_{-1}^{0}\right) + \left(2\tan^{-1}x\Big|_0^1 - \frac{x^2}{2}\Big|_0^1\right)$$

$$= \left(\frac{\pi}{2} - \frac{1}{2}\right) + \left(\frac{\pi}{2} - \frac{1}{2}\right)$$

$$= \frac{2\pi}{2} - \frac{2}{2} = \pi - 1$$

16. $y = \dfrac{1}{\sqrt{1-x^2}}$, $y = 2$.

$$\left(\frac{1}{\sqrt{1-x^2}}\right)^2 = 4 \Rightarrow 4(1-x^2) = 1$$

$$4 - 4x^2 = 1$$

$$4x^2 = 3$$

$$x^2 = 3/4$$

$$x = \pm\sqrt{3/4} = \pm\sqrt{3}/2$$

⋮

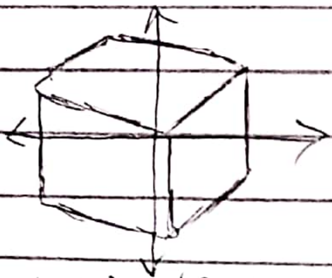
(Not completed)

1 Exercise 11.1

1- a) $(0,0,0), (3,0,0), (0,5,0), (0,0,4), (3,5,0), (3,0,4), (0,5,4), (3,5,4)$.

b) $(0,1,0), (0,6,0), (4,1,0), (4,6,0), (0,1,-2), (0,6,-2), (4,6,-2), (4,1,-2)$.

Q2 -



$(2,2,2), (2,2,-2), (2,-2,2), (-2,2,2), (-2,-2,-2), (-2,-2,2), (-2,2,-2), (2,-2,-2)$.

Q10- $(4,5,+2), (1,7,3), (2,4,5)$.

Show that it is a equilateral Δ.

$d = \sqrt{(x_0-x_p)^2+(y_0-y_p)^2+(z_0-z_p)^2}$

distance b/w $(4,5,2)$ & $(1,7,3)$ is

$\sqrt{(4-1)^2+(5-7)^2+(2-3)^2} = \sqrt{9+4+1}$

$= \sqrt{14}$

Distance b/w $(4,5,2)$ & $(2,4,5)$ is

$\sqrt{(4-2)^2+(5-4)^2+(2-5)^2} = \sqrt{4+1+9}$

$= \sqrt{14}$

Distance b/w $(1,7,3)$ & $(2,4,5)$ is

$\sqrt{(1-2)^2+(7-4)^2+(3-5)^2} = \sqrt{1+9+4} = \sqrt{14}$

Since, all three sides are equal so proved.

Q11 a) $(2,1,6), (4,7,9), (8,5,-6)$ are vertices of a right Δ.

$\bar{AB} = d(A,B) = \sqrt{(4-2)^2+(7-1)^2+(9-6)^2}$

$= \sqrt{4+36+9} = \sqrt{49} = 7$

$\bar{BC} = d(B,C) = \sqrt{(8-4)^2+(5-7)^2+(-6-9)^2}$

$= \sqrt{16+4+225} = \sqrt{245} = 7\sqrt{5}$

$\bar{AC} = d(A,C) = \sqrt{(8-2)^2+(5-1)^2+(-6-6)^2}$

$= \sqrt{36+16+144} = \sqrt{196} = 14$

b) $(\bar{BC})^2 = (\bar{AB})^2 + (\bar{AC})^2$

$(7\sqrt{5})^2 = (7)^2 + (14)^2$

$245 = 245$

So, ABC is a square Δ which is square at $A(2,1,6)$.

c) Area $= \frac{1}{2}(\bar{AC})(\bar{AB})$

$= \frac{1}{2}(14)(7)$

$= 49\ (\text{unit})^2$

Q12 - $(-5,2,-3)$

a) xy plane: z' is zero in this plane

$d_z = |z - z'| = |-3-0|$

$= |-3| = 3$

b) xz plane: y' is zero in this plane

$d_y = |y - y'| = |-0+2| = |2| = 2$

c) yz plane: x' is zero in this plane

$d_x = |x - x'| = |-5-0| = |-5| = 5$

d) x-axis:

$d = \sqrt{dy^2 + dz^2} = \sqrt{2^2+3^2} = \sqrt{13}$

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e) y-axis:

$d = \sqrt{dx^2 + dz^2} = \sqrt{5^2 + 3^2} = \sqrt{34}$

f) z-axis:

$d = \sqrt{dx^2 + dy^2} = \sqrt{5^2 + 2^2} = \sqrt{29}$

Q13- a) $O(7, 1, 1)$, $R = 4$

$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2$ — (1).

$(x-7)^2 + (y-1)^2 + (z-1)^2 = 16$

b) $O(1, 0, -1)$, diameter $= 8 \Rightarrow r = 4$

$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2$ (1).

$(x-1)^2 + (y-0)^2 + (z+1)^2 = 16$

c) $O(-1, 3, 2)$ and passing through origin

$R = \sqrt{(x_0-0)^2 + (y_0-0)^2 + (z_0-0)^2}$ bcz

Through origin means $(0, 0, 0)$.

$= \sqrt{(-1-0)^2 + (3-0)^2 + (2-0)^2} = \sqrt{14}$

Equation $= (x+1)^2 + (y-3)^2 + (z-2)^2 = 14$

d) Diameter has endpoints $(-1, 2, 1)$ and $(0, 2, 3)$.

Diameter $= \sqrt{(-1-0)^2 + (2-2)^2 + (1-3)^2}$

$= \sqrt{(-1)^2 + 0^2 + (-2)^2} = \sqrt{1+0+4} = \sqrt{5}$

Diameter $=$ Radius $= \frac{\sqrt{5}}{2}$

Center (x, y, z).

$x = \frac{-1+0}{2} = \frac{-1}{2}$, $y = \frac{2+2}{2} = 2$, $z = \frac{1+3}{2} = 2$

Equation: $\left(x + \frac{1}{2}\right)^2 + (y-2)^2 + (z-2)^2 = \frac{5}{4}$

Q14- $O(0, 0, 0)$ center $(3, -2, 4)$.

radius $= 1$.

$(x-3)^2 + (y+2)^2 + (z-4)^2 = 1$

Distance from C to origin

$d = \sqrt{(-3)^2 + (2)^2 + (-4)^2} = \sqrt{9+4+16} = \sqrt{29}$

To closet on sphere to origin:

$d_{min} = d - r = d - 1 = \sqrt{29} - 1$

To furthest point:

$d_{max} = d + r = d + 1 = \sqrt{29} + 1$

$r = \sqrt{29} \pm 1$ are tangent to sphere of radius 1 centered at $(3, -2, 4)$.

Q15- $(2, -1, -3)$

a) Tangent to xy-plane.

$R = |z - z'| = |-3-0| = |-3| = 3$

$(x-2)^2 + (y+1)^2 + (z+3)^2 = 9$

b) Tangent to xz-plane.

$R = |y - y'| = |-1-0| = |-1| = 1$

$(x-2)^2 + (y+1)^2 + (z+3)^2 = 1$

c) Tangent to yz-plane.

$R = |x - x'| = |2-0| = |2| = 2$

$(x-2)^2 + (y+1)^2 + (z+3)^2 = 4$

Q16- a) $O(-2, 1, 3)$, $l = 1$.

$R = l/2 = 1/2$.

$(x+2)^2 + (y-1)^2 + (z-3)^2 = 1/4$

b) diagonal $= \sqrt{a^2 + a^2 + a^2}$

$= \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$R = \frac{d}{2} = \frac{\sqrt{3}}{2}$

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$(x+2)^2+(y-1)^2+(z-3)^2=\frac{3}{4}$.

c) $x=6, x=2,$

$y=5, y=9,$

$z=0, z=4.$

Center points $=(x_0, y_0, z_0)$.

$x_0=\frac{6+2}{2}=4$ $\quad z_0=\frac{4+0}{2}=2.$

$y_0=\frac{5+9}{2}=7$

length of side $=6-2=4$

Radius $=\frac{\text{length}}{2}=\frac{4}{2}=2.$

$(x-4)^2+(y-7)^2+(z-2)^2=4$

d) diagonal $=\sqrt{l^2+l^2+l^2}$

$=\sqrt{4^2+4^2+4^2}$

$=4\sqrt{3}$

$R=\frac{d}{2}=\frac{4\sqrt{3}}{2}=2\sqrt{3}$

$(x-4)^2+(y-7)^2+(z-2)^2=12$

Q23. $x^2+y^2+z^2+10x+4y+2z-19=0$

$(x^2+10x+25)+(y^2+4y+4)+(z^2+2z+1)$
$-19-25-4-1=0$

$(x+5)^2+(y+2)^2+(z+1)^2=49$

$(x+5)^2+(y+2)^2+(z+1)^2=7^2$

$C(-5,-2,-1)$ and radius $=7$

Q24- $x^2+y^2+z^2-y=0$

$x^2+(y^2-y)+z^2=0$.

$x^2+\left(y^2-2\left(\frac{1}{2}\right)y+(1/2)^2\right)+z^2=(1/2$

$x^2+\left(y-\frac{1}{2}\right)^2+z^2=(1/2)^2$

$C(0, 1/2, 0)$ and radius $=1/2$

Q25- $2x^2+2y^2+2z^2-2x-3y+5z-2=0$.

Multiplying by 1/2.

$x^2+y^2+z^2-x-3/2y+5/2z=1$

$(x^2-x+1/4)+(y^2-3/2y+9/16)+(z^2+\frac{5}{2}z+$

$=1+\frac{1}{4}+\frac{9}{16}+\frac{25}{16}$

$(x-1/2)^2+(y+3/4)^2+(z+5/4)^2=\left(\frac{3\sqrt{3}}{2\sqrt{2}}\right)^2$

$=\left(\frac{3\sqrt{6}}{4}\right)^2$

$C(1/2, -3/4, -5/4)$ and radius $=\frac{3\sqrt{6}}{4}$

Q26- $x^2+y^2+z^2+2x-2y+2z+3=0$

$(x^2+2x)+(y^2-2y)+(z^2+2z)=-3$

$(x^2+2x+1)+(y^2-2y+1)+(z^2+2z+1)=-3+1+1+1$

$(x+1)^2+(y-1)^2+(z+1)^2=0$

$C(-1,1,-1)$ and radius $=0$

Q27- $x^2+y^2+z^2-3x+4y-8z+25=0$

$x^2+y^2+z^2-3x+4y-8z=-25$

$\left(x^2-3x+\frac{9}{4}\right)+(y^2+4y+4)+(z^2-8z+16)$

$=-25+\frac{9}{4}+4+16$

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$$\left(x-\frac{3}{2}\right)^2+(y+2)^2+(z-4)^2=\frac{-11}{4}$$

radius can't be less than 0. so it is not satisfied.

Q28- $x^2+y^2+z^2-2x-6y-8z+1=0.$

$(x^2-2x)+(y^2-6y)+(z^2-8z)=-1$

$(x-2x+1)^2+(y^2-6y+9)+(z^2-8z+16=-1+1+9+16$

$(x-1)^2+(y-3)^2+(z-4)^2=25$

$(x-1)^2+(y-3)^2+(z-4)^2=5^2$

$C(1,3,4)$ and radius $=5$.

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$$x - \frac{3}{2})^2 + (y+2)^2 + (z-4)^2 = \frac{-11}{4}$$

radius can't be less than 0, so it is not satisfied.

)28. $x^2+y^2+z^2-2x-6y-8z+1=0$

$(x^2-2x)+(y^2-6y)+(z^2-8z)=-1$

$x-2x+1)^2+(y^2-6y+9)+(z^2-8z+16=-1+1+9+16$

$(x-1)^2+(y-3)^2+(z-4)^2=25$

$(x-1)^2+(y-3)^2+(z-4)^2=5^2$

$C(1,3,4)$ and radius $=5$.

Exercise 3.1

$- x+xy-2x^3=2$

$)\frac{d}{dx}[x+xy-2x^3]=\frac{d}{dx}[2]$. interms of x

$= 1+x\frac{dy}{dx}+y-6x^2=0$

$$x\frac{dy}{dx}=6x^2-y-1$$

$$\frac{dy}{dx}=\frac{6x^2-y-1}{x}$$

) $xy=2+2x^3-x$

$$y=\frac{2+2x^3-x}{x}$$ in Terms of y.

$$\frac{y}{x}=\frac{x(6x^2-1)-(2+2x^3-x)}{x^2}$$

$$=\frac{6x^3-\cancel{x}-2-2x^3+x}{x^2}=\frac{4x^3-2}{x^2}$$

c) $$\frac{dy}{dx}=\frac{6x^2-y-1}{x}$$

$$y=\frac{2+2x^3-x}{x}$$

$$\frac{dy}{dx}=\frac{6x^2-(2+2x^3-x)-1}{x\cdot x}$$

$$=\frac{6x^3-2-2x^3+x-x}{x^2}$$

$$=\frac{4x^3-2}{x^2}$$

The results are consistent.

$2-\sqrt{y}-\sin x=2$

a) $\frac{d}{dx}(\sqrt{y}-\sin x)=\frac{d}{dx}[2]$.

$$\frac{1}{2\sqrt{y}}\frac{dy}{dx}-\cos x=0.$$

$$\frac{dy}{dx}=\cos x[2\sqrt{y}] \Rightarrow 2\sqrt{y}\cos x$$

b) $\sqrt{y}=2+\sin x$

$y=(2+\sin x)^2$

$$\frac{dy}{dx}=2(2+\sin x)\frac{d}{dx}(2+\sin x)$$

$$=2\cos x(2+\sin x)$$

c) $$\frac{dy}{dx}=2\cos x\sqrt{y}=2\sqrt{(2+\sin x)^2}\cos x$$

$$=2(2+\sin x)\cos x$$

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3. $x^2 + y^2 = 100$

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(100)$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2\left(x + y\frac{dy}{dx}\right) = 0$$

$$x + y\frac{dy}{dx} = 0$$

$$y\frac{dy}{dx} = -x \Rightarrow \frac{dy}{dx} = \frac{-x}{y}$$

4. $x^3 + y^3 = 3x^2y^2$

$$3x^2 + 3y^2\frac{dy}{dx} = 3y^2 + 6xy\frac{dy}{dx}$$

$$3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} = 3y^2 - 3x^2$$

$$\cancel{3}\frac{dy}{dx}\left[y^2 - 2xy\right] = \cancel{3}\left[y^2 - x^2\right]$$

$$\frac{dy}{dx} = \frac{[y^2 - x^2]}{y(y - 2x)}$$

5. $x^2y + 3xy^3 - x = 3$

$$2xy + x^2\frac{dy}{dx} + 3y^3 + 9xy^2\frac{dy}{dx} - 1 = 0$$

$$x^2\frac{dy}{dx} + 9xy^2\frac{dy}{dx} = -2xy - 3y^3 + 1$$

$$\frac{dy}{dx}\left[x^2 + 9xy^2\right] = 1 - 2xy - 3y^3$$

$$\frac{dy}{dx} = \frac{1 - 2xy - 3y^3}{x^2 + 9xy^2}$$

6. $x^3y^2 - 5x^2y + x = 1$

$$3x^2y^2 + 2x^3y\frac{dy}{dx} - 10xy - 5x^2\frac{dy}{dx} + 1 =$$

$$2x^3y\frac{dy}{dx} - 5x^2\frac{dy}{dx} = 10xy - 3x^2y^2 -$$

$$\frac{dy}{dx}\left[2x^2y - 5x^2\right] = 10xy - 3x^2y^2 -$$

$$\frac{dy}{dx} = \frac{10xy - 3x^2y^2 - 1}{x^2(2xy - 5)}.$$

7. $\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = 1$

$$x^{-1/2} + y^{-1/2} = 1$$

$$-\frac{1}{2}x^{-3/2} - \frac{1}{2}y^{-3/2}\frac{dy}{dx} = 0.$$

$$-\frac{\cancel{1}}{\cancel{2}}y^{-3/2}\frac{dy}{dx} = \frac{\cancel{1}}{\cancel{2}}x^{-3/2}$$

$$\frac{dy}{dx} = -\frac{x^{-3/2}}{y^{-3/2}} \Rightarrow -\frac{y^{3/2}}{x^{3/2}}$$

8. $x^2 = x + y / x - y$

$$x^2(x - y) = x + y$$

$$x^3 - x^2y = x + y$$

$$3x^2 - 2xy - x^2\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$3x^2 - 2xy - 1 = \frac{dy}{dx} + x^2\frac{dy}{dx}$$

$$\frac{3x^2 - 2xy - 1}{1 + x^2} = \frac{dy}{dx}$$

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9- $\sin(x^2y^2) = x$.

$$\frac{d}{dx}[\sin[x^2y^2]] = x$$

$$\cos(x^2y^2)\frac{d}{dx}[x^2y^2] = 1$$

$$\cos(x^2y^2)[2x^1y^2 + x^2 \cdot 2y\frac{dy}{dx}] = 1$$

$$2xy^2 + 2x^2y\frac{dy}{dx} = \frac{1}{\cos(x^2y^2)}$$

$$2x^2y\frac{dy}{dx} = \frac{1}{\cos(x^2y^2)} - 2xy^2$$

$$\frac{dy}{dx} = \frac{\frac{1}{\cos(x^2y^2)} - 2xy^2}{2x^2y}$$

$$\frac{dy}{dx} = \frac{1 - 2xy^2(\cos(x^2y^2))}{2x^2y\cos(x^2y^2)}$$

10- $\cos(xy^2) = y$

$$-\sin(xy^2)[y^2 + 2xy\frac{dy}{dx}] = \frac{dy}{dx}$$

$$-y^2\sin(xy^2) - 2xy\sin(xy^2)\frac{dy}{dx} = \frac{dy}{dx}$$

$$-y^2\sin(xy^2) = 2xy\sin(xy^2)\frac{dy}{dx} + \frac{dy}{dx}$$

$$-y^2\sin(xy^2) = \frac{dy}{dx}[2xy\sin(xy^2) + 1]$$

$$\frac{-y^2\sin(xy^2)}{2xy\sin(xy^2) + 1} = \frac{dy}{dx}$$

11- $\tan^3(xy^2+y) = x$

$$3\tan^2(xy^2+y)\sec^2(xy^2+y)$$
$$(y^2 + 2xy\frac{dy}{dx} + \frac{dy}{dx}) = 1$$

Divide $3\tan^2(xy^2+y)\sec^2(xy^2+y)$ on both sides.

$$y^2 + 2xy\frac{dy}{dx} + \frac{dy}{dx} = \frac{1}{(3\tan^2(xy^2)+y)\sec^2(xy^2+y)}$$

$$2xy\frac{dy}{dx} + \frac{dy}{dx} = \frac{1}{(3\tan^2(xy^2)+y)\sec^2(xy^2+y)} - y^2$$

$$\frac{dy}{dx}(2xy+1) = \frac{1}{(3\tan^2(xy^2)+y)\sec^2(xy^2+y)} - y^2$$

$$\frac{dy}{dx} = \frac{\frac{1}{3\tan^2(xy^2+y)\sec^2(xy^2+y)} - y^2}{2xy+1}$$

$$= \frac{1 - 3y^2\tan^2(xy^2+y)\sec^2(xy^2+y)}{3(2xy+1)\tan^2(xy^2+y)\sec^2(xy^2+y)}$$

12. $\frac{xy^3}{1+\sec y} = 1 + y^4$

$$\frac{xy^3}{1+\sec y} - y^4 = 0$$

$$\frac{xy^3(1+\sec y) - xy^3\sec y\tan y\frac{dy}{dx}}{(1+\sec y)^2} - 4y^3\frac{dy}{dx} = 0$$

$$\frac{y^3 + 3xy^2\frac{dy}{dx}(1+\sec y) - xy^3\sec y\tan y\frac{dy}{dx}}{(1+\sec y)^2} - 4y^3\frac{dy}{dx} = 0$$

$$\frac{y^3 + 3xy^2 dy/dx(1+\sec y)}{(1+\sec y)^2} - \frac{xy^3\sec y\tan y\frac{dy}{dx}}{(1+\sec y)^2} - 4y^3\frac{dy}{dx} =$$

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13- $\left[\frac{xy^3\sec y\tan y - 3xy^2(1+\sec y)}{(1+\sec y)^2}\right]\frac{dy}{dx} - \frac{4y^3(1+\sec y)^2}{(1+\sec y)^2}\frac{dy}{dx}$

$\left[\frac{xy^3\sec y\tan y - 3xy^2(1+\sec y) + 4y^3(1+\sec y)^2}{(1+\sec y)^2}\right]\frac{dy}{dx}$

$= \frac{-y^3}{1+\sec y}$

$\frac{dy}{dx} = \frac{y^3}{1+\sec y} \cdot \frac{(1+\sec y)^{\cancel{2}}}{xy^3\sec y\tan y - 3xy^2(1+\sec y) + 4y^3(1+\sec y)^2}$

$= \frac{y^3(1+\sec y)}{y^2(xy\sec y\tan y - 3x(1+\sec y) + 4y(1+\sec y)^2)}$

$= \frac{y(1+\sec y)}{xy\sec y\tan y - 3x(1+\sec y) + 4y(1+\sec y)^2}$

- $2x^2 - 3y^2 = 4$

$4x - 6y\frac{dy}{dx} = 0.$

$6y\frac{dy}{dx} = 4x$

$\frac{dy}{dx} = \frac{4x}{6y} = \frac{2x}{3y}$

$\frac{d^2y}{dx^2} = \frac{3y(2) - 3\,dy/dx\,(2x)}{(3y)^2}$

$= \frac{6y - 6x\,dy/dx}{(3y)^2} = \frac{{}^2\cancel{6}\left(y - x\frac{dy}{dx}\right)}{{}_3\cancel{9}y^2}$

$= \frac{2}{3}\left[\frac{y - x\,dy/dx}{y^2}\right] \quad \therefore \frac{dy}{dx} = \frac{2x}{3y}$

$= \frac{2}{3}\left[\frac{y - 2x^2/3y}{y^2}\right]$

$= \frac{2}{3}\left[\frac{3y^2 - 2x^2}{3y} \cdot \frac{1}{y^2}\right]$

$2x^2 = 4 + 3y^2$

$x^2 = (4+3y^2)/2$

Substitute,

$= \frac{2}{3}\left[\frac{3y^2 - \cancel{2}\left(\frac{4+3y^2}{\cancel{2}}\right)}{3y} \cdot \frac{1}{y^2}\right]$

$= \frac{2}{3}\left[\frac{-4}{3y^3}\right] = \frac{-8}{9y^3}$

14- $x^3 + y^3 = 1$

$3x^2 + 3y^2\frac{dy}{dx} = 0.$

$3\left[x^2 + y^2\frac{dy}{dx}\right] = 0$

$y^2\frac{dy}{dx} = -x^2 \Rightarrow \frac{dy}{dx} = \frac{-x^2}{y^2}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{-x^2}{y^2}\right) = -\left(\frac{y^2(2x) - x^2(2y)\,dy/dx}{y^4}\right)$

$= -\left(\frac{2xy^2 - 2x^2y\,dy/dx}{y^4}\right)$

$= -2x\left[\frac{y^2 - xy(-x^2/y^2)}{y^4}\right]$

$= -2x\left[\frac{y^3 + x^3}{y^5}\right]$

Substitute:

$y^3 = 1 - x^3$

$+x^3 + y^3 = 1$

$= -2x\left[\frac{1 - \cancel{x^3} + \cancel{x^3}}{y^5}\right]$

$= \frac{-2x}{y^5}.$

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17. $y + \sin y = x$

$\frac{dy}{dx} + \cos y \frac{dy}{dx} = 1$

$\frac{dy}{dx}\left[1 + \cos y\right] = 1$

$\frac{dy}{dx} = \frac{1}{1+\cos y}$

$\frac{d^2y}{dx^2} = \frac{-(-\sin y)}{(1+\cos y)^2} \times \frac{dy}{dx}$

$= \frac{\sin y}{(1+\cos y)^3}$

18. $x\cos y = y$.

$\cos y - x \sin y \frac{dy}{dx} = \frac{dy}{dx}$

$\cos y = \frac{dy}{dx}(1 + x\sin y)$.

$\frac{dy}{dx} = \frac{\cos y}{1+x\sin y}$

$\frac{d^2y}{dx} = \frac{-\sin y \frac{dy}{dx}(1+x\sin y) - \cos y(\sin y + x\cos y\frac{dy}{dx})}{(1+x\sin y)^2}$

$= \frac{-\sin y\frac{dy}{dx} - x\sin^2 y\frac{dy}{dx} - \cos y\sin y - x\cos^2 y\frac{dy}{dx}}{(1+x\sin y)^2}$

$= \frac{-\sin y\frac{dy}{dx} - x\frac{dy}{dx}[\sin^2 y - \cos^2 y] - \cos y\sin y}{(1+x\sin y)^2}$

$= \frac{-\frac{dy}{dx}[\sin y + x] - \cos y\sin y}{(1+x\sin y)^2}$

$= \frac{-\left(\frac{\cos y}{1+x\sin y}\right)(x+\sin y) - \cos y\sin y}{(1+x\sin y)^2}$

$= \frac{-x\cos y - \sin y\cos y - \cos y\sin y(1+x\sin y)}{(1+x\sin y)^3}$

$= \frac{-x\cos y - \sin y\cos y - \cos y\sin y - x\cos y\sin^2 y}{(1+x\sin y)^3}$

$= \frac{-x\cos y - 2\sin y\cos y - x\cos y\sin^2 y}{(1+x\sin y)^3}$

$= \frac{-x\cos y(1+\sin^2 y) - \sin 2y}{(1+x\sin y)^3}$

$= \frac{-y(1+\sin^2 y) - \sin 2y}{(1+x\sin y)^3}$

19. $x^2+y^2=1$; $(1/2, \sqrt{3}/2), (1/2, -\sqrt{3}/2)$

$y^2 = 1-x^2$

$y = \pm\sqrt{1-x^2}$

$\frac{dy}{dx} = \frac{d}{dx}\left((1-x^2)^{1/2}\right)$.

$f(u) = u^{1/2}$

$u = g = 1-x^2$

$= \frac{1}{2}u^{-1/2}\frac{d}{dx}\left[1-x^2\right]$

$= \frac{1}{2u^{1/2}}(-2x)$

$= \frac{-x}{\sqrt{u}} = \frac{-x}{\sqrt{1-x^2}}$

$m = \frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$

$(x,y) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$: $\frac{-1/2}{\sqrt{1-(1/2)^2}} = \frac{-1}{2\cdot\sqrt{3}/2} =$

$$\frac{dy}{dx} = \frac{d}{dx}\left(-\sqrt{1-x^2}\right).$$

$$= -\frac{d(1-x^2)^{1/2}}{dx} = -\left(\frac{-x}{\sqrt{1-x^2}}\right)$$

$$m = \frac{x}{\sqrt{1-x^2}}$$

$$(x,y) = \left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right) = \frac{1/2}{\sqrt{1-(1/2)^2}} = \frac{1}{\sqrt{3}}$$

$$\frac{d}{dx}(y^2+x^2) = \frac{d}{dx}(1).$$

$$2y\frac{dy}{dx} + 2x = 0. \Rightarrow 2[ydy + x] = 0$$

$$\frac{dy}{dx} = -\frac{x}{y} \Rightarrow m = \frac{-x}{y}.$$

$(x,y) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$: $\frac{-1/2}{\sqrt{3}/2} = -1/\sqrt{3}$

$(x,y) = \left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$: $-1/2 / -\sqrt{3}/2 = 1/\sqrt{3}$

slope of tangent line at point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is $m = -1/\sqrt{3}$

slope of tangent line at point $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ is $m = 1/\sqrt{3}$

25. $x^4 + y^4 = 16$; $(1, \sqrt[4]{15})$

$$4x^3 + 4y^3\frac{dy}{dx} = 0.$$

$$\frac{dy}{dx} = \frac{-x^3}{y^3}$$

$$\frac{dy}{dx} = \frac{-1^3}{(\sqrt[4]{15})^3} \approx -0.1312$$

equation of tangent.

$$y - \sqrt[4]{15} = -0.1312(x-1)$$

$$y = -0.1312x + 0.1312 + \sqrt[4]{15}$$

$$y = -0.1312x + 2.0991$$

31. $a^2w^2 + b^2\lambda^2 = 1$; $dw/d\lambda$

$$2a^2w\frac{dw}{d\lambda} + 2b^2\lambda = 0$$

$$2a^2w\frac{dw}{d\lambda} = -2b^2\lambda$$

$$\frac{dw}{d\lambda} = \frac{-\cancel{2}b^2\lambda}{\cancel{2}a^2w} = \frac{-b^2\lambda}{a^2w}$$

Exercise 3.2

1. $y = \ln 5x$

$$y = \ln 5 + \ln x$$

$$\frac{dy}{dx} = 0 + 1/x = 1/x$$

2- $y = \ln x/3$

$$y = \ln x - \ln 3$$

$$\frac{dy}{dx} = 1/x - 0 = 1/x$$

3- $y = \ln|1+x|$

$$\frac{dy}{dx} = \frac{1}{1+x}\frac{d}{dx}(x+1)$$

$$= \frac{1}{1+x} \cdot 1 = \frac{1}{1+x}$$

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4- $y = \ln(2+\sqrt{x})$

$$= \frac{1}{2+\sqrt{x}} \frac{d}{dx}(2+\sqrt{x})$$

$$= \frac{1}{2+\sqrt{x}} \left(\frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{x}(2+\sqrt{x})}$$

5- $y = \ln|x^2-1|$

$$\frac{dy}{dx} = \frac{1}{x^2-1} \frac{d}{dx}(x^2-1)$$

$$= \frac{1}{x^2-1}(2x) = \frac{2x}{x^2-1}$$

6- $y = \ln|x^3-7x^2-3|$

$$\frac{dy}{dx} = \frac{1}{x^3-7x^2-3} \frac{d}{dx}(x^3-7x^2-3)$$

$$= \frac{1}{x^3-7x^2-3}(3x^2-14x)$$

$$= \frac{3x^2-14x}{x^3-7x^2-3}$$

7- $y = \ln\left(\frac{x}{1+x^2}\right)$

$$\frac{dy}{dx} = \frac{1+x^2}{x} \frac{d}{dx}\left(\frac{x}{1+x^2}\right)$$

$$= \frac{1+x^2}{x} \cdot \frac{(1+x^2)-x(2x)}{(1+x^2)^2}$$

$$= \frac{1+x^2}{x} \cdot \frac{(1+x^2)-2x^2}{(1+x^2)^2}$$

$$= \frac{\cancel{(1+x^2)}}{x} \cdot \frac{1-x^2}{(1+x^2)^{\cancel{2}}}$$

$$= \frac{1-x^2}{x(1+x^2)}$$

8- $y = \ln\left|\frac{1+x}{1-x}\right|$

$$y = \ln|1+x| - \ln|1-x|$$

$$\frac{dy}{dx} = \frac{1}{1+x}(1) - \frac{1}{1-x}(-1)$$

$$= \frac{1}{1+x} + \frac{1}{1-x}$$

$$= \frac{1-x+1+x}{1-x^2} = \frac{2}{1-x^2}$$

9- $y = \ln x^2$

$$\frac{dy}{dx} = \frac{1}{x^2}(2x) = \frac{2}{x}$$

10- $y = (\ln x)^3$

$$\frac{dy}{dx} = 3(\ln x)^2 \cdot \frac{1}{x}$$

$$= \frac{3(\ln x)^2}{x}$$

11- $y = \sqrt{\ln x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x} = \frac{1}{2x\sqrt{\ln x}}$$

12- $y = \ln\sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x}$$

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13- $y = x\ln x$.

$$\frac{dy}{dx} = \ln x + x\left(\frac{1}{x}\right)$$
$$= \ln x + 1.$$

14- $y = x^3 \ln x$

$$\frac{dy}{dx} = \ln x(3x^2) + x^3\left(\frac{1}{x}\right).$$
$$= 3x^2\ln x + x^2$$
$$= x^2(3\ln x + 1).$$

15- $y = x^2 \log_2(3-2x)$.

$$\frac{dy}{dx} = \log_2(3-2x)(2x) + x^2\left(\frac{-2}{(3-2x)\ln 2}\right)$$
$$= 2x\log_2(3-2x) - \frac{2x^2}{(3-2x)\ln 2}$$

16- $y = x[\log_2(x^2-2x)]^3$

$$\frac{dy}{dx} = [\log_2(x^2-2x)]^3 + x[3(\log_2(x^2-2x))]^2 \cdot \frac{(2x-2)}{\ln 2(x^2-2x)}$$
$$= [\log_2(x^2-2x)]^3 + \frac{6x[\log_2(x^2-2x)]^2(x-1)}{x(x-2)\ln 2}$$

17- $y = \frac{x^2}{1+\log x}$

$$\frac{dy}{dx} = \frac{2x(1+\log x) - 4x^2\left(\frac{1}{x\ln 10}\right)}{(1+\log x)^2}.$$
$$= \frac{2x(1+\log x)4 - (x/\ln 10)}{(1+\log x)^2}$$

18- $y = \frac{\log x}{1+\log x}$.

$$\frac{dy}{dx} = \frac{(1+\log x)\cdot\frac{1}{x} - \log x\left(0+\frac{1}{x}\right)}{(1+\log x)^2}$$
$$= \frac{1+\log x - \log x}{x(1+\log x)^2}$$

19- $y = \ln(\ln x)$.

$$\frac{dy}{dx} = \frac{1}{\ln x}\cdot\frac{1}{x} = \frac{1}{x\ln x}$$

20- $y = \ln(\ln(\ln x))$.

$$\frac{dy}{dx} = \frac{1}{\ln(\ln x)} \cdot \frac{d}{dx}(\ln(\ln x)).$$
$$\frac{dy}{dx}(\ln(\ln x)) = \frac{1}{\ln x}\cdot\frac{1}{x}.$$
$$= \frac{1}{\ln(\ln x)}\cdot\frac{1}{\ln x}\cdot\frac{1}{x}$$
$$= \frac{1}{(\ln(\ln x)(\ln x)(x))}$$

21- $y = \ln(\tan x)$.

$$\frac{dy}{dx} = \frac{1}{\tan x}(\sec^2 x) = \frac{\sec^2 x}{\tan x}$$
$$= \sec x \csc x$$

22- $y = \ln(\cos x)$

$$\frac{dy}{dx} = \frac{1}{\cos x}(-\sin x) = -\tan x$$

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23- $y = \cos(\ln x)$.

$\frac{dy}{dx} = -\sin(\ln x) \cdot \frac{d}{dx}(\ln x)$

$= -\frac{\sin(\ln x)}{x}$

24- $y = \sin^2(\ln x)$

$\frac{dy}{dx} = 2\sin(\ln x) \cdot \frac{\cos x(\ln x)}{x}$

$= \frac{2\sin(\ln x)\cos(\ln x)}{x}$

$= \frac{\cancel{2}\sin 2\ln x}{x} = \frac{\sin(\ln x^2)}{x}$

25- $y = \log(\sin^2 x)$.

$\frac{dy}{dx} = \frac{1}{\sin^2 x \cdot \ln 10} \cdot (2\sin x)(\cos x)$

$= \frac{2\sin x\cos x}{\sin^2 x \cdot \ln 10} = \frac{2\cos x}{\sin x \ln 10}$

$= \frac{2\cot x}{\ln 10}$

26- $y = \log(1-\sin^2 x)$

$\frac{dy}{dx} = \frac{1}{(1-\sin^2 x)(\ln 10)}(-2\sin x)(\cos x)$

$= \frac{1}{\cos^2 x(\ln 10)}(-2\sin x)(\cos x)$

$= \frac{-2\sin x}{\cos x(\ln 10)} = \frac{-2\tan x}{\ln 10}$

27- $\frac{d}{dx}\left[\ln (x-1)^3(x^2+1)^4\right]$

$= \frac{d}{dx}\left[\ln(x-1)^3 + \ln(x^2+1)^4\right]$.

$= \frac{d}{dx}\left[3\ln(x-1)\right] + \frac{d}{dx}\left[4\ln(x^2+1)\right]$

$= \frac{3}{x-1}(1) + \frac{4}{x^2+1}(2x)$.

$= \frac{3}{x-1} + \frac{8x}{x^2+1} = \frac{3x^2+3+8x^2-8x}{(x-1)(x^2+1)}$

$= \frac{11x^2-8x+3}{(x-1)(x^2+1)}$

28- $\frac{d}{dx}\left[\ln((\cos^2 x)\sqrt{1+x^4})\right]$.

$= \frac{d}{dx}\left[\ln(\cos^2 x)\right] + \frac{d}{dx}\left[\ln\sqrt{x^4+1}\right]$

$= 2 \cdot \frac{1}{\cos x}(-\sin x) + \frac{1}{2} \cdot \frac{1}{1+x^4}(4x^3)$

$= \frac{-2\sin x}{\cos x} + \frac{4x^3}{2(x^4+1)} = -2\tan x + \frac{4x^3}{2(x^4+1)}$

29- $\frac{d}{dx}\left[\ln \frac{\cos x}{\sqrt{4-3x^2}}\right]$.

$= \frac{d}{dx}\left[\ln\cos x - \ln\sqrt{4-3x^2}\right]$.

$= \frac{1}{\cos x}(-\sin x) - \frac{1}{2} \cdot \frac{1}{4-3x^2}(-6x)$

$= -\tan x + \frac{\overset{3}{\cancel{6}}x}{\cancel{2}(4-3x^2)} = -\tan x + \frac{3x}{4-3x^2}$

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30. $\frac{d}{dx}\left[\ln\sqrt{\frac{x-1}{x+1}}\right]$

$= \frac{d}{dx}\left[\ln\sqrt{x-1} - \ln\sqrt{x+1}\right]$

$= \frac{1}{2}\cdot\frac{1}{x-1} - \frac{1}{2}\frac{1}{x+1}$

$= \frac{1}{2(x-1)} - \frac{1}{2(x+1)} = \frac{x+1-x+1}{2(x^2-1)}$

$= \frac{2}{2(x^2-1)} = \frac{1}{x^2-1}$

35. $y = x\sqrt[3]{1+x^2}$

$\ln y = \ln(x\sqrt[3]{1+x^2})$

$\ln y = \ln x + \ln(1+x^2)^{1/3}$

$\ln y = \ln x + \frac{1}{3}\ln(1+x^2)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{3}\cdot\frac{1}{1+x^2}(2x)$.

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{2x}{3(1+x^2)}$

$\frac{dy}{dx} = y\left[\frac{1}{x} + \frac{2x}{3(1+x^2)}\right]$

$= x\sqrt[3]{1+x^2}\left[\frac{1}{x} + \frac{2x}{3(1+x^2)}\right]$.

37. $y = \frac{(x^2-8)^{1/3}\sqrt{x^3+1}}{x^6-7x+5}$

$\ln y = \ln(x^2-8)^{1/3} + \ln(x^3+1)^{1/2} - \ln(x^6-7x+5)$

$\ln y = \frac{1}{3}\ln(x^2-8) + \frac{1}{2}\ln(x^3+1) - \ln(x^6-7x+5)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{3}\cdot\frac{1}{x^2-8}(2x) + \frac{1}{2}\cdot\frac{1}{x^3+1}(3x^2) - \frac{1}{x^6-7x+5}(6x^5-7)$

$\frac{1}{y}\frac{dy}{dx} = \frac{2x}{3(x^2-8)} + \frac{3x^2}{2(x^3+1)} - \frac{6x^5-7}{x^6-7x+5}$

$\frac{dy}{dx} = \frac{(x^2-8)^{1/3}\sqrt{x^3+1}}{x^6-7x+5}\left[\frac{2x}{3(x^2-8)} + \frac{3x^2}{2(x^3+1)} - \frac{6x^5-7}{x^6-7x+5}\right]$.

38. $y = \frac{\sin x\cos x\tan^3 x}{\sqrt{x}}$

$\ln y = \ln\sin x + \ln\cos x + \ln\tan^3 x - \ln\sqrt{x}$

$\ln y = \ln\sin x + \ln\cos x + 3\ln\tan x - \frac{1}{2}\ln x$

$\frac{1}{y}\frac{dy}{dx} = \frac{1(\cos x)}{\sin x} + \frac{1(-\sin x)}{\cos x} + \frac{3(\sec^2 x)}{\tan x} - \frac{1}{2}\cdot\frac{1}{x}$

$= \cot x - \tan x + \frac{3\sec^2 x}{\tan x} - \frac{1}{2x}$

$\frac{dy}{dx} = \left[\frac{\sin x\cos x\tan^3 x}{\sqrt{x}}\right]\left[\cot x - \tan x + \frac{3\sec^2 x}{\tan x} - \frac{1}{2x}\right]$

Exercise 3.6

1. a) $\lim_{x\to 2}\frac{x^2-4}{x^2+2x-8}$

$= \lim_{x\to 2}\frac{(x+2)\cancel{(x-2)}}{(x+4)\cancel{(x-2)}}$

$= \frac{2+2}{2+4} = \frac{4}{6} = \frac{2}{3}$

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b) $\lim_{x\to\infty} \frac{2x-5}{3x+7}$

$= \lim_{x\to\infty} \frac{2-\frac{5}{x}}{3+\frac{7}{x}} = \frac{2}{3}$

using L'Hopital's rule,

$\lim_{x\to\infty} \frac{2x-5}{3x+7} = \lim_{x\to\infty} \frac{2}{3} = \frac{2}{3}$

2- a) $\lim_{x\to 0} \frac{\sin x}{\tan x}$

$= \lim_{x\to 0} \frac{\sin x}{\sin x/\cos x} = \lim_{x\to 0} \cos x$.

$= 1$.

By using L'Hopital's rule.

$\lim_{x\to 0} \frac{\cos x}{\sec^2 x} = \lim_{x\to 0} \cos^3 x = 1$.

b) $\lim_{x\to 1} \frac{x^2-1}{x^3-1}$

$= \lim_{x\to 1} \frac{(x-1)(x+1)}{(x-1)(x^2+x+1)}$

$= \lim_{x\to 1} \frac{x+1}{x^2+x+1}$

$= \frac{2}{3}$

By using L'Hopital's rule,

$= \lim_{x\to 1} \frac{2x}{3x^2} = \frac{2(1)}{3(1)^2} = \frac{2}{3}$

7- $\lim_{x\to 0} \frac{e^x-1}{\sin x}$

By using L'Hopital's Rule

$= \lim_{x\to 0} \frac{e^x}{\cos x} = 1$.

8- $\lim_{x\to 0} \frac{\sin 2x}{\sin 5x}$

$= \lim_{x\to 0} \frac{2\cos 2x}{5\cos 5x} = \frac{2}{5}$.

9- $\lim_{\theta\to 0} \frac{\tan\theta}{\theta}$

$= \frac{\sec^2\theta}{1} = 1$.

10- $\lim_{t\to 0} \frac{te^t}{1-e^t}$

$= \lim_{t\to 0} \quad e^t$

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