C:/Users/torsten/GitHub/colore/ontologies/multidim_space_physcont/ fullphyscont_full.clif

1.
$$\forall x \ \forall y \ \left[\text{osurroundsvoid}(y, x) \leftrightarrow \left(\text{surroundsvoid}(y, x) \land \neg \left(\text{enclosesvoid}(y, x) \right) \land \neg \left(\text{isurroundsvoid}(y, x) \right) \right] \right]$$

2.
$$\forall x \ \forall y \ \left[\left[\text{isurroundsvoid}(y, x) \leftrightarrow \left(\text{surroundsvoid}(y, x) \land \neg \left(P(r(x), \text{voidspaceall}(y)) \right) \right) \right] \right]$$

3.
$$\forall x \ \forall y \ \left[\left[\text{hostsvany}(y, x) \leftrightarrow \left(\text{hostsv}(y, x) \lor \text{hostsv1}(y, x) \lor \text{hostsv2}(y, x) \lor \text{hostsv3}(y, x) \right) \right] \right]$$

4.
$$\forall x \ \forall y \ \left[\left[\operatorname{PO}(y, \operatorname{voidspaceall}(x)) \leftrightarrow \exists v \ \left[\left(\operatorname{hostsvany}(x, v) \land \operatorname{PO}(y, \operatorname{r}(v)) \right) \right] \right] \right]$$

5.
$$\forall x \left[\left[\left(\operatorname{mat}(x) \land \neg \left(\operatorname{ZEX}(\operatorname{voidspaceall}(x)) \right) \right) \rightarrow \exists y \ \exists h \ \left[\left(= (\operatorname{r}(y), \operatorname{voidspaceall}(x)) \land \operatorname{hostsv}(h, y) \right) \right] \right] \right]$$

6.
$$\forall x \ \forall y \ \left[\left[VS(x,y) \leftrightarrow \left(PED(x) \land S(y) \land Cont(y, ch(x)) \land \neg \left(PO(y, r(x)) \right) \right) \right] \right]$$

7.
$$\forall x \ \forall y \ \left| \left[\text{SimpleV}(y) \leftrightarrow \left(\text{ICon}(\mathbf{r}(y)) \land \exists x \ \left[\text{hostsv}(x,y) \right] \right) \right] \right|$$

8.
$$\forall x \ \forall y \ \left[\left[\text{ComplexV}(y) \leftrightarrow \left(\neg \left(\text{ICon}(\mathbf{r}(y)) \right) \wedge \exists x \ \left[\text{hostsv}(x,y) \right] \right) \right] \right]$$

9.
$$\forall x \left[\left[V(x) \leftrightarrow \left(SimpleV(x) \lor ComplexV(x) \right) \right] \right]$$

10.
$$\forall x \ \forall y \ \left[\left[\text{hostsv}(x,y) \rightarrow \left(\text{hosts}(x,y) \land \text{VS}(x,\text{r}(y)) \land \text{StrongC}(\text{r}(x),\text{r}(y)) \right) \right] \right]$$

11.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\text{hostsv}(x,y) \land \text{ComplexV}(y) \land \text{PO}(\text{r}(z),\text{r}(y)) \right) \rightarrow \exists v \ \left[\left(\text{hostsv}(x,v) \land \text{SimpleV}(v) \land \text{PO}(\text{r}(z),\text{r}(y)) \right) \right] \right] \right]$$

12.
$$\forall x \ \forall y \ \left[\left[\left(\text{hosts}(x, y) \land V(y) \right) \rightarrow \neg \left(V(x) \right) \right] \right]$$

13.
$$\forall x \ \forall y \ \left[\left[\left(\text{hostsv}(x,y) \land \text{RPF}(x) \right) \rightarrow \exists z \ \left[\left(\text{hosts}(z,x) \land \neg \left(\text{F}(z) \right) \land \text{hostsv}(z,y) \right) \right] \right] \right]$$

14.
$$\forall x \ \forall y \ \left[\left[\left(\text{hostsv}(x,y) \land \neg \left(F(x) \right) \right) \rightarrow \exists z \ \left[\left(\text{hosts}(x,z) \land \text{RPF}(z) \land \text{hostsv}(z,y) \right) \right] \right] \right]$$

15.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\text{hostsv}(x,y) \land \text{hostsv}(x,z) \land \text{PO}(\text{r}(y),\text{r}(z)) \right) \rightarrow \left(\text{Cont}(\text{r}(y),\text{r}(z)) \lor \text{Cont}(\text{r}(z),\text{r}(y)) \right) \right] \right]$$

16.
$$\forall x \ \forall y \ \forall v \ \left[\left[\left(\text{hostsv}(x, v) \land P(\mathbf{r}(x), \mathbf{r}(y)) \land PED(y) \land \neg \left(\text{DPF}(y) \right) \land \neg \left(\text{Cont}(\mathbf{r}(v), \mathbf{r}(y)) \right) \right) \rightarrow \exists u \ \left[\left(\text{Cont}(\mathbf{r}(v), \mathbf{r}(y)) \land \neg \left(\text{Cont}(\mathbf{r}(v), \mathbf{r}(y)) \right) \right) \right] \right]$$

17.
$$\forall x \ \forall y \ \forall v \ \left[\left[\left(\text{hostsv}(x, v) \land P(r(y), r(x)) \land PED(y) \land \neg \left(DPF(y) \right) \land PO(r(v), \text{ch}(y)) \right) \rightarrow \exists u \ \left[\left(= (r(u), \text{interpolation}) \right) \right] \right] \right]$$

18.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\operatorname{mat}(y) \land \operatorname{mat}(z) \land \operatorname{hostsv}(y, x) \land \operatorname{P}(\operatorname{r}(y), \operatorname{r}(z)) \land \neg \left(\operatorname{PO}(\operatorname{r}(z), \operatorname{r}(x)) \right) \right) \rightarrow \operatorname{hostsv}(z, x) \right] \right]$$

19.
$$\forall x \left[\left[\max(x) \leftrightarrow \left(\text{POB}(x) \lor M(x) \lor \text{RPF}(x) \right) \right] \right]$$

20.
$$\forall x \left[\left[PED(x) \to \left(POB(x) \lor M(x) \lor F(x) \right) \right] \right]$$

21.
$$\forall x \ \left[\left[POB(x) \to PED(x) \right] \right]$$

22.
$$\forall x \left[\left[M(x) \to PED(x) \right] \right]$$

23.
$$\forall x \left[\left[F(x) \to PED(x) \right] \right]$$

24.
$$\forall x \left[\left[POB(x) \rightarrow \neg \left(M(x) \right) \right] \right]$$

25.
$$\forall x \left[\left[POB(x) \rightarrow \neg \left(F(x) \right) \right] \right]$$

26.
$$\forall x \left[\left[M(x) \to \neg \left(F(x) \right) \right] \right]$$

27.
$$\forall x \ \left[\left[NAPO(x) \to POB(x) \right] \right]$$

28.
$$\forall x \ \forall y \ \left[\left[\text{hosts}(x, y) \to \left(\text{PED}(x) \land \text{F}(y) \right) \right] \right]$$

29.
$$\forall x \left[\left[F(x) \leftrightarrow \exists y \left[hosts(y, x) \right] \right] \right]$$

30.
$$\forall x \ \forall y \ \left[\left[\text{hosts}(x,y) \to \neg \left(\text{hosts}(y,x) \right) \right] \right]$$

31.
$$\forall x \left[\left[F(x) \leftrightarrow \left(RPF(x) \lor DPF(x) \right) \right] \right]$$

32.
$$\forall x \left[\left(\neg \left(\text{RPF}(x) \right) \lor \neg \left(\text{DPF}(x) \right) \right) \right]$$

33.
$$\forall x \left[S(\operatorname{ch}(x)) \right]$$

34.
$$\forall x \left[= (\operatorname{ch}(x), \operatorname{ch}(\mathbf{r}(x))) \right]$$

35.
$$\forall x \left[\left[\neg \left(\text{ZEX}(x) \right) \rightarrow \text{ICon}(\text{ch}(x)) \right] \right]$$

36.
$$\forall x \left[= (\operatorname{ch}(x), \operatorname{ch}(\operatorname{ch}(x))) \right]$$

37.
$$\forall x \left[\left[\neg \left(\text{ZEX}(x) \right) \rightarrow \text{Cont}(\mathbf{r}(x), \text{ch}(x)) \right] \right]$$

38.
$$\forall x \left[\left[\left(\neg \left(\operatorname{ZEX}(x) \right) \land \neg \left(\operatorname{Closed}(\mathbf{r}(x)) \right) \right) \to \operatorname{TCont}(\mathbf{r}(x), \operatorname{ch}(x)) \right] \right]$$

39.
$$\forall x \ \forall y \ \left[\left[\operatorname{Cont}(\mathbf{r}(x),\mathbf{r}(y)) \to \operatorname{Cont}(\operatorname{ch}(x),\operatorname{ch}(y)) \right] \right]$$

40.
$$\forall x \ \forall y \ \left[\left[\left(= (\operatorname{ch}(x), \operatorname{ch}(y)) \land \neg \left(\operatorname{ZEX}(x) \right) \right) \to \operatorname{C}(\operatorname{r}(x), \operatorname{r}(y)) \right] \right]$$

41.
$$\forall x \ \forall y \ \Big| \Big[\big(\operatorname{MaxDim}(\mathbf{r}(x)) \land \operatorname{MaxDim}(\mathbf{r}(y)) \land = (\mathbf{r}(x), \operatorname{ch}(x)) \land = (\mathbf{r}(y), \operatorname{ch}(y)) \big) \to = (\operatorname{intersection}(\operatorname{ch}(x), \operatorname{ch}(y))) \Big] \Big] \Big| \Big[\Big(\operatorname{MaxDim}(\mathbf{r}(x)) \land \operatorname{MaxDim}(\mathbf{r}(y)) \land = (\mathbf{r}(x), \operatorname{ch}(x)) \land = (\mathbf{r}(y), \operatorname{ch}(y)) \big) \Big] \Big] \Big| \Big[\Big(\operatorname{MaxDim}(\mathbf{r}(x)) \land \operatorname{MaxDim}(\mathbf{r}(y)) \land = (\mathbf{r}(x), \operatorname{ch}(x)) \land = (\mathbf{r}(y), \operatorname{ch}(y)) \big) \Big] \Big] \Big] \Big| \Big[\Big(\operatorname{MaxDim}(\mathbf{r}(x)) \land \operatorname{MaxDim}(\mathbf{r}(y)) \land = (\mathbf{r}(x), \operatorname{ch}(x)) \land = (\mathbf{r}(y), \operatorname{ch}(y)) \big) \Big] \Big] \Big] \Big| \Big[\Big(\operatorname{MaxDim}(\mathbf{r}(x)) \land \operatorname{MaxDim}(\mathbf{r}(y)) \land = (\mathbf{r}(x), \operatorname{ch}(x)) \land = (\mathbf{r}(x), \operatorname{ch}$$

42.
$$\forall x \ \forall y \ \left[\left[\left(\mathrm{ICont}(\mathbf{r}(x), \mathbf{r}(y)) \land \neg \left(\mathrm{Closed}(\mathrm{difference}(\mathbf{r}(y), \mathbf{r}(x))) \right) \right) \rightarrow \neg \left(= (\mathrm{difference}(\mathbf{r}(y), \mathbf{r}(x)), \mathrm{ch}(\mathrm{difference}(\mathbf{r}(y), \mathbf{r}(x))) \right) \right] \right]$$

43.
$$\forall x \ \forall y \ \left[\left[\left(\mathrm{ICont}(\mathbf{r}(x), \mathbf{r}(y)) \land \mathrm{MaxDim}(\mathbf{r}(x)) \land \mathrm{MaxDim}(\mathbf{r}(y)) \right) \rightarrow \mathrm{Cont}(\mathbf{r}(x), \mathrm{ch}(\mathrm{difference}(\mathbf{r}(y), \mathbf{r}(x)))) \right] \right]$$

44.
$$\forall x \ \forall y \ \left[\left[\left(\neg \left(\operatorname{ZEX}(x) \right) \lor \neg \left(\operatorname{ZEX}(y) \right) \right) \to \exists z \ \exists v \ \left[\left(\operatorname{TSum}(\mathbf{r}(x), \mathbf{r}(y), \mathbf{r}(z)) \land \operatorname{TSum}(\mathbf{ch}(x), \mathbf{ch}(y), \mathbf{r}(v)) \land \operatorname{Co}(\mathbf{r}(x), \mathbf{r}(y), \mathbf{r}(z)) \right] \right] \right]$$

45.
$$\forall x \ \forall y \ \forall z \ \forall v \ \forall w \ \left[\left(\operatorname{EqDim}(x,y) \wedge \operatorname{EqDim}(y,z) \wedge \operatorname{SC}(x,y) \wedge \operatorname{SC}(y,z) \wedge \neg \left(\operatorname{C}(x,z) \right) \wedge \operatorname{TSum}(x,y,v) \wedge \operatorname{TS}(x,y,z) \right] \right]$$

46.
$$\forall x \ \forall y \ \forall z \ \forall v \ \left[\left[\left(\mathrm{TSum}(x,y,z) \land \mathrm{TSum}(x,y,v) \right) \rightarrow \left(\mathrm{S}(x) \land \mathrm{S}(y) \land \mathrm{S}(z) \land = (v,z) \right) \right] \right]$$

47.
$$\forall x \ \forall y \ \forall z \ \left[\left[\mathrm{TSum}(x,y,z) \to \mathrm{TSum}(y,x,z) \right] \right]$$

48.
$$\forall x \ \forall y \ \left[\left[\operatorname{lt}(x,y) \to \operatorname{TSum}(x,y,y) \right] \right]$$

49.
$$\forall x \ \forall y \ \forall z \ \forall v \ \left[\left[\left(\mathrm{TSum}(x,y,z) \land \mathrm{leq}(x,y) \land \mathrm{Cont}(v,y) \right) \rightarrow \mathrm{Cont}(v,z) \right] \right]$$

50.
$$\forall x \ \forall y \ \forall z \ \forall v \ \left[\left[\left(\mathrm{TSum}(x,y,z) \land \mathrm{Cont}(v,z) \land \neg \left(\mathrm{Cont}(v,x) \right) \right) \rightarrow \mathrm{Cont}(\mathrm{difference}(v,x),y) \right] \right]$$

51.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\mathrm{S}(z) \wedge \mathrm{EqDim}(x,y) \wedge \forall v \ \left[\left[\mathrm{PO}(v,z) \leftrightarrow \left(\mathrm{PO}(v,x) \vee \mathrm{PO}(v,y) \right) \right] \right] \right) \rightarrow \mathrm{TSum}(x,y,z) \right] \right]$$

52.
$$\forall x \ \forall y \ \left[\left(\operatorname{EqDim}(x,y) \land \forall z \ \left[\left(\operatorname{Cont}(z,x) \land \operatorname{Cont}(z,y) \land \operatorname{Min}(z) \right) \rightarrow \exists u \ \exists v \ \left[\left(\operatorname{P}(u,x) \land \operatorname{P}(v,y) \land \operatorname{BCont}(z,y) \land \operatorname{Min}(z) \right) \right] \right] \right]$$

53.
$$\forall x \left[\left[\left(S(x) \land \neg \left(ZEX(x) \right) \right) \rightarrow Cont(x, CUni) \right] \right]$$

54.
$$\forall x \left[\left[\left(S(x) \land \neg \left(ZEX(x) \right) \right) \rightarrow \exists y \left[\left(P(y, x) \land Min(y) \right) \right] \right] \right]$$

55.
$$\forall x \ \forall y \ \left[\left[\left(S(x) \land S(y) \land BCont(x,y) \right) \rightarrow \left(Cont(x,y) \land Inc(x,y) \right) \right] \right]$$

56.
$$\forall x \ \forall y \ \forall v \ \forall z \ \left[\left[\left(\mathbf{S}(x) \land \mathbf{S}(y) \land \mathbf{S}(v) \land \mathbf{S}(z) \land \mathbf{SC}(x,y) \land \mathbf{Min}(x) \land \mathbf{P}(x,v) \land \mathbf{Cont}(y,v) \land \mathbf{Cont}(z,x) \land \mathbf{Cont}(x,v) \land \mathbf{Cont}(x,$$

57.
$$\forall x \ \forall y \ \forall z \ \forall v \ \left[\left[\left(\mathbf{S}(x) \land \mathbf{S}(y) \land \mathbf{S}(v) \land \mathbf{S}(z) \land \mathbf{SC}(x,y) \land \mathbf{P}(x,v) \land \mathbf{P}(y,v) \land \mathbf{Cont}(z,x) \land \mathbf{Cont}(z,y) \land \mathbf{Cover}(x,y) \land \mathbf{Cover}(x,y$$

58.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(S(x) \land S(y) \land S(z) \land BCont(x,y) \land P(y,z) \land \forall v \ \forall w \ \left[\left[\left(S(v) \land S(w) \land P(v,z) \land \neg \left(PO(v,y) \land \neg (PO(v,y) \land (PO(v,y) \land \neg (PO(v,y) \land \neg (PO(v,y) \land \neg (PO(v,y) \land (PO(v,y)$$

59.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(S(x) \land S(y) \land S(z) \land BCont(x,y) \land Cont(z,x) \right) \rightarrow BCont(z,y) \right] \right]$$

60.
$$\forall x \ \forall y \ \left[\left(\mathbf{S}(x) \land \mathbf{S}(y) \land \mathbf{BCont}(x,y) \right) \leftrightarrow \left(\mathbf{S}(x) \land \mathbf{S}(y) \land \neg \left(\mathbf{ZEX}(x) \right) \land \forall z \ \left[\left[\left(\mathbf{P}(z,x) \land \mathbf{Min}(z) \right) \rightarrow \mathbf{BCont}(x,y) \right) \right] \right] \right]$$

61.
$$\forall x \ \forall y \ \left[\left[\mathrm{TCont}(x,y) \leftrightarrow \left(\mathrm{Cont}(x,y) \land \exists z \ \left[\left(\mathrm{Cont}(z,x) \land \mathrm{BCont}(z,y) \right) \right] \right) \right] \right]$$

62.
$$\forall x \ \forall y \ \left[\left[SC(x,y) \to C(x,y) \right] \right]$$

63.
$$\forall x \ \forall y \ \left[\left[SC(x,y) \to \neg \exists z \ \left[\left(Cont(z,x) \land P(z,y) \right) \right] \right] \right]$$

64.
$$\forall x \ \forall y \ \left[\left[\mathrm{SC}(x,y) \to \neg \exists z \ \left[\left(\mathrm{P}(z,x) \wedge \mathrm{Cont}(z,y) \right) \right] \right] \right]$$

65.
$$\forall x \ \forall y \ \left[\left(\mathrm{C}(x,y) \land \forall z \ \left[\left(\neg \left(\mathrm{Cont}(z,x) \right) \lor \neg \left(\mathrm{Cont}(z,y) \right) \lor \left(\neg \left(\mathrm{P}(z,x) \right) \land \neg \left(\mathrm{P}(z,y) \right) \right) \right) \right] \right) \to \mathrm{SC}(x,y) \right]$$

66.
$$\forall x \left[\neg \left(SC(x, x) \right) \right]$$

67.
$$\forall x \ \forall y \ \left[\left[SC(x, y) \to SC(y, x) \right] \right]$$

68.
$$\forall x \ \forall y \ \left[\left[\mathrm{SC}(x,y) \to \exists z \ \left[\left(\mathrm{lt}(z,x) \wedge \mathrm{lt}(z,y) \wedge \mathrm{Cont}(z,x) \wedge \mathrm{Cont}(z,y) \right) \right] \right] \right]$$

69.
$$\forall x \ \forall y \ \left[SC(x,y) \leftrightarrow \left(\exists z \ \left[\left(Cont(z,x) \land Cont(z,y) \right) \right] \land \forall z \ \left[\left[\left(Cont(z,x) \land Cont(z,y) \right) \rightarrow \left(leq(z,x) \land Cont(z,y) \right) \right] \right] \right] \right]$$

70.
$$\forall x \ \forall y \ \left[\left[\operatorname{EqDim}(x, y) \leftrightarrow \left(\operatorname{leq}(x, y) \land \operatorname{leq}(y, x) \right) \right] \right]$$

71.
$$\forall x \ \forall y \ \left[\left[\operatorname{leq}(x, y) \to S(x) \right] \right]$$

72.
$$\forall x \ \forall y \ \left[\left[\operatorname{leq}(x, y) \to S(y) \right] \right]$$

73.
$$\forall x \left[\left[ZEX(x) \to S(x) \right] \right]$$

74.
$$\forall x \left[\left[S(x) \to leq(x, x) \right] \right]$$

75.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\operatorname{leq}(x,y) \land \operatorname{leq}(y,z) \right) \rightarrow \operatorname{leq}(x,z) \right] \right]$$

76.
$$\forall x \ \forall y \ \left[\left[\left(\operatorname{ZEX}(x) \wedge \operatorname{ZEX}(y) \right) \to = (x, y) \right] \right]$$

77.
$$\forall x \ \forall y \ \left[\left[\left(\operatorname{ZEX}(x) \land \operatorname{S}(y) \right) \to \operatorname{leq}(x, y) \right] \right]$$

78.
$$\forall x \ \forall y \ \left[\left[\text{Cont}(x,y) \to \text{leq}(x,y) \right] \right]$$

79.
$$\exists x \ [\text{MinDim}(x)]$$

80.
$$\forall x \left[\left[\operatorname{MaxDim}(x) \leftrightarrow \left(\operatorname{S}(x) \land \neg \left(\operatorname{ZEX}(x) \right) \land \forall y \left[\left[\left[\operatorname{S}(y) \rightarrow \operatorname{leq}(y, x) \right] \right] \right) \right] \right]$$

81.
$$\forall x \left[\left[\operatorname{MinDim}(x) \leftrightarrow \left(\operatorname{S}(x) \land \neg \left(\operatorname{ZEX}(x) \right) \land \forall y \left[\left[\left(\operatorname{S}(y) \land \neg \left(\operatorname{ZEX}(y) \right) \right) \rightarrow \operatorname{leq}(x,y) \right] \right] \right) \right] \right]$$

82.
$$\forall x \left[\left[\left(S(x) \land \neg \left(ZEX(x) \right) \right) \leftrightarrow Cont(x, x) \right] \right]$$

83.
$$\forall x \ \forall y \ \left[\left[\left(\operatorname{Cont}(x,y) \wedge \operatorname{Cont}(y,x) \right) \to =(x,y) \right] \right]$$

84.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\operatorname{Cont}(x,y) \wedge \operatorname{Cont}(y,z) \right) \to \operatorname{Cont}(x,z) \right] \right]$$

85.
$$\forall x \ \forall y \ \left[\left[\operatorname{ZEX}(x) \to \left(\operatorname{S}(x) \land \neg \left(\operatorname{Cont}(y, x) \right) \land \neg \left(\operatorname{Cont}(x, y) \right) \right) \right] \right]$$

86.
$$\forall x \, \forall y \, \left[\left[\left(\operatorname{ZEX}(x) \wedge \operatorname{ZEX}(y) \right) \to =(x,y) \right] \right]$$

87.
$$\forall x \ \forall y \ \left[\left[P(x,y) \leftrightarrow \left(Cont(x,y) \land EqDim(x,y) \right) \right] \right]$$

88.
$$\forall x \ \forall y \ \left[\left[\mathrm{C}(x,y) \leftrightarrow \exists z \ \left[\left(\mathrm{Cont}(z,x) \land \mathrm{Cont}(z,y) \right) \right] \right] \right]$$

89.
$$\forall x \left[\neg \left(\operatorname{Inc}(x, x) \right) \right]$$

90.
$$\forall x \ \forall y \ \left[\left[\operatorname{Inc}(x,y) \to \operatorname{Inc}(y,x) \right] \right]$$

91.
$$\forall x \ \forall y \ \left[\left[\operatorname{EqDim}(x, y) \to \neg \left(\operatorname{Inc}(x, y) \right) \right] \right]$$

92.
$$\forall x \, \forall y \, \left[\left[\operatorname{Inc}(x,y) \to \left(\operatorname{lt}(x,y) \vee \operatorname{lt}(y,x) \right) \right] \right]$$

93.
$$\forall x \ \forall y \ \left[\left[\left(\operatorname{Cont}(x,y) \wedge \operatorname{lt}(x,y) \right) \to \operatorname{Inc}(x,y) \right] \right]$$

94.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\operatorname{Inc}(x,y) \land P(y,z) \right) \rightarrow \operatorname{Inc}(x,z) \right] \right]$$

95.
$$\forall x \ \forall y \ \left[\operatorname{Inc}(x,y) \leftrightarrow \left(\exists z \ \left[\left(\operatorname{leq}(z,x) \land \neg \left(\operatorname{EqDim}(z,x) \right) \land \operatorname{Cont}(z,x) \land \operatorname{P}(z,y) \right) \right] \lor \exists z \ \left[\left(\operatorname{leq}(z,y) \land \neg \left(\operatorname{EqDim}(z,x) \land \operatorname{P}(z,y) \right) \right) \right] \lor \exists z \ \left[\left(\operatorname{leq}(z,y) \land \neg \left(\operatorname{EqDim}(z,x) \land \operatorname{P}(z,y) \right) \right) \right] \lor \exists z \ \left[\left(\operatorname{leq}(z,y) \land \neg \left(\operatorname{EqDim}(z,x) \land \operatorname{P}(z,y) \right) \right) \right] \lor \exists z \ \left[\left(\operatorname{leq}(z,y) \land \neg \left(\operatorname{EqDim}(z,x) \land \operatorname{P}(z,y) \right) \right) \right] \lor \exists z \ \left[\left(\operatorname{leq}(z,y) \land \neg \left(\operatorname{EqDim}(z,x) \land \operatorname{P}(z,y) \right) \right) \right] \lor \exists z \ \left[\left(\operatorname{leq}(z,y) \land \neg \left(\operatorname{EqDim}(z,y) \land \neg \left(\operatorname{EqDim}(z,y) \land \operatorname{P}(z,y) \right) \right) \right] \lor \exists z \ \left[\left(\operatorname{leq}(z,y) \land \neg \left(\operatorname{EqDim}(z,y) \land$$

96.
$$\forall x \left[\left[\left(S(x) \land \neg \left(ZEX(x) \right) \right) \rightarrow PO(x, x) \right] \right]$$

97.
$$\forall x \ \forall y \ \left[\left[PO(x, y) \to PO(y, x) \right] \right]$$

98.
$$\forall x \ \forall y \ \left[\left[PO(x, y) \to EqDim(x, y) \right] \right]$$

99.
$$\forall x \ \forall y \ \left[\left[PO(x, y) \leftrightarrow \exists z \ \left[\left(P(z, x) \land P(z, y) \right) \right] \right] \right]$$

100.
$$\forall x \left[\left[\operatorname{Max}(x) \leftrightarrow \left(\operatorname{S}(x) \land \neg \left(\operatorname{ZEX}(x) \right) \land \forall y \left[\neg \left(\operatorname{PP}(x, y) \right) \right] \right) \right] \right]$$

101.
$$\forall x \left[\left[\text{Min}(x) \leftrightarrow \left(\text{S}(x) \land \neg \left(\text{ZEX}(x) \right) \land \forall y \left[\neg \left(\text{PP}(y, x) \right) \right] \right) \right] \right]$$

102.
$$\forall x \ \forall y \ \left[\left[PP(x,y) \leftrightarrow \left(P(x,y) \land \neg \left(=(x,y) \right) \right) \right] \right]$$

103.
$$\forall x \ \forall y \ \left[\left[\mathrm{BCont}(x,y) \to \left(\mathrm{Cont}(x,y) \land \neg \left(\mathrm{EqDim}(x,y) \right) \right) \right] \right]$$

104.
$$\forall x \ \forall y \ \forall v \ \forall z \ \left[\left[\left(\operatorname{SC}(x,y) \land \operatorname{Min}(x) \land \operatorname{P}(x,v) \land \operatorname{Cont}(y,v) \land \operatorname{Cont}(z,x) \land \operatorname{Cont}(z,y) \right) \rightarrow \operatorname{BCont}(z,x) \right] \right]$$

105.
$$\forall x \ \forall y \ \forall z \ \forall v \ \left[\left[\left(\operatorname{SC}(x,y) \land \operatorname{P}(x,v) \land \operatorname{P}(y,v) \land \operatorname{Cont}(z,x) \land \operatorname{Cont}(z,y) \land \operatorname{Covers}(v,z) \right) \rightarrow \neg \left(\operatorname{BCont}(z,v) \right) \right] \right]$$

106.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\operatorname{BCont}(x,y) \land \operatorname{P}(y,z) \land \forall v \ \forall w \ \left[\left[\left(\operatorname{P}(v,z) \land \neg \left(\operatorname{PO}(v,y) \right) \land \operatorname{P}(w,x) \right) \rightarrow \neg \left(\operatorname{Cont}(w,v) \right) \right] \right] \right] \right]$$

107.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\mathrm{BCont}(x,y) \wedge \mathrm{Cont}(z,x) \right) \to \mathrm{BCont}(z,y) \right] \right]$$

108.
$$\forall x \ \forall y \ \left[\left[\text{Covers}(x,y) \leftrightarrow \left(\text{lt}(y,x) \land \forall z \ \left[\left[\text{S}(z) \rightarrow \neg \left(\text{lt}(y,z) \land \text{lt}(z,x) \right) \right] \right] \right) \right] \right]$$

109.
$$\forall x \ \forall y \ \left[\left[\operatorname{gt}(x,y) \leftrightarrow \operatorname{lt}(y,x) \right] \right]$$

110.
$$\forall x \ \forall y \ \left[\left[\gcd(x, y) \leftrightarrow \deg(y, x) \right] \right]$$

111.
$$\forall x \ \forall y \ \left[\left[\operatorname{lt}(x,y) \leftrightarrow \left(\operatorname{leq}(x,y) \land \neg \left(\operatorname{EqDim}(x,y) \right) \right) \right] \right]$$

112.
$$\forall x \ \forall y \ \left[\left[\mathrm{ICont}(x,y) \leftrightarrow \left(\mathrm{Cont}(x,y) \land \forall z \ \left[\left[\mathrm{Cont}(z,x) \rightarrow \neg \left(\mathrm{BCont}(z,y) \right) \right] \right] \right) \right] \right]$$

113.
$$\forall x \left[\left[\text{Closed}(x) \leftrightarrow \forall y \left[\neg \left(\text{BCont}(y, x) \right) \right] \right] \right]$$

114.
$$\forall x \ \forall y \ \left[\left[\left(S(x) \land S(y) \land \neg \left(C(x,y) \right) \right) \leftrightarrow ZEX(intersection(x,y)) \right] \right]$$

115.
$$\forall x \ \forall y \ \left[\left[\left(S(x) \land S(y) \land \neg \left(ZEX(intersection(x,y)) \right) \right) \rightarrow Cont(intersection(x,y),x) \right] \right]$$

116.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\operatorname{Cont}(z, x) \wedge \operatorname{Cont}(z, y) \right) \to \operatorname{leq}(z, \operatorname{intersection}(x, y)) \right] \right]$$

117.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\operatorname{Cont}(z, x) \wedge \operatorname{Cont}(z, y) \wedge \operatorname{EqDim}(z, \operatorname{intersection}(x, y)) \right) \leftrightarrow \operatorname{P}(z, \operatorname{intersection}(x, y)) \right] \right]$$

118.
$$\forall x \ \forall y \ \left[\left[\left(S(x) \land S(y) \land \neg \left(ZEX(difference(x,y)) \right) \right) \rightarrow EqDim(x, difference(x,y)) \right] \right]$$

119.
$$\forall x \ \forall y \ \left[\left[\operatorname{lt}(y, x) \to = (x, \operatorname{difference}(x, y)) \right] \right]$$

120.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\operatorname{leq}(x,y) \land \operatorname{Cont}(z,x) \land \operatorname{lt}(\operatorname{intersection}(z,y),z) \right) \rightarrow \operatorname{Cont}(z,\operatorname{difference}(x,y)) \right] \right]$$

121.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\operatorname{leq}(x,y) \wedge \operatorname{Cont}(z,\operatorname{difference}(x,y)) \right) \to \operatorname{Cont}(z,x) \right] \right]$$

122.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\operatorname{leq}(x,y) \land \mathrm{P}(z,\operatorname{difference}(x,y)) \right) \to \operatorname{lt}(\operatorname{intersection}(z,y),z) \right] \right]$$

123.
$$\forall x \ \forall y \ \left[\left[\text{ZEX}(\text{difference}(x,y)) \leftrightarrow \left(\text{ZEX}(x) \lor \text{Cont}(x,y) \right) \right] \right]$$

124.
$$\forall x \left[\left(\neg \left(\text{PED}(x) \right) \lor \neg \left(\text{S}(x) \right) \right) \right]$$

125.
$$\forall x \ [S(\mathbf{r}(x))]$$

126.
$$\forall x \left[\left[S(x) \leftrightarrow =(x, r(x)) \right] \right]$$

127.
$$\forall x \ \forall y \ \left[\left[\text{Cont}(x,y) \to \left(S(x) \land S(y) \right) \right] \right]$$

128.
$$\forall x \ \forall y \ \left[\left[\operatorname{lt}(x,y) \to \left(\operatorname{S}(x) \wedge \operatorname{S}(y) \right) \right] \right]$$

129.
$$\forall x \left[\left[ZEX(x) \to S(x) \right] \right]$$

130.
$$\forall x \ \forall y \ \left[\left[\mathrm{BCont}(x,y) \to \left(\mathrm{S}(x) \land \mathrm{S}(y) \right) \right] \right]$$

131.
$$\forall x \left[\left[PED(x) \to MaxDim(r(x)) \right] \right]$$

132.
$$\forall x \ \forall y \ \left[\left[\text{hosts}(x,y) \to \left[\text{RPF}(y) \leftrightarrow \text{P}(\text{r}(y),\text{r}(x)) \right] \right] \right]$$

133.
$$\forall x \ \forall y \ \left[\left[\text{hosts}(x,y) \to \left[\text{DPF}(y) \leftrightarrow \neg \left(\text{PO}(\mathbf{r}(x), \mathbf{r}(y)) \right) \right] \right] \right]$$

134.
$$\forall x \left[\left[\text{ICon}(x) \leftrightarrow \left(\text{Con}(x) \land \forall y \left[\left[\text{PP}(y, x) \rightarrow \text{Covers}(x, \text{intersection}(y, \text{difference}(x, y))) \right] \right] \right) \right] \right]$$

135.
$$\forall x \left[\left[\operatorname{Con}(x) \leftrightarrow \left(\operatorname{S}(x) \land \forall y \left[\left[\operatorname{PP}(y, x) \rightarrow \operatorname{SC}(y, \operatorname{difference}(x, y)) \right] \right] \right) \right] \right]$$

136.
$$\forall v \ \forall o \ \left[\left[\text{PO}(v, \text{porespace}(o)) \leftrightarrow \exists m \ \left[\left(\text{DK1}(m, o) \land \forall u \ \left[\left[\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right] \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right) \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right] \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right] \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right] \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right] \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v, \text{r}(u)) \right) \right] \right] \land \exists u \ \left[\left(\text{hostsv}(o, u) \rightarrow \neg \left(\text{PO}(v,$$

137.
$$\forall v \ \forall o \ \left[\left[\text{PO}(v, \text{voidspace}(o)) \leftrightarrow \left(\text{PO}(v, \text{porespace}(o)) \lor \exists u \ \left[\left(\text{hostsv}(o, u) \land \text{PO}(v, \text{r}(u)) \right) \right] \right) \right] \right]$$

138.
$$\forall o \left[\left[\neg \left(\operatorname{ZEX}(\operatorname{porespace}(o)) \right) \rightarrow \exists v \ \exists m \ \left[\left(= (\operatorname{r}(v), \operatorname{porespace}(o)) \land \operatorname{hostsv}(m, v) \land \operatorname{DK1}(m, o) \right) \right] \right] \right]$$

139.
$$\forall o \left[\left[\neg \left(\operatorname{ZEX}(\operatorname{voidspace}(o)) \right) \rightarrow \exists v \ \exists m \ \left[\left(= (\operatorname{r}(v), \operatorname{voidspace}(o)) \land \operatorname{hostsv}(m, v) \land \operatorname{DK1}(m, o) \right) \right] \right] \right]$$

140.
$$\forall v \ \forall o \ \left[\text{PO}(v, \text{convoidspace}(o)) \leftrightarrow \exists u \ \left[(\text{PO}(v, u) \land \text{ICon}(u) \land \text{Cont}(u, \text{voidspace}(o)) \land \text{StrongC}(u, \text{differential options of the property o$$

141.
$$\forall v \ \forall o \ \left[\text{PO}(v, \text{conporespace}(o)) \leftrightarrow \exists u \ \left[\text{PO}(v, u) \land \text{ICon}(u) \land \text{Cont}(u, \text{porespace}(o)) \land \text{StrongC}(u, \text{differential properties of the properti$$

142.
$$\forall x \left[\left(\neg \left(\text{PED}(x) \right) \lor \neg \left(\text{S}(x) \right) \right) \right]$$

143.
$$\forall x \left[S(r(x)) \right]$$

144.
$$\forall x \left[\left[S(x) \leftrightarrow =(x, r(x)) \right] \right]$$

145.
$$\forall x \ \forall y \ \left[\left[\operatorname{Cont}(x, y) \to \left(\operatorname{S}(x) \wedge \operatorname{S}(y) \right) \right] \right]$$

146.
$$\forall x \ \forall y \ \left[\left[\mathrm{lt}(x,y) \to \left(\mathrm{S}(x) \wedge \mathrm{S}(y) \right) \right] \right]$$

147.
$$\forall x \left[\left[ZEX(x) \to S(x) \right] \right]$$

148.
$$\forall x \ \forall y \ \left[\left[\mathrm{BCont}(x,y) \to \left(\mathrm{S}(x) \land \mathrm{S}(y) \right) \right] \right]$$

149.
$$\forall x \left[\left[PED(x) \to MaxDim(r(x)) \right] \right]$$

150.
$$\forall x \ \forall y \ \left[\left[\mathrm{DK1}(x,y) \to \mathrm{P}(\mathrm{r}(x),\mathrm{r}(y)) \right] \right]$$

151.
$$\forall x \ \forall y \ \left[\left[\text{hosts}(x,y) \rightarrow \left[\text{RPF}(y) \leftrightarrow \text{P}(\text{r}(y),\text{r}(x)) \right] \right] \right]$$

152.
$$\forall x \ \forall y \ \forall m \ \forall n \ \left[\left[\left(\text{hosts}(x,y) \land \text{RPF}(y) \land \text{DK1}(m,x) \land \text{DK1}(n,y) \right) \rightarrow \text{P}(\text{r}(n),\text{r}(m)) \right] \right]$$

153.
$$\forall x \ \forall y \ \left[\left[\text{hosts}(x,y) \to \left[\text{DPF}(y) \leftrightarrow \neg \left(\text{PO}(\mathbf{r}(x), \mathbf{r}(y)) \right) \right] \right] \right]$$

154.
$$\forall x \ \forall y \ \left[\left[\mathrm{DK1}(x,y) \to \mathrm{M}(x) \right] \right]$$

155.
$$\forall x \, \forall y \, \left[\left[\mathrm{DK1}(x, y) \to \left(\mathrm{POB}(y) \vee \mathrm{RPF}(y) \right) \right] \right]$$

156.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\mathrm{DK1}(x,y) \wedge \mathrm{DK1}(z,y) \right) \to =(x,z) \right] \right]$$

157.
$$\forall y \left[\left[\left(\text{POB}(y) \vee \text{RPF}(y) \right) \to \exists x \left[\text{DK1}(x, y) \right] \right] \right]$$

158.
$$\forall x \ \forall v \ \left[\left[\text{hostsv}(x,v) \rightarrow = (\text{op}(x,v), \text{intersection}(\mathbf{r}(v), \text{difference}(CUni, \text{sum}(\mathbf{r}(x), \mathbf{r}(v))))) \right] \right]$$

159.
$$\forall x \ \forall y \ \left[= (\text{sum}(x, y), \text{sum}(y, x)) \right]$$

160.
$$\forall x \ \forall y \ \left[\left[\operatorname{lt}(x,y) \to = (y, \operatorname{sum}(x,y)) \right] \right]$$

161.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\operatorname{leq}(x,y) \wedge \operatorname{Cont}(z,y) \right) \to \operatorname{Cont}(z,\operatorname{sum}(x,y)) \right] \right]$$

162.
$$\forall x \ \forall y \ \forall z \ \left[\left[\left(\operatorname{Cont}(z, \operatorname{sum}(x, y)) \land \neg \left(\operatorname{Cont}(z, x) \right) \right) \rightarrow \operatorname{Cont}(\operatorname{difference}(z, x), y) \right] \right]$$

163.
$$\forall x \ \forall y \ \left[\left[\text{surroundsvoid}(y, x) \leftrightarrow \left(\text{surrounds}(y, x) \land \text{V}(x) \right) \right] \right]$$

164.
$$\forall x \ \forall y \ \left[\left[\text{surrounds}(y, x) \leftrightarrow \left(\text{detcont}(y, x) \land \text{mat}(y) \right) \right] \right]$$

165.
$$\forall x \ \forall y \ \left[\left[\operatorname{detcont}(y, x) \leftrightarrow \left(\operatorname{fullphyscont}(y, x) \land \neg \left(\operatorname{matdep}(y, x) \right) \right) \right] \right]$$

166.
$$\forall x \ \forall y \ \left[\left[\operatorname{matdep}(x, y) \to \operatorname{dep}(x, y) \right] \right]$$

167.
$$\forall x \ \forall y \ \left[\left[\operatorname{matdep}(x, y) \to \left(\operatorname{mat}(x) \lor \operatorname{V}(x) \right) \right] \right]$$

168.
$$\forall x \ \forall y \ \left[\left[\operatorname{matdep}(x, y) \to \left(\operatorname{mat}(y) \lor \operatorname{V}(y) \right) \right] \right]$$

169.
$$\forall x \ \forall y \ \left[\left[\left(\operatorname{matdep}(x, y) \land \operatorname{V}(x) \land \operatorname{mat}(y) \right) \rightarrow \left(\operatorname{PO}(\operatorname{r}(x), \operatorname{r}(y)) \lor \exists v \ \left[\left(\operatorname{hostsv}(y, v) \land \operatorname{PO}(\operatorname{r}(x), \operatorname{r}(v)) \right) \right] \right) \right] \right]$$

170.
$$\forall x \ \forall y \ \left[\left[\left(\mathrm{matdep}(x,y) \wedge \mathrm{V}(x) \wedge \mathrm{V}(y) \right) \to \mathrm{PO}(\mathrm{r}(x),\mathrm{r}(y)) \right] \right]$$

171.
$$\forall x \ \forall y \ \left[\left[\text{matdep}(x, y) \to \left(\text{PO}(\mathbf{r}(x), \mathbf{r}(y)) \lor \text{StrongC}(\mathbf{r}(x), \mathbf{r}(y)) \right) \right] \right]$$

172.
$$\forall x \ \forall y \ \left[\left[\left(\operatorname{matdep}(x, y) \land \operatorname{mat}(x) \land \operatorname{mat}(y) \right) \to \operatorname{PO}(\operatorname{r}(x), \operatorname{r}(y)) \right] \right]$$

173.
$$\forall x \ \forall y \ \left[\left(\operatorname{mat}(x) \land \operatorname{V}(y) \right) \rightarrow \left[\operatorname{matdep}(x,y) \leftrightarrow \exists z \ \exists hy \ \left[\left(\operatorname{submat}(z,x) \land \operatorname{submat}(z,hy) \land \operatorname{hostsv}(hy,y) \land \right) \right] \right] \right]$$

174.
$$\forall x \ \forall y \ \left[\left[(V(x) \land V(y)) \rightarrow \left[\text{matdep}(x,y) \leftrightarrow \left(PO(r(x),r(y)) \land \exists hx \ \exists hy \ \exists z \ \left[(\text{hostsv}(hx,x) \land \text{hostsv}(hy,x)) \right] \right] \right] \right] \right]$$

175.
$$\forall x \ \forall y \ \left[\left[\text{StrongC}(x,y) \leftrightarrow \left(\text{SC}(x,y) \land \text{EqDim}(x,y) \land \text{Covers}(x, \text{intersection}(x,y)) \right) \right] \right]$$

176.
$$\forall x \, \forall y \, \left[\left[\operatorname{dep}(x, y) \to \operatorname{PED}(x) \right] \right]$$

177.
$$\forall x \ \forall y \ \left[\left[\operatorname{dep}(x, y) \to \operatorname{PED}(y) \right] \right]$$

178.
$$\forall x \ \forall y \ \left[\left[\operatorname{dep}(x, y) \to \operatorname{dep}(y, x) \right] \right]$$

179.
$$\forall x \ \forall y \ \left[\left[\left(\operatorname{mat}(x) \land \operatorname{mat}(y) \right) \rightarrow \left[\operatorname{dep}(x,y) \leftrightarrow \operatorname{PO}(\operatorname{r}(x),\operatorname{r}(y)) \right] \right] \right]$$

180.
$$\forall x \ \forall y \ \left[\left[\left(V(x) \land \text{mat}(y) \right) \rightarrow \left[\text{dep}(y, x) \leftrightarrow \text{hostsvany}(y, x) \right] \right] \right]$$

181.
$$\forall x \ \forall y \ \left[\left(\mathbf{V}(x) \land \mathbf{V}(y) \right) \rightarrow \left[\mathbf{dep}(x,y) \leftrightarrow \left(\mathbf{PO}(\mathbf{r}(x),\mathbf{r}(y)) \land \exists hx \ \exists hy \ \left[\left(\mathbf{hostsv}(hx,x) \land \mathbf{hostsv}(hy,y) \land \mathbf{r}(y) \right) \right] \right] \right] \right]$$

182.
$$\forall x \left[\left[PED(x) \to dep(x, x) \right] \right]$$

183.
$$\forall x \ \forall y \ \left[\left[\text{fullphyscont}(y, x) \leftrightarrow \left(\text{PED}(x) \land \text{PED}(y) \land \text{P}(\text{r}(x), \text{ch}(y)) \land \left[\neg \left(\text{mat}(y) \right) \rightarrow \text{P}(\text{r}(x), \text{r}(y)) \right] \right) \right] \right]$$

184.
$$\forall x \ \forall y \ \left| \begin{array}{c} \text{enclosesvoid}(y,x) \leftrightarrow \left(\text{surroundsvoid}(y,x) \land \exists v \ \left[\left(\text{hostsvany}(y,v) \land \text{hostscavity}(y,v) \land \text{P}(\textbf{r}(x),\textbf{r}(x$$

185.
$$\forall x \ \forall y \ \left[\left[\text{hostsh}(x,y) \leftrightarrow \left(\text{hostsv}(x,y) \land \text{ICon}(\textbf{r}(x)) \right) \right] \right]$$

186.
$$\forall x \ \forall y \ \left[\left[\text{hostsg}(x, y) \leftrightarrow \left(\text{hostsv}(x, y) \land \neg \left(\text{ICon}(\mathbf{r}(x)) \right) \right) \right] \right]$$

187.
$$\forall y \ \left[\left[\operatorname{Hole}(y) \leftrightarrow \exists x \ \left[\operatorname{hostsh}(x, y) \right] \right] \right]$$

188.
$$\forall y \left[\left[\operatorname{Gap}(y) \leftrightarrow \exists x \left[\operatorname{hostsg}(x, y) \right] \right] \right]$$

189.
$$\forall x \ \forall y \ \left[\left[\text{hostscavity}(x,y) \leftrightarrow \left(\text{hostsv}(x,y) \land \neg \left(\text{Covers}(\mathbf{r}(x), \text{op}(x,y)) \right) \right) \right] \right]$$

190.
$$\forall y \ \left[\left[\text{CAVITY}(y) \leftrightarrow \exists x \ \left[\text{hostscavity}(x, y) \right] \right] \right]$$

191.
$$\forall x \ \forall y \ \left[\left[\text{hostscavityi}(x,y) \leftrightarrow \left(\text{hostscavity}(x,y) \land \text{ZEX}(\text{op}(x,y)) \right) \right] \right]$$

192.
$$\forall x \ \forall y \ \left[\left[\text{hostscavityt}(x,y) \leftrightarrow \left(\text{hostscavity}(x,y) \land \neg \left(\text{ZEX}(\text{op}(x,y)) \right) \right) \right] \right]$$

193.
$$\forall x \ \forall y \ \left[\left[\text{hostshollow}(x,y) \leftrightarrow \left(\text{hostsv}(x,y) \land \text{Covers}(\textbf{r}(x), \text{op}(x,y)) \land \text{ICon}(\text{op}(x,y)) \right) \right] \right]$$

194.
$$\forall y \ \left[\left[\mathrm{HOL}(y) \leftrightarrow \exists x \ \left[\mathrm{hostshollow}(x,y) \right] \right] \right]$$

195.
$$\forall x \ \forall y \ \left[\left[\text{hoststunnel}(x, y) \leftrightarrow \left(\text{hostsv}(x, y) \land \text{Covers}(\mathbf{r}(x), \text{op}(x, y)) \land \neg \left(\text{ICon}(\text{op}(x, y)) \right) \right) \right] \right]$$

196.
$$\forall y \ \left[\left[\text{TUN}(y) \leftrightarrow \exists x \ \left[\text{hoststunnel}(x, y) \right] \right] \right]$$

197.
$$\forall x \ \forall y \ \left[\text{hostsve}(x,y) \leftrightarrow \left(\text{hostsv}(x,y) \land \exists z \ \left[\left(P(z,\text{op}(x,y)) \land \forall u \ \left[\left[\left(\text{hostsv}(x,u) \land \text{EqDim}(\text{intersection}) \right) \right] \right] \right] \right] \right] \right] \right]$$

198.
$$\forall x \ \forall y \ \left[\left[\text{hostsvi}(x,y) \leftrightarrow \left(\text{hostsv}(x,y) \land \neg \left(\text{hostsve}(x,y) \right) \right) \right] \right]$$

200.
$$\forall x \ \forall y \ \left[\left[\text{isurroundsmat}(y, x) \leftrightarrow \left(\text{surroundsmat}(y, x) \land \neg \left(P(r(x), \text{voidspaceall}(y)) \right) \right) \right] \right]$$

201.
$$\forall x \ \forall y \ \left[\left[\text{surroundsmat}(y, x) \leftrightarrow \left(\text{surrounds}(y, x) \land \text{mat}(x) \right) \right] \right]$$

202.
$$\forall x \ \forall y \ \left[\text{enclosesmat}(y, x) \leftrightarrow \left(\text{surroundsmat}(y, x) \land \exists v \ \left[\left(\text{hostsvany}(y, v) \land \text{hostscavity}(y, v) \land P(\mathbf{r}(x), \mathbf{r}(x), \mathbf{r}(x),$$

203.
$$\forall x \ \forall y \ \left[\left[\text{voidinside}(x,y) \leftrightarrow \left(\text{inside}(x,y) \land \text{V}(x) \right) \right] \right]$$

204.
$$\forall x \ \forall y \ \left[\left[\text{inside}(x, y) \leftrightarrow \left(\text{detcont}(y, x) \land V(y) \right) \right] \right]$$

205.
$$\forall x \ \forall y \ \left[\left[\text{matfillsinside}(x,y) \leftrightarrow \left(\text{matinside}(x,y) \land = (\text{r}(x),\text{r}(y)) \right) \right] \right]$$

206.
$$\forall x \ \forall y \ \left[\left[\operatorname{matinside}(x, y) \leftrightarrow \left(\operatorname{inside}(x, y) \land \operatorname{mat}(x) \right) \right] \right]$$

207.
$$\forall x \ \forall y \ \left[\left[\text{matsplitinside}(x,y) \leftrightarrow \left(\text{matinside}(x,y) \land \text{PP}(\textbf{r}(x),\textbf{r}(y)) \land \text{ICon}(\textbf{r}(y)) \land \neg \left(\text{ICon}(\text{difference}(\textbf{r}(y),\textbf{r}(y)) \land \neg \left(\textbf{r}(y) \land \neg (y) \land ($$

208.
$$\forall x \ \forall y \ \left[\left[\text{depimmatcontains}(y, x) \leftrightarrow \left(\text{depcont}(y, x) \land V(y) \right) \right] \right]$$

209.
$$\forall x \ \forall y \ \left[\left[\operatorname{depcont}(y, x) \leftrightarrow \left(\operatorname{fullphyscont}(y, x) \land \operatorname{matdep}(y, x) \right) \right] \right]$$

210.
$$\forall x \ \forall y \ \left[\left[\operatorname{depmatcont}(y, x) \leftrightarrow \left(\operatorname{depcont}(y, x) \land \operatorname{mat}(y) \right) \right] \right]$$

211.
$$\forall x \ \forall y \ \left[\left[\text{subvoid}(x,y) \leftrightarrow \left(\text{immatcont}(y,x) \land \exists h \ \left[\left(\text{hostsv}(h,y) \land \text{hostsv}(h,x) \right) \right] \right) \right] \right]$$

212.
$$\forall x \ \forall y \ \left[\left[\text{immatcont}(y, x) \leftrightarrow \left(\text{depcont}(y, x) \land V(x) \land V(y) \right) \right] \right]$$

213.
$$\forall x \ \forall y \ \left[\left[\text{submaterial}(x,y) \leftrightarrow \left(\text{matcont}(y,x) \land \text{P}(\textbf{r}(x),\textbf{r}(y)) \right) \right] \right]$$

214.
$$\forall x \ \forall y \ \left[\left[\operatorname{matcont}(y, x) \leftrightarrow \left(\operatorname{depcont}(y, x) \land \operatorname{mat}(x) \land \operatorname{mat}(y) \right) \right] \right]$$