From data to vectors & NLP 101
Methods in AI research

Roxana Rădulescu
September 2025

Utrecht University

Credit: Dong Nguyen

Practicalities

Literature for today:

- Hal Daumé III, A Course in Machine Learning, 3.1-3.3 (Geometry and Nearest Neighbors; http://ciml.info/dl/v0_99/ciml-v0_99-ch03.pdf)
  - Description in text for Figure 3.4 is wrong (+ and – are switched)

- Jurafsky & Martin SLP3, 6.3 (Words and vectors) and 6.4 (Cosine for measuring similarity) https://web.stanford.edu/~jurafsky/slp3/old_jan25/6.pdf

- Noah A. Smith (2020), Contextual Word Representations: Putting Words into Computers https://cacm.acm.org/magazines/2020/6/245162-contextual-word-representations/fulltext

So far

- **ML concepts:**
  - Supervised learning (how to frame your task as a supervised learning problem)
  - Inductive bias
  - Overfitting and underfitting
  - Decision boundaries
  - Evaluation of supervised learning systems (don’t touch your test data!)

- **Methods**
  - Decision Trees

Let's say you work at a bank. You're asked to make a system to detect whether a credit card transaction is fraudulent or genuine. What kind of features would you use? List at least 5 features

- Characteristics of the transaction
  - Amount, time, location, type (online, retail shop), how it was verified (signature, or…) etc.
- Characteristics of the receiver/sender? Maybe there is some blacklist?
- Deviations from previous transaction patterns
  - E.g. How much does the amount differ from previous/average transactions
  - Unusual location?
- Time (and location?) between two consecutive transactions

If the input features don’t capture the necessary information, even a complex model won’t be able to do well.

So…. the more features the better?

If the input features don’t capture the necessary information, even a complex model won’t be able to do well.

So…. the more features the better?

No:

- More features increases the risk of overfitting
- Sometimes there are features that we don’t want to use (demographics)
- Interpretability

Questions via the last quiz

- How do we split the dataset into train/dev/*test*?
- Training and test errors, and expected loss

Formalizing the Learning Problem

Loss function – measure of error of the current system's predictions in comparison to the ground truth

$$l(y, \hat{y})$$

true label predicted

$\mathcal{D}$ - the true underlying data distribution, unknown, all we get is a random sample from it (training data)

We get access to training error (**sample error**), but not to the **true/expected error** over $\mathcal{D}$

Estimating error (not for the exam)

If S is training set, $error_S(h)$ is optimistically biased

$$bias \equiv E[error_S(h)] - error_D(h)$$

How well does $error_S(h)$ estimate $error_D(h)$?

$S1 \Rightarrow error_{S1}(h)$

$S2 \Rightarrow error_{S2}(h)$

……….

$Sn \Rightarrow error_{Sn}(h)$

Estimating error (not for the exam)

If S is training set, $error_S(h)$ is optimistically biased

$$bias \equiv E[error_S(h)] - error_D(h)$$

How well does $error_S(h)$ estimate $error_D(h)$?

$S1$ => $error_{S1}(h)$

$S2$ => $error_{S2}(h)$

……….

Sn => $error_{Sn}(h)$

With approximately 95% probability, $error_D(h)$ lies in interval

$$error_S(h) \pm 1.96\sqrt{\frac{error_S(h)(1-error_S(h))}{n}}$$

Today

- **ML concepts:**
  - Vector spaces
  - Distance metrics

- **ML method:**
  - K-nearest neighbours

- **NLP 101**
  - How to represent documents as vectors
  - How to represent words as vectors

RECAP !

Supervised learning

Learn a machine learning model using **labeled example instances:**

features   target

$\{<x^{(1)}, y^{(1)}>, \ldots, <x^{(N)}, y^{(N)}>\}$

Goal: Predict the target using the features

Need to define **features**, characteristics of the instances that the model uses for predictions (words in a document, movie ratings, etc..)

Features for house price prediction:

- Neighborhood
- Number of bedrooms
- First floor square meters
- Number of schools within 2 km
- Police Label Safe Housing
- ..

Representing instances using feature vectors

Number of bedrooms: 2
Plot size (square meters): 110
Number of schools within 2 km: 2

`[2, 110, 2]`

vector

Representing instances using feature vectors

Number of bedrooms: 2
Plot size (square meters): 110
Number of schools within 2 km: 2
Police Label Safe Housing: yes

`[2, 110, 2, 1]`

vector

We encode binary features as 1 (yes) and 0 (no)

Representing instances using feature vectors

Number of bedrooms: 2
Plot size (square meters): 110
Number of schools within 2 km: 2
Police Label Safe Housing: yes
Property type: House

vector

```
[2, 110, 2, 1, ?]
```

```
Apartment: 0
House: 1
Tiny home: 2
Storage Space: 3
```

Representing instances using feature vectors

Number of bedrooms: 2
Plot size (square meters): 110
Number of schools within 2 km: 2
Police Label Safe Housing: yes
Property type: House

vector

`[2, 110, 2, 1, 0, 1, 0, 0]`

```
Apartment: 0
House: 1
Tiny home: 2
Storage Space: 3
```

One Hot Encoding

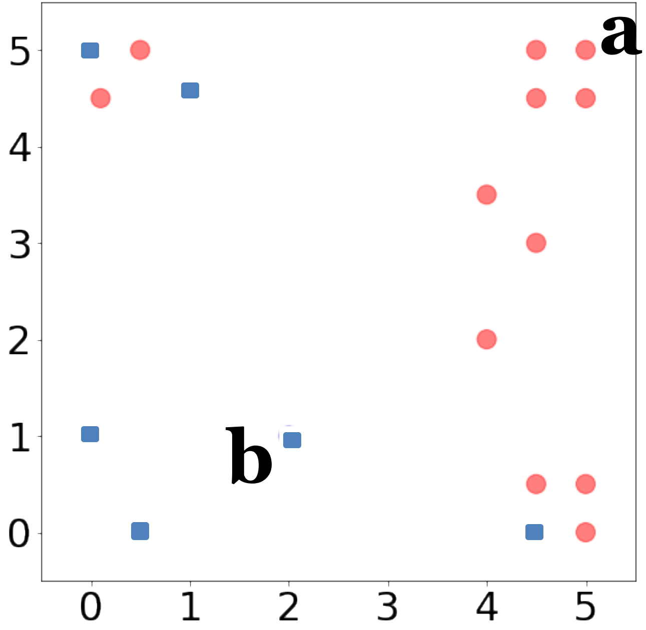
```
Apartment? Yes = 1, No = 0
House? Yes = 1, No = 0
Tiny home? Yes = 1, No = 0
Storage Space: Yes = 1, No = 0

Property type feature: [0,1,0,0]
```

Types of features

- A **numerical** feature (a real number)
  - Sentence length
  - Number of likes
  - Temperature
- A **binary** feature: a yes vs. no distinction (usually 1 vs. 0)
  - Is the text capitalized?
  - Are A and B friends?
  - Employed?
  - Like Chinese restaurants?
- **Categorical** feature:
  - Country
  - Genre of a text

Vector space




```
a = [5, 5]
b = [2, 1]
```

Your instances are now represented as points in *vector space*. Each dimension represents a feature (e.g. whether the user liked a certain restaurant)

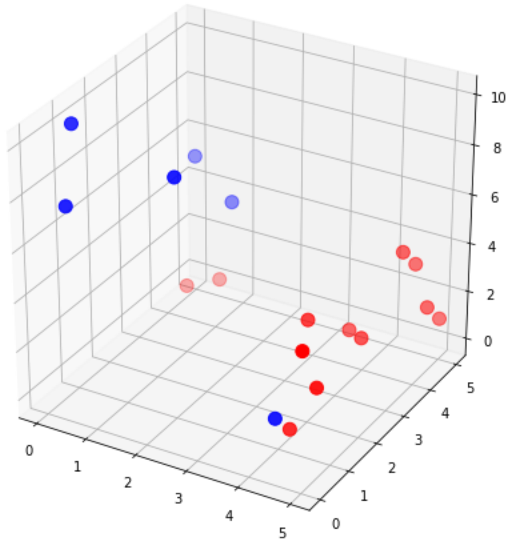
Usually *thousands of features*

Vectors & vector spaces

a = [5, 5]
b = [2, 1]

a is a two-dimensional vector, i.e. **a** $\in \mathbb{R}^2$

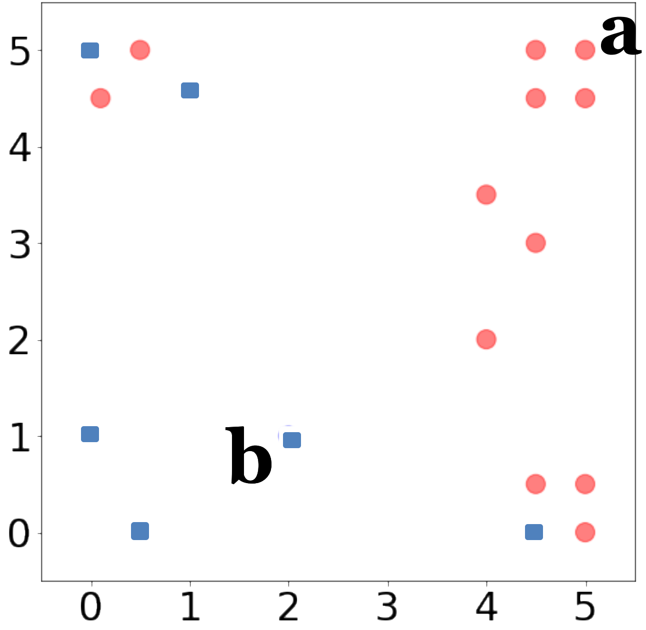
a + **b** = [5 + 2, 5 + 1] = [7, 6]



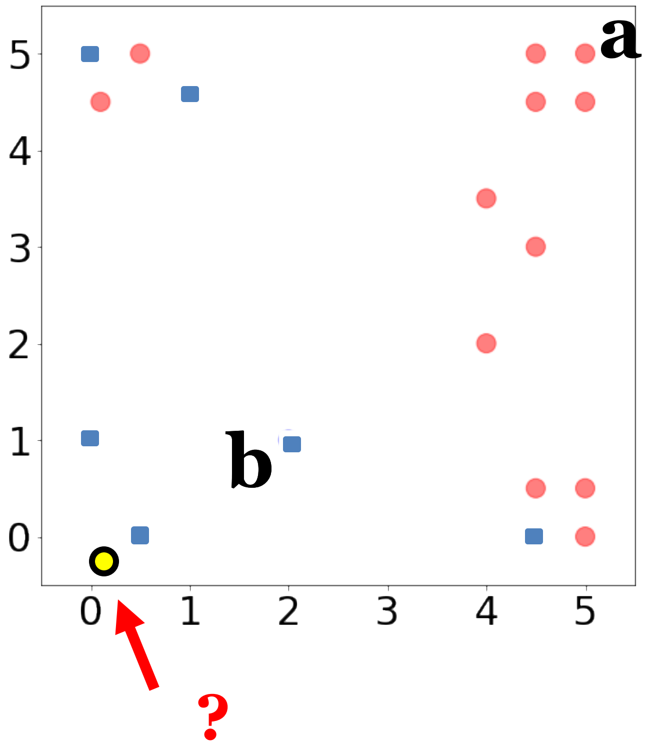
c = [c_1, ..., c_d]
c is a d-dimensional vector, i.e. **c** $\in \mathbb{R}^d$

What is a_1? *5*
What is b_2? *1*

Vector space



Vector space



Q: How would you classify this point? (red or blue?)

Nearest neighbor

*This "**rule of nearest neighbour**" has considerable elementary intuitive appeal and probably corresponds to practice in many situations. For example, it is possible that much medical diagnosis is influenced by the doctor's **recollection** of the subsequent history of an earlier patient whose symptoms **resemble in some way** those of the current patient. (Fix and Hodges, 1952)*

Idea: Classify new examples based on the most similar training examples

[CIML: chapter 3]

Memory-based learner

This is **memory-based** learning (also called instance-based learning): look for similar instances in the training data (stored in **memory**) and fit with the local points.

Four components:

- A distance metric
- How many neighbors to look at?
- A weighting function (*optional*)
- How to fit with the local points?

Memory-based learner

This is **memory-based** learning (also called instance-based learning): look for similar instances in the training data (stored in **memory**) and fit with the local points.

Four components: 1-nearest neighbors

- A distance metric

  Many options, e.g. Euclidian

- How many neighbors to look at?

  1

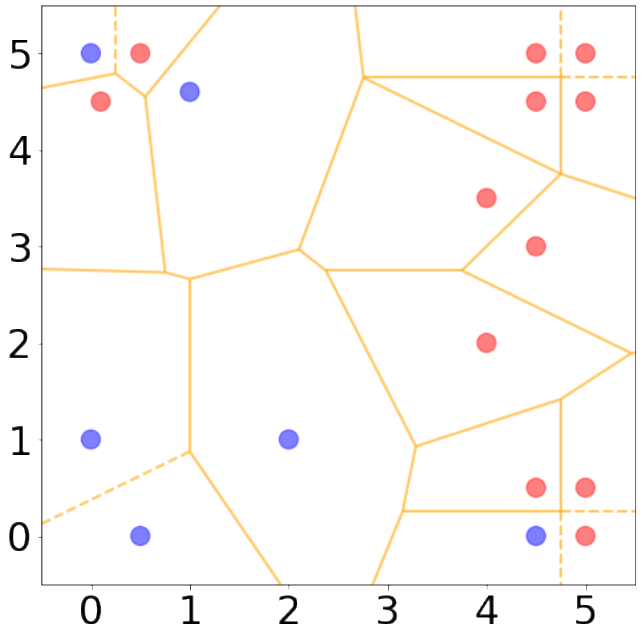
- A weighting function (*optional*)

  Unused

- How to fit with the local points?

  Just return the label of the nearest point

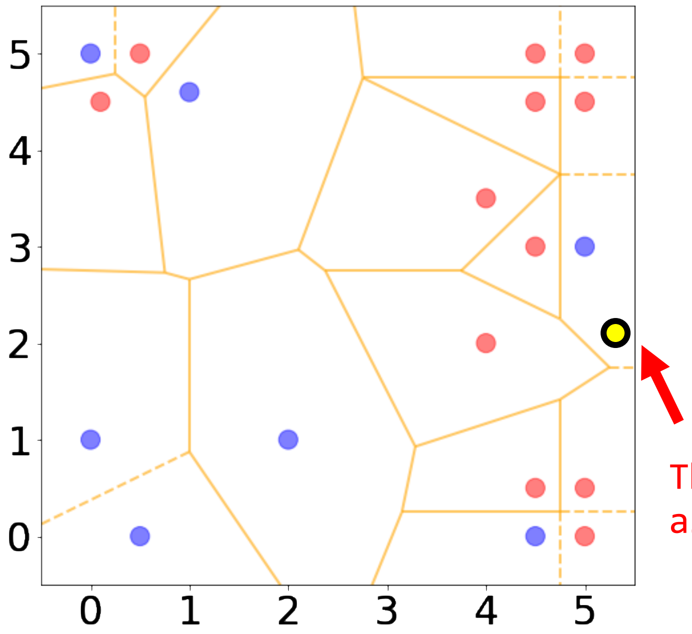
1-nearest neighbors decision boundaries



Every training example has its own neighborhood

For any point x in a training set, the Voronoi cell of x consists of all points closer to x than any other points in the training set.

1-nearest neighbors decision boundaries



1-nearest neighbors is sensitive to outliers!

Small changes in the training set can lead to large differences in the decision boundary

This point now gets classified as blue, is this what we want?

Memory-based learner

This is **memory-based** learning (also called instance-based learning): look for similar instances in the training data (stored in **memory**) and fit with the local points.

Four components: K-nearest neighbors

- A distance metric
  Many options, e.g. Euclidian
- How many neighbors to look at?
  K
- A weighting function (*optional*)
  Unused
- How to fit with the local points?
  Just predict the majority label

K-nearest neighbor

Training data *number of neighbors* *test instance*

Algorithm 3 KNN-PREDICT($\mathbf{D}$, K, $\hat{x}$)

$S \leftarrow [\,]$
for $n = 1$ **to** N **do**
  $S \leftarrow S \oplus \langle \mathrm{d}(x_n, \hat{x}), n \rangle$ // store distance to training example n
end for
$S \leftarrow \textsc{sort}(S)$ // put lowest-distance objects first
$\hat{y} \leftarrow 0$
for $k = 1$ **to** K **do**
  $\langle dist, n \rangle \leftarrow S_k$ // n this is the kth closest data point
  $\hat{y} \leftarrow \hat{y} + y_n$ // vote according to the label for the nth training point
end for
return $\textsc{sign}(\hat{y})$ // return $+1$ if $\hat{y} > 0$ and -1 if $\hat{y} < 0$

[CIML: chapter 3]

K-nearest neighbor

Training data *number of neighbors* *test instance*

Algorithm 3 KNN-PREDICT($\mathbf{D}$, K, $\hat{x}$)

$S \leftarrow [\,]$

for $n = 1$ **to** N **do**

  $S \leftarrow S \oplus \langle \mathrm{d}(\boldsymbol{x}_n, \hat{\boldsymbol{x}}), n \rangle$ // store distance to training example n

end for

$S \leftarrow \mathrm{SORT}(S)$ // put lowest-distance objects first

$\hat{y} \leftarrow 0$

for $k = 1$ **to** K **do**

  $\langle dist,n \rangle \leftarrow S_k$ // n this is the kth closest data point

  $\hat{y} \leftarrow \hat{y} + y_n$ // vote according to the label for the nth training point

end for

return $\mathrm{SIGN}(\hat{y})$ // return $+1$ if $\hat{y} > 0$ and -1 if $\hat{y} < 0$

[CIML: chapter 3]

K-nearest neighbor

Training data *number of neighbors* *test instance*

Algorithm 3 KNN-PREDICT($\mathbf{D}$, K, $\hat{x}$)

$S \leftarrow [\,]$
for $n = 1$ **to** N **do**
  $S \leftarrow S \oplus \langle \mathrm{d}(x_n, \hat{x}), n \rangle$ // store distance to training example n
end for
$S \leftarrow \textsc{sort}(S)$ // put lowest-distance objects first
$\hat{y} \leftarrow 0$
for $k = 1$ **to** K **do**
  $\langle dist, n \rangle \leftarrow S_k$ // n this is the kth closest data point
  $\hat{y} \leftarrow \hat{y} + y_n$ // vote according to the label for the nth training point
end for
return $\textsc{sign}(\hat{y})$ // return $+1$ if $\hat{y} > 0$ and -1 if $\hat{y} < 0$

Example:
[+1,+1,-1]

[CIML: chapter 3]

Choosing *K*

How many instances should we consider when making the classification?

K = 1

Sensitive to outliers

Could lead to overfitting

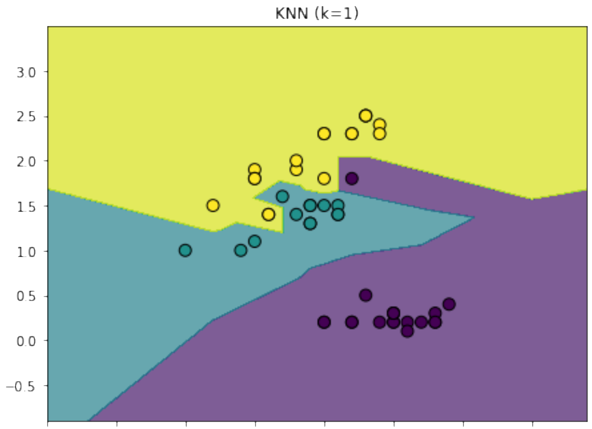
K = N (number of instances)

Always predict the majority label

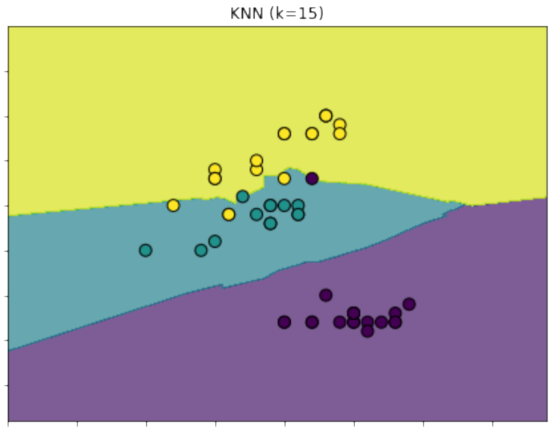
Leads to underfitting!

Decision boundary

K=1

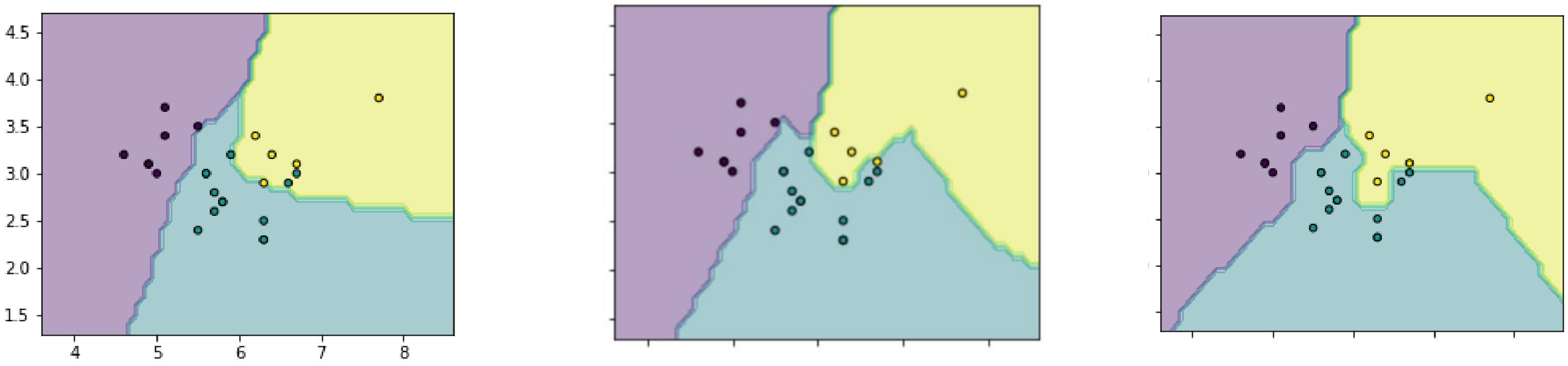


K=15



Larger K values lead to smoother decision boundaries

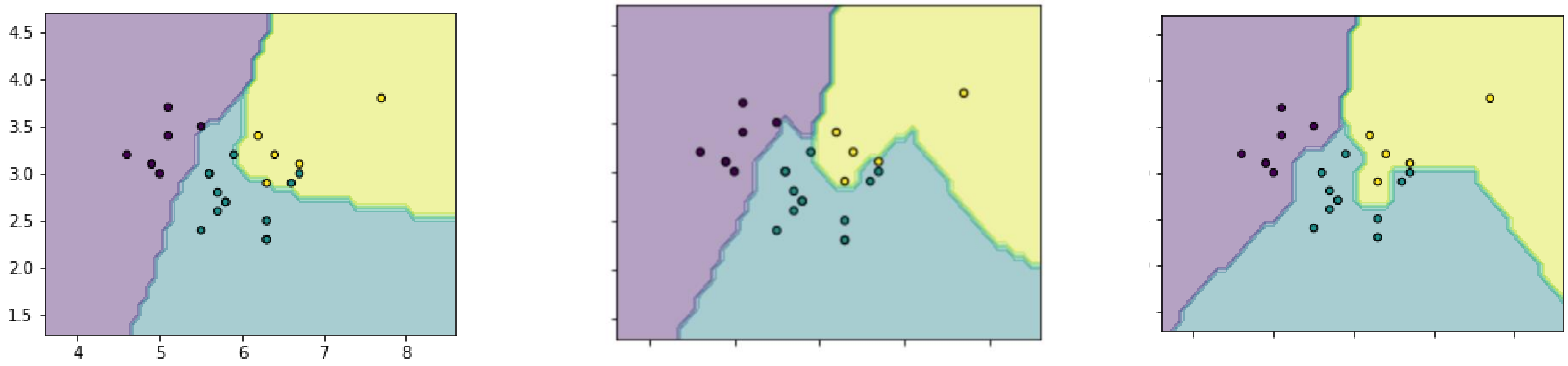
Decision boundary



Questions:

- Which one is: K=1, K=3, K=5?
- K is usually an odd number, why?
- How would you set K?

Decision boundary



Questions:

- Which one is: K=1, K=3, K=5?
- K is usually an odd number, why?
- How would you set K?

K=5, K=3, K=1

To not have ties (with binary classification)

K is a hyper parameter

K-nearest neighbor

Training data *number of neighbors* *test instance*

Algorithm 3 KNN-PREDICT($\mathbf{D}$, K, $\hat{x}$)

How do we compute the distance between examples?

$S \leftarrow [\,]$
for $n = 1$ **to** N **do**
  $S \leftarrow S \oplus \langle \mathrm{d}(x_n, \hat{x}), n \rangle$ // store distance to training example n
end for
$S \leftarrow \textsc{sort}(S)$ // put lowest-distance objects first
$\hat{y} \leftarrow 0$
for $k = 1$ **to** K **do**
  $\langle dist, n \rangle \leftarrow S_k$ // n this is the kth closest data point
  $\hat{y} \leftarrow \hat{y} + y_n$ // vote according to the label for the nth training point
end for
return $\textsc{sign}(\hat{y})$ // return $+1$ if $\hat{y} > 0$ and -1 if $\hat{y} < 0$

[CIML: chapter 3]

Distance measure

By representing data points as vectors, we can measure their distance (or similarity) in the vector space.

If a and b are scalars, the most straightforward way to define distance is:

$$| a - b | \qquad \text{(e.g., } |3\text{-}5| = 2\text{)}$$

Let's generalize this idea to vectors

Length (or, norm) of a vector

Length of vector `[3, 4]`:

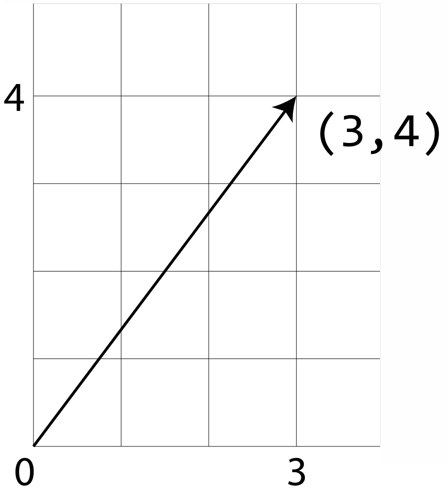
$$\sqrt{x^2+y^2}=\sqrt{4^2+3^2}$$

Length of vector `[3, 4, 1]`:

$$\sqrt{4^2+3^2+1^2}$$

Length of vector **a** :

$$\|\boldsymbol{a}\|_2 = \sqrt{\square\, a_i^2}$$



This is also called the l_2- norm or the Euclidian norm

Euclidian distance

Distance between endpoint of vectors

Question:
What is the Euclidian distance between a & b

```
a = [0, 3, 5]
b = [3, 1, 2]
```

Euclidian distance:

$$\|\boldsymbol{a} - \boldsymbol{b}\|_2 = \sqrt{\sum (a_i - b_i)^2}$$

Euclidian distance

Distance between endpoint of vectors

```
a-b     = [(0-3), (3-1), (5-2)]
        = [-3, 2, 3]
```

`Euclidian distance:`

$$\|\boldsymbol{a}-\boldsymbol{b}\|_2 = \sqrt{\sum (a_i - b_i)^2}$$

Question:
What is the Euclidian distance between a & b

```
a = [0, 3, 5]
b = [3, 1, 2]
```

Euclidian distance

Distance between endpoint of vectors

```
a-b     = [(0-3), (3-1), (5-2)]
        = [-3, 2, 3]
```

`Euclidian distance:`

$$\|\boldsymbol{a}-\boldsymbol{b}\|_2 = \sqrt{\sum (a_i - b_i)^2}$$

$= \mathrm{sqrt}((-3)^2 + 2^2 + 3^2)$

$= \mathrm{sqrt}(9 + 4 + 9)$

$= 4{,}690$

Question:
What is the Euclidian distance between a & b

```
a = [0, 3, 5]
b = [3, 1, 2]
```

Euclidian distance

Distance between endpoint of vectors

```
a-b     = [(0-3), (3-1), (5-2)]
        = [-3, 2, 3]
```

Euclidian distance:

$$\|\boldsymbol{a}-\boldsymbol{b}\|_2 = \sqrt{\sum (a_i - b_i)^2}$$

also called L2 distance

```
= sqrt((-3)² + 2² + 3²)
= sqrt(9 + 4 + 9)
= 4,690
```

Question:
What is the Euclidian distance between a & b

```
a = [0, 3, 5]
b = [3, 1, 2]
```

Manhattan distance

Distance between endpoint of vectors

```
a-b     = [(0-3), (3-1), (5-2)]
        = [-3, 2, 3]
```

```
Manhattan distance:
```

$$\|\boldsymbol{a}-\boldsymbol{b}\|_1 = \sum |a_i - b_i|$$

Question:
What is the Manhattan distance between a & b

```
a = [0, 3, 5]
b = [3, 1, 2]
```

Manhattan distance

Distance between endpoint of vectors

```
a-b     = [(0-3), (3-1), (5-2)]
        = [-3, 2, 3]
```

```
Manhattan distance:
```

$$\|\boldsymbol{a}-\boldsymbol{b}\|_1 = \sum |a_i - b_i|$$

$$= |-3| + |2| + |3|$$

$$= 8$$

Question:
What is the Manhattan distance between a & b

```
a = [0, 3, 5]
b = [3, 1, 2]
```

Manhattan distance

Distance between endpoint of vectors

```
a-b      = [(0-3), (3-1), (5-2)]
         = [-3, 2, 3]
```

Manhattan distance:

$$\|\boldsymbol{a} - \boldsymbol{b}\|_1 = \sum |a_i - b_i|$$

also called L1 distance

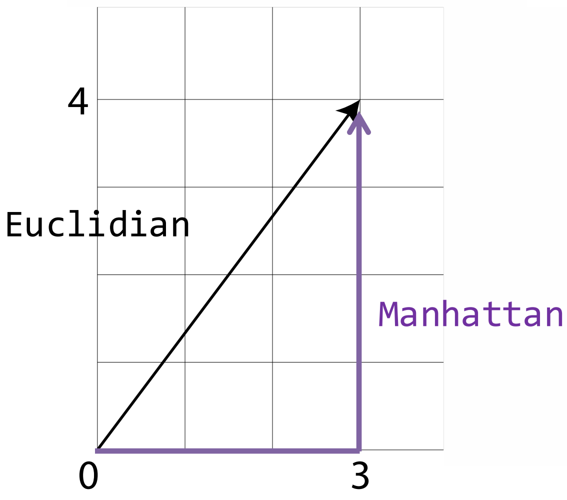
$$= |-3| + |2| + |3|$$

$$= 8$$

Question:
What is the Manhattan distance between a & b

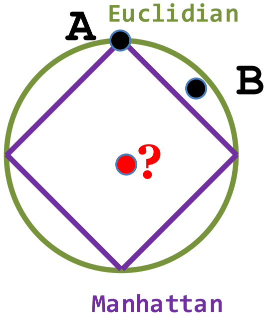
```
a = [0, 3, 5]
b = [3, 1, 2]
```

Comparison distances



Minkowski distance:
Generalization of the Euclidian (p=2) and Manhattan distance (p=1)

$$\sqrt[p]{\sum |a_i - b_i|^p}$$

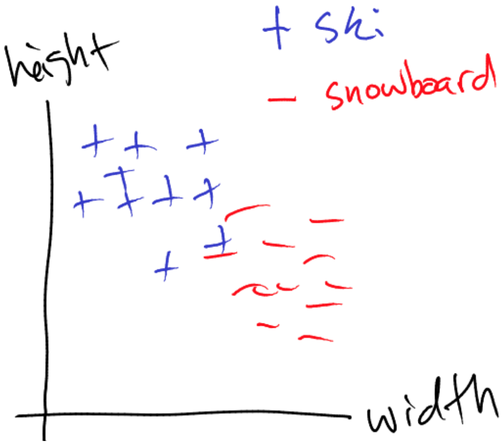


Many options!

Different distance measures → different neighborhoods

Inductive bias of k-nearest neighbors

- The label of a data point should be similar to labels of nearby points
- All features are equally important (but weighted variants exist!)



Feature Scaling

Centering

the ith training instance

$$x_j^{(i)} = x_j^{(i)} - \mu_j$$

the jth feature

with $\mu_j = \frac{1}{N} \sum_i \; x_j^{(i)}$

N training examples
1 ≤ i ≤ N
d features
1 ≤ j ≤ d

Variance Scaling

$$x_j^{(i)} = \frac{x_j^{(i)}}{\sigma_d}$$

[CIML: 3.6 and 3.7]

Practical considerations

- How can we use K-Nearest Neighbors for **regression?**
  - Just predict the mean of the neighbors

- What about **ties?**
  - Random
  - Use 1 -nearest neighbor to decide
  - Use the class that is most frequent in the training data (highest prior probability)

Computational Considerations

training data *number of neighbors* *test instance*

Algorithm 3 KNN-PREDICT($\mathbf{D}$, K, $\hat{x}$)

$S \leftarrow [\,]$
for $n = 1$ **to** N **do**
  $S \leftarrow S \oplus \langle \mathrm{d}(x_n, \hat{x}), n \rangle$
end for
$S \leftarrow$ SORT(S) // put lowest-distance objects first
$\hat{y} \leftarrow 0$
for $k = 1$ **to** K **do**
  $\langle dist, n \rangle \leftarrow S_k$ // n this is the kth closest data point
  $\hat{y} \leftarrow \hat{y} + y_n$ // vote according to the label for the nth training point
end for
return SIGN($\hat{y}$) // return $+1$ if $\hat{y} > 0$ and -1 if $\hat{y} < 0$

Q: How does the classification speed (for test instances) depend on the **number of instances** in the training data?

[CIML: chapter 3]

Computational Considerations

training data *number of neighbors* *test instance*

Algorithm 3 KNN-PREDICT($\mathbf{D}$, K, $\hat{x}$)

$S \leftarrow [\,]$
for $n = 1$ **to** N **do**
  $S \leftarrow S \oplus \langle d(x_n, \hat{x}), n \rangle$
end for
$S \leftarrow$ SORT(S) // put lowest-distance objects first
$\hat{y} \leftarrow 0$
for $k = 1$ **to** K **do**
  $\langle dist, n \rangle \leftarrow S_k$ // n this is the kth closest data point
  $\hat{y} \leftarrow \hat{y} + y_n$ // vote according to the label for the nth training point
end for
return SIGN($\hat{y}$) // return $+1$ if $\hat{y} > 0$ and -1 if $\hat{y} < 0$

Q: How does the classification speed (for test instances) depend on the **number of instances** in the training data? → linearly ☹

[CIML: chapter 3]

Computational Considerations

- Training is fast
- It's easy to add new training data
  (no 'retraining' needed)

- Making predictions is slow
  - Takes N comparisons (number of instances) × d (number of dimensions) operations
- Need to *store* the training data

There are techniques to speed up K-nearest neighbors, such as kd-trees

Computational Considerations

- Training is fast
- It's easy to add new training data (no 'retraining' needed)

- Making predictions is slow
  - Takes N comparisons (number of instances) × d (number of dimensions) operations
- Need to *store* the training data

Usually we prioritize the speed of making predictions over the speed of training models.

There are techniques to speed up K-nearest neighbors, such as kd-trees

Decision trees vs. nearest neighbors

Decision Trees

- Learn a model from the training data
- Apply model to new data

- Features are selected

- Decision boundaries are axis-aligned cuts

Nearest neighbors

- Store data
- Compare new data to stored data

- All features have equal weight → Sensitive to irrelevant features

- Decision boundaries can be complex

Up next: How can we represent words and documents as vectors?

Natural Language Processing 101

Natural Language Processing (NLP)

Automatic processing and analysis of
natural language

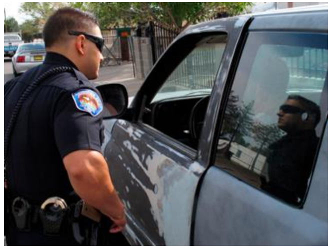
Dutch, English, Spanish, Hindi, Frisian, Turkish, ...

7,111 known living languages.
https://www.ethnologue.com/

machine translation

IBM Watson (question answering)

https://www.youtube.com/watch?v=WFR3lOm_xhE



uncovering racial disparities in analyzing language from police body camera's (PNAS 2017)

ChatGPT

What makes language understanding hard? Polysemy

Today, I went to the **bank** to deposit a check.

The hut is located near the **bank** of the river.

Ardougne North **Bank** is near the **bank** of the River Dougne in the north part of the city.

Words can have multiple meanings

What makes language understanding hard? Syntactic ambiguity

"One morning, I shot an elephant in my pajamas."

Who wore the pajamas?

What makes language understanding hard?
Syntactic ambiguity

“One morning, I shot an elephant in my pajamas.”

“How he got in my pajamas, I don't know.”

Who wore the pajamas?

(by Groucho Marx)

What makes language understanding hard?

ikr smh he asked fir yo last name

so he can add u on fb lololol

http://www.cs.cmu.edu/~ark/TweetNLP/

What makes language understanding hard?

i know right	shake my head			for	your		
ikr	smh	he	asked	fir	yo	last	name
inter-jection	acronym	pronoun	verb	preposition	determiner	adjective	noun

				you		facebook	
so	he	can	add	u	on	fb	lololol
preposition	pronoun	verb	verb	pronoun	preposition	proper noun	inter-jection

http://www.cs.cmu.edu/~ark/TweetNLP/

What makes language understanding hard?

A string may have many possible interpretations in different contexts, and resolving ambiguity correctly may rely on knowing a lot about the world. (Noah Smith)

- Linguistic **diversity**: languages, dialects, registers, styles
- Language is constantly **changing**

ML for NLP: What assumptions should the machine learning model have? (‘*How does language work?*’) How should we represent language data in our models?

Tokenization

```
A tokenizer segments text into a
sequence of tokens
```

```
A
tokenizer
segments
text
into
a
sequence
of
tokens
```

We often take words as the basic level of analysis

Tokenization: tokens vs types

```
A tokenizer segments text into a
sequence of tokens
```

Token:

```
A
tokenizer
segments
text
into
a
sequence
of
tokens
```

Type:

```
a
tokenizer
segments
text
into
sequence
of
tokens
```

Tokenization challenges

始めまして。お元気ですか。

Japanese: No spaces between words

Note: Many libraries provide tokenization methods (e.g. nltk, spacy, scikit-learn)

New York, European Union

Multiword expressions

I'm, don't

Contractions

Pre-processing steps

Stop word removal

- *a, an, the, it, ..*
- Using a stop word list or by filtering words that appear in many documents

Lemmatization

- *sang, sung, sings* → *sing*

Stemming (strip endings of the word)

- E.g., *running* to *run*

Lowercasing

- E.g., *Running* to *running*

Removing infrequent words

- E.g., occurring in less than 10 (or 25, or …) documents

Overall goal: reduce the number of *types* (usually thousands to millions in a large dataset!)

But could remove useful signals!

Example: function words (the, a, and, however, on, etc..) are not so important for topic classification, but strong signals for authorship identification.

Edit distance

The minimum **edit distance** between two strings:

the minimum number of editing operations (insertion, deletion, substitution) needed to transform one string into another

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
d s s   i s
```

```
d = deletion
s = substitution
i = insertion
```

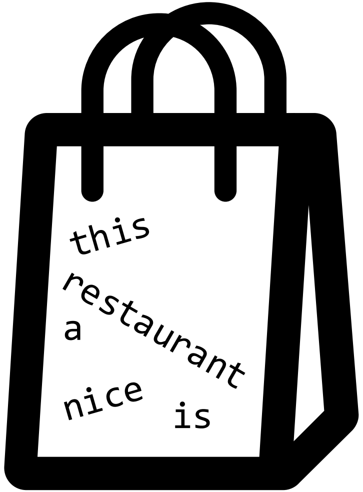
J&M: Fig. 2.14

Levenshtein distance:
each of these operators has cost 1

Bag of words representation

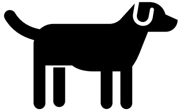
Only look at the presence (or frequency) of words, i.e. ignore the order. (*Often a surprisingly hard baseline to beat!*)

`this is a nice restaurant`



But..

`Dog bites man`



`Man bites dog`

Question: Come up with a task for which bag of words is probably sufficient. And another task for which it is not.

Jaccard similarity

Jaccard similarity

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|}$$

```
this is not bad

this is really bad
```

```
Common (3): this, is, bad
Union (5):  this, is, not, bad, really

Jaccard similarity: 3/5
```

Jaccard similarity

Jaccard similarity

Simple 😃

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|}$$

```
this is not bad

this is really bad
```

```
Common (3): this, is, bad
Union (5):  this, is, not, bad, really

Jaccard similarity: 3/5
```

Jaccard similarity

Jaccard similarity

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|}$$

```
this is not bad

this is really bad
```

```
Common (3): this, is, bad
Union (5):  this, is, not, bad, really

Jaccard similarity: 3/5
```

Simple 😃

Infrequent words (e.g. *bad*) versus frequent words (e.g. *is*) 😔

Frequency and order of words are ignored

Today: Vector representations of words and documents

Represent *documents*, *words*, phrases, sentences, etc.. as vectors

 → `[0.20, 0.80, 0.10, ..., -0.10]`

dog → `[0.10, 0.90, -0.20, ..., -0.40]`

- What are the dimensions of the vector space?
- How do we measure distances in the vector space?

Today: Vector representations of words and documents

Represent *documents*, *words*, phrases, sentences, etc.. as vectors

 → `[0.20, 0.80, 0.10, ..., -0.10]`

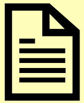
dog → `[0.10, 0.90, -0.20, ..., -0.40]`

- What are the dimensions of the vector space?
- How do we measure distances in the vector space?

To find similar documents, find similar words, as input to ML models, ...

Today: Vector representations of words and documents

Represent *documents*, *words*, phrases, sentences, etc.. as vectors

 → [0.20, 0.80, 0.10, ..., -0.10]

dog → [0.10, 0.90, -0.20, ..., -0.40]

- What are the dimensions of the vector space?
- How do we measure distances in the vector space?

To find similar documents, find similar words, as input to ML models, ...

Today: Vector representations of words and documents

Represent *documents*, *words*, phrases, sentences, etc.. as vectors

 → `[0.20, 0.80, 0.10, ..., -0.10]`

dog → `[0.10, 0.90, -0.20, ..., -0.40]`

- What are the dimensions of the vector space?

`One dimension for each word.`

- How do we measure distances in the vector space?

To find similar documents, find similar words, as input to ML models, …

Vector representations of documents

word-document matrix

	doc_1	doc_2	doc_3	doc_4	doc_5	doc_6	doc_7
cat	5	2	0	1	4	0	0
dog	7	3	1	0	2	0	0
car	0	0	1	3	2	1	1

Vector representations of documents

word-document matrix

	doc_1	doc_2	doc_3	doc_4	doc_5	doc_6	doc_7
cat	5	2	0	1	4	0	0
dog	7	3	1	0	2	0	0
car	0	0	1	3	2	1	1

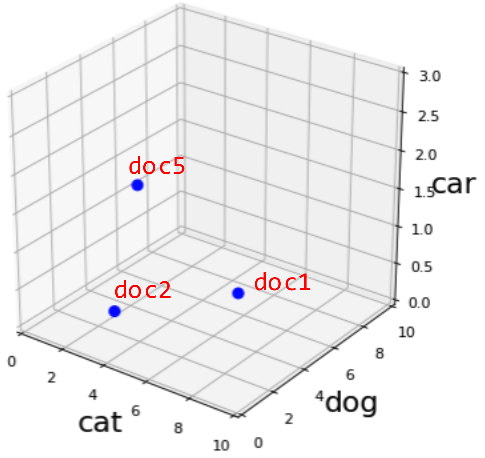
vector

Vector representations of documents

word-document matrix

	doc_1	doc_2	doc_3	doc_4	doc_5	doc_6	doc_7
cat	5	2	0	1	4	0	0
dog	7	3	1	0	2	0	0
car	0	0	1	3	2	1	1

vector



Vector representations of documents

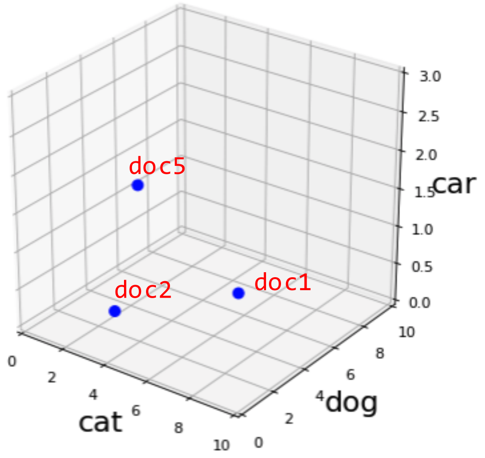
word-document matrix

	doc_1	doc_2	doc_3	doc_4	doc_5	doc_6	doc_7
cat	5	2	0	1	4	0	0
dog	7	3	1	0	2	0	0
car	0	0	1	3	2	1	1

vector

Euclidian distance

$$\sqrt{\sum (a_i - b_i)^2}$$



Question: Calculate the Jaccard Similarity and the Euclidian distance between doc_1 and doc_2, and between doc_1 and doc_5

Vector representations of documents

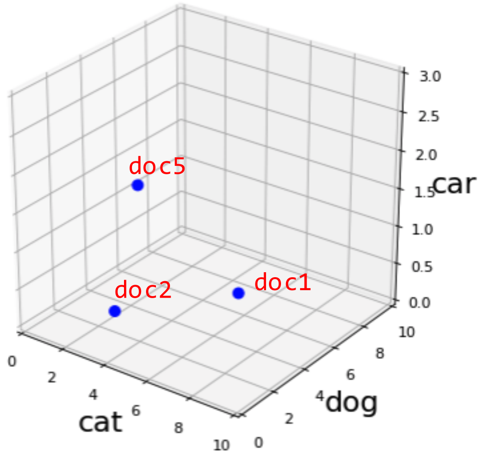
word-document matrix

	doc_1	doc_2	doc_3	doc_4	doc_5	doc_6	doc_7
cat	5	2	0	1	4	0	0
dog	7	3	1	0	2	0	0
car	0	0	1	3	2	1	1

vector

Euclidian distance

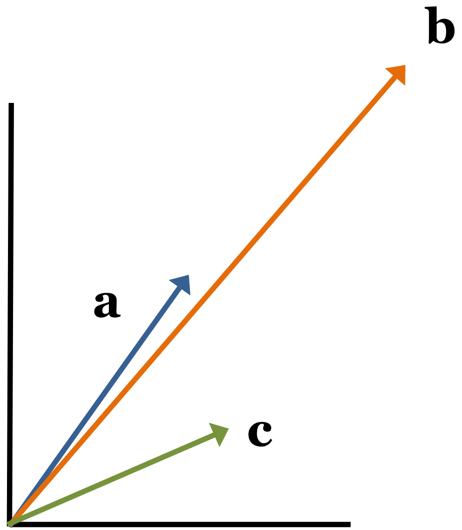
$$\sqrt{\sum (a_i - b_i)^2}$$



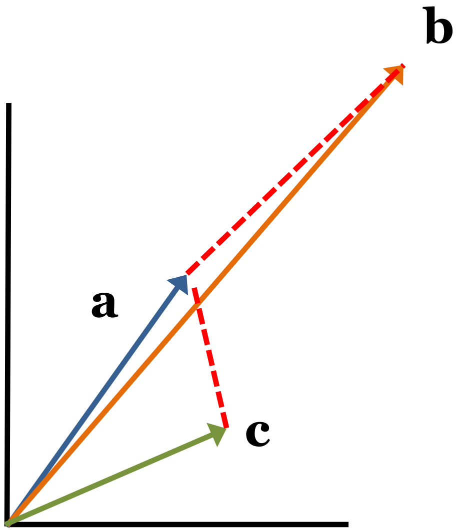
doc_1 and doc_2
Jaccard similarity = 1
Euclidian distance = 5

doc_1 and doc_5
Jaccard similarity = 2/3
Euclidian distance = 5.477

How similar are two documents?

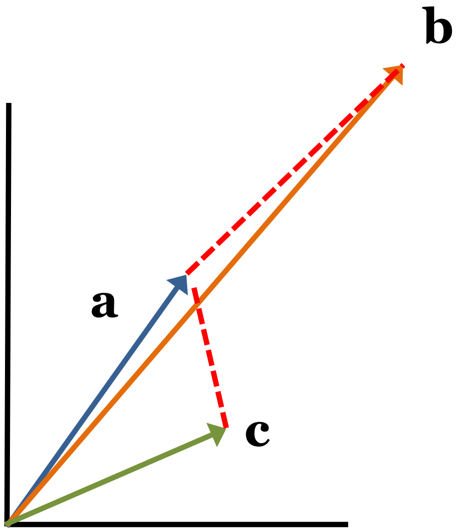


How similar are two documents?



First attempt:
the magnitude of the vector difference between two document vectors (L2/Euclidian distance)

How similar are two documents?

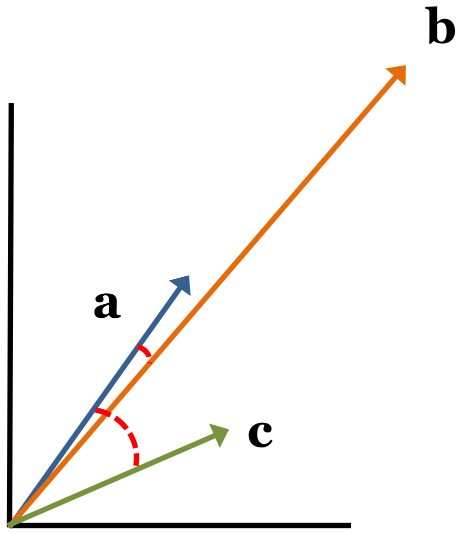


First attempt:
the magnitude of the vector difference between two document vectors (L2/Euclidian distance)

	doc_1	doc_2
cat	5	10
dog	7	14
car	0	0

Extreme example:
Concatenate two documents.

How similar are two documents?



First attempt:
the magnitude of the vector difference between two document vectors (L2/Euclidian distance)

Second attempt:
Look at the angle (cosine similarity)

Dot product

Dot product:

$$\boldsymbol{a} \bullet \boldsymbol{b} = \sum a_i b_i$$

```
a = [0, 3, 5]    a•b= 0 × 3 + 3 × 1
b = [3, 1, 2]             + 5 × 2 = 13

c = [0, 1, 0]    c•d= 0 × 1 + 1 × 1
d = [1, 1, 1]             + 0 × 1 = 1
```

If we have a **set-based** representation, i.e. ***a*** and ***b*** are **binary** vectors (1 = word is present, 0=absence), then the dot product is the number of words present in both documents (intersection)

Compare to Jaccard Similarity! (normalization by dividing by the union of words)

Dot product

Dot product:

$$\boldsymbol{a} \bullet \boldsymbol{b} = \sum a_i b_i$$

Length of a vector:

RECAP!

$$\|\boldsymbol{a}\|_2 = \sqrt{\sum a_i^2}$$

Therefore:

$$\|\boldsymbol{a}\|_2 = \sqrt{\boldsymbol{a} \bullet \boldsymbol{a}}$$

Normalization of a vector to unit length

$$\frac{\boldsymbol{a}}{\|\boldsymbol{a}\|_2}$$

$\boldsymbol{a}$ = [0, 3, 5]

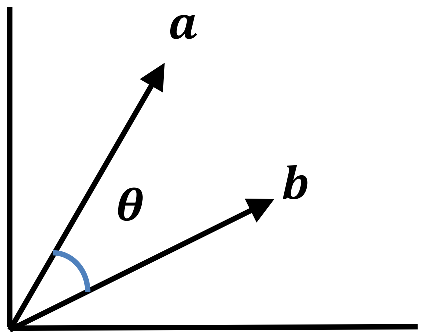
$\|\boldsymbol{a}\|_2$ = sqrt(9 + 25) = 5.831

$\frac{\boldsymbol{a}}{\|\boldsymbol{a}\|_2}$ = [0, 0.5145, 0.8575]

Cosine similarity

Cosine similarity

$$\cos(\boldsymbol{\theta}) = \frac{\boldsymbol{a} \bullet \boldsymbol{b}}{\|\boldsymbol{a}\|\|\boldsymbol{b}\|} = \frac{\sum \boldsymbol{a}_i \, \boldsymbol{b}_i}{\sqrt{\sum \boldsymbol{a}_i^2} \, \sqrt{\sum \boldsymbol{b}_i^2}}$$



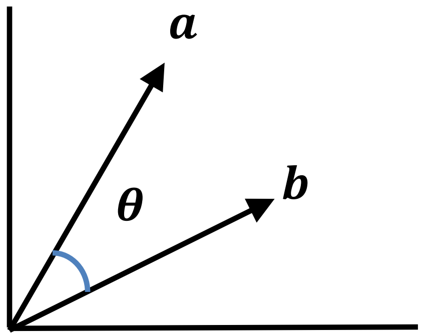
Question:
What is the cosine similarity between ***a*** & ***b***

a = `[0, 3, 5]`
b = `[3, 1, 2]`

Cosine similarity

Cosine similarity

$$\cos(\boldsymbol{\theta}) = \frac{\boldsymbol{a} \bullet \boldsymbol{b}}{\|\boldsymbol{a}\|\|\boldsymbol{b}\|} = \frac{\sum \boldsymbol{a}_i \, \boldsymbol{b}_i}{\sqrt{\sum \boldsymbol{a}_i^2}\sqrt{\sum \boldsymbol{b}_i^2}}$$



Question:
What is the cosine similarity between ***a*** & ***b***

a `= [0, 3, 5]`

b `= [3, 1, 2]`

$\boldsymbol{a} \bullet \boldsymbol{b} = 13$

$\|\boldsymbol{a}\|$ = sqrt($0^2 + 3^2 + 5^2$) = sqrt(34)

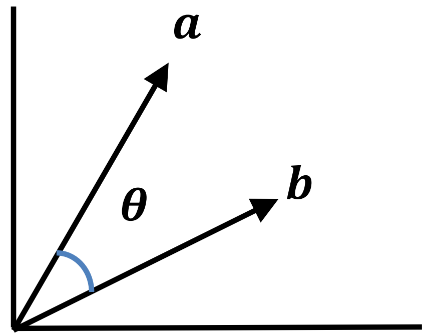
$\|\boldsymbol{b}\|$ = sqrt($3^2 + 1^2 + 2^2$) = sqrt(14)

→ 13/(sqrt(34) * sqrt(14)) ≈ 0.60

Cosine similarity

Cosine similarity

$$\cos(\boldsymbol{\theta}) = \frac{\boldsymbol{a} \bullet \boldsymbol{b}}{\|\boldsymbol{a}\|\|\boldsymbol{b}\|} = \frac{\sum \boldsymbol{a}_i \, \boldsymbol{b}_i}{\sqrt{\sum \boldsymbol{a}_i^2} \, \sqrt{\sum \boldsymbol{b}_i^2}}$$



When $\boldsymbol{a}$ and $\boldsymbol{b}$ are normalized:

$$\cos(\boldsymbol{\theta}) = \boldsymbol{a} \bullet \boldsymbol{b}$$

If the vectors are orthogonal:

$$\boldsymbol{a} \bullet \boldsymbol{b} = 0$$

Cosine ranges from -1 (vectors pointing in opposite directions) to 0 (orthogonal) to 1 (vectors pointing in the same direction).

Raw frequencies (non-negative): 0-1

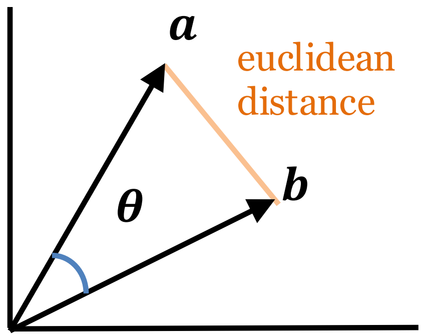
Comparisons

Cosine similarity

$$\cos(\boldsymbol{\theta}) = \frac{\boldsymbol{a} \bullet \boldsymbol{b}}{\|\boldsymbol{a}\|\|\boldsymbol{b}\|} = \frac{\sum \boldsymbol{a}_i \boldsymbol{b}_i}{\sqrt{\sum \boldsymbol{a}_i^2}\sqrt{\sum \boldsymbol{b}_i^2}}$$

Euclidean distance

$$\|\boldsymbol{a} - \boldsymbol{b}\|_2 = \sqrt{\sum (a_i - b_i)^2}$$



If $\boldsymbol{a}$ and $\boldsymbol{b}$ have unit length, then Euclidean distance and cosine similarity will result in the same ordering (but reversed)

Today: Vector representations of words and documents

Represent *documents*, *words*, phrases, sentences, etc.. as vectors

 → [0.20, 0.80, 0.10, ..., -0.10]

dog → [0.10, 0.90, -0.20, ..., -0.40]

Document representations

- What are the dimensions of the vector space?

One dimension for each word.
Values: binary (presence or absence), frequency, weighting schemes (e.g. tf-idf, not discussed)

- How do we measure distances in the vector space?

Cosine similarity

Today: Vector representations of words and documents

Represent *documents*, *words*, phrases, sentences, etc.. as vectors

 → `[0.20, 0.80, 0.10, ..., -0.10]`

dog → `[0.10, 0.90, -0.20, ..., -0.40]`

- What are the dimensions of the vector space?
- How do we measure distances in the vector space?

Today: Vector representations of words and documents

Represent *documents*, *words*, phrases, sentences, etc.. as vectors

 → `[0.20, 0.80, 0.10, ..., -0.10]`

dog → `[0.10, 0.90, -0.20, ..., -0.40]`

- What are the dimensions of the vector space?
- How do we measure distances in the vector space?

```
dog and puppy,
Monday and Tuesday,
buy and purchase
```

Word overlap is not enough!
Can we also map
words to vectors??

One hot encoding

Map each word to a unique identifier
e.g. cat (3) and dog (5).
→ Vector representation: all zeros, except 1 at the ID

cat	0	0	1	0	0	0	0
dog	0	0	0	0	1	0	0
car	0	0	0	0	0	0	1

What are limitations of one hot encodings?

One hot encoding

Map each word to a unique identifier
e.g. cat (3) and dog (5).
→ Vector representation: all zeros, except 1 at the ID

cat	0	0	1	0	0	0	0
dog	0	0	0	0	1	0	0
car	0	0	0	0	0	0	1

Even related words have distinct vectors!

High number of dimensions

Word representations

wampos

Four species of	**wampos**	can be found in Africa
some believe that	**wampos**	scales have medicinal qualities
approach to fighting	**wampos**	(and general wildlife) trafficking
Even though	**wampos**	scales are made of exactly the

What is a wampos?

Word representations

Photo by Piekfrosch / CC-BY-SA-3.0 / Wikipedia

wampos

Four species of	**wampos**	can be found in Africa
some believe that	**wampos**	scales have medicinal qualities
approach to fighting	**wampos**	(and general wildlife) trafficking
Even though	**wampos**	scales are made of exactly the

Word representations

Photo by Piekfrosch / CC-BY-SA-3.0 / Wikipedia

You shall know a word by the company it keeps (Firth, J. R. 1957:11)

wampos

Four species of	**wampos**	can be found in Africa
some believe that	**wampos**	scales have medicinal qualities
approach to fighting	**wampos**	(and general wildlife) trafficking
Even though	**wampos**	scales are made of exactly the

To assign similar vectors to similar words a notion of similarity is needed.

The distributional hypothesis: Words that occur in similar contexts tend to have similar meanings.

Vector representations of words

documents as context

word-document matrix

	doc_1	doc_2	doc_3	doc_4	doc_5	doc_6	doc_7
cat	5	2	0	1	4	0	0
dog	7	3	1	0	2	0	0
car	0	0	1	3	2	1	1

vector

Vector representations of words

documents as context
word-document matrix

	doc_1	doc_2	doc_3	doc_4	doc_5	doc_6	doc_7
cat	5	2	0	1	4	0	0
dog	7	3	1	0	2	0	0
car	0	0	1	3	2	1	1

neighboring words as context
word-word matrix

	cat	dog	car	bike	book	house	tree
cat	0	3	1	1	1	2	3
dog	3	0	2	1	1	3	1
car	1	2	0	3	2	2	0

Vector representations of words

documents as context
word-document matrix

	doc$_1$	doc$_2$	doc$_3$	doc$_4$	doc$_5$	doc$_6$	doc$_7$
cat	5	2	0	1	4	0	0
dog	7	3	1	0	2	0	0
car	0	0	1	3	2	1	1

Properties:

- Vectors are sparse: Many zero entries
- Values are based on frequencies (sometimes weighted)

neighboring words as context
word-word matrix

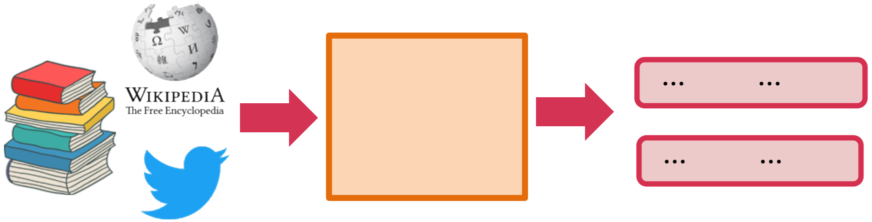
	cat	dog	car	bike	book	house	tree
cat	0	3	1	1	1	2	3
dog	3	0	2	1	1	3	1
car	1	2	0	3	2	2	0

Word embeddings (Dense word vectors)

Word embeddings:
- Vectors are dense
- Individual dimensions are less interpretable

Dense real-valued vectors

cat	0.52	0.84	0.01	....	0.23
dog	0.40	0.90	0.10	....	0.40



How are word embeddings learned?

(the skipgram model)

Target word

The domestic **cat** is a small, typically furry carnivorous mammal.

Context words

Context words

target word	context word	label
cat	small	1
cat	furry	1
cat	car	0
…	…	….

See also:
Word2vec (Mikolov et al. 2013), skipgram & continuous bag of words. (https://code.google.com/archive/p/word2vec/, another implementation in https://radimrehurek.com/gensim/)

Properties of word embeddings

We can use cosine similarity to find similar words in the vector space.

```
san francisco

los_angeles        0.666175
golden_gate        0.571522
oakland            0.557521
california         0.554623
```

https://code.google.com/archive/p/word2vec/
https://en.wikipedia.org/wiki/San_Francisco

Tokens vs. types

RECAP!

```
The hut is located near the bank of the river
```

Tokens	**Types**
The	the
hut	hut
is	is
located	located
near	near
the	bank
bank	of
of	river
the	
river	

Contextualized word representations

So far: an embedding **for each word (type)**

`Today, I went to the` **`bank`** `to deposit a check.`

bank | 0.92 | 0.24 | -0.01 | .... | 0.53

`The hut is located near the` **`bank`** `of the river.`

bank | 0.22 | 0.91 | 0.50 | .... | 0.23

See models such as BERT

Final words

- How to represent instances in your data is one of the most important parts of building a ML model
- It used to be based on manually crafting features
- Nowadays: many ML systems (especially deep learning) learn vector representations from data

- We can use vector representations:
    - kNN
    - in neural networks
    - etc…

Quiz

I posted **a short quiz (optional) on Brightspace** for you to practice with the material.

Do the quiz before **Monday 9am**, so I have time to take a look before the next lecture.

What do you need to know

- K-nearest neighbor method (algorithm, pros and cons, effect of K, distance measures)
- You should be able to compute Manhattan/L1 and Euclidian/L2 distance, cosine similarity, dot product
- The frequently used preprocessing steps in natural language processing
- What makes processing and analyzing language difficult?
- Representing documents (from documents to vectors)
- Representing words (from words to vectors)

NLP tools

- **Natural Language Toolkit (NLTK)** http://www.nltk.org/

python™
Natural Language Analyses with NLTK

- **Spacy** https://spacy.io/

- Many machine learning libraries (e.g. scikit-learn) also implement basic NLP methods.

spaCy

Books (drafts online)

- Speech & language processing (3rd edition draft) by Jurafsky and Martin
  - https://web.stanford.edu/~jurafsky/slp3/old_jan25/
- Introduction to Natural Language Processing (1st edition, to appear in 2019) by Jacob Eisenstein
  - https://github.com/jacobeisenstein/gt-nlp-class/blob/master/notes/eisenstein-nlp-notes.pdf