Quantum Computing Module Student Questions

Qubits and the Bloch Sphere - Guided Inquiry Questions

1. Looking at the two Bloch spheres above, which of the six qubit states (|0⟩, |1⟩, |+i⟩, |−i⟩, |+⟩, and |−⟩) correspond to the spin-up (|α⟩) and spin-down (|β⟩) states in the spin-½ Bloch sphere? What do the remaining qubit states correspond to in the spin-½ Bloch sphere?
   Ans: |0⟩ corresponds to the spin-up state and |1⟩ corresponds to the spin-down state. All the other qubit states correspond to superposition states in the x-y plane with 50% probability of being measured in either the spin-up or spin-down state.

2. If the qubit were in one of the following qubit states, what would be the probability of measuring the qubit in the |0⟩ state?
   a. |0⟩
      Ans: 100% or 1
   b. |1⟩
      Ans: 0% or 0
   c. |+i⟩
      Ans: 50% or 0.5
   d. |−i⟩
      Ans: 50% or 0.5
   e. |+⟩
      Ans: 50% or 0.5
   f. |−⟩
      Ans: 50% or 0.5

3. If spin-½ particles were to be used for qubits, what spin-½ state would you use for the qubit initialization state, |0⟩?
   Ans: Spin-up state, |α⟩. This is a natural choice because the system will thermally equilibrate to this lowest-energy state.

4. If spin-½ particles were to be used for qubits, and all qubits were initialized in the quantum state |0⟩, list the pulses (e.g. 90_X, 180_Y) you could use to get a qubit in the following states:
   a. |1⟩
      Ans: 180_X or 180_Y

   b. |−i⟩

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Ans: 90_X

c. |+⟩
   Ans: 90_Y

5. Why do you think what you have learned thus far about NMR may be useful in understanding how to control, store, and recall information from qubits?
   Ans: The manipulation of superposition states of spin-½ particles using magnetic fields is a way we can control the state of a two-state qubit.

Quantum Circuits and Single-Qubit Quantum Gates - Guided Inquiry Questions

6. Fill in the table below using the Bloch sphere simulator.

Input	Gate	Output
|0⟩	X	|1⟩
|0⟩	Y	|1⟩
|0⟩	Z	|0⟩
|0⟩	H	|+⟩

7. Notice the path the qubit's state vector (the blue arrow in the simulator) takes when operated on by the various gates. If you were using a spin-1/2 particle as a qubit, what pulses could you use to mimic the following gates:
   a. X
      Ans: 180_X

   b. Y
      Ans: 180_Y

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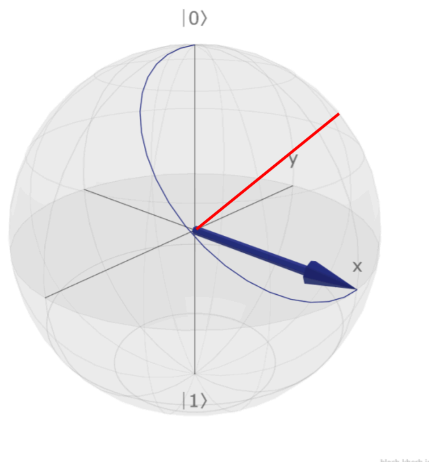
c. Z
Ans: 180_Z

d. H (This is trickier than the others - look carefully at what axis the state vector is being rotated around!)
Ans: Get to the same state using 90_X followed by 90_Z, but for the actual path taken along the block sphere, it is a 180-degree pulse around an axis that is 45 degrees from the z-axis in the x-z plane.



8. Using your NMR pulse analogues for the Pauli and Hadamard gates found in question 7, what outputs would you predict if the inputs in the table for question 6 were |1⟩ instead of |0⟩. Check your answer using the simulator. Note: For checking your answer in the simulator, you need to now start the qubit in |1⟩, so consider what gate/s you can first apply to get the qubit into |1⟩ from its initialized state.

Input	Gate	Output
|1⟩	X	|0⟩
|1⟩	Y	|0⟩
|1⟩	Z	|1⟩

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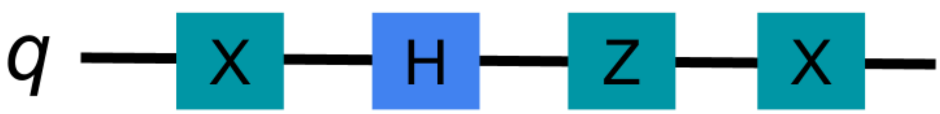
|1⟩	H	|-⟩

9. In your opinion, why might the Hadamard gate be particularly useful for quantum computations?
Ans: It outputs a state that has an equal probability of being in |1⟩ and |0⟩ (which includes the superposition |+⟩, |-⟩, |+i⟩, |-i⟩ states).

10. Using your NMR pulse analogues for the Pauli and Hadamard gates found in question 7, what would be the output for the quantum circuit model below if the qubit is initialized in |0⟩? Check your answer using the simulator.



Ans: |+ ⟩

Multiple Qubits - Guided Inquiry Questions

11. Suppose a two-qubit state is in the state:

$$|\psi\rangle = \frac{1}{\sqrt{4}}|00\rangle + \frac{1}{\sqrt{4}}|01\rangle + \frac{1}{\sqrt{4}}|10\rangle + \frac{1}{\sqrt{4}}|11\rangle.$$

What is the probability of measuring |ψ⟩ in the following two-qubit states:

a. |00⟩
Ans: 1/4 OR 25%
b. |01⟩
Ans: 1/4 OR 25%
c. |10⟩
Ans: 1/4 OR 25%
d. |11⟩
Ans: 1/4 OR 25%

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12. Suppose a two-qubit state is in the state |ψ⟩ given in Question 11.
    a. If you were to measure just q_1, what is the probability you would measure it to be in the state |0⟩? |1⟩?
    Ans: The probability is 1/2 or 50% for both states.

    b. If you now measured qubit 1 to be in the state |0⟩, what is the probability that you would measure q_2 in the state |0⟩?
    Ans: The probability is 1/2 or 50% for both states.

13. Suppose a two-qubit state is in the Bell state, $|\Phi^+\rangle$. What is the probability of measuring the two-qubit system in the following states:
    a. |00⟩
    Ans: 1/2 OR 50%

    b. |01⟩
    Ans: 0%

    c. |10⟩
    Ans: 0%

    d. |11⟩
    Ans: 1/2 OR 50%

14. Suppose a two-qubit state is in the Bell state, $|\Phi^+\rangle$ as in Question 13.
    a. If you were to measure just q_1, what is the probability you would measure it to be in the state |0⟩? |1⟩?
    Ans: 1/2 or 50% for both states.

    b. If you now measured qubit 1 to be in the state |0⟩, what is the probability that you would measure q_2 in the state |0⟩?
    Ans: 1 OR 100%

Two-Qubit Quantum Gates and Entangled States

15. In your own words, use the logic table for the CNOT gate above to explain what the CNOT gate does.
    Ans: The CNOT gate flips the state of q_2 if q_1 is in the state |1⟩, or leaves it in its original state if q_1 is in the state |0⟩.

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16. If the control qubit is in a state such as $|+\rangle$ and the target qubit is in $|0\rangle$, what can you say about the output state of the target qubit after being operated on by the CNOT gate? Would this be an entangled state for the two-qubit system?
Ans: The output state would be a superposition state, because there would be a 50% chance it is flipped from $|0\rangle$ to $|1\rangle$, and a 50% chance it is not. More specifically, the system would be in the entangled state of $|\Phi^+\rangle$, because if you measure the control qubit, you would know, with 100% certainty, the state of the target qubit, however the probability of measuring the control qubit in states $|0\rangle$ or $|1\rangle$ is still 50/50.

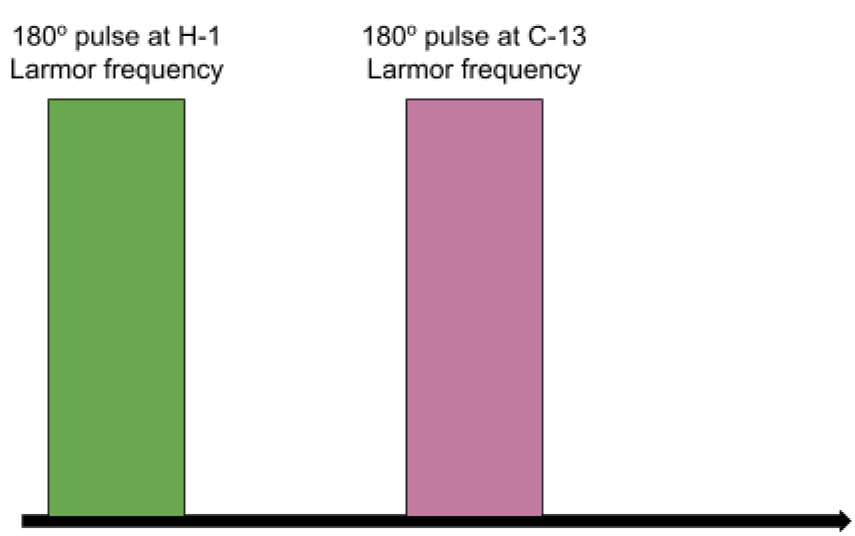
17. Determine the final state of the following circuit if q_1 and q_2 are both initially set to $|0\rangle$. Anything special about this final state?
Ans: The final state would be the Bell state $|\Phi+\rangle$ as described in question 16, which is special because it is an entangled state.

Making Quantum Computing a Reality - Guided Inquiry Questions

18. Which two spin-1/2 nuclei in chloroform (and their isotopes) do you think would be good candidates for our two qubits?
Ans: H-1 and C-13 isotope.

19. How would you initialize both of these nuclei into the $|0\rangle$ state?
Ans: Place the sample in an external magnetic field and let the spins settle into their lowest energy state, $|\alpha\rangle$, which corresponds to the $|0\rangle$ state.

20. If you wanted to put the qubits into the two-qubit state $|11\rangle$, draw the pulse sequence diagram you would use.



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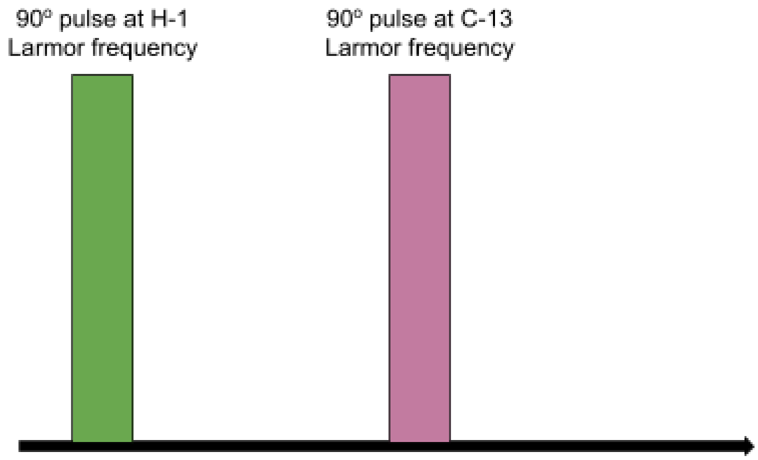
21. In order to entangle the spins through J-coupling, you would first need to get both spins precessing in the xy-plane of the Bloch sphere. Draw the pulse sequence you would use to achieve this.



Reflection Questions

1. Read again the quote by Seth Lloyd at the beginning of this module. In your opinion, can the universe be considered a quantum computer?
   Ans: (Obviously will vary!) Insomuch as everything in the universe is made up of quantum particles interacting under the rules of quantum mechanics, I think the universe could be considered a quantum computer, with the laws of physics dictating the algorithm being followed.

2. Out of the various applications of quantum computing discussed, which are you most excited about? Why?
   Ans: (Obviously will vary!) I am most interested in doing quantum simulations and what possible scientific breakthroughs can occur when we can better understand complex quantum systems using a quantum computer to model those systems.

3. Quantum circuit models have some similarities with pulse sequence diagrams. Coincidence or not? Make your case.
   Ans: (Obviously will vary!) I think it is likely not a coincidence, because pulse sequence diagrams were around well before quantum computers were theorized and widely used as a way to control two-level spin-½ systems once pulsed NMR became standard practice. It would make sense for scientists trying to control qubits to use the diagrams already understood by scientists. It was also not surprising that NMR quantum computing was then one of the first experimental systems to demonstrate a quantum algorithm.

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4. A helpful review of what you learned in this module can be found in this [Quantum Enigmas introductory video](). Describe three concepts mentioned in the video and their NMR analogues covered in this module.
   Ans: Some examples of NMR analogues in quantum computing include:
   - Qubit two-state superposition → Nuclear spin two-state superposition
   - Qubit state probability → Nuclear spin state probability
   - Qubit Bloch sphere state vector representation → NMR spin Bloch sphere
   - Quantum gates → Pulses

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