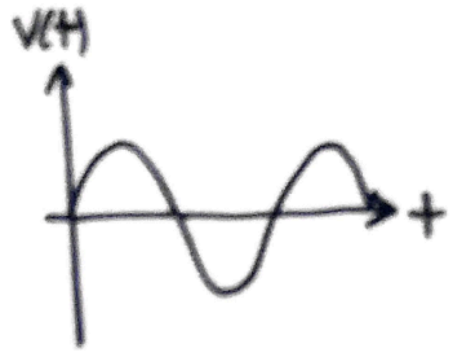
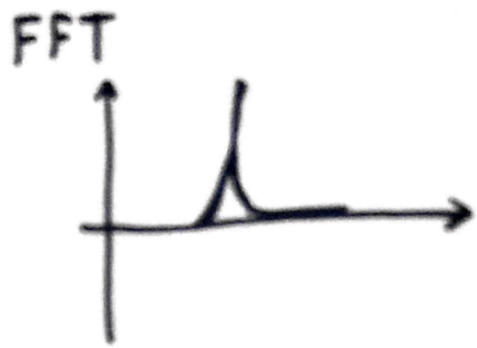
Module 8 Student Questions

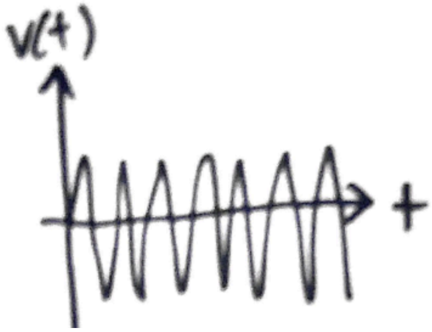
Observation Experiments: Looking at FFTs of Sinusoidal Time-Domain Signals - Guided Inquiry Questions

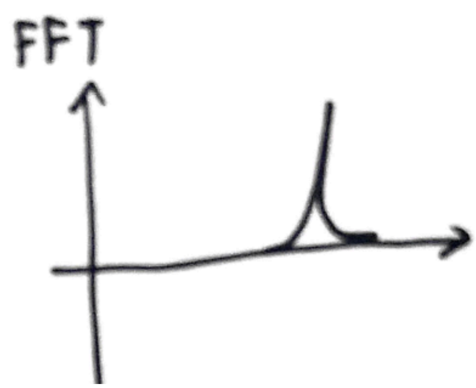
1. Have a lab partner set up any sinusoidal function they desire with the function generator and view it on the oscilloscope. Sketch the time-domain signal (i.e., the voltage versus time plot you see on the scope). Then have the scope show the FFT of this signal and sketch what you see to the right of your time-domain plot.





2. Have a lab partner increase the frequency of the sinusoidal function. Below your previous sketches, sketch the time-domain signal and the resulting FFT.





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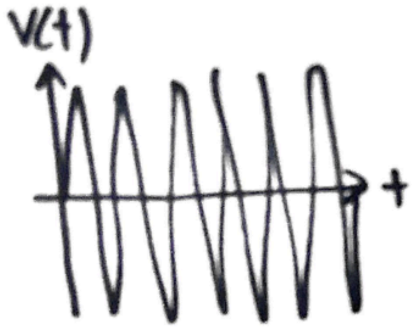
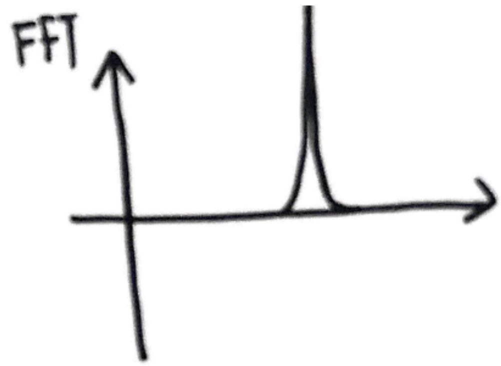
3. Have a lab partner increase the amplitude of the sinusoidal function. Below your previous sketches, sketch the time-domain signal and the resulting FFT.





4. From your observations, what do you think the FFT is plotting (i.e., which parameters do the x- and y-axes seem to correspond to?)

   Looks like the x-axis is related to the frequency of the time-domain signal, and the y-axis is related to the amplitude.

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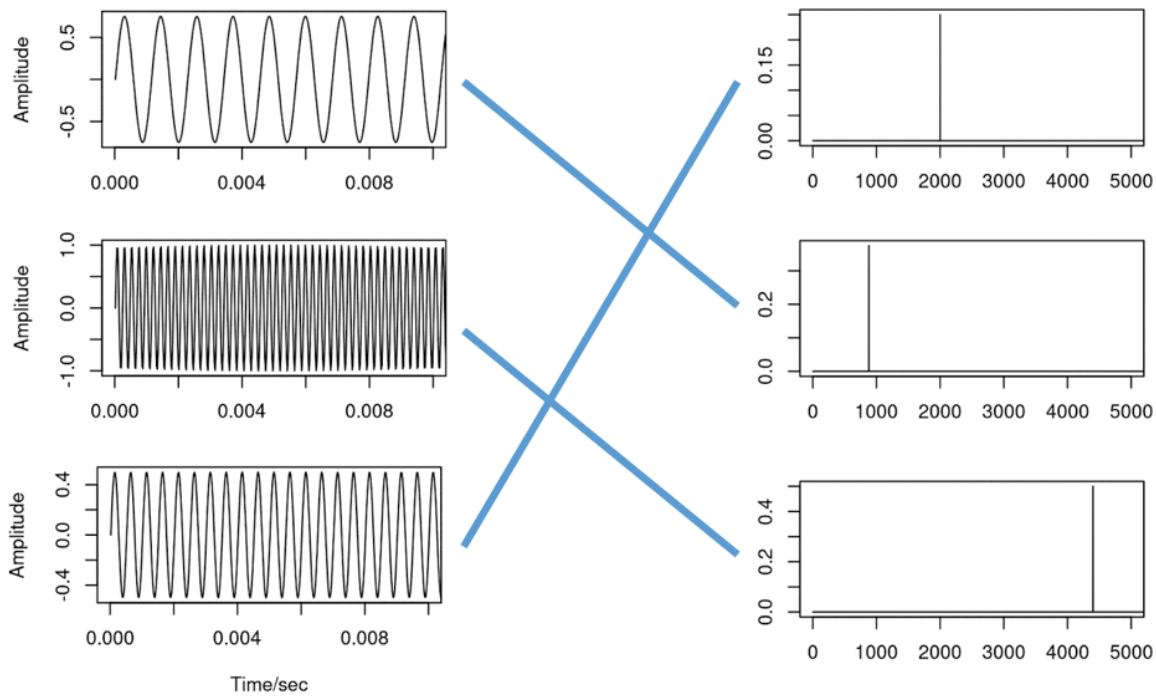
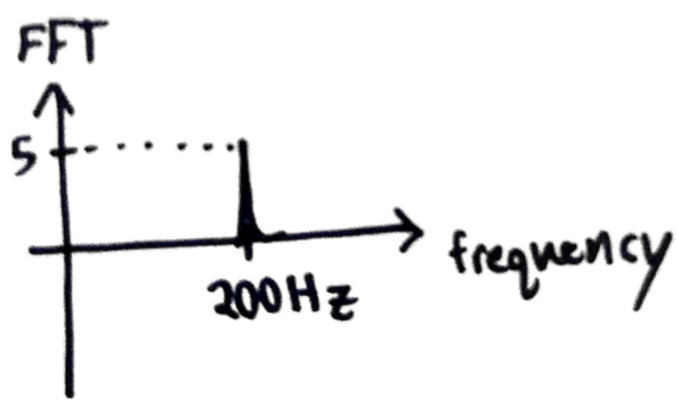
5. Match up the three time-domain signals below with their corresponding FFTs.



Testing Experiment: Expanding Beyond Sinusoidal Functions - Guided Inquiry Questions

6. If the time-domain signal is the sinusoidal function given below, and the hypothesis above is true, sketch your prediction of the resulting FFT.

$$s(t) = 5\sin(2\pi \cdot 200\ \mathrm{Hz} \cdot t)$$



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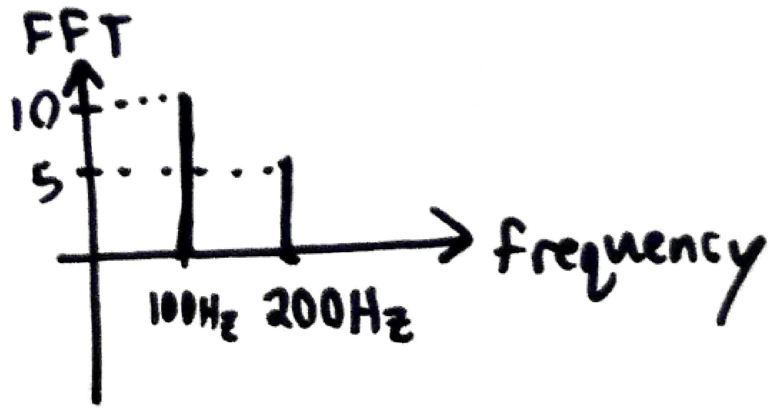
7. If the time-domain signal is instead a sum of two sinusoidal functions given below, sketch your prediction of the resulting FFT. *This case isn't exactly covered in our hypothesis above, so you may need to give an expanded hypothesis to explain your prediction.*

$$s(t) = 5\sin(2\pi \cdot 200\ \mathrm{Hz} \cdot t) + 10\sin(2\pi \cdot 100\ \mathrm{Hz} \cdot t)$$



8. Design and perform an experiment that can test the hypothesis given above. Write a brief summary of the experiment you performed, your results, and your conclusions regarding this hypothesis.

   We can generate a signal that is the sum of two different sinusoidal functions (with different amplitudes and frequencies) and then look at the resulting FFT. If this hypothesis is correct, then I would expect to see two peaks in the FFT corresponding to the two frequencies with two different amplitudes corresponding to the different amplitudes of the sinusoidal functions.

   We used an online tone generator and played that, and looked at the resulting FFT using the phyphox app. If you have an audio jack to BNC connector, you can analyze the resulting electronic signal using the oscilloscope. You should see two distinct peaks, and the separation of the peaks depends on how far apart you put the two frequencies. This supports our hypothesis.

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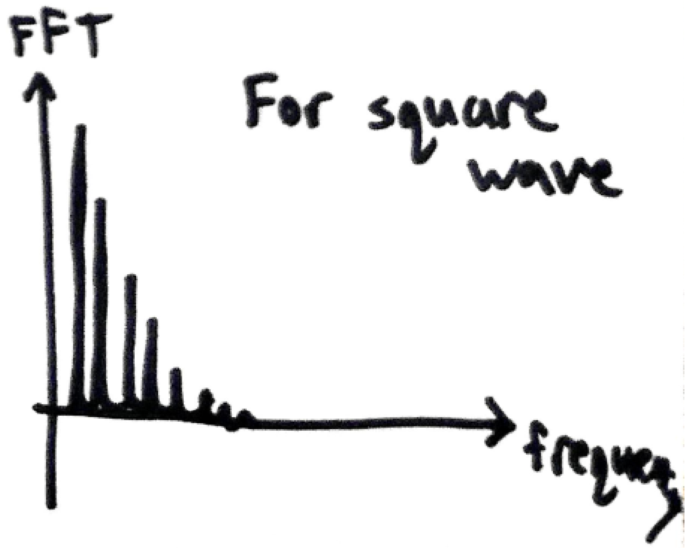
Observation Experiment: Exploring the FFTs of Periodic Time-Domain Signals - Guided Inquiry Questions

9. Sketch both the time-domain signal and the resulting FFT of your periodic but non-sinusoidal waveform.



10. What does the FFT of this periodic waveform suggest about how these periodic functions may be related to sinusoidal functions?

    Suggests that the squarewave waveform is made up of multiple sinusoidal functions with different frequencies and different amplitudes. (The larger the frequency, the smaller the amplitude).

To further explore the relationship between periodic functions and sinusoidal functions, we will use the following Desmos calculator: https://www.desmos.com/calculator/gjpuxeuwbj. There are two variables (*N* and *L*) that you can set by clicking on the sliders below.

11. Observe what happens when you increase *N*. What do you think *N* is controlling?

    As you increase N, more and more blue sinusoids are added to the plot. The red trace (which appears to be the sum of all these sinusoids) gets more complicated and appears to approach becoming a squarewave. N is controlling how many sinusoids are being added together to form the red trace.

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12. What is happening to the frequency and amplitude of each of the blue sinusoidal functions that get added as you increase *N*?

    As you increase N, each new sinusoid has a higher frequency and lower amplitude.

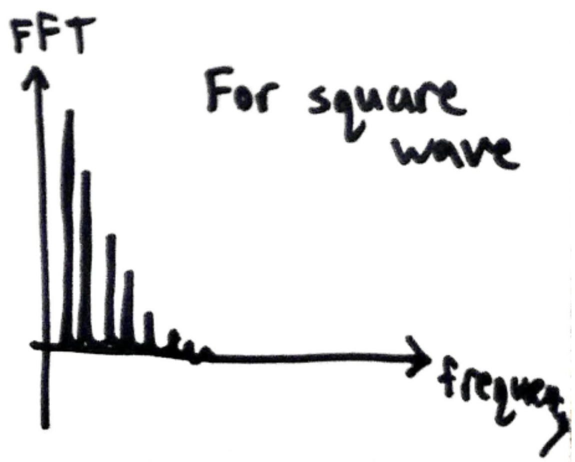
13. Observe what happens when you increase *L.* What do you think *L* is controlling?

    As you increase L, the width or period of the square wave increases, and the frequencies of all the blue sinusoidal functions decrease (wavelength increases). L is controlling the width/period of the square wave we are trying to produce.

14. The red trace is the sum of all the sinusoidal blue traces shown. How can you adjust the settings to make the red trace better approximate a square wave?

    Maximizing N is the best way to better approximate a square wave, along with reducing L so that higher frequencies are present.

15. Based on your answer above, sketch your prediction of the FFT of a square wave.



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16. Alice and Sayed have come up with the following hypothesis regarding periodic functions:

    All periodic functions *P(t)* can be written as an infinite sum of sinusoidal functions with different frequencies and amplitudes, such as

$$P(t) = \sum_{n=1}^{\infty} A_n \sin(2\pi n f t)$$

    Is this hypothesis supported or disproved by what you have observed experimentally and learned from the Desmos calculator for the square wave?

    What we have observed supports this hypothesis. It looks like the squarewave can be written as a sum of sinusoidal functions, so I would not be too surprised to learn that all periodic functions can be written as an infinite sum of sinusoidal functions. All the different frequencies would cover the different possible variations in the signal: long-range variations (low frequencies) or short-range variations (high frequencies).

What Information Does the Fourier Transform Provide? - Guided Inquiry Questions

17. How does the exponential decay of a sinusoidal time-domain signal appear to impact the height and width of the Fourier transform peak?

    The exponential decay appears to increase the width and decrease the amplitude of the Fourier transform peak. This makes sense because the area underneath the peak should be the same for both peaks, since we are assuming they start with the same maximum amplitude in the time domain.

18. As the signal decays faster (i.e., shorter T_2* relaxation time), does the width of the peak in the frequency spectrum decrease or increase? Does this match with the following description of the cause of T_2* relaxation - the spins in the sample are seeing different magnetic field environments and thus have slightly different precession frequencies that cause them to dephase. Explain your reasoning.

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The faster the decay (the smaller the time constant), the width of the corresponding peak in the FFT increases. This matches the description because the broader width of the Fourier transform peak suggests there is a wider range of frequencies present in the signal, which explains the dephasing of the spins and faster decay of the signal.

Reflection Questions

1. Where else might being able to gather frequency information from time-domain signals be useful? *You can watch this video, linked in the Example Real-World Application, to get some ideas.*

Beyond being very helpful for NMR and MRI, the Fourier transform appears to be important for digital signal processing, particularly where information might be more compactly represented by the frequency information instead of saving the complete time-domain signal. This can apparently help with digital file compression, both audio and visual. It also appears to be useful in machine learning algorithms like principal component analysis (PCA) and singular value decomposition (SVD). Some mathematical operations are much easier in the frequency domain compared with the time domain (like the convolution of time-domain signals). This is being explored in applications like convolutional neural networks.

2. Why might MR scientists primarily look at frequency spectra instead of the directly acquired time-domain data?

The frequency spectra have all the information we want in a nice, compact form that is easier for us to interpret!

The following questions make use of the figure below.

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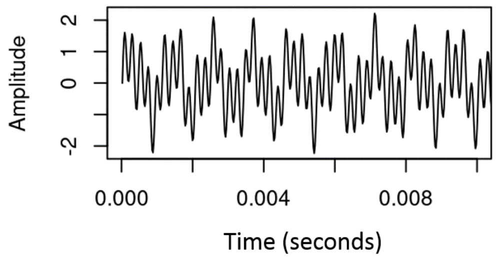

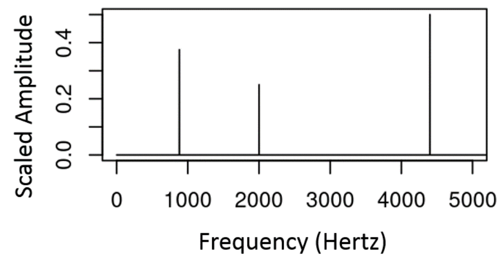

3. What approximate frequencies are present in the periodic signal shown in the figure above?

Approximately 900 Hz, 2000 Hz, and 4300 Hz.

4. Do all the frequencies contribute equally to the signal? Why or why not?

No, not all the frequencies contribute equally to the signal, as evident by their different amplitudes. There appears to be more 4300 Hz than 900 Hz, and the least amount of 2000 Hz.

5. Write a possible mathematical equation for the time-domain signal, assuming it is a sum of multiple sinusoidal functions. Please use a form similar to the many $s(t)$ given in the text.

$$s(t) = 0.38\sin(2\pi \cdot 900\ \mathrm{Hz} \cdot t) + 0.25\sin(2\pi \cdot 2000\ \mathrm{Hz} \cdot t) + 0.49\sin(2\pi \cdot 4300\ \mathrm{Hz} \cdot t)$$

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