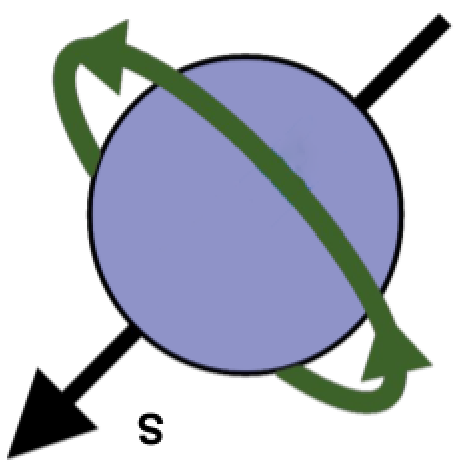
Module 3 - Answers

The Spinning Aspects of Quantum Spin - Guided Inquiry Questions

1. Many people use 'spin' to refer to either the spin quantum number or the spin angular momentum vector. If I were to tell you the spin of a particular electron is $\hbar/2$, which aspect of spin am I talking about?

   Ans: Since we know that $\hbar$ corresponds with spin angular momentum it is evident to assume that spin angular momentum is the aspect of spin that is being referred to.

2. Draw a picture of a spin rotating in the opposite direction to the one shown above. *Make sure to draw the S vector pointing in the correct direction using the right-hand rule!*

   Ans:

3. Based on the behavior observed in our physical model of a quantum spin, do you think it is safe to say that it has some angular momentum and that angular momentum is an important factor in explaining the dynamical behavior observed?

   Ans: Yes, it is safe to say our physical model of a quantum spin has angular momentum because it appears to behave like a spinning object and spinning objects have angular momentum. Presumably, angular momentum is important

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for explaining the dynamic behavior we observe of both a gyroscope and our physical model of a quantum spin.

Testing Experiment: What causes the quantum spin to interact with the magnetic field? - Guided Inquiry questions

4. Describe (using both words and pictures) what you observe of the behavior of both the white cue ball and gyroscope in the presence of a magnetic field without any spinning.

   Ans: The white cue ball oscillates back and forth (like an upside-down pendulum) with an equilibrium point aligned with the magnetic field. The gyroscope falls towards the direction of gravity.

5. List some different explanations for why our physical model of a quantum spin (white cue ball) can interact with a magnetic field. *Some explanations may seem more plausible than others, but list all the explanations you can think of since we don't know what the correct answer may turn out to be, and it may not be the most obvious one!*

   Ans: The white cue ball can interact with the magnetic field due to: (1) having a magnet within the ball itself or (2) the ball being made out of a (strong) magnetic material.

6. For your list of explanations (this will become your different *hypotheses*), design an experiment whose outcome you can predict using all the hypotheses that you constructed. *Note that when there are multiple explanations, the best-designed experiment will give different predicted outcomes, allowing us to determine which explanation best explains the observed phenomenon*.

   Ans: We need to perform an experiment that differentiates whether the cue ball contains an actual magnet or is made of a magnetic material that strongly reacts to magnetic fields (but not a permanent magnet itself). Some possible experiments would be: (1) see how iron filings, magnetic materials, and compasses interact with the cue ball (i.e. are they deflected/attracted) - would interact if magnet but not if electrically charged or magnetic material.

7. For each different hypothesis: write down what you would predict to observe if you performed your chosen experiment and that particular hypothesis were correct. For example, "If [hypothesis] is correct and we perform [experiment], then we would predict [predicted outcome for that hypothesis]."

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Ans: If the cue ball has a magnet inside the cue ball and we place a compass near the cue ball at different positions, then we would predict the compass needle will move around in response to the magnetic field being produced by the magnet inside the cue ball.

If the cue ball is made of some magnetic material and we place a compass near the cue ball at different positions then we would predict the compass needle will NOT move around when placed near the cue ball at different positions.

8. If you ultimately observed something different than your prediction for a particular hypothesis, what would that tell you about that hypothesis?

   Ans: If the outcome is different from your prediction then the hypothesis is disproved.

9. Perform your experiment and/or watch some of the videos of the different experiments students have performed. Write down a brief description of the experiment being performed, and the observed results of that experiment. Based on the experimental results, what is your judgment about your different hypotheses?

   Experiment #1: The cue ball interacted with the compass. The needle pointed towards the compass when placed very close to the cue ball. Once the compass was placed farther away from the cue ball, the compass went back to working as intended.

10. Based on the experimental results, is there a particular hypothesis that provides the best explanation of why the white cue ball interacts with a magnetic field? Please explain by referencing the experimental results.

    Ans: The compass changes direction when placed near the cue ball. The only hypothesis that matches the experimental results is that the cue ball has a magnet inside of it.

Spin Magnetic Moment - Guided Inquiry Questions

11. In the visualization of a quantum spin given above with both the spin magnetic moment (as a bar magnet) and the spin angular momentum, is the gyromagnetic ratio positive or negative? How can you tell?

    Ans: Positive gyromagnetic ratio. Based on the bar magnet being aligned with the angular momentum vector.

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12. Draw your own visualization of a quantum spin with a negative gyromagnetic ratio. *Feel free to have it rotate in any direction, but make sure to draw the S vector pointing in the correct direction using the right-hand rule!*

    Ans: DRAW (flip the magnet and keep the spin in the same direction or vice versa)

Exploratory Experiment - What determines the frequency of precession of a quantum spin? - Guided Inquiry Questions

13. Consider the different possible ways we can set up the precessional motion of our physical model of a quantum spin (the white cue ball), including the different apparatus controls highlighted in the diagram given in the **Background Information section**. List all the possible variables you can think of that might influence the precession frequency of our physical model of a quantum spin.

    Ans: The frequency of our physical model can be affected by the strength of the magnetic current, the direction of the magnetic field, how fast the ball is spinning, and the angle of the axis of rotation with respect to the magnetic field.

14. Perform some experiments and/or watch some of the videos of the different experiments students have performed. *Try to only change one variable at a time! If a particular variable is hard to reliably reproduce, then test that particular variable first so you can potentially rule out its influence on future experiments.* For each experiment, write down what independent variable was being changed and your observations of the impacts on the precession frequency.

    Experiment #1: Independent variable: How fast the ball is spinning (magnitude of angular momentum). The precession frequency changes slightly. Slower precession frequency for faster spinning (higher angular momentum).

    Experiment #2: Independent variable: Angle of the axis of rotation with respect to the magnetic field (direction of angular momentum). The precession frequency may change very slightly. (Slightly higher precession frequency for smaller angles, i.e. rotation axis is closer to straight up and down.)

    Experiment #3: Independent variable: Magnet current. If we increase the magnetic current then the precession frequency will increase. If we decrease the magnetic current then the precession frequency will decrease.

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Experiment #4: The direction of the magnetic field is the independent variable. The precession frequency of the cue ball does not change depending on the direction of the magnetic field, but the direction of the precession does change (i.e. from CW to CCW).

15. Based on the experiments above, what variables influence the precession frequency?

    Ans: The strength of the magnetic field clearly influences the precession frequency. How fast the ball is spinning and the angle looks like it might influence the precession frequency, but not as dramatic as magnetic field strength. The direction of the magnetic field did NOT influence the precession frequency.

Larmor Precession - Guided Inquiry Questions

16. In the apparatus we have been using, the magnet current in the magnet coils are directly proportional to the magnetic field strength (e.g. if you took the current value and multiplied it by a particular constant, you would get the magnetic field strength, B.) If you doubled the magnet current, what would you expect to happen to the magnetic field strength? What would happen to the precession frequency?

    Ans: If you doubled the magnet current the magnet field strength should also double, which would also double the precession frequency.

17. Do your conclusions from your precession experiments above appear to agree with the Larmor precession frequency equation given for a quantum spin? Explain.

    Ans: Yes, they agree with the Larmor precession frequency equation. We see that the Larmor precession depends on the strength of the magnetic field and the gyromagnetic ratio. The precession frequency increasing with the increasing magnetic field strength is a direct match with the Larmor precession frequency equation.

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The observation that the precession frequency decreased for faster spinning (higher angular momentum) also matches with the Larmor precession frequency equation. Since spinning the cue ball faster increases the angular momentum without changing the magnetic moment of the cue ball, thus it actually *decreases* the gyromagnetic ratio, since $\vec{\mu}_s = \gamma \vec{S}$. This matches what we would expect from the Larmor precession frequency equation - decreasing the gyromagnetic ratio (by increasing the angular momentum while keeping the magnetic moment constant) would decrease the precession frequency. (I like to think of it as the gyro- part of the gyromagnetic ratio is increasing, and the magnetic component is not increasing to compensate, so the precessional frequency goes down.)

18. Are there any differences with the behavior of our physical model of a quantum spin and the theoretical quantum behavior given by the Larmor precession frequency equation? What does this suggest about the possible limitations of our physical model?

    Ans: There are some differences between our physical model of a quantum spin and the theoretical quantum behavior given by the Larmor precession frequency equation. The angle of the spin relative to the magnetic field should not change the precession frequency according to the Larmor precession frequency equation. However, the precession frequency of the cue ball in the magnetic torque apparatus can change somewhat with the angle. One possible explanation could be if the center of mass is not exactly in the center of the cue ball, then we expect to see a bit of impact from classical gravitational torque acting on the cue ball along with the magnetic torque, which could vary with angle. Our physical model also has friction and other drag forces at play, which is not a factor for actual quantum spins, so possibly that plays a role in the angle dependency as well. Of course, it could also be some other possible explanation. This would be an interesting phenomenon to explore further!

19. What precession frequency would you expect for ^{1}H in a 2-T magnetic field? What precession frequency would you expect for an electron in the same magnetic field? What does the negative sign mean?

    Ans: 85.16 MHz precession frequency for the hydrogen. The electron precession frequency would be -54,408 MHz. The negative sign means that the electron would be moving in the opposite direction.

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20. If you observed a Larmor frequency of 80.1 MHz in a 2 T magnetic field, which nucleus are you likely observing?

    Ans: We are most likely looking at ^{19}F.

Reflection Questions

1. What is the independent variable (i.e. the variable the experimenter was controlling) in the data given? What is the dependent variable (i.e. the variable that was measured).

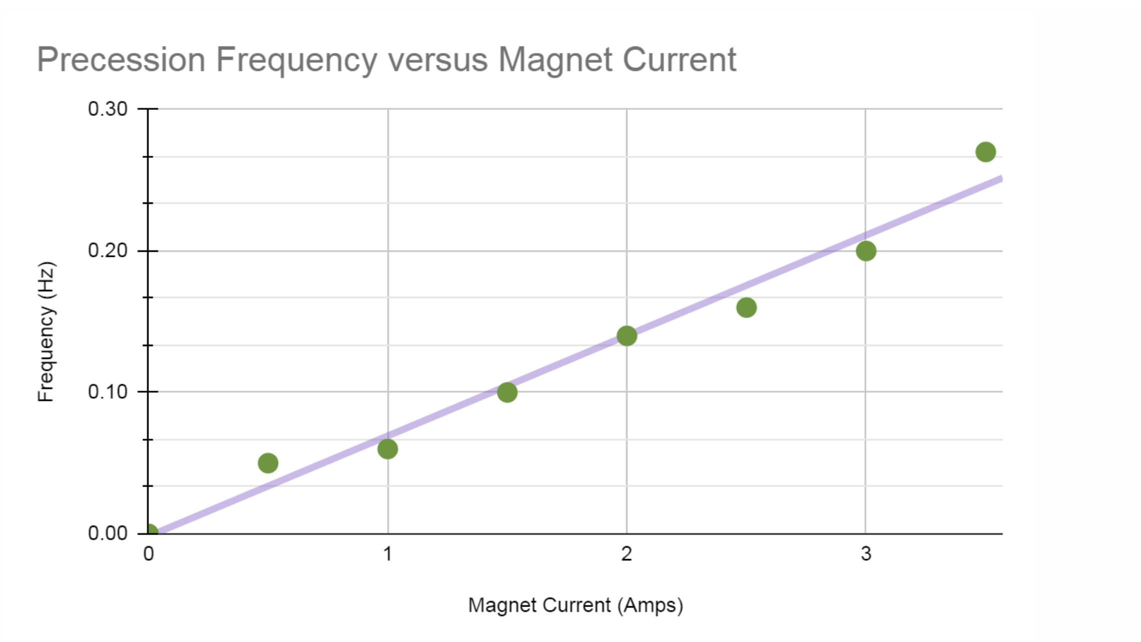
    Ans: Independent variable: Magnet Current

    Dependent variable: Precession Frequency

2. What experiment was being performed?

    Ans: Measuring the precession frequency change when applying different magnet currents (effectively changing the magnetic field strength).

3. Neatly plot the data, with the independent variable on the x axis and the dependent variable on the y axis.



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4. What type of relationship do these variables appear to have with each other (e.g. completely independent from each other, linear dependence, or some other dependence?)

   Ans: This is a clear relationship between the two variables (e.g. the precession frequency increases as the magnet current increases), and it looks pretty linear (like we would expect for a true quantum spin!)

5. Does this data match what we expect given the equation for Larmor precession of a quantum spin? Why or why not?

   Ans: This data does follow the Larmor precession trend where the frequency increases linearly as the magnetic field strength increases.

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