Module 4 - Answers

Bloch Sphere - Guided Inquiry Questions

1. If the quantum spin is in the green state shown in the figure and then is measured along the z-direction (meaning it can be found in either α or β), which state will it more likely be found in?
   Ans: It could be found in either state because it is in a quantum superposition state, but it is more likely to be in the **α** state as it is in the upper hemisphere.

2. If the quantum spin is in a quantum state represented by an arrow aligned with the +y axis and then is measured along the z-direction (meaning it can be found in either α or β), what is the percent probability of it being found in α? Being found in β? Hint: Since there are only two allowed states, the percent probability of being found in α plus the percent probability of being found in β must equal 100%.
   Ans: There is 50% probability of it being in α and a 50% probability that it is in β.

Initializing Spin States - Guided Inquiry Questions

3. A common initialization state for quantum experiments would be the lowest energy level of the system. Give an argument for why you think that is a common choice.
   Ans: This could be a common choice as the system will naturally go towards that state.

4. For our spin-½ particles in the presence of an external magnetic field, what spin state would be the lowest energy level?
   Ans: The spin up, **α** state.

5. If we wanted to initialize our spins into the lowest energy state, what do you suggest we do?
   Ans: Wait a long time without adding more energy to the system (and allow interactions with the environment that enable energy to leave the system) and the system will naturally transition into the lowest energy state.

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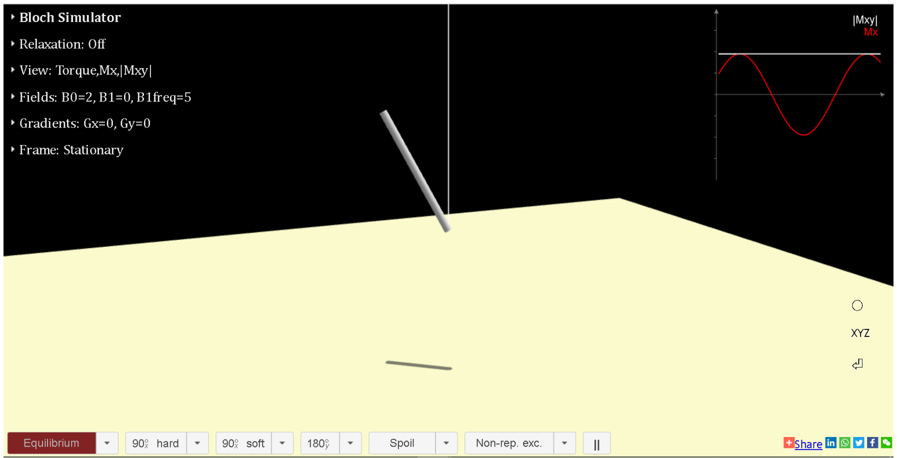
Getting Started with the Bloch Simulator

6. Open the Bloch Simulator (https://www.drcmr.dk/BlochSimulator/).



   a. Describe what you see.
      Ans: a singular rod moving in circles about the z axis, casting a shadow on the ground. There is a menu section to the left and a graph to the right
   b. What motion do you think is being depicted?
      Ans: precession about the z (vertical) axis
   c. What is being shown in the plot in the upper righthand corner?
      Ans: |Mxy| versus time (white) is being plotted as well as Mx versus time (red). The white trace is remaining constant while the red is showing a sinusoidal curve.

7. Is there a magnetic field being applied? If so, what direction is this magnetic field being applied?
   Ans: Yes, there is a magnetic field applied in the vertical (z) direction.

8. In the upper left corner there are several drop down menus for changing different parameters of the simulation. Click on the dropdown arrow for ‘Fields’. B0 is typically reserved for the large external magnetic field being applied in the +z direction. Observe what happens when you increase or decrease B0. Does this match with what we saw happen with our physical model of quantum spin?
   Ans: As you increase the B0, the rod starts precessing faster about the z axis and when you decrease the B0, the rod starts precessing slower. When the B0 is at 0 the rod no longer precesses. This observation follows the trends we saw with the physical model of the quantum spin with the cue ball.

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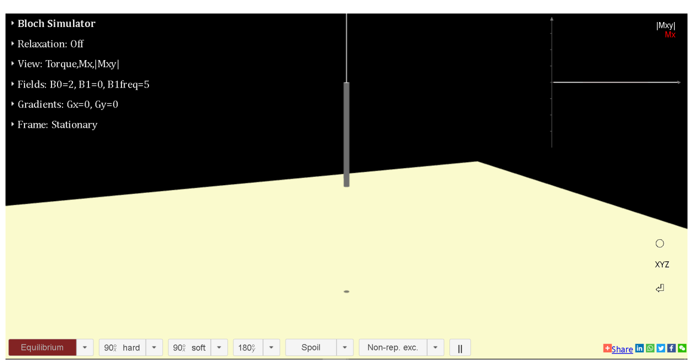
Side drop down menus

9. Click on the dropdown arrow for ‘Frame’ and change from ‘Stationary’ to ‘B0’.
    a) Describe what you observe now. What is happening when you switch from ‘Stationary’ to ‘B0’?
    Ans: The rod precession along the axis has stopped and the background plane is showing movement in the form of rotation about the z axis in the opposite sense of the rod precessing in the stationary frame.
    b) In NMR, scientists often reference the ‘lab frame’ and the ‘rotating frame’. Which of the possible frames in the simulator do you think would correspond to the lab frame? Which do you think would correspond to the rotating frame?
    Ans: I think the lab frame would be the stationary frame and the rotating frame would be the B0 frame.

10. Try clicking on the red ‘Equilibrium’ button in the lower left-hand corner. Describe what happens. What quantum state is the spin put in?
    (Note: This is a good way to initialize the simulator so that the system starts in a known quantum state!)



Ans: The rod stops spinning/rotating/precessing and aligns with the z axis. It is put in a useful initialization quantum state.

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Controlling Transitions Between Spin States - Guided Inquiry Questions

11. Consider how the dynamics of the Bloch Simulator are very similar to the dynamics of the physical model of quantum spin we saw in the previous module.
    a) If we were to introduce another magnetic field to cause a spin-flip, what direction of the magnetic field should we be using (e.g. should the additional magnetic field be oriented in the X, Y, or Z direction?)
    Ans: Any direction perpendicular to the z axis (e.g. X, Y, or any direction along the XY plane) will cause a spin flip
    b) Would you want the additional magnetic field to remain 'on' indefinitely, or would you need to turn it 'off' at some point? Explain your reasoning.

    (Hint: Consider the spin-flip has a precession that only goes half-way around.)
    Ans:. It would be preferable to turn it off once it reaches the desired spin state. If it is left on, it will continue to rotate and alternate between the two spin states indefinitely.

12. In the Bloch Simulator, initialize the quantum state (using 'Equilibrium' button) and turn off all the 'Fields' to 0 (B0, B1, and B1Freq). Under the menu 'View' you can **check** the checkbox next to 'B1' to show the B1 direction and **uncheck** the checkbox next to 'Torque/B1eff'. Now increase the B1 field and observe what happens to the quantum state in the lab frame. Describe what happens. What direction is the B1 field pointing in? Will this potentially be helpful for causing a spin-flip?

    Ans: the rod changes direction and does a flip of sorts and goes in the downwards direction. The rotation is changed and it goes down then up in a circular motion. The B1 is pointing in the horizontal direction and this will be helpful in causing a spin flip.

13. For actual NMR experiments, we cannot simply turn off the large external field B0, so make B0 nonzero as well (but keep B1Freq set at 0) and observe what happens to the quantum state in the lab frame. Can you explain why that might be happening?
    Ans: The rod is precessing about an axis between z axis and B1 axis. It is precessing about the net magnetic field which is the vector sum of B0 and B1.

14. Now keep B0 and B1 at some non-zero value and observe what happens when you increase B1Freq. Explain what you think B1Freq is controlling.
    Ans: The precession changes and the B1 field starts to precess around the xy plane. B1 frequency controls the precession of the B1 field.

15. Your goal is to try to replicate a spin-flip (starting from the spin-up/equilibrium state) with a non-zero B0 using B1. You can play with any of the settings in the 'Fields' menu. Record the values that you use in order to replicate a spin-flip.
    Ans: Lots of potential options, but B1 Freq needs to be equal to B0 in order to get the spin to flip completely from an initial spin-up state.

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16. In order to induce a spin-flip using B1, did you have to change B1Freq? What did you have to do to get the desired result? The units in the simulator are dimensionless, simply numbers showing the relative strength of the different variables. In actual NMR experiments, what do you think the frequency of B1 would be (if you know the strength of the B0 field and the gyromagnetic ratio of the spin)? *Hint: MR experiments are making use of **resonance** and the natural frequency of spins in a external magnetic is the Larmor frequency.*

    Ans: Yes, the B1 Freq had to be equal to B0. The frequency of B1 would be the same as the Larmor frequency = gyromagnetic ratio*B0.

17. In order to induce a spin-flip using B1, did you have to keep B1 'on' for a limited time? Usually the sources of these B1 fields in NMR are called 'pulses' and the different lengths of pulses are labeled by the angle of rotation they will cause the quantum state to undergo and the axis about which they will rotate the quantum state. For example, $90°_y$ would provide a short-lived B1 field in the y-direction that causes a 90° rotation about the y-axis. In the bottom row, to the right of the 'Equilibrium' button, you will see some options of 'hard' and 'soft' pulses. Try some of them out and observe what happens to the B1 field during the pulse and the resulting motion of the spin. What is the difference between 'hard' and 'soft' pulses?
    Ans: Yes, B1 does have to be on for a limited time if you want the spin to stay in the spin-down state. During a hard pulse, B1 has a large magnitude and causes the spin to rotate very quickly, so the pulse length is very short. During a soft pulse, B1 has a smaller magnitude and the spin takes much longer to rotate to its final position, so the pulse length is longer.

18. Scientists typically like to view the spin dynamics in the rotating frame. View some of the pulses in the rotating frame. Why might looking at the spin dynamics in this frame be more convenient?
    Ans: It allows you to view the rod at the same speed and direction as it is precessing. It makes it easier to view what is going on (essentially seeing everything from the spin's POV).

Nutation and Pulses - Guided Inquiry Questions

19. Scientists typically consider a very short pulse duration as a 'hard' pulse. What parameter would you adjust in order to generate a 'hard' pulse? Give an explanation for your answer using the nutation frequency, $f_1 = \gamma B_1$.

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Ans: I would alter the magnitude of B1. The smaller the B1 field the lower the nutation frequency, and the longer the pulse for a given rotation angle.

20. With the addition of pulses, do we have the ability to effectively 'control' the quantum state? Why or why not?
    Ans: Yes, we can effectively "control" the quantum state, since we have a method of initialization and putting the quantum spin into a particular state along the Bloch sphere using pulses. In reality, it is not possible to do both these things 100% accurately, but you can never expect that with experimental work in the physical world (and especially not in the quantum world!)

Reflection Questions

1. Why might scientists prefer the Bloch sphere representation for two-level quantum systems instead of the energy-level representation?
   Ans: Scientists prefer the Bloch sphere system as the quantum state of a two-level system is represented by an arrow that starts at the origin of a sphere and points at a particular point on the sphere. All of the possible quantum states - including superposition states - are represented in the Bloch sphere, and you can visualize the transition from one state to another through these superposition states, unlike the energy-level representation. It is also helpful that the dynamics of a quantum state on the Bloch sphere matches the motion we observed in our physical model of spin using the magnetic torque apparatus.

2. How is utilizing resonance an important aspect of quantum spin control?
   Ans: You can't do a true spin-flip without the B1 field being in resonance (B1freq = B0 precession frequency) and this allows you to put the spin in *any* of the allowed Bloch sphere quantum states. Also, resonance allows for the most efficient transfer of energy from the environment to the quantum system, optimizing our control.

3. View this video using the magnetic torque apparatus from our previous module. Explain what you think is going on.
   Ans: There is an applied horizontal field in addition to the vertical magnetic field, causing the cue ball to precess around the new vector sum of both magnetic fields.

4. Write down the pulse sequence that led to this video.
   (Note: this video was taken in the rotating frame, so you want to use that frame when trying to replicate this pulse sequence using the simulator!)
   Ans: $90°_y$ (hard) -> $90°_y$ (soft) -> $180°_y$ (hard)

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