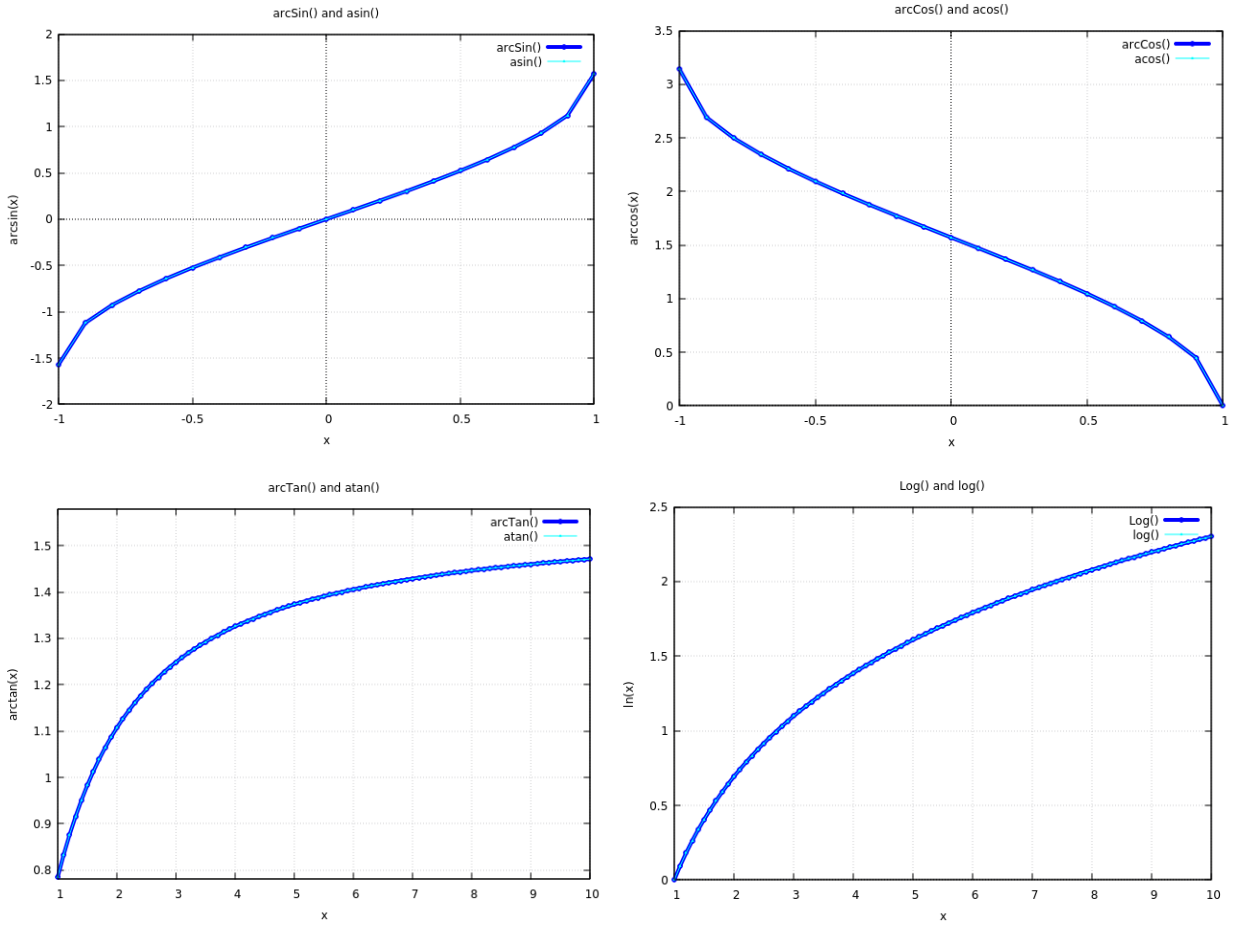
Merilyn Kuo
CSE 13S
Spring 2021

Assignment 2: A Small Numerical Library

Writeup

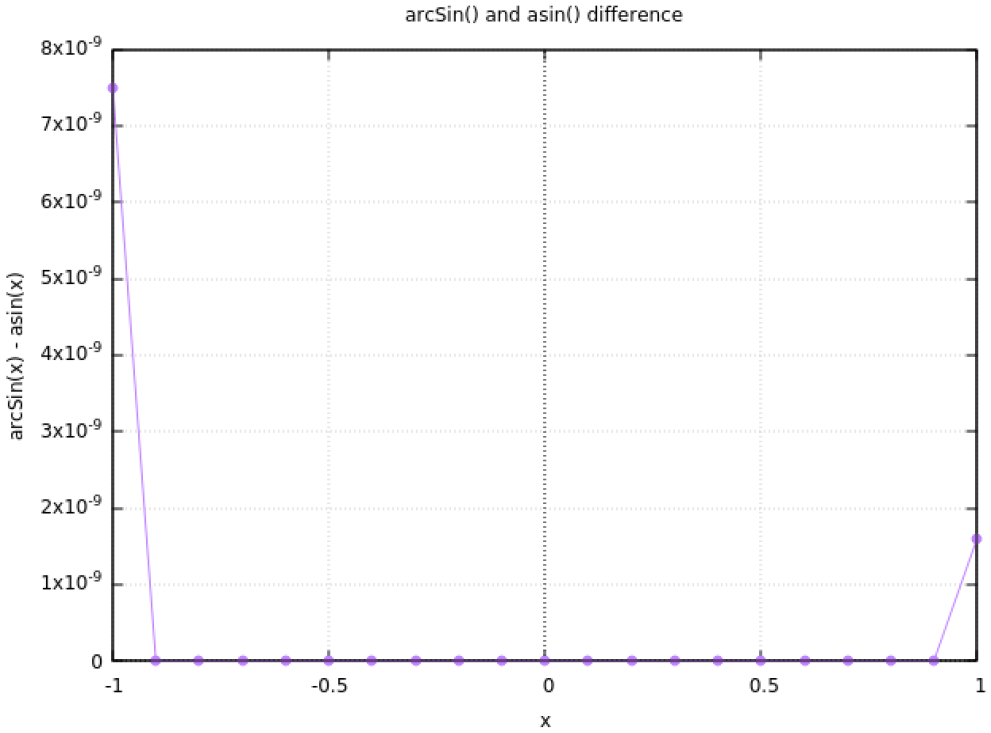
For the most part, the differences between my functions and the library's functions are zero with the few exceptions of endpoints and even then, the difference is extremely small. The following graphs show the outputs of my functions and the library's functions plotted. Just by looking at them, it seems that the two functions are the same and create quite a nice curve in the arcsin and arccos functions and even smoother curves for arctan and log.

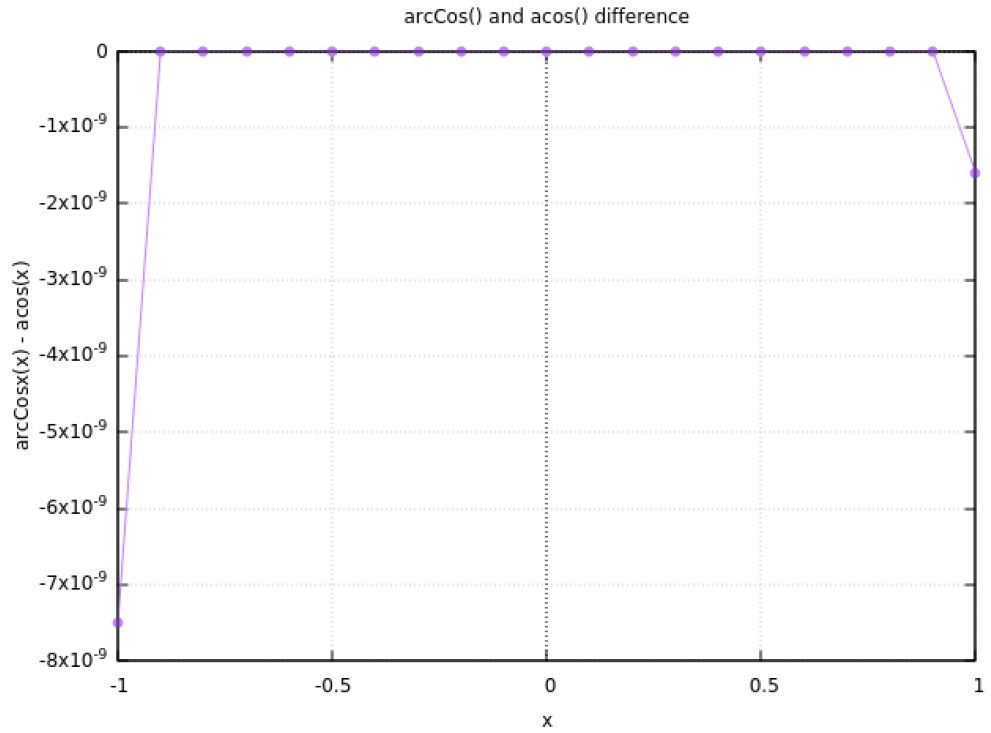


*Larger graphs can be viewed at the end of the document

Using an epsilon of 1e-10, the difference between the arctan and log functions is zero for all x in [1, 10]. This can be shown by running the command `./mathlib-test -t -l`. For the arcsin and arccos functions, there are only differences at the two endpoints for x in [-1, 1]. At x = -1, the

difference for both arcsin and arccos is 7.50 x 10^{-9} and at x = 1, the difference is 1.60 x 10^{-9}. Everywhere else, the difference is zero. A graph plotting the differences between functions for arcsin and arccos are shown below.

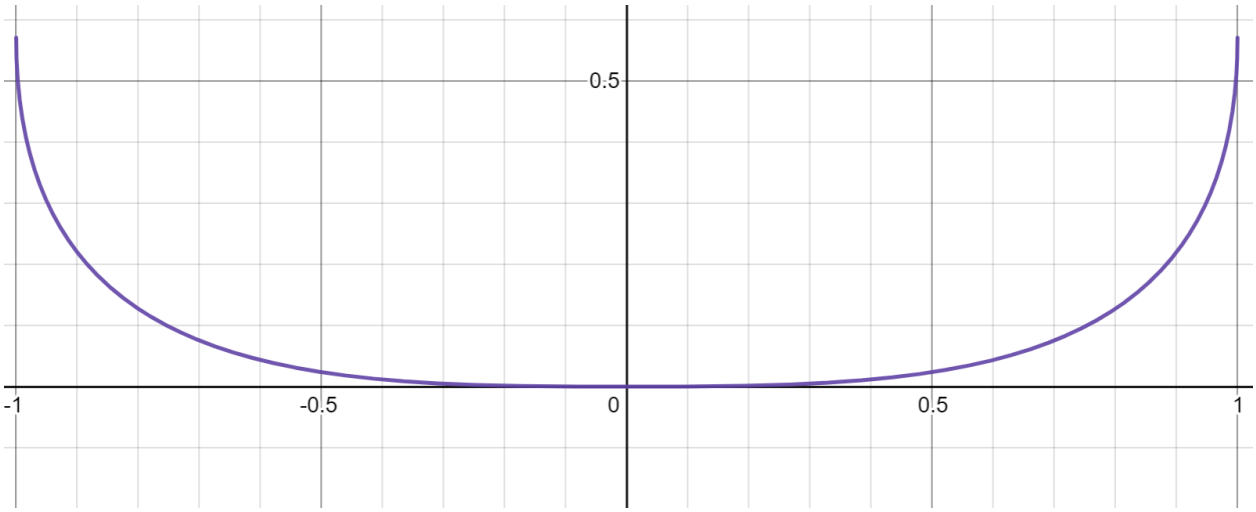




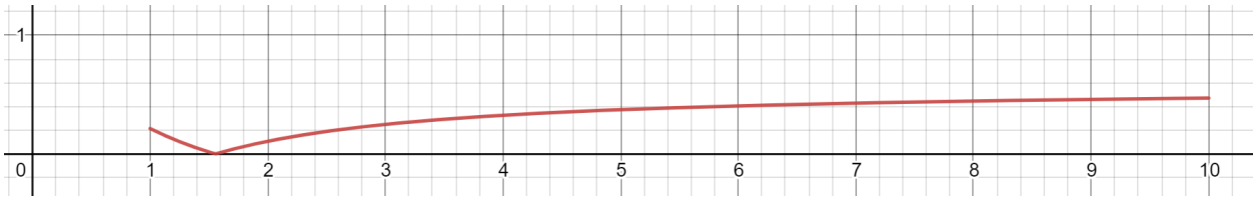
For the arcsin function, I used Newton's method of approximation. For the arccos function I used the identity, $arccos(x) = \frac{\pi}{2} - arcsin(x)$. Newton's method is highly dependent on the initial guess for a good approximation. The closer the initial guess is to the actual value, the better the approximation Newton's method will give. In the arcsin and arccos graph, there appears to be a large spike in difference when x is 0.9 and 1 and when x is -0.9 and 0.1. The same goes for arccos. For arcsin, I used x itself as the initial guess since I saw that the actual value of arcsin(x) was close to x itself. Between -0.9 and 0.9, the difference between the actual value and the initial guess is no more than0.2198. However, when x is -1 or 1, the difference between the actual value of arcsin(x) and the initial guess, x is about 0.5708. This explains the spike in inaccuracy between arcSin() and asin() at the endpoints. It also explains the spike in the inaccuracy between arcCos() and acos() at the endpoints since my arccos function is correlated to arcsin(x).

In the arctan and log functions there were no points where the difference between the actual value and initial guess (x =1) was drastically different from the rest.

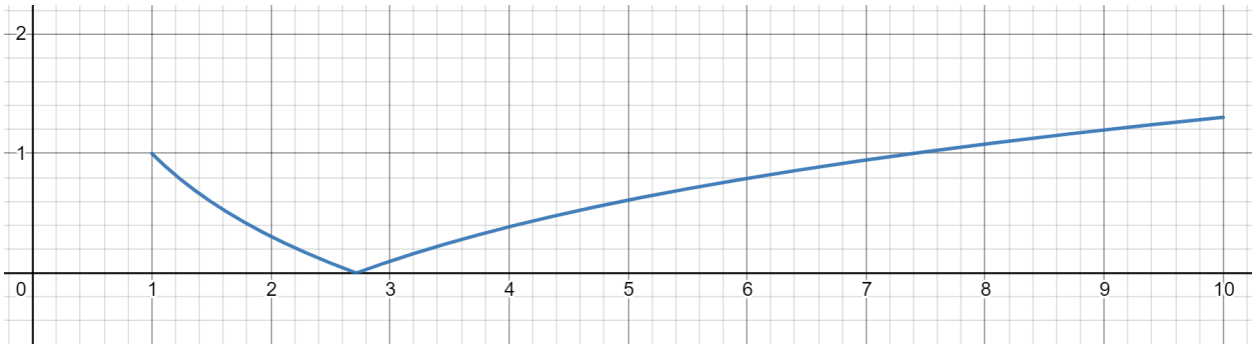
The following graphs show the differences between the initial guess and the actual value.



$y = |x - arcsin(x)|$

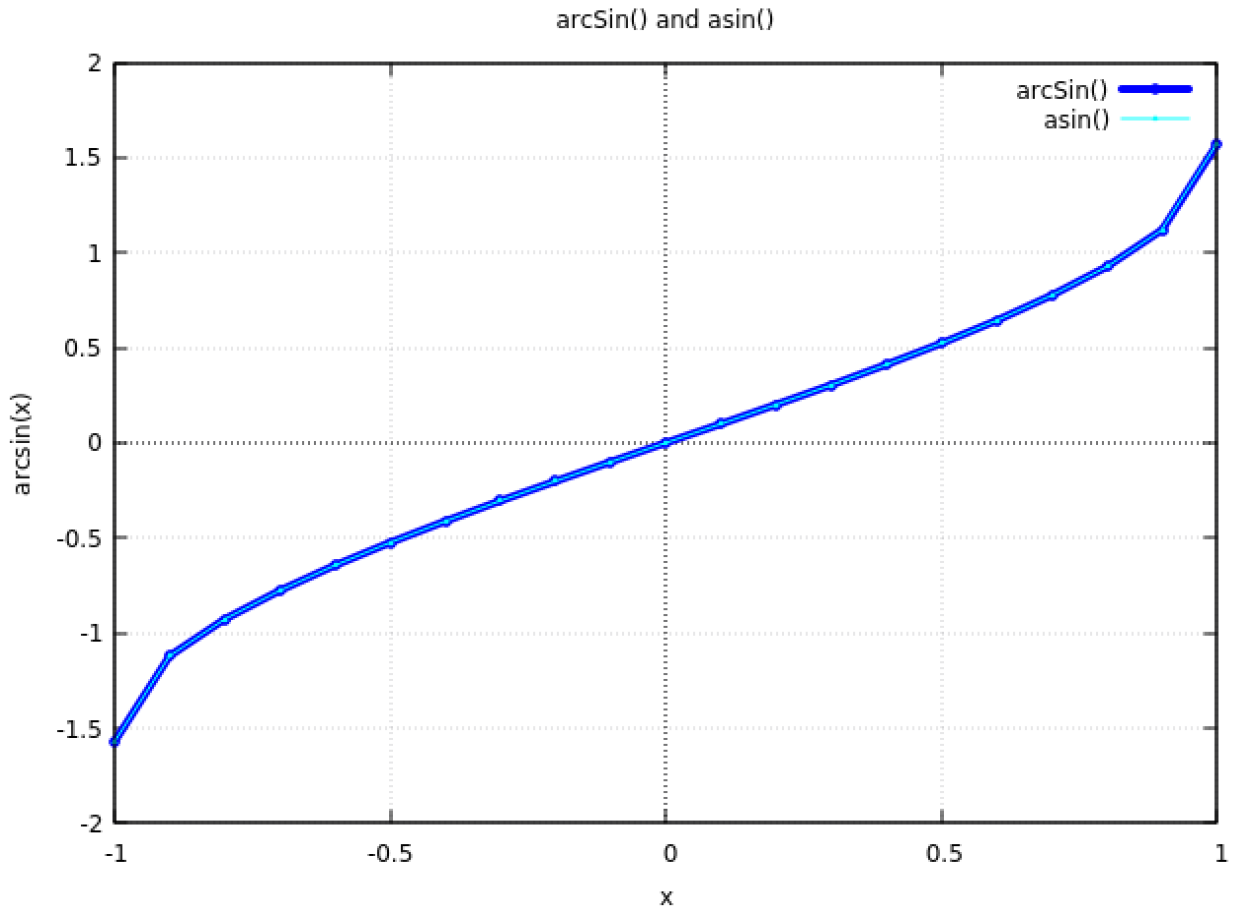


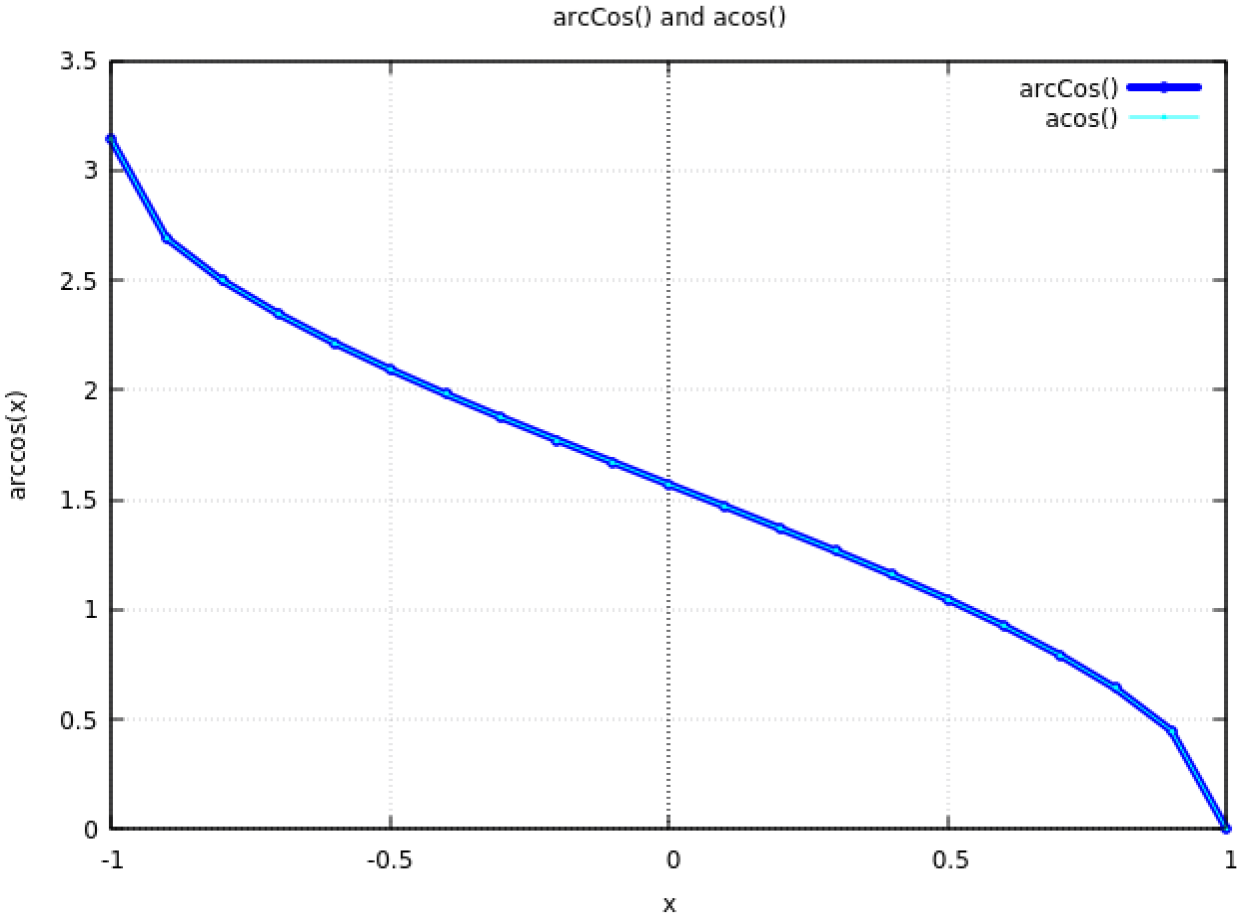
$y = |1 - arctan(x)|$

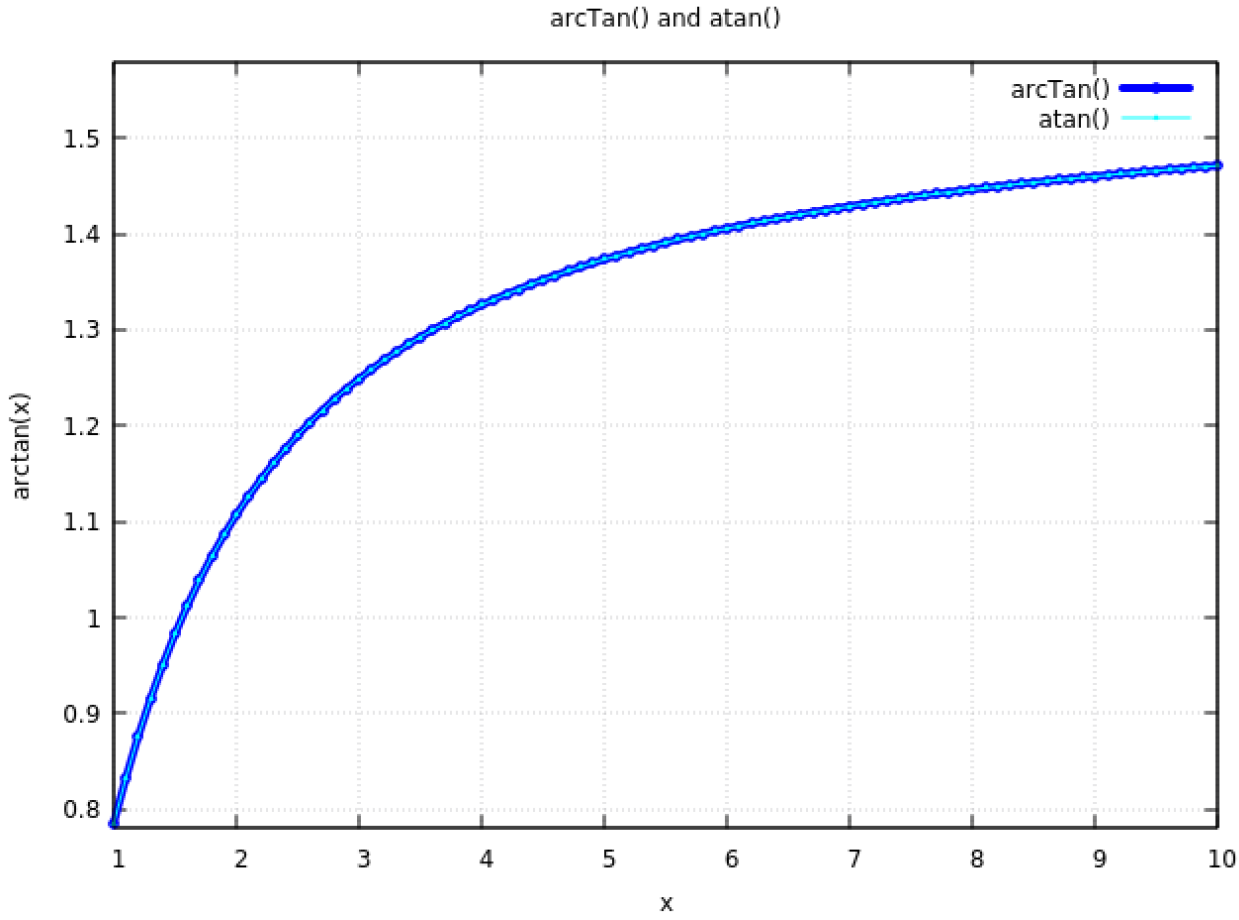


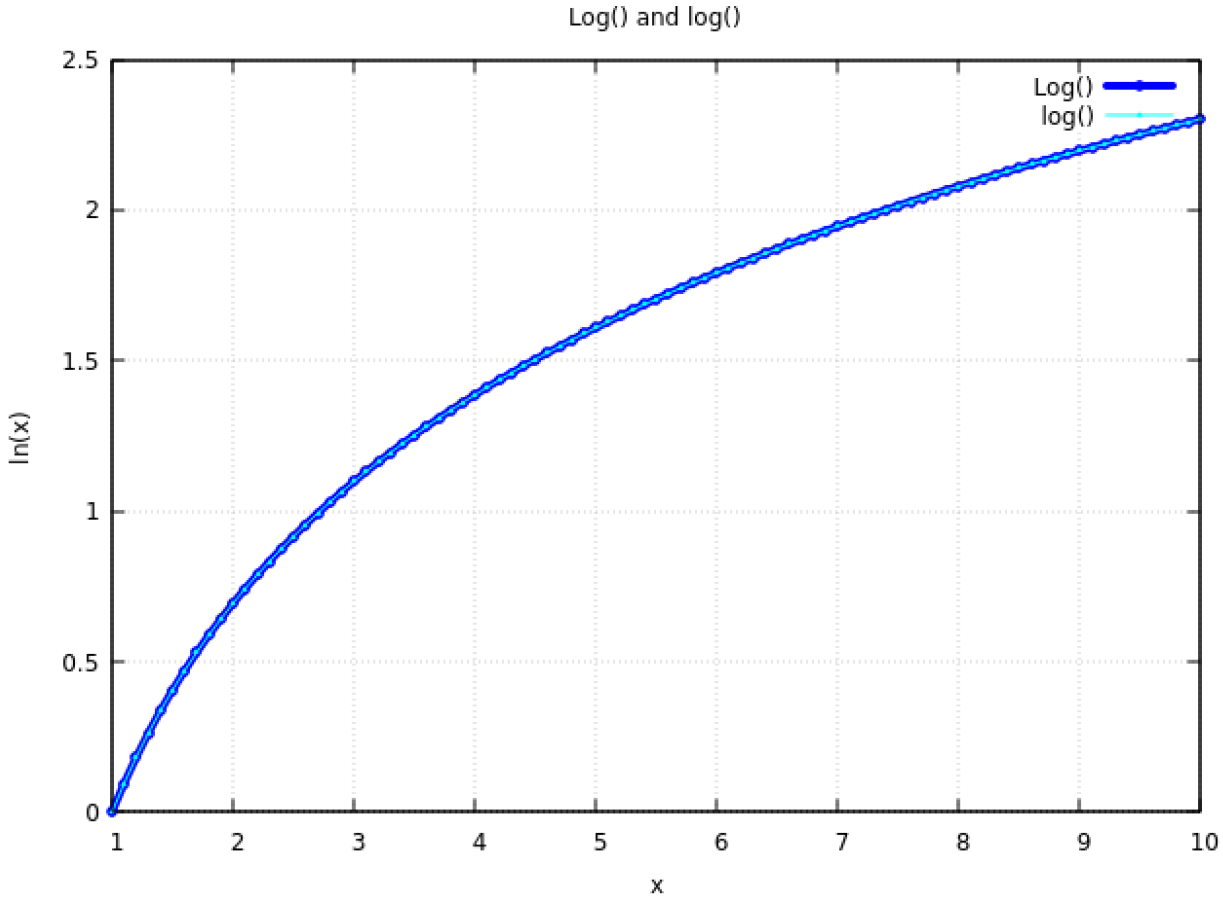
$$y = |\, 1 - ln(x) \,|$$

Enlarged Graphs:




arcCos() and acos()
arcCos()
acos()
arccos(x)
x
3.5
3
2.5
2
1.5
1
0.5
0
-1
-0.5
0
0.5
1


arcTan() and atan()
arcTan()
atan()
arctan(x)
x
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
1
2
3
4
5
6
7
8
9
10


Log() and log()
Log()
log()
ln(x)
x
0
0.5
1
1.5
2
2.5
1
2
3
4
5
6
7
8
9
10