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CSE 13S
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Assignment 2: A Small Numerical Library

Design Document

Introduction:

Purpose:

The purpose of this assignment is to learn how libraries are created and understand how the functions that libraries have are not made of magic.

Concepts Used:

Functions
Loops
Switch Statements
If Statements

Program Files:

`mathlib.c`: This file will contain the function implementation for arcSin, arcCos, arcTan, and Log as well as any supporting functions for square root and absolute value. The functions will be implemented with Newton's Method.

`mathlib-test.c`: This file is the test harness. The program will allow us to use command line options to print out results from testing the math functions. The program should loop through a specified range for x and print out the outputs of arcsin, arccos, arctan, and log using the functions created in `mathlib.c` and that of `math.h` library. It should also print out the computed difference between them.

Program Setup:

`mathlib.c`:

Include:

- stdio.h
- unistd.h
- mathlib.h

Define:

- options: asctl

`mathlib-test.c`:

Include:

- stdio.h
- math.h
- mathlib.h

Define:

- epsilon: 1e-10

Program Implementation:

All of the functions can be computed with Newton's Method. The general iterative formula for Newton's Method is:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

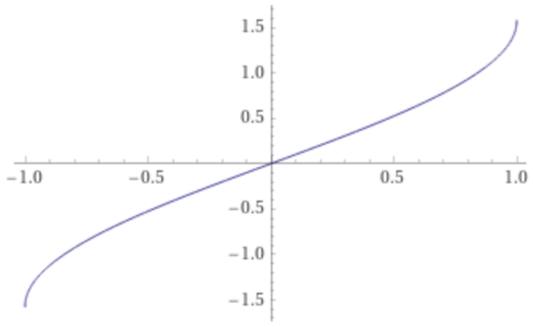
Note that $f(x_k)$must be a differentiable function and $x_k$is the initial approximation for $f(x) = 0$. However for the program, I want to find some y that is the result of $f(x)$ so $f(x) = y$. Therefore the iterative formula I will use is:

$$y_{k+1} = y_k - \frac{f(y_k)}{f'(y_k)}$$

Now, to write my arcsin function, I can use $y = arcsin(x)$. Sine we can't use arcsin in our algorithm, we need to manipulate the formula to be $x = sin(y)$. Since we are finding an approximation for when the function is equal to 0, we solve $x = sin(y)$ for 0 and get $f(y) = sin(y) - x$. Following this the iterative algorithm for the arcSin function is:

$$y_{k+1} = y_k - \frac{sin(y_k) - x}{cos(y_k)}$$

To determine the initial guess, y_k, we can plot the arcsin function with the domain [-1,1].



The actual value of arcsin(x) is always close to x itself so I will use x as the initial guess.

With all this, the pseudocode for the arcSin function is as follows:

```
arcSin (x):
    yk = 0, yk_1 = x
    while(|yk_1 - yk| > epsilon):
        yk = yk_1
        yk_1 = yk - (sin(yk) - x)/cos(yk)
    return yk_1
```

For the arccos function, I can use the identity, $arccos(x) = \frac{\pi}{2} - arcsin(x)$. Using such, the pseudocode for arcCos is as follows:

```
arcCos (x):
    return (pi/2) - arcSin(x)
```

Arctan can also be computed with the identity, $arctan(x) = arcsin(\frac{x}{\sqrt{x^2+1}})$. So, the pseudocode for arcTan is:

```
arcTan (x):
    return arcSin(x/√(x*x) + 1))
```

The implementation of the Log function will be very similar to the implementation of the arcSin function. I used the same Newton's Method formula:

$$y_{k+1} = y_k - \frac{f(y_k)}{f'(y_k)}$$

The equation we are using is $y = log_e(x)$ and converted to exponent form, $e^y = x$. Solving for zero we get $e^y - x$ which will be $f(y_k)$. Therefore our iterative algorithm is:

$$y_{k+1} = y_k - \frac{x - e^{y_k}}{e^{y_k}}$$

The initial guess will be 1.0. The following is the pseudocode for Log:

```
Log (x):
    yk = 0, yk_1 = x
    while(|yk_1 - yk| > epsilon):
        yk = yk_1
        yk_1 = yk + (x - Exp(yk))/Exp(yk)
        return yk_1
```

Note that some of these functions need to use square root, e^x, and/or absolute value. I will be using the Sqrt() function and Exp() function provided on Piazza. For absolute value, I will be writing my own function with this pseudocode:

```
AbsVal(x):
    if x < 0:
        x = x * -1
    return x
```

Test Harness Pseudocode:

Since the program will need a table header a few times, I will use a function for creating the table header.

```
printTableHeader(function name):
    print("  x        function name        Library        Difference")
    print("-----      -------------        -------        ----------")
```

The code for the main function is based on the example program for parsing options with getopt() in the assignment PDF and the code Sahiti went through in her section. The formatting of the print statement for printing the outputs was also given in the assignment PDF.

```
main (argc, **argv):
    opt = 0
    all
    while opt = getopt(argc, argv, options) != -1:
        switch (opt):
        case a:
            arcsin = 1
            arccos = 1
            arctan = 1
            log = 1
        case s:
            arcsin = 1
        case c:
            arccos = 1
        case t:
            arctan = 1
        case l:
            log = 1
        default:
            print error message
            return 1
    if arcsin == 1:
        printTableHeader(arcSin)
        for i in range -1 to 1 with step of 0.1:
            printf(" %7.4lf %16.8lf % 16.8lf % 16.10lf\n",
            i, arcSin(i), asin(i), arcSin(i) - asin(i))
    if arccos == 1:
        printTableHeader(arcCos)
        for i in range -1 to 1 with step of 0.1:
            printf(" %7.4lf %16.8lf % 16.8lf % 16.10lf\n",
            i, arcCos(i), acos(i), arcCos(i) - acos(i))
    if arctan == 1:
        printTableHeader(arcTan)
        for i in range 1 to 10 with step of 0.1:
            printf(" %7.4lf %16.8lf % 16.8lf % 16.10lf\n",
            i, arcTan(i), atan(i), arcTan(i) - atan(i))
    if log == 1:
        printTableHeader(Log)
        for i in range 1 to 10 with step of 0.1:
            printf(" %7.4lf %16.8lf % 16.8lf % 16.10lf\n",
            i, Log(i), log(i), Log(i) - log(i))
```

Makefile:

The makefile should have the following commands:

- `all`: builds `mathlib-test` program
- `clean`: remove files the compiler made
- `clang-format`: formats all .h and .c files

Process and Changes to Design:

Credits and Resources Used:

As the Assignment 2 PDF states, the Taylor series for arcsin(x) centered about zero is:

$$\arcsin(x) = \sum_{k=0}^{\infty} \frac{(2k)!}{2^{2k}(k!)^2} \frac{x^{2k+1}}{2k+1}, \quad |x| \leq 1.$$

To compute *(2k)!* I used the formula: *(2k)! = (2k - 2)! * (2n - 1) * 2n*. Using this, I found that you can calculate (2k)! recursively. The following table demonstrates this for the first few integers, *k*.

k	current *(2k)*	mid *(2k - 1)*	previous *(2k - 2)!*	*(2k)!*
1	2	1	null	2 * 1 * null = 2
2	4	3	2	4 * 3 * 2 = 24
3	6	5	24	6 * 5 * 24 = 720
4	8	7	720	8 * 7 * 720 = 40320

From the table we can see with each iteration, *(2k)!* will just be the product of the three calculated values (current, mid, previous) in the interaction before. For the purpose of initializing the variable, in the pseudocode, the previous value when k is 1 will be initialized to 1 since it will not affect the calculation.

Similarly, I can compute recursively *k!* using the formula: *k! = (k - 1)! * k*. Here are the first few:

k	previous *(k - 1)!*	*k!*
1	null	1 * null = 1
2	1	2 * 1 = 2
3	2	3 * 2 = 6
4	6	4 * 6 = 24

5	24	5 * 24 = 120

In short, *k!* is the product of the previous *k!* and the current *k* value. Again, for the purpose of creating variables, the variable for the previous *k!* will be initialized as 1 since anything multiplied by 1 is itself.