

TRIGONOMETRIJSKI IDENTITETI

Zadatak 1. Dokazati identitet: $\operatorname{tg}^2 x - \sin^2 x = \operatorname{tg}^2 x \cdot \sin^2 x$

$$\begin{aligned}
 \operatorname{tg}^2 x - \sin^2 x &= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x = \\
 &= \frac{\sin^2 x - \sin^2 x \cdot \cos^2 x}{\cos^2 x} = \\
 &= \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x} = \\
 &= \frac{\sin^2 x (\overbrace{\sin^2 x + \cos^2 x}^1 - \cos^2 x)}{\cos^2 x} = \\
 &= \frac{\sin^2 x}{\cos^2 x} \cdot \sin^2 x = \\
 &= \operatorname{tg}^2 x \cdot \sin^2 x
 \end{aligned}$$

$\sin^2 x + \cos^2 x = 1$
 $\operatorname{tg} x = \frac{\sin x}{\cos x}$

Zadatak 2. Dokazati identitet: $\operatorname{ctg}^2 x - \cos^2 x = \operatorname{ctg}^2 x \cdot \cos^2 x$

$$\begin{aligned}
 \operatorname{ctg}^2 x - \cos^2 x &= \frac{\cos^2 x}{\sin^2 x} - \cos^2 x = \\
 &= \frac{\cos^2 x - \cos^2 x \cdot \sin^2 x}{\sin^2 x} = \\
 &= \frac{\cos^2 x (1 - \sin^2 x)}{\sin^2 x} = \\
 &= \frac{\cos^2 x (\overbrace{\sin^2 x + \cos^2 x}^1 - \sin^2 x)}{\sin^2 x} = \\
 &= \frac{\cos^2 x}{\sin^2 x} \cdot \cos^2 x = \\
 &= \operatorname{ctg}^2 x \cdot \cos^2 x
 \end{aligned}$$

$\sin^2 x + \cos^2 x = 1$
 $\operatorname{tg} x = \frac{\sin x}{\cos x}$

Zadatak 3. Dokaži identitet: $\frac{1 - \sin^2 x}{\cos^2 x - 1} = -\operatorname{ctg}^2 x$

$$\begin{aligned}\frac{1 - \sin^2 x}{\cos^2 x - 1} &= \frac{\overbrace{\sin^2 x + \cos^2 x}^1 - \cancel{\sin^2 x}}{\cos^2 x - \underbrace{(\sin^2 x + \cos^2 x)}_1} = \\ &= \frac{\cos^2 x}{\cancel{\cos^2 x} - \sin^2 x - \cancel{\cos^2 x}} = \\ &= \frac{\cos^2 x}{-\sin^2 x} = \\ &= -\operatorname{ctg}^2 x\end{aligned}$$

Zadatak 4. Dokaži identitet: $(1 + \operatorname{tg}^2 x) \cdot \cos^2 x = 1$

$$\begin{aligned}(1 + \operatorname{tg}^2 x) \cdot \cos^2 x &= \left(1 + \frac{\sin^2 x}{\cos^2 x}\right) \cdot \cos^2 x = \\ &= \frac{\cos^2 x + \sin^2 x}{\cancel{\cos^2 x}} \cdot \cancel{\cos^2 x} = \\ &= \sin^2 x + \cos^2 x = \\ &= 1\end{aligned}$$

Zadatak 5. Dokaži identitet: $\frac{1 + \operatorname{tg} x + \operatorname{tg}^2 x}{1 + \operatorname{ctg} x + \operatorname{ctg}^2 x} = \operatorname{tg}^2 x$

$$\begin{aligned}\frac{1 + \operatorname{tg} x + \operatorname{tg}^2 x}{1 + \operatorname{ctg} x + \operatorname{ctg}^2 x} &= \frac{1 + \frac{\sin x}{\cos x} + \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\cos x}{\sin x} + \frac{\cos^2 x}{\sin^2 x}} = \\ &= \frac{\cancel{\cos^2 x} + \cancel{\cos x} \sin x + \sin^2 x}{\sin^2 x + \cancel{\sin x} \cos x + \cancel{\cos^2 x}} = \\ &= \frac{\cos^2 x}{\sin^2 x} = \\ &= \frac{1}{\frac{\sin^2 x}{\cos^2 x}} = \\ &= \frac{\sin^2 x}{\cos^2 x} = \\ &= \operatorname{tg}^2 x\end{aligned}$$

Zadatak 6. Dokaži identitet: $\frac{\sin^3 x + \cos^3 x}{1 - \sin x \cos x} = \sin x + \cos x$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\begin{aligned} \frac{\sin^3 x + \cos^3 x}{1 - \sin x \cos x} &= \frac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{1 - \sin x \cos x} = \\ &= \frac{(\sin x + \cos x)(\overbrace{\sin^2 x + \cos^2 x}^1 - \sin x \cos x)}{1 - \sin x \cos x} = \\ &= \frac{(\sin x + \cos x)(1 - \cancel{\sin x \cos x})}{\cancel{1 - \sin x \cos x}} = \\ &= \sin x + \cos x \end{aligned}$$