LOGIC EXERCISES: WEEK 5

QUESTION 1

Formalize the following claims as sequents of L_2 by considering an arbitrary relation R^2.

    i.   Asymmetric relations are irreflexive.

    ii.   Intransitive relations are irreflexive.

    iii.   Transitive relations that are not asymmetric are not irreflexive.

    iv.   Relations with no "two-step journeys" are transitive.

    v.   The only relation that is both symmetric and asymmetric is the null relation.

QUESTION 2

(i) What is the difference between the claim that $G \vDash f$ and the claim that $G \vdash f$? (ii) What does it mean to say that a given proof system is sound? (iii) What does it mean to say that a proof system is complete?

QUESTION 3

Calculate the truth-value of the following L_2-sentences in the given L_2-structure S.

$D_S = \{ 1, 2, 3 \}$	$|P|_S = F$
$|a|_S = 1$	$|Q|_S = \{ 2 \}$
$|b|_S = 3$	$|R|_S = \{ <1, 2>, <2, 3>, <1, 3> \}$

    i.   Qa

    ii. $(P \rightarrow Qb)$

    iii.   Rab

    iv.   Rba

    v. $(Rab \leftrightarrow Rba)$

    vi. $(Rbb \lor (\neg Qa \land \neg Raa))$

    vii. $\exists xQx$

   viii. $\exists xRax$

    ix. $(Qb \lor \exists xRxx)$

    x. $\exists x(Rax \land Rxb)$

    xi. $\forall xQx$

    xii. $(\forall xRxa \leftrightarrow P)$

    xiii. $\forall x(Rxx \rightarrow Qx)$

    xiv. $\forall x\exists yRxy$

    xv. $\forall x(Qx \rightarrow (\exists yRxy \land \exists yRyx))$

QUESTION 4

Produce L_2-structures that are counterexamples to the following incorrect claims of validity.

i. ⊨ ∀xPx
ii. ∃xPx ⊨ ∀xPx
iii. Pa ⊨ ∃x(Px ∧ Qx)
iv. ∀xRxx ⊨ ∀xRax
v. ∃x(Px ∧ Qx) ⊨ R
vi. ∀x(Px ∨ ¬Qx) ⊨ R
vii. Pa, ∀xQx ⊨ P
viii. ∀y(Py → ∃xRyx) ⊨ ∀x(Px → ∃yRyx)