Relocation Strategies and Algorithms for Free-Floating Car Sharing Systems

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Abstract—During the last years so-called free-floating Car Sharing Systems became very popular. These systems in comparison to the conventional Car Sharing Systems allow short one-way trips. Today, the spatial distribution of vehicles within free-floating Car Sharing Systems is either self-organized, which means it is only dependent on the customer's demand or in a few cases the positioning is manually controlled by system operators. None of the real-life free-floating Car Sharing Systems has a clear defined relocation strategy or is even online optimized based on the current demand. Within this paper several relocation strategies are introduced and categorized. For each category known relocation algorithms are described and evaluated. Also a new integrated two-step model for optimal vehicle positioning and relocation is described in detail. This new approach consists of an offline demand clustering that allows for the prediction of demand and thus the prediction of the optimal future state of spatially available vehicles. The online module of the approach measures the differences between optimal vehicle positioning and current positioning. An optimization algorithm finds optimal relocation strategies if necessary. The main focus of this paper is on the description of the Offline Demand Module.

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I. Introduction

During the last decades, new types of mobility services like *Carpooling* and *Car Sharing* emerged based on the idea of *sharing a car*. This idea was first realized by *Carpooling*, which was used as a rationing tactic in the U.S. during World War II and later on organized professionally by the first carpooling organization SEFAGE in Zurich in 1948 [1]. A second wave followed due to the oil and energy crisis in the 1970s, when the first employee vanpools were established and the sharing of private vehicles was organized in car clubs and neighborhood groups.

Today, the whole process is simplified by the internet and mobile technology and hence, new forms of sharing cars emerge. For instance, now Carpooling is organized on internet platforms operating as bulletin boards for (one-way-) trips offered by car owners/drivers. Internet platforms are also used by car owners for providing their private cars to others. The latter form of sharing vehicles allows round-trips only.

Most of the developments can be observed in the area of professionally organized *Car Sharing* by so called mobility providers, who offer different vehicles at different places assuring a high variety and a good maintenance. At first, only station based systems were implemented with vehicles available at fixed stations, e.g. at rented parking lots or garages. The booking of the vehicles had to be carried out before usage and the

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mandatory return of the vehicles to the initial station didn't allow one-way-trips. A disadvantage compared to rental cars, which also offer one-way-trips, remained. Therefore, later more flexibility was provided by *one-way* Car Sharing Systems, which also accepted returns at other parking spots in a specific parking zone or in a given region. Those developments and the growth of Car Sharing are e.g. covered in detail in [2] and [3].

During the last years, new *free-floating* systems came up (cf. [4]). The provider usually defines a certain business district, mostly the city center and therein buys on-street parking licenses for each vehicle of his fleet. Thus, the return of the vehicles is possible on any parking spot within the district and the customer is not bound to stations anymore allowing for short one-way-trips. Mostly, the pricing model includes a starting fee and a time-dependent price function. Some providers have established additional charges, if the journey is finished outside of the district and reduced prices for parking periods. Those new systems, which don't require a booking in advance, make the usage of Car Sharing more flexible and spontaneous. Nevertheless, they still have their weaknesses. The principle of having a *car to go* and providing mobility on demand is not yet guaranteed due to bottlenecks in supply, which are currently not eliminated. Vehicles sometimes get stuck in areas of lower individual mobility demand (*cold spots*) while needed in zones of higher demand (*hot spots*). To make the system even more efficient and more profitable, this imbalance of supply and demand could be adjusted by applying different intervention strategies. A similar problem also emerges in the field of bike sharing where relocation methods were developed in the past (cf. e.g. [5]–[7]). However, the relocation of bicycles is easier than relocating cars. Up to 60 bicycles can be transported altogether to hot spots on a bicycle carrier, which saves costs and is relatively uncomplicated. In contrast, Car Sharing vehicles have to be reallocated separately or on a car transporter incurring high costs. This makes effective strategies to relocate cars even more important.

During the last few years, a vision among a group of scientists of completely substituting car ownership by other efficient mobility concepts developed. This idea emerged above all due to the exploding number of vehicles on the streets, the resulting shortages in parking space, congestion and environmental problems. In this context, the term *cloud commuting* (cf. [8]) was established according to cloud computing. Cars were compared to other goods like vinyl's or CD's, which have been replaced by real-time and on-demand access in the past. Since providing vehicles is much more complex than e.g. providing music via the internet, efficient and cost-effective ways to manage the system have to be invented.

This paper at first describes different relocation strategies for the last-mentioned free-floating Car Sharing Systems. They are grouped into two different approaches: user-based and operator-based approaches. Later on, specific known relocation algorithms for Car Sharing Systems are introduced and evaluated. Finally, a new two-step relocation model for free-floating Car Sharing Systems is introduced.

II. Relocation Strategies for Free-Floating Car Sharing Systems

In the following, different relocation strategies for free-floating Car Sharing Systems are described and compared. On the one hand, the so-called user-based approaches are based on users' (re)actions and the relocation process is carried out by the users. On the other hand, operator-based approaches are based on the interventions made by an operator or system manager. A relocation strategy should be taken into account, if several constraints are fulfilled. Those are explained in the first section of this chapter as well as the whole decision process for or against a relocation strategy.

A. Constraints

The intervention in a Car Sharing System is always accompanied by costs. As the whole system has to be profitable for the provider, additional costs and benefits have to be compared. If the relocation costs compensate the additional earnings, a relocation strategy may be applied. Exceptions could be made shortly after implementation of the system, when the main objective is getting visible to the users and outperforming the competitors by a better supply.

Controlling the system can lead to higher total earnings and an optimized operation of the system. On the other hand, relocation strategies might neglect areas of very low demand, where in the worst case not a single Car Sharing vehicle would be available.

Leaving the system on its own without intervention saves costs and working time, but has serious disadvantages. In a self-organized system, the few frequent users might adapt the system to their behavior, so that it is difficult to gain new customers. Furthermore, the vehicles might get stuck in areas of low demand causing a loss of money. Finally, if no intervention is carried out to improve the system, no knowledge of the optimal conditions for both the users and the system owner is available.

The relocation should especially focus on spots of high demand caused by a poor public transport connection, because Car Sharing Systems should not substitute existing efficient public transport systems. A good possibility for relocation is by night, when the demand is lower. If the current utilization of the Car Sharing vehicles is high, no intervention is needed.

B. User-Based Relocation Strategies

User-based relocation strategies make use of different incentives or bonus models for the customers. One way of compensating the imbalance of supply and demand is the adjustment of the prize model. Trips to specific under supplied areas could be offered at a lower rate or even for free when needed. Suggestions for alternative destinations basically only make sense prior to or shortly after departure. As longer the trip the harder to influence the user's destination choice and the lower the willingness to change. The special-prized Car Sharing rides could be offered for instance via social communities or e-mail and are particularly appropriate for trips to recreational facilities, which are usually less sensitive concerning the destination choice. Another incentive for users to drive to under supplied hot spots is free parking in parking garages. If the number of vehicles leaving a hot spot is too high and there is a risk of future insufficient supply, trips of several Car Sharing customers could be combined and executed with one vehicle. The system informs the customers prior to departure, that a joint trip at a lower rate is possible. The informed customers with similar destinations then depart in one vehicle altogether. It is also possible, that an already driving user is asked to pick up another customer in a hot spot area, who has a similar destination. The prize for the Car Sharing is then divided among the users. This approach of combining Carpooling and Car Sharing does not or only to a small extent increase the earnings of the provider, but at least prevents an under-supply in a hot spot and assures customer's satisfaction. Theoretically, also the contrary is possible. Joint trips departing from cold spots could be splitted in order to regulate the over-supply in those areas.

User-based relocation strategies are especially advantageous from a financial point of view. Letting customers relocate the vehicles is free of costs except for free rides. Furthermore, those methods are environmentally sustainable. Vehicles are only driven by customers and no additional vehicle trips without customers are conducted.

Nevertheless, there are also some disadvantages. The customer can only be influenced by these incentives to a certain degree. The customer acceptance and decision is difficult to control and predict. Furthermore, the explanation to the customer is difficult, because he might not understand why the system is trying to influence his routes/destinations and might thus refuse to participate. Additionally, the customer loses some privacy, because he/she has to specify his/her destination before usage. Moreover, it can be stated that the described strategies are rather suitable for long-term than for short-term adjustments to the system. Finally, a user-based relocation process can hardly be combined with vehicle maintenance. This task can only be shifted to the users by stipulations and enforcement. However, this is not beneficial to customer satisfaction. Maintenance and cleaning thus mostly have to be carried out separately by the system operator under high costs.

C. Operator-Based Relocation Strategies

Operator-based relocation strategies are based on interventions initiated by the system manager and executed by the Car Sharing provider itself. There are basically two different possibilities of relocation for the system provider.

On the one hand, the maintenance personnel or additional staff can relocate the vehicles. This can happen either separately for each car or for up to three cars by a car transporter. An advantage of this kind of intervention is its clear definition. Vehicles are in any case brought to the specific desired positions. Additionally, maintenance and gas filling can be combined with the relocation rides and thus cost can be reduced. However, one negative effect is the occurrence of additional rides without customers, which originally weren't necessary. This aspect is ecologically questionable. Moreover, high costs are generated by additional staff, additional gas consumption and the employment of car transporters.

On the other hand, buffer depots with a specific number of vehicles could be arranged at known hot spots, which could be released for utilization when the demand is high. This method has several advantages. First of all, no additional costly rides are necessary. Secondly, the depots can be filled a priori according to the expected demand while during the period of high demand nothing has to be done. Finally, those depots could be helpful for the integration of electric vehicles into Car Sharing Systems, because they could easily be equipped with charging stations. Vehicles that have to be loaded or maintained could be collected in the depot. If the charge of battery falls below a certain threshold, customers should be instructed to bring the vehicle to the nearest depot and eventually exchange it with another already loaded vehicle. Nevertheless, for filling the depot vehicles have to be taken from the existing fleet or additional vehicles are necessary, which have to be removed again in normal operation. Furthermore, the depot incurs additional costs which have to be taken into account.

D. Comparison

In general, there are two main relocation strategies (see Table 1). The first user-based category shifts the relocation process to the user. Incentives are created for the customer to deviate from the actual destination or vehicle occupancy in favor of a better balance of supply and demand. As mentioned before, the explanation of those strategies to the user and the prediction of the acceptance rate is rather difficult. Therefore, they should be primarily used in case of long-term events with high destination traffic like fair trades or festivals.

The operator-based strategies consist of decisions made by the system manager, which can be immediately executed by the Car Sharing staff. Those methods are more reliable and thus more suitable for spontaneous or short-term events with a one-time punctual high demand like football matches. Especially the buffer depots are suitable for those events with clear beginning and end, because they can be filled in advance and released whenever needed.

Compared to operator-based relocation algorithms, approaches that shift the relocation to the user are relatively rare.

III. Relocation Algorithms for Car Sharing Systems

The following section gives an overview on existing relocation algorithms for Car Sharing vehicles. Those are grouped into operator-based and user-based algorithms.

A. Operator-Based Relocation Algorithms

During the last few years, the research in the area of Car Sharing vehicle relocation focused on operator-based vehicle movements. Most algorithms were designed for station-based one-way systems, where vehicle stock imbalances occurred.

Barth and Todd [9] designed a queuing-based transportation simulation model in order to evaluate the overall system performance of a multiple station shared vehicle system allowing one-way-rides. Several measures of effectiveness like vehicle availability, vehicle distribution and energy management have been determined and three different relocation mechanisms were tested. Those were static, historic and exact predictive relocations which are based on immediate, expected and exact Car Sharing demand respectively.

Correia and Antunes [10] developed an optimization approach on how to determine the best number, location and size of depots in one-way station-based Car Sharing systems. The model accounts for vehicle stock imbalances by relocating vehicles at the end of the day.

Fan et al. [11] proposed a multi-stage stochastic linear integer programming model with recourse for dynamic vehicle allocation and trip selection that integrates operator-based vehicle relocations. The dynamic Car Sharing demand for the next day is determined at the end of each day.

Kek et al. [12] propose a three-phase decision support system to find operator-based relocation strategies for station-based Car Sharing Systems that allow one-way-rides. At first, the vehicle relocation problem is formulated as a mixed integer linear problem similar to the typical pickup and delivery problem. Afterwards, heuristics are introduced for translating the optimization outputs into near-optimal manpower and operating parameters. Finally, a time-stepping Relocation Simulator is applied that evaluates the improvements in the system. The model was validated by using data from a local Car Sharing company in Singapore.

Mukai and Watanabe [13] describe an operator-based relocation algorithm based on virtual spring forces. Finally, Jorge et al. [14] recently developed an approach that optimizes operator-based relocation operations for station-based one-way Car Sharing systems. The real-time relocation policies were tested by applying a simulation model.

B. User-Based Relocation Algorithm

Compared to operator-based relocation algorithms, approaches that shift the relocation to the user are relatively rare.

Febbraro et al. [15] use a rolling horizon method that uses Discrete Event Systems for defining a user-based relocation algorithm for one-way Car Sharing systems. The relocation is carried out by those users, whose destinations are close to an area/station with an insufficient supply of vehicles. The algorithm is based on the minimization of the rejection ratio of reservations for Car Sharing cars and on a linear integer programming problem. It is assumed by the authors that the allowed delay for the user is limited and the vehicle is either returned in a zone defined by the user or in a zone proposed by the system. The areas of the customers' approximate destinations have to be known in advance for the forecasting of the vehicles' positions and the optimization of the vehicle distribution in those areas.

Uesugi et al. [16] suggest a simulation-based method for optimal vehicle assignment for station-based one-way Car

Table 1. Comparison of user-based and operator-based relocation strategies.

Relocation Strategy	Advantages	Disadvantages	Major Application Area
User-based	Low costs due to staff savings; no additional vehicle movements (environmentally sustainable)	Customers difficult to influence; user acceptance difficult to predict; loss of customer's privacy	Long-term events with high destination traffic
Operator-based	Reliability due to clear definition; combination with maintenance; a priori intervention in case of depots; easy integration of electric vehicles in case of depots	Costs for staff, rides, car transporters and depots; additional vehicle movements in case of relocation by staff; vehicles have to be taken from existing fleet in case of depots	Short-term events with defined beginning and end and one-time high demand

Sharing systems. The proposed user-based strategies are trip splitting and trip joining.

C. Algorithms Combining Operator-Based and User-Based Relocation

Only one relocation algorithm could be found that integrates both operator-based as well as user-based relocations. Todd et al. [17] proposed an algorithm for station-based one-way systems that tries to reduce operator-based relocations by additionally considering user-based redistribution. This model was implemented on a real-world university campus Car Sharing system as well as in a computer simulation model. The method uses trip splitting and trip joining as in [16].

Summing up, the literature review on relocation algorithms showed, that—in the past—little research was done on finding relocation models for fully free-floating Car Sharing systems without stations. The focus has been mostly on station-based one-way Car Sharing systems with relocations at the end of the day, similar to Bike Sharing Systems.

However, the new systems have different dynamics that should be taken into account. Trips are usually shorter than within station-based systems and the number of trips per vehicle is thus higher. Imbalances might occur more often and relocations have to be conducted dynamically throughout the whole day. Future demand has to be known for very small time steps like one hour and Car Sharing demand models have to be reliable. A demand prediction at the end of the day is not sufficient for free-floating systems. None of the studied papers offers a microscopic spatial-temporal demand prediction for free-floating Car Sharing systems. Additionally, user-based algorithms are still relatively rare and the question on how and when customers are willing to accept incentives for deviations from their destinations etc. is not yet covered or even investigated in practice. User-based relocations are even more difficult to implement in free-floating systems due to spontaneous trips without a-priori reservations. This makes the reliable and high-resolution prediction of Car Sharing demand even more important. Little literature covers the combination of user-based and operator-based relocation. The decision on which strategies to apply depends on lots of factors like time of day, number and type of events in the city, etc. Thus, demand-sensitive recommendations are necessary on when to apply which strategies or combinations of strategies. Integrating both the demand model and the strategy recommendations into an online algorithm adds lots of complexity to the problem. Therefore, a two-step modeling approach consisting of an offline cluster analysis and an online optimization as described in this paper is preferred. This approach allows the implementation of the model in a real-life free-floating Car Sharing System.

IV. Two-Step Model for The Relocation of Vehicles Within Free-Floating Car Sharing Systems

To cover the above mentioned new aspects of free-floating Car Sharing systems an integrated two-step model for optimal vehicle positioning and relocation was developed. This algorithm consists of an offline demand clustering and demand prediction module (Step 1). The results enable spatial-temporal predictions of the optimal future state of spatially available vehicles. The online module (Step 2) measures the differences between optimal future vehicle positioning and current positioning and applies an optimization algorithm for finding the best relocation strategy in case of a certain deviation.

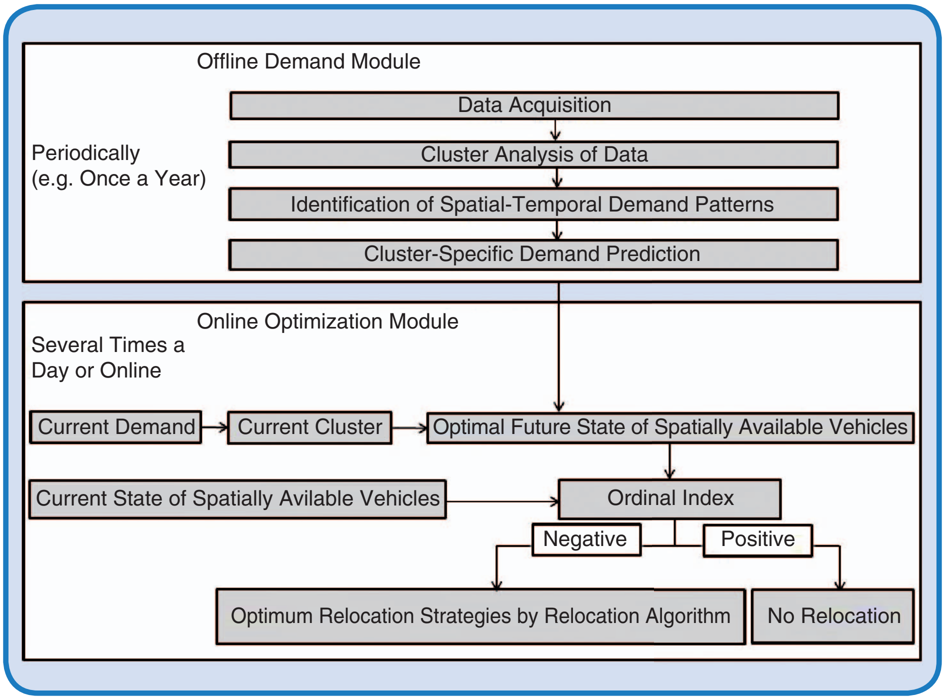

FIG 1 Two-step model for the relocation of vehicles in free-floating car sharing systems.

The idea behind this new approach is to provide a continuous monitoring of the Car Sharing System. The system is however not fully controlled in a closed-loop. In case of a certain deviation of the actual spatial vehicle distribution from the future optimal state, a demand-sensitive relocation strategy is determined by an optimization algorithm in step 2 and could be implemented manually. Fig. 1 shows the whole two-step controlling process.

A. Offline Demand Module

The first step is an *Offline Demand Module* which is carried out periodically, e.g. once a year. The idea of this offline analysis is the identification of certain periodically repeating spatial-temporal demand patterns within the free-floating Car Sharing System of a specific test site/city. Those are the essential requirement for an optimal relocation of Car Sharing vehicles. The demand patterns could be daily (time of the day, peak hours), weekly (weekend or weekday) or even longer (seasons) or based on specific events (football matches, festivals etc.). During this offline step, real vehicle positioning and booking data of a running Car Sharing System are at first collected and analyzed. So-called spatial-temporal "hot spots" and "cold spots" are identified. The periodical repetition of the module guarantees that the demand model remains up-to-date and adjusts to changing user behavior.

The empirical basis of this work are real historical vehicle data of a real-life free-floating Car Sharing system in Munich, Germany. Vehicles are distributed all over the business district and can be returned on any parking spot within this district. The prize model is composed by a fixed registration fee and a fixed time-dependent usage rate as well as a lower time-dependent rate for parking. The historical data consists of the geo-referenced start and end locations of the conducted trips, booking times and booking durations, satisfied and unsatisfied booking requests (online requests/searches by mobile phone or internet which did not lead to a booking), trip distances and parking durations. For macroscopic modeling purposes the business district of the Car Sharing system is divided into 10 homogenous segments that consist of 5 or 6 traffic zones each (see Fig. 2).

At first, characteristic indicators for spatial-temporal Car Sharing demand have to be defined. The main indicator for a "hot spot" segment of Car Sharing demand is a very high frequency of booking or booking requests per time interval. Let $N = 366$ be the number of days contained in the data set of Car Sharing bookings and let $S = 10$ be the number of segments of the business district. Each of the analyzed N days is divided into 6 three-hour time slices from 6 a.m. to 12 p.m. and 1 six-hour time slice from 12 p.m. to 6 a.m. For the identification of spatial-temporal demand patterns in a specific time interval, the demand indicators have to be calculated for each segment on each day in the considered time interval. The first indicator for Car Sharing demand is defined by the number of bookings $n_{i,j,k}$ in segment i in the jth time interval on day k, $i = 1{:}S$, $j = 1{:}7$ and $k = 1{:}N$. The second indicator is the mean of the vehicles' standstill times in segment i, $E(s_{i,j,k})$. This contains information on whether the segment had a shortage or a surplus of vehicles in the current time interval. When standstill times are high, it is likely that the number of available vehicles exceeds the demand whereas for low standstill times the contrary can be assumed. This information is very helpful for deciding on whether Car Sharing vehicles should be relocated between segments.

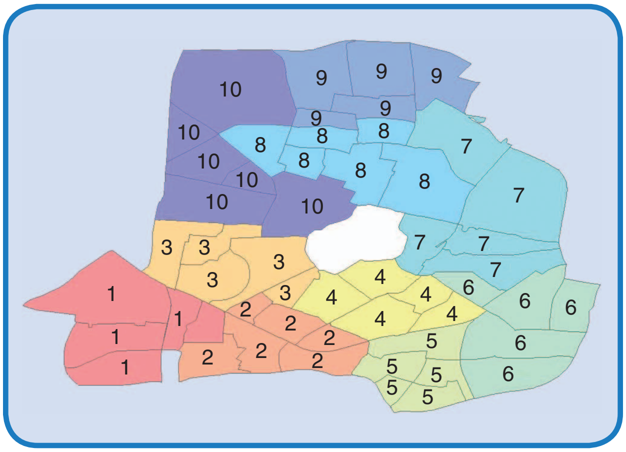

FIG 2 Business district of the studied car sharing system and partition into 10 segments.

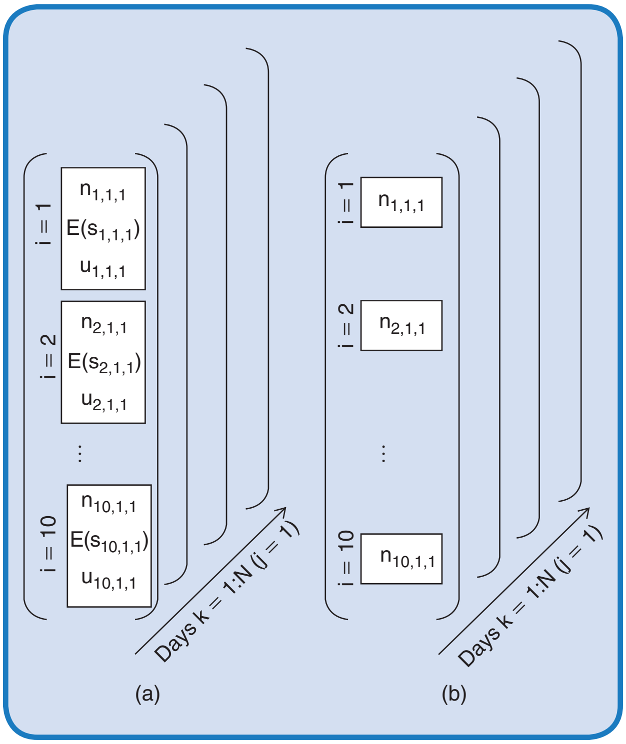

FIG 3 (a) Original and (b) reduced matrix of car sharing demand indicators for time slice 1 (6 a.m.–9 p.m.).

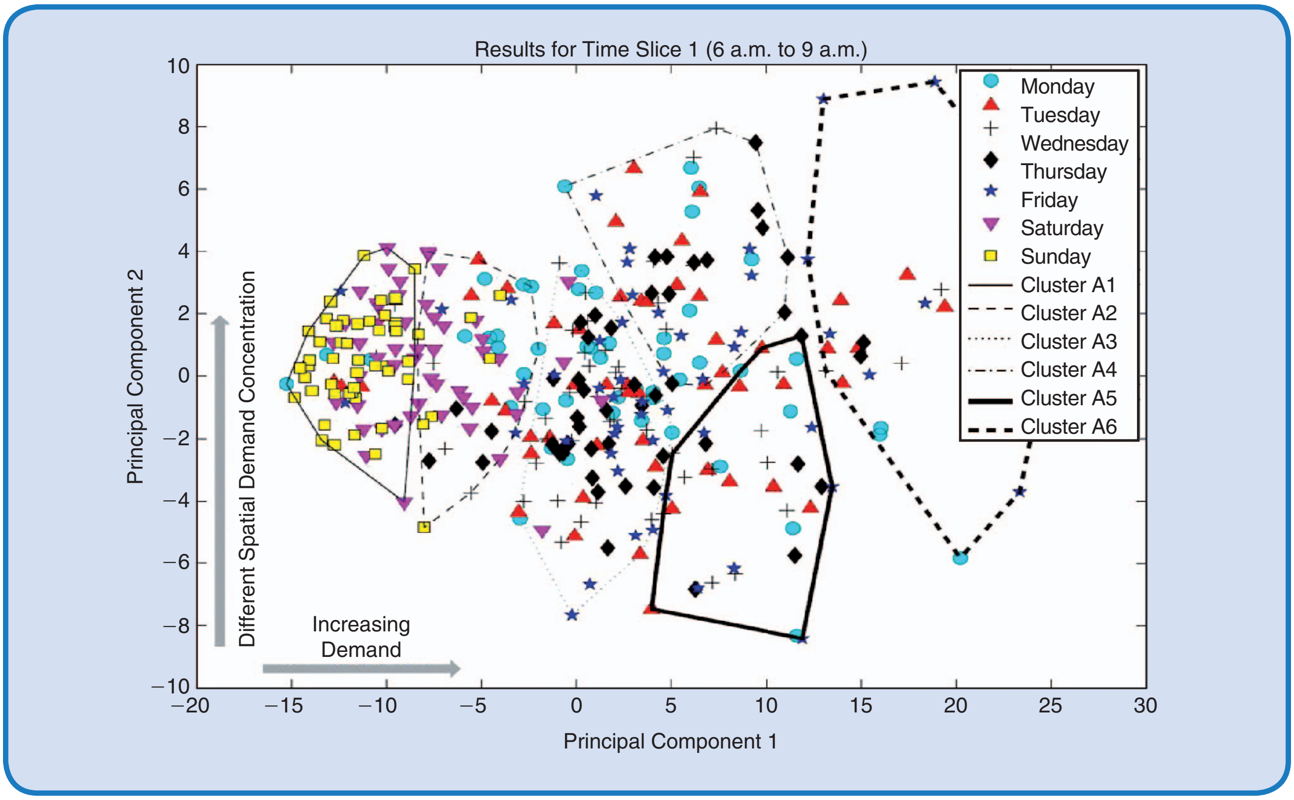

FIG 4 Results of the cluster method k-means on the first two principal components for the considered N days in time slice 1 (6 a.m. to 9 a.m.).

Finally, the number of unsatisfied booking requests $u_{i,j,k}$ also indicates whether the number of vehicles in segment i was sufficient or not. Additional indicators that were neglected in our model due to complexity reasons could be the vehicle in- and outflow of segment per time interval.

Out of those three demand indicators a $(3 * S) \times N$ demand matrix is created for each of the 7 time slices. Fig. 3(a) shows such a demand matrix for the first time slice (6 a.m. to 9 a.m.). The rows represent the different demand indicators and the columns represent the different days. The goal is the collection of days that have similar demand patterns during the considered time slice. This can be done by means of a cluster method that builds groups of spatially related two-dimensional points. The demand vectors for each of the N days (the columns in the demand matrix) are of dimension 30. They can only be represented as two-dimensional points if their dimensions are reduced. This is done, by finding the first two principal components of the demand matrix thus reducing the demand matrix to dimension $2 \times N$. The principal components are calculated using the software MATLAB. The considered two principal components should account for a certain percentage of the variance between the considered demand vectors. During the analysis, it became obvious that the demand indicators 2 and 3 do not result in a significantly higher percentage. For simplicity and computational reasons, the number of bookings $n_{i,j,k}$ was later used as the only demand indicator and a reduced $S \times N$ demand matrix shown in Fig. 3(b) was used for the principal component analysis. The percentage of variance that is described by the derived principal components is between 61% and 74% depending on the time slice. The first two principal components of the considered N days are shown for time slice 1 (6 a.m. to 9 a.m.) in Fig. 4. Different weekdays are represented in different forms and colors.

Table 2. Characterization of the demand clusters for time slice 1 (6 a.m. to 9 a.m.).

Cluster	No. of Days	Relative Deviation from Overall Average Booking Number in Cluster	Description
A1	81	−69%	Weekend or official holiday with minimal demand
A2	57	−31%	Reduced demand, mostly Sat to Mon
A3	111	+8%	Weekday with average demand
A4	52	+30%	Weekday with increased demand
A5	38	+55%	Weekday with strongly increased demand
A6	27	+101%	Weekday with maximal demand

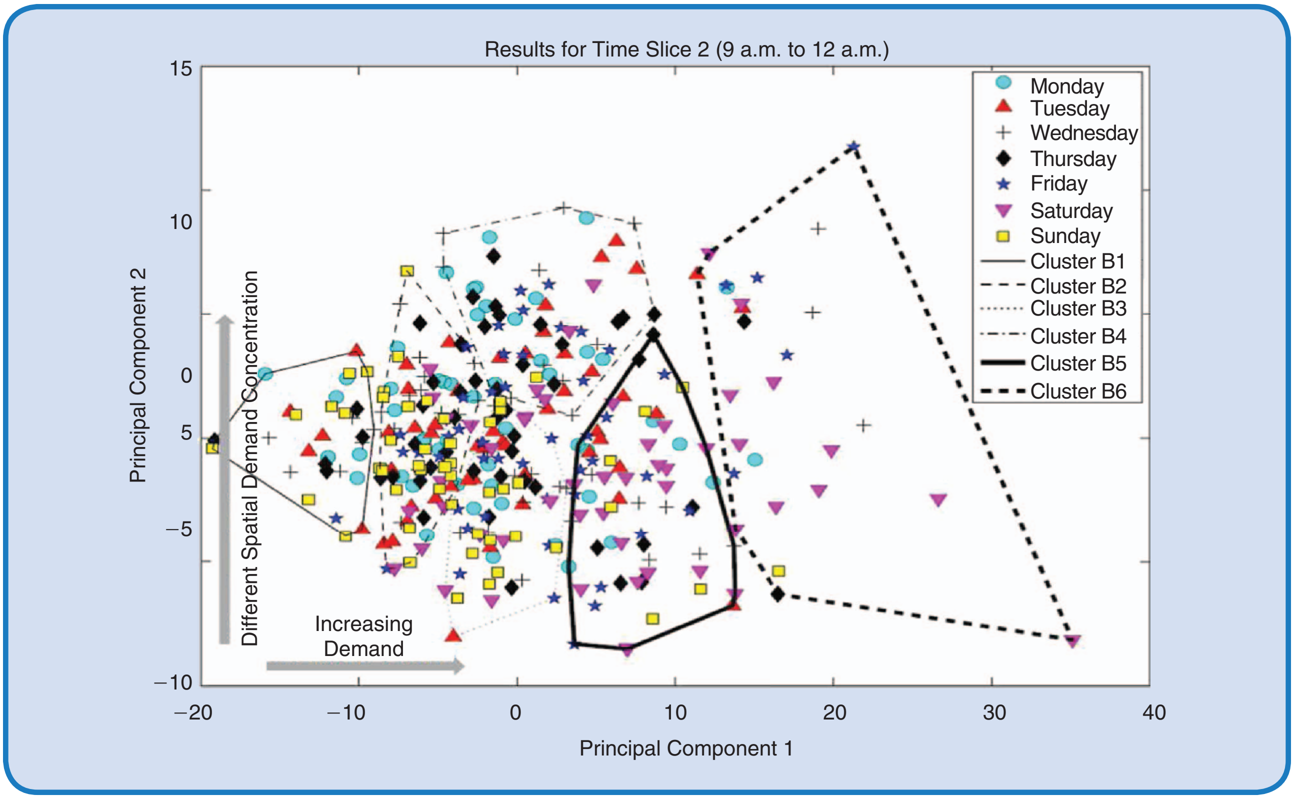

FIG 5 Results of the cluster method k-means on the first two principal components for the considered N days in time slice 2 (9 a.m. to 12 a.m.).

Horizontal displacements between points illustrate different magnitudes of demand (increasing overall demand from left to right) whereas vertical displacements show different spatial concentrations of demand concerning the defined 10 segments.

The two-dimensional points describing the demand of each specific day are now combined into groups by applying the cluster method k-Means in MATLAB. This method partitions the points in the demand matrix into k clusters by iteratively minimizing the sum of the within-cluster sums of point-to-cluster-centroid Euclidean distances. The number of clusters k was set to 6 for all time slices as it was identified during the analysis process as the most appropriate number for representing the points' distribution. The results of the cluster method are shown for time slice 1 (6 a.m. to 9 a.m.) in Fig. 4.

After the spatial identification of the six clusters, their demand characteristics were derived by analyzing the days they contain. The clusters were distinguished according to their days' distribution over the different weekdays and seasons, the number of official holidays, the number of days with events like football matches, concerts and festivals, and the number of days with disruptions of public transport and sudden, unexpected changes of weather. However, in most of the cases the characterization by different weekdays was the only significant one. Furthermore, for each cluster the average number of bookings in the specific three-hour time interval and its deviation from the overall mean booking number in this time slice were calculated. This gives information on which clusters exhibit reduced, average or increased demand in the considered time slice. The results of the cluster characterization for time slice 1 (6 a.m. to 9 a.m.) are shown in Table 2. It is obvious, that weekdays exhibit higher demands in this early time slice than weekends.

The objective of the cluster analysis is a prediction of future demand that is necessary as an input for the optimal relocation of Car Sharing vehicles. For each time slice, the development of the clusters in the next time slice is thus identified. This enables the response to questions like:

"The Car Sharing demand currently belongs to cluster A6 (weekday with maximal demand). Will the demand stay maximal in the next time slice? What is the predicted number of bookings (overall and per segment) for the next time slice?"

Let us consider as an example the development of the 6 clusters from time slice 1 to time slice 2 (9 a.m. to 12 a.m.). The results of the cluster method for time slice 2 are shown in Fig. 5. The results of the cluster characterization for time slice 2 are summarized in Table 3.

For each cluster in time slice 1, the distribution of its days over the clusters in time slice 2 was calculated. Those distributions are shown in a From-To-Matrix in Table 4. The most frequent clusters in time slice 2 are represented in bold for each cluster in time slice 1. Furthermore, high percentages are marked in red and low percentages in green.

Weekends or official holidays with minimum demand (cluster A1) have with a probability of 66% reduced or average demand (cluster B2 and B3) in time slice 2. The predicted overall demand for those days is thus calculated as the average of the characteristic demands for the clusters B2 and B3 in time slice 2. This calculation is also done for all of the other cluster developments. For days (mostly Saturdays to Mondays) with reduced demand (cluster A2), it is difficult to identify the demand development due to a relatively homogeneous distribution of the days to all clusters in time slice 2. Most of those days have increased demand in time slice 2 (cluster B5). Weekdays with average demand in time slice 1 (cluster A3) tend to have reduced or average demand in time slice 2 (cluster B4 and B2). Weekdays with increased demand in time slice 1 (cluster A4) are with a probability of 63% weekdays with average demand in time slice 2 (cluster B4 and B3). Weekdays with strongly increased demand (cluster A5) tend to have increased or average demand in time slice 2 (cluster B5 and B4). Finally, weekdays with maximum demand in time slice 1 (cluster A6) most likely have increased or even strongly increased demand in time slice 2 (cluster B5 and B6). To sum up, most of the clusters devolve to clusters of similar rank in time slice 2. This is also illustrated by the red entries in the From-To-Matrix that are mostly in vicinity of the diagonal. An exception is cluster A2 (reduced demand, mostly Sat to Mon) with an almost homogenous distribution to all clusters. This is due to the distribution of the different weekdays in cluster A2 to different clusters in time slice 2. Table 5 shows for each cluster in time slice 1 the most probable cluster memberships in time slice 2. The resulting relative change in the number of bookings occurring for the cluster by the transition from time slice 1 to time slice 2 is also listed.

Table 3. Characterization of the demand clusters for time slice 2 (9 a.m. to 12 a.m.).

Cluster	No. of Days	Relative Deviation from Overall Average Booking Number in Cluster	Description
B1	32	−49%	Strongly reduced demand, mostly Sun to Thu
B2	99	−25%	Reduced demand, mostly Sun to Thu
B3	75	−2%	Weekday or Weekend with average demand
B4	66	+/−0%	Weekday with average demand
B5	68	+34%	Increased demand, mostly Fri and Sat
B6	26	+69%	Strongly increased demand, mostly Sat

Table 4. Temporal development of the demand clusters.

From/To Cluster	B1	B2	B3	B4	B5	B6
A1	16%	**40%**	**26%**	5%	14%	0%
A2	14%	**19%**	12%	11%	**28%**	**16%**
A3	9%	**40%**	16%	**27%**	5%	3%
A4	2%	15%	**40%**	**23%**	17%	2%
A5	0%	11%	18%	**29%**	**34%**	8%
A6	0%	0%	4%	11%	**48%**	**37%**

Table 5. Most probable demand cluster memberships in time slice 2 and according relative change in booking number.

Cluster in Time Slice 1	Predicted Cluster in Time Slice 2	Relative Change in Booking Number
A1	B2 or B3	+300%
A2	B5 or B2 or B6	+165%
A3	B2 or B4	+16%
A4	B3 or B4	+8%
A5	B5 or B4	+9%
A6	B5 or B6	+8%

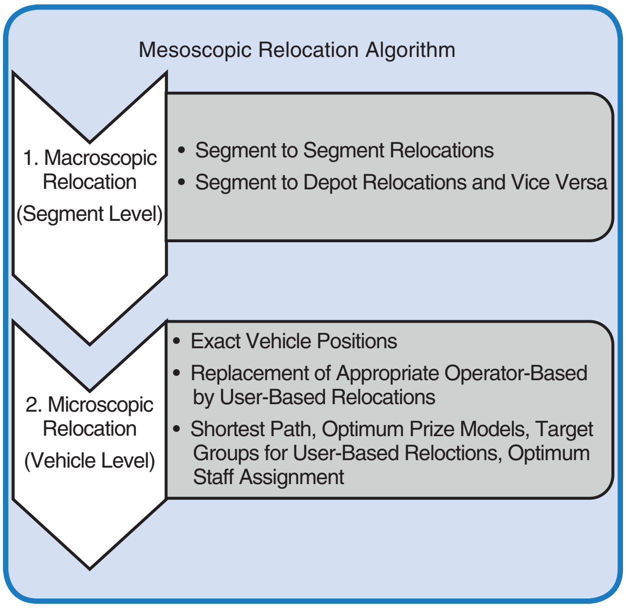

FIG 6 Mesoscopic two-step relocation algorithm.

We have identified the demand developments for the transition between time slice 1 and 2. This approach is repeated for all of the other transitions between different time slices. The macroscopic results are predictions of the number of bookings in the next time interval. For the relocation of Car Sharing vehicles, booking predictions should be available on a microscopic (segment) level. For each cluster, the average booking number per segment can also be easily calculated. As vertical displacements between the principal component points represent different spatial demand concentrations, each cluster could be divided into days with positive second principal component and days with negative second principal component. This would lead to an improvement of the microscopic results. For time slice 1, the number of clusters would increase to 10 because the clusters A1, A2, A3 and A6 would be divided into two clusters each. This has the effect that the different clusters also contain days with similar demand concentrations and not only days with similar overall demand. For each of the 10 clusters in time slice 1 the average booking number is then calculated on a segment level. As before, a From-To-Matrix is created for the 10 clusters.

$$
\begin{aligned}
\min c(R,D,P) = & \underbrace{\sum_{i=1}^{k}\sum_{j=1}^{k} \mathbb{1}_{\{r_{ij}>0\}} r_{ij} * (\mathrm{dist}_{ij} * (f_p + l_v) + tt_{ij} * w)}_{\text{Relocation Costs from Segment to Segment}} \\
& + \underbrace{\sum_{p=1}^{l}\sum_{i=1}^{k} \mathbb{1}_{\{d_{pi}>0\}} d_{pi} * (\mathrm{dist}_{pi} * (f_p + l_v) + tt_{pi} * w - h_p)}_{\text{Relocation Costs from Depot to Segment}} \\
& + \underbrace{\sum_{p=1}^{l}\sum_{i=1}^{k} \mathbb{1}_{\{d_{pi}<0\}} |d_{pi}| * (\mathrm{dist}_{pi} * (f_p + l_v) + tt_{pi} * w + h_p)}_{\text{Relocation Costs from Segment to Depot}} \\
& + \underbrace{\sum_{i=1}^{k} p_i * g_i}_{\text{Penalty Costs for Demand That Can't Be Met}}
\end{aligned}
$$

FIG 7 Cost function for the macroscopic relocation problem.

B. Online Optimization Module

A second Online Optimization Module is carried out online, at least several times a day. The current demand of the Car Sharing system as well as the current state of the spatial vehicle distribution is continuously measured and monitored. The demand indicators defined in section IV.A are calculated for the current demand and the corresponding demand vector is reduced to the first two principal components according to step 1. This representation of the current demand is then assigned to one of the 6 demand clusters identified in step 1 by minimizing the point-to-cluster-centroid Euclidean distance. The temporal development of the several clusters derived in step 1 enables a prediction of the demand in the next time slice based on the current demand cluster. Knowing the future demand, the corresponding optimal spatial vehicle distribution for the next time slice can be easily calculated.

For this module, an ordinal index has to be defined, that measures the deviation between the optimal future state and the current state and indicates whether a relocation intervention should be conducted or not. In case of a positive ordinal index, a relocation algorithm is applied immediately for finding cost-efficient relocation strategies. Otherwise, supply and demand are relatively balanced and no relocation is currently taken into account.

The relocation algorithm in the Online Optimization Module is mesoscopic, i.e. it consists of a macroscopic and a microscopic optimization algorithm (see Fig. 6). This optimization algorithm is not explained in detail in this paper as the focus is on the Offline Demand Module (step 1). First tests of the relocation algorithm however already show promising results. Of course further research needs to be carried out to proof these first findings.

$$
\begin{gathered}
\underbrace{r_{ij} = -r_{ji}, \ \forall \text{ Segments } i, j = 1:k}_{\text{Vehicle Conservation Between Segments}} \\
\underbrace{\sum_{j=1}^{k} r_{ij} + p_i + \sum_{p=1}^{l} d_{pi} - a_i = dev_i \ \forall \text{ Segments } i, i = 1:k}_{\text{Eliminate Vehicle Shortage/Exploit Vehicle Surplus}} \\
\underbrace{\sum_{j=1}^{k} \mathbb{1}_{\{r_{ij}>0\}} r_{ij} + \sum_{p=1}^{l} \mathbb{1}_{\{d_{pi}<0\}} |d_{pi}| \le \mathbb{1}_{\{dev_i<0\}} |dev_i| \ \forall \text{ Segments } i, i = 1:k}_{\text{Limited Vehicle Surplus in Segments}} \\
\underbrace{-\sum_{i=1}^{k} d_{pi} \le cap_{d_p} - cur_{d_p} \ \forall \text{ Depots } p, p = 1:l}_{\text{Limited Depot Capacity}} \\
\underbrace{\sum_{i=1}^{k} \mathbb{1}_{\{d_{pi}>0\}} d_{pi} \le cur_{d_p} \ \forall \text{ Depots } p, p = 1:l}_{\text{Limited Vehicle Number in Depots}} \\
\underbrace{p_i, a_i \ge 0 \ \forall \text{ Segments } i, i = 1:k}_{\text{Positive Variables}} \\
\underbrace{r_{ij}, d_{pi}, p_i, \text{ and } a_i \text{ integer } \forall \ i, j = 1:k, p = 1:l}_{\text{Integer Variables}}
\end{gathered}
$$

FIG 8 Constraints for the macroscopic relocation problem.

Table 6. Parameters and variables for the macroscopic relocation algorithm.

Variable/ Parameter	Description
r_{ij}	$=$ n, if n vehicles are moved from segment i to segment j $= -$ n, if n vehicles are moved from segment j to segment i
d_{pi}	$=$ n, if n vehicles are moved from depot i to segment j $= -$ n, if n vehicles are moved from segment j to depot i
P_i	$=$ n, if a demand of n vehicles can't be met in segment i
$dist_{ij}$	distance between segments i and j in kilometers
g_i	gain of an average car sharing trip out of segment i for the operator
fp	fuel prize per kilometer in Euros
1v	loss of value of a vehicle per kilometer in Euros
tt_{ij}	travel time between segments i and j in hours
w	hourly wage for the relocation personnel in Euros
h_p	holding cost per vehicle in Euros
a_i	$=$ n, if n "superfluous" vehicles in segment i cannot be stored in a depot
dev_i	deviation between supply and demand in segment i, positive: shortage of vehicles, negative: surplus of vehicles
cap_{d_p}	capacity of depot p
cur_{d_p}	current number of vehicles in depot p

The macroscopic algorithm acts on a segment level. It answers the question on how many vehicles have to be relocated from which segment to which segment or from which segment to which buffer depot and vice versa. This algorithm considers operator-based relocations only. The cost function of the optimization model consists of time-related and distance-related costs like fuel, wages for the relocation personnel, etc. Additionally, holding costs for the buffer depots are included, if they occur on a per-vehicle basis. Finally, penalty costs are added for demand that can't be met due to a deviation between supply and demand. The optimization problem has several constraints that guarantee that vehicle shortage is eliminated and vehicle surplus is exploited, that the number of vehicles in the depots is limited etc. The cost function of the macroscopic relocation problem and the corresponding constraints are shown in Fig. 7 and 8. The corresponding parameters and variables are explained in Table 6 for a Car Sharing system with segments and depots.

The microscopic relocation algorithm aims at solving the relocation problem on an individual vehicle level. First, it answers the question on which vehicle to relocate to which exact position based on the optimal macroscopic movements between segments. Second, it generates recommendations on which macroscopic operator-based relocation could be possibly replaced by a user-based relocation. As user-based relocations are mostly more cost-efficient than operator-based relocation, this might lead to cost savings. Third, the microscopic algorithm also optimally assigns the relocations to the several relocation staff and computes the shortest path for those relocation plans. Finally, optimal prize models and optimal user target groups are generated in case of user-based relocations.

To sum up, the objective of the mesoscopic relocation algorithm is to reach the estimated optimal future spatial vehicle distribution for the current cluster. Contrary to most of the algorithms described in section III, operator-based as well as user-based relocations are considered.

V. Conclusion and Outlook

In this paper, different relocation algorithms for Car Sharing Systems have been described, categorized and evaluated. Those were primarily pure user-based or pure operator-based strategies. The literature review showed that little research was done on finding relocation models for fully free-floating Car Sharing systems without any stations. The focus has been mostly on station-based one-way systems with relocations at the end of the day. However, the dynamics of the new systems require dynamic relocations throughout the whole day. Future demand has to be known for very small time steps and Car Sharing demand models thus have to be reliable.

To cover those new aspects of free-floating Car Sharing systems, an integrated two-step model for optimal vehicle positioning and relocation was developed. The idea behind this new approach is to provide a continuous monitoring of a Car Sharing System. The system is however not fully controlled in a closed-loop. The focus of this paper was on the Offline Demand Module. This step is conducted periodically, e.g. once a year. It is based on the representation of demand by demand indicators in a demand matrix and the reduction of the multidimensional demand matrix to principal components. Finally, a two-dimensional cluster method combines different days with similar demand patterns. The Demand Module offers a high-resolution (per segment) spatial-temporal demand prediction for three-hour time slices. Some results of the offline cluster analysis were shown for a real-life free-floating Car Sharing system in Munich, Germany.

In contrast, the Online Optimization Module is carried out several times a day or online. It is based on the permanent measurement of the current demand and the current vehicle distribution. The results of the Offline Demand Module enable the prediction of future demand and thus the prediction of the optimum future spatial vehicle distribution. If an ordinal index expressing the deviation between current and future state is negative, a relocation algorithm is applied. A mesoscopic relocation algorithm is introduced in this paper. It consists of two parts: the relocation on a macroscopic segment level and the relocation on a microscopic individual vehicle level.

The introduced two-step model has several advantages. First of all, the partition into two independent phases, one offline and the other online, simplifies the relocation problem to a certain degree. The periodically executed data collection and analysis guarantees that the conducted relocation strategies remain up-to-date and adjust to changing user behavior and demand. Since the Demand Module is carried out offline, a sufficient amount of time can be invested in analyzing the data and thus generating a reliable and realistic demand model. In comparison to most of the relocation algorithms known from literature, the Online Optimization Modules integrates user-based relocation strategies. The additional microscopic relocation algorithm checks if some of the optimal operator-based movements computed in the macroscopic part can be replaced by more cost-efficient user-based strategies.

For future work, the identification of demand patterns based on historical vehicle positioning data (Offline Demand Module) has to be automated using appropriate software. This simplifies the periodical repetition of the cluster analysis with new data sets. Second, the Online Optimization module has to be developed further. An ordinal index has to be defined that best reflects if relocations make sense at present. This ordinal index should be a reliable decision support tool for the utilization of the relocation algorithm. Third, the application of the macroscopic relocation algorithm to first example scenarios shows promising results. However, those have to be analyzed further. Next, the microscopic relocation algorithm has to be developed in detail and has to be modeled mathematically. Finally, the whole two-step relocation model should be evaluated by applying it to test scenarios of the real-life free-floating Car Sharing system in Munich, Germany.

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