Edge Rotation of Polyhedrons on Plane

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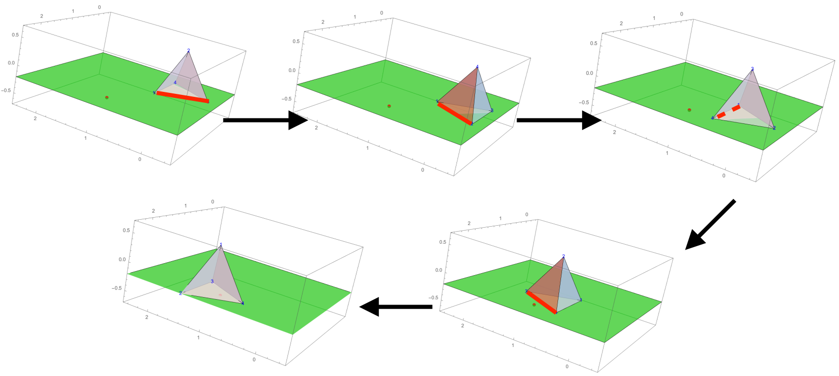
Problem 1
Given a convex polyhedron and an aim point on the same plane, determine a path to rotate the polyhedron to the aim along edges.

Algorithm 1 (Greedy)

Take the **biggest step** possible towards the aim **at every rotation**.

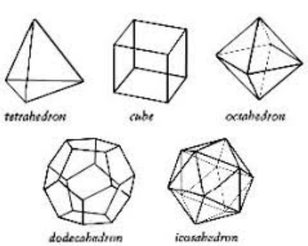


Run Time

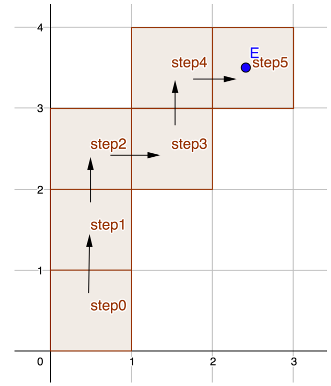
O(nd) in total, where n is the maximum number of edges on polyhedron faces and d is the number of steps taken

Now, instead of just a path, we want to find the shortest one.

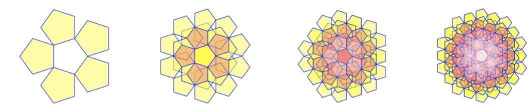
Problem 2
Given a **Platonic solid** and an aim point on the same plane, determine **the shortest path** to rotate the polyhedron to the aim along edges.



Platonic Solid:
Regular Convex Polyhedron
We mainly focused on platonic solids: tetrahedron, cube, octahedron, dodecahedron, isocahedron.



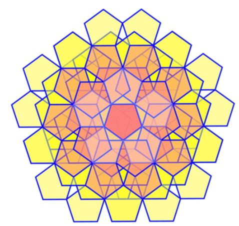
All except dodecahedron
equilateral triangle and square faces
→ regular tessellation
→ bases during rotations correspond to triangular or square grids
→ **Greedy Algorithm works**



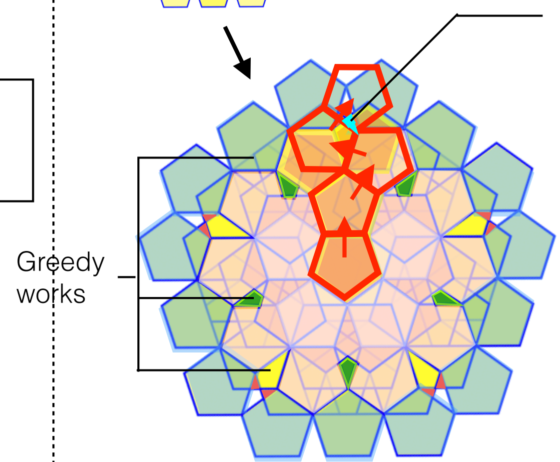
(a) (b) (c) (d)

Figure 2: Graph of rotations. Rings R_1 R_4 are colored yellow in each graph, and the previous ring is colored pink. The rest "fade away".

Dodecahedron It is different because its faces are pentagons which does not tile the plane. For them, we consider the **growth of covered areas** during rotation, as shown in above.



denote:
dd(p) = #rotations necessary to get to p;
S(i) = {p:dd(p)=i}, i.e., set of points at least i rotations away from the dodecahedron.
Consider the graph for S(4) (colored in yellow).



Algorithm 2 (Modified)

Greedy path(red) has more rotations than an optimal path(orange) and thus **Greedy is not optimal**.

However, we notice the **difference only happens at third-to-last rotation** of Greedy path.

We can fix it by **receding three rotations** after finding Greedy path and try an alternative rotation. And we return the better one between Greedy and Modified path.