

# CS554 Introduction to Machine Learning HW1 Report

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## 1 Introduction

In this study multi degree polynomial regression models fitted from degree 0 to 8 by given data and calculated errors. According to error plots, the best performing model is decided and explained why.

## 2 Background

The goal is to fit a model to given data that generalizes the problem and predicts next outcomes when new input is given. To be able to fit the model correctly polynomial equations have been used. Degree of 2 polynomial equation can be seen below in [1c](#) from [\[1\]](#). NumPy library in Python has a function called polyfit to fit the given data (X,R) with specific degree of polynomial with minimum sum of squared error can be seen in equation [2](#) from [\[2\]](#).

$$y = w_0x^2 + w_1x + b, \tag{1}$$

where  $y$  is the function that I will fit,  $w_0$ ,  $w_1$  and  $b$  are the coefficients that will be settled in polyfit function and  $x$  is the input of the function [1](#).

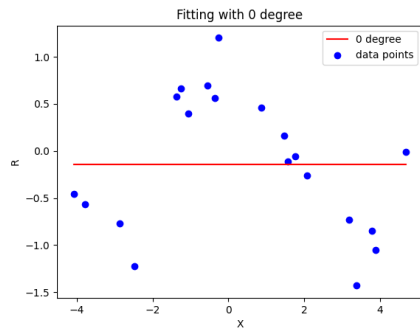
$$SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \tag{2}$$

## 3 Methodology

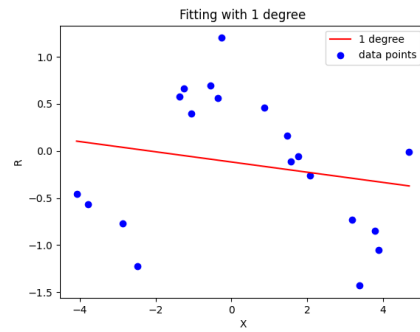
To complete the project Python version 3.11 has been used as a programming language and two libraries: NumPy and Matplotlib. The structure is follows, Read the train and test data, fit the polynomial regression models using numpy, evaluate them on test data by calculating sum of squared errors. At last the errors plotted as a function of degree and every fit visualized on the training set.

## 4 Results

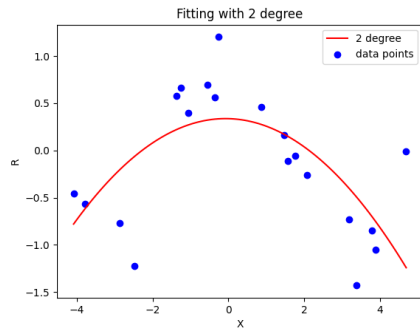
In the figures below each degree of polynomial models fitted on the training data can be seen.



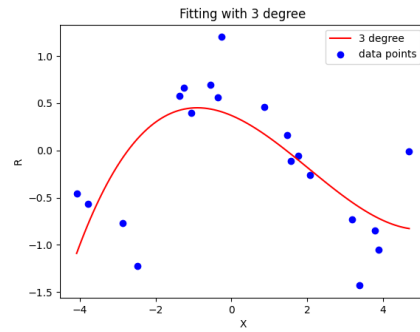
(a) Polynomial fitted with degree of 0



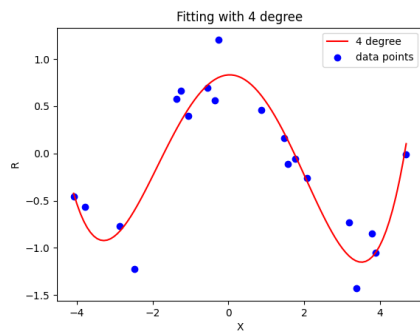
(b) Polynomial fitted with degree of 1



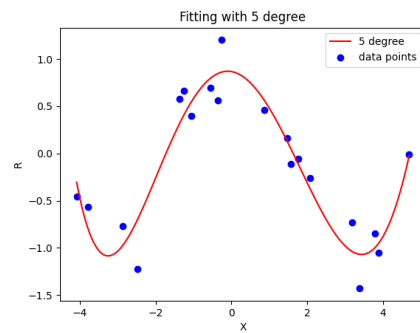
(c) Polynomial fitted with degree of 2



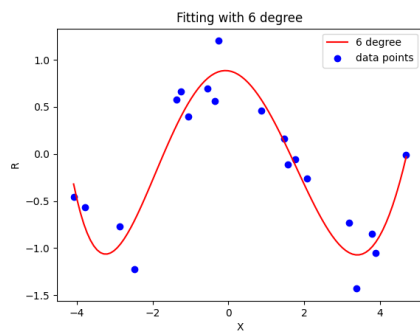
(d) Polynomial fitted with degree of 3



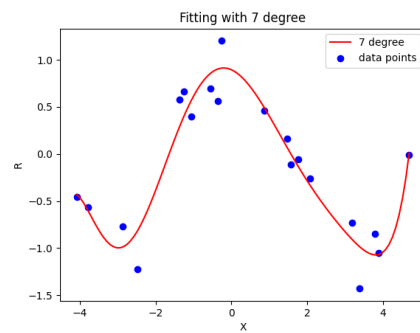
(e) Polynomial fitted with degree of 4



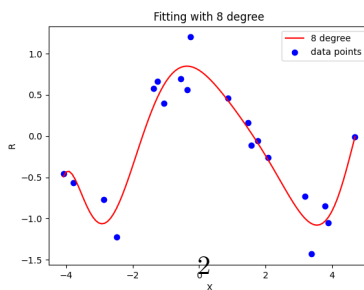
(f) Polynomial fitted with degree of 5



(g) Polynomial fitted with degree of 6



(h) Polynomial fitted with degree of 7



(i) Polynomial fitted with degree of 8

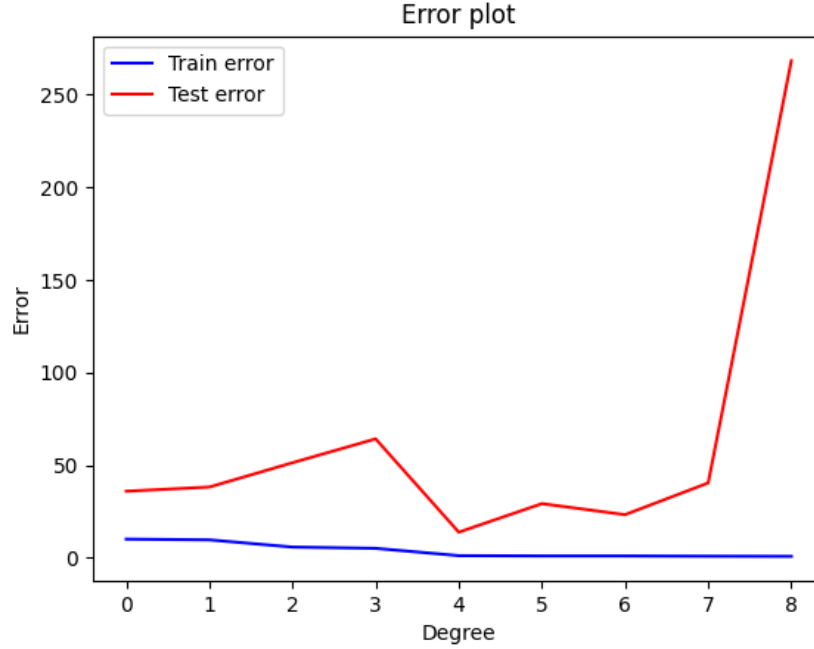


Figure 2: Train and Test errors plotted as a function of polynomial degree

## 5 Discussion

According to the results section it is possible to find an optimum degree for the model within the given dataset. To be able to decide the optimum degree for the polynomial fit it is not enough to just see the drawn plots on the training set (figs 1a to 1i). SSE needs to be calculated at each dataset and models with lowest errors should be evaluated to select the most suitable model. As clearly seen in the fig 2 SSE on each dataset seems to be decreasing but not in every increase in the polynomial degree. Inside the trained models, fourth degree model is the best performing model because it has one of the lowest test error compared to other models. The test errors in fifth and sixth polynomial models are also very low but they are not suitable models because the model should also have the power to generalize the problem. They have very similar error values so there is no need to increase the complexity of the model. After degree of seven test error increases hugely that indicates the model started to overfit the dataset even though training error stays the same or decreasing. To solve the overfitting problem training samples may be increased in the training set.

## 6 Conclusion

In this study several different degrees of polynomial models have been fitted to the dataset and evaluated with error functions and multiple datasets. This study indicates the importance of finding the sweet spot between underfitting and overfitting regions and selecting the best model that generalizes the problem.

## References

- [1] Wikipedia. *Curve fitting* — *Wikipedia, The Free Encyclopedia*. 2024. URL: [https://en.wikipedia.org/wiki/Curve\\_fitting](https://en.wikipedia.org/wiki/Curve_fitting).
- [2] Wikipedia. *Residual sum of squares* — *Wikipedia, The Free Encyclopedia*. 2024. URL: [https://en.wikipedia.org/wiki/Residual\\_sum\\_of\\_squares](https://en.wikipedia.org/wiki/Residual_sum_of_squares).