The improved LCD-SN algorithm demonstrates remarkable computational efficiency, making it particularly suitable for large-scale social networks.

Phase 1: Initial Community Formation

In this phase, the importance scores of all nodes are initialized and iteratively updated. Each iteration involves traversing the neighbors of every node, resulting in a time complexity of

$$O(\gamma \cdot n \cdot k)$$

where n is the number of nodes, k is the average degree, and γ is the number of iterations. Although sorting the nodes by their importance incurs an additional

$$O(nlogn)$$

complexity, this phase is primarily dominated by

$$O(\gamma \cdot n \cdot k).$$

Phase 2: Overlapping Node Assignment

This phase focuses on calculating similarity scores between overlapping nodes and their respective communities. For each overlapping node v, the similarity checks across its associated communities require

$$O(v \cdot C \cdot k)$$

where C is the average number of communities containing the node.

Phase 3: Community Integration

This phase ensures effective merging of small and weak communities. Small communities are combined with larger neighboring ones, resulting in a complexity of

$$O(L \cdot k)$$

where L is the number of small communities. Weak communities are merged based on intra- and inter-community edge densities, which requires

$$O(n \cdot k).$$

Overall Complexity

Combining the complexities of all three phases, the overall time complexity of the algorithm becomes

$$O(n \cdot k),$$

ensuring linear scalability with the size of the network. The space complexity is

$$O(n+m)$$

where m is the number of edges, accounting for the storage of graph structures, importance scores, and community assignments.

With its efficient handling of overlapping nodes and fragmented communities while maintaining low computational overhead, the improved LCD-SN algorithm provides a highly scalable and practical solution for real-world social networks.