STA721 Homework 3

1. Condsider the linear model  $\mathbf{Y} \sim \mathsf{N}(\boldsymbol{\mu}, \sigma^2 \mathbf{I}_n)$  with  $\boldsymbol{\mu} = \mathbf{1}\beta_0 + \mathbf{X}\boldsymbol{\beta}$  and  $\mathbf{X}$  a full rank matrix with rank p

(a) Show that the projection, P, on the column space spanned by  $\mathbf{1}$  and  $\mathbf{X}$  may be written as  $P = P_1 + P_{\mathbf{X} - \mathbf{1}\bar{\mathbf{x}}^T}$ . Show that diagonal elements are

$$h_{ii} = \frac{1}{n} + (\mathbf{x}_i - \bar{\mathbf{x}})^T \left( (\mathbf{X} - \mathbf{1}\bar{\mathbf{x}}^T)^T (\mathbf{X} - \mathbf{1}\bar{\mathbf{x}}^T) \right)^{-1} (\mathbf{x}_i - \bar{\mathbf{x}})$$

(Recall all vectors are column vectors). The  $h_{ii}$  are known as the leverage values.

- (b) Find the sampling distribution of  $\hat{\mu}_i$  (the mean of  $Y_i$  at  $\mathbf{x}_i^T$  as a function of  $h_{ii}$  and provide an expression for a 95% confidence interval. For what values of  $\mathbf{x}$  will the interval be the narrowest? Explain.
- (c) Given  $\sigma^2$ , find the distribution of  $\mathbf{e}_i$  as a function of  $h_{ii}$ . Explain (rigorously) why  $\mathbf{e}_i$  unconditional on  $\sigma^2$  does not have a student t distribution with n-p-1 degrees of freedom.
- 2. Refer to the Prostate data from library(lasso2); data(Prostate)
  - (a) Fit a linear model using 1cavol (log cancer volume) as the response and include all covariates. Construct 95% confidence intervals for each coefficient and provide a meaningful interpretations for changes in the cancer volume (not log cancer volume) include any units etc in your interpretation. See Wakefield page 1.3.1 for details on variables. Note "a 1 unit" change may or may not be meaningful for interpretation.
  - (b) Fit the regression model with response lcavol, and variables svi and lpsa as predictors. Plot the cancer volume versus PSA on the log scale. Add the fitted regression function for svi = 1 and svi = 0, with lines representing the (pointwise) 95% confidence intervals for each.