Checking Assumptions

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STA721 Linear Models

Duke University

October 8, 2012

Linear Model

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If $\mu_i \neq \mathbf{x}_i^T \beta$ then expected value of $e_i = Y_i - \hat{Y}_i$ is not zero;

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- Variance is also almost 0, so standardize value may not flag "outliers"

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Estimates without Case (i):

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Standardized predicted residual is

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Can show that these are the same as standardized residual!

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Estimate $\hat{\sigma}^2_{(i)}$ using data with case i deleted

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$$\text{SSE}_{(i)} = \text{SSE} - \frac{e_i^2}{1 - h_{ii}}$$

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May still miss extreme points with high leverage, but will pick up unusual y_is

Distribution of Externally Studentized Residual

$$t_i = \frac{e_{(i)}}{\sqrt{\hat{\sigma}^2_{(i)}/(1-h_{ii})}} = \frac{y_i - \mathbf{x}_i^T \hat{\beta}_{(i)}}{\sqrt{\hat{\sigma}^2_{(i)}/(1-h_{ii})}} \sim \mathrm{St}(n-p-1)$$

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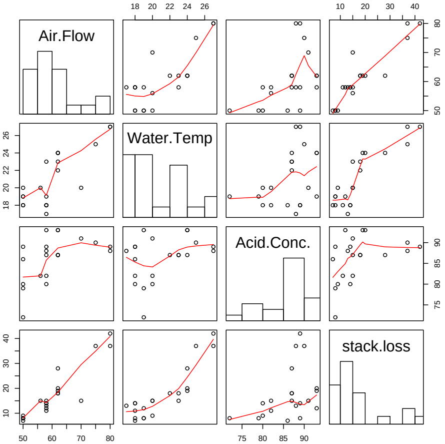
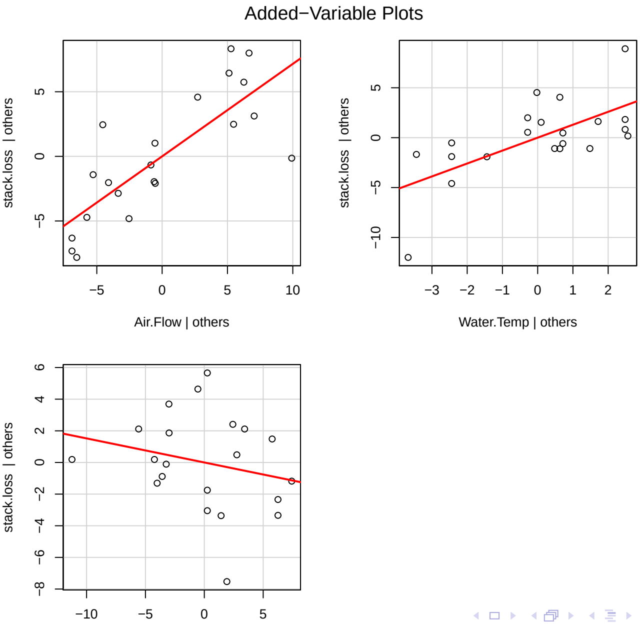
Influential Cases

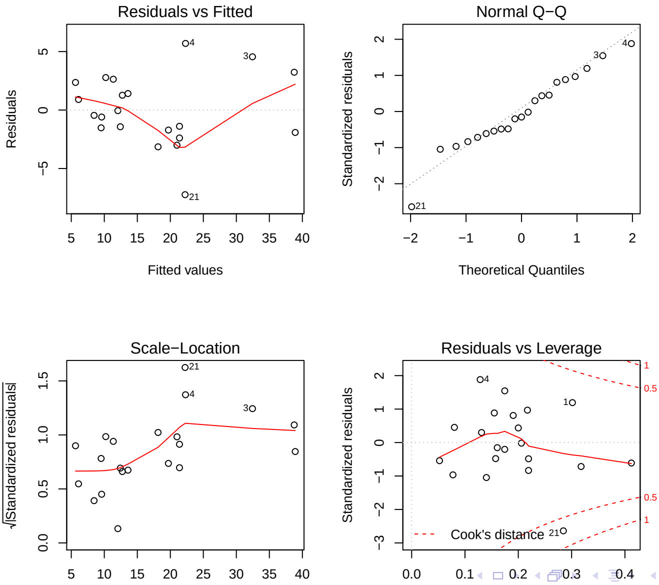
Stackloss Data



Stackloss Added Variable Plot



Stackloss Data Again



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Other analyses have suggested that cases (1, 2, 3, 4, 21) are outliers

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Using BAS

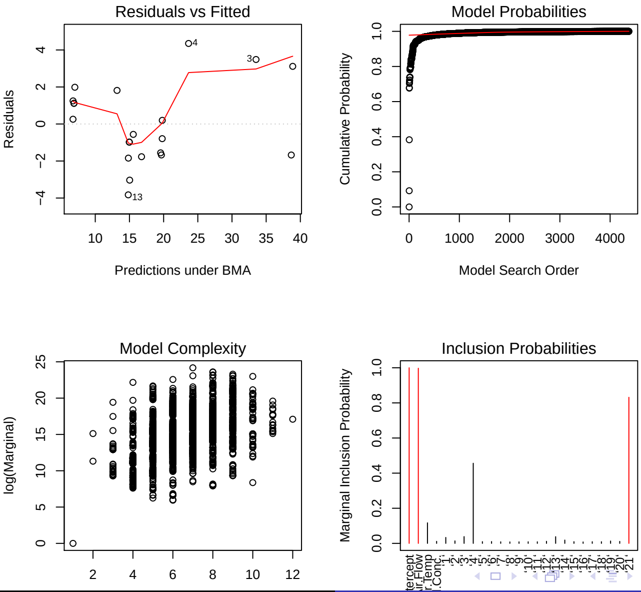
```
library(MASS)
data(stackloss)
n = nrow(stackloss)
stack.out = cbind(stackloss, diag(n))

library(BAS)
BAS.stack = bas.lm(stack.loss ~ ., data=stack.out,
                   prior="hyper-g-n", a=3,
                   modelprior=beta.binomial(1, ncol(stack.c
                   n.models=2^20,
                   method="MCMC", MCMC.it=2^21)
```

Output

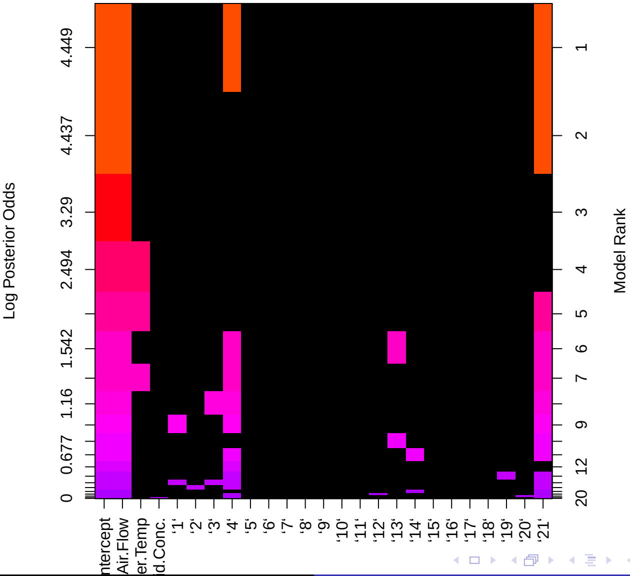
```
> summary(BAS.stack)
     Intercept Air.Flow Water.Temp Acid.Conc. `1` `2` `3` `4` `5` `6` `7` `8`
[1,]         1        1          0          0   0   0   0   1   0   0   0   0
[2,]         1        1          0          0   0   0   0   0   0   0   0   0
[3,]         1        1          0          0   0   0   0   0   0   0   0   0
[4,]         1        1          1          0   0   0   0   0   0   0   0   0
[5,]         1        1          1          0   0   0   0   0   0   0   0   0
     `9` `10` `11` `12` `13` `14` `15` `16` `17` `18` `19` `20` `21`
[1,]   0    0    0    0    0    0    0    0    0    0    0    0    1
[2,]   0    0    0    0    0    0    0    0    0    0    0    0    1
[3,]   0    0    0    0    0    0    0    0    0    0    0    0    0
[4,]   0    0    0    0    0    0    0    0    0    0    0    0    0
[5,]   0    0    0    0    0    0    0    0    0    0    0    0    1
               BF PostProbs     R2 dim  logmarg
[1,] 1.0000000000    0.2940 0.9605   4 22.16893
[2,] 0.0644223528    0.2904 0.9271   3 19.42664
[3,] 0.0008704905    0.0922 0.8458   2 15.12248
[4,] 0.0092248619    0.0416 0.9088   3 17.48308
[5,] 0.0845728129    0.0249 0.9466   4 19.69879
>
```

BAS



Residuals vs Fitted
Residuals
Predictions under BMA
4
3
13
Model Probabilities
Cumulative Probability
Model Search Order
Model Complexity
log(Marginal)
Inclusion Probabilities
Marginal Inclusion Probability

BAS



Log Posterior Odds
4.449
4.437
3.29
2.494
1.542
1.16
0.677
0
Model Rank
1
2
3
4
5
6
7
9
12
20
'1'
'2'
'3'
'4'
'5'
'6'
'7'
'8'
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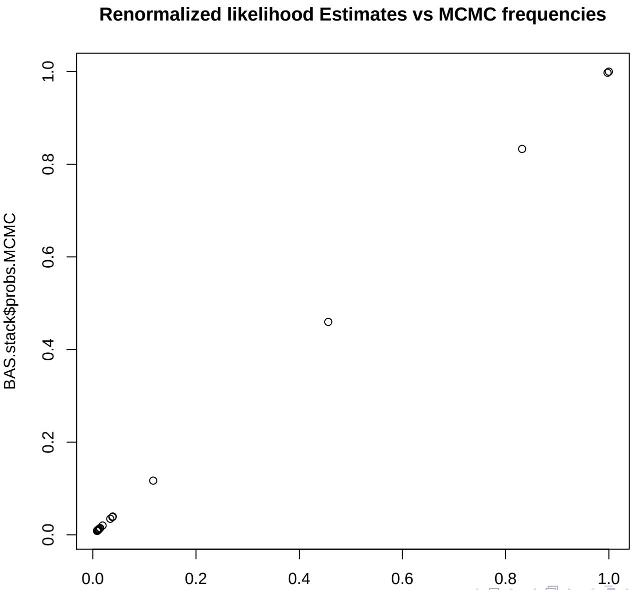
Diagnostics



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- Investigate need for transformations (use EDA at several stages)
- Influential cases - report results with and without cases (results may change - are differences meaningful?)

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