Horseshoe, Lasso and Related Shrinkage Methods

Readings Chapter 15 Christensen

STA721 Linear Models Duke University

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Scale Mixture of Normals (Andrews and Mallows 1974)

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Homework Nextweek: Derive the full conditionals for $\boldsymbol{\beta}^s$, ϕ, $1/\tau^2$
see `http://www.stat.ufl.edu/~casella/Papers/Lasso.pdf`

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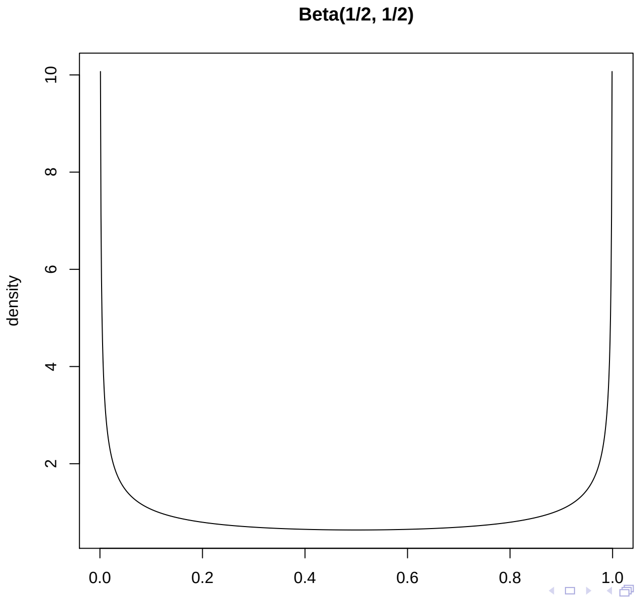
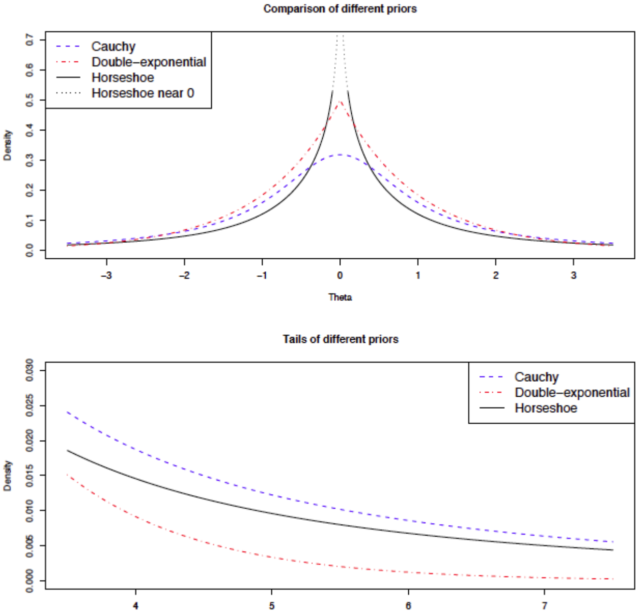
Half-Cauchy prior induces a Beta(1/2, 1/2) distribution on κ_i a priori

Horseshoe



Prior Comparison (from PSC)



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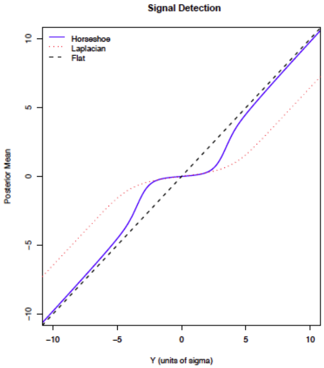


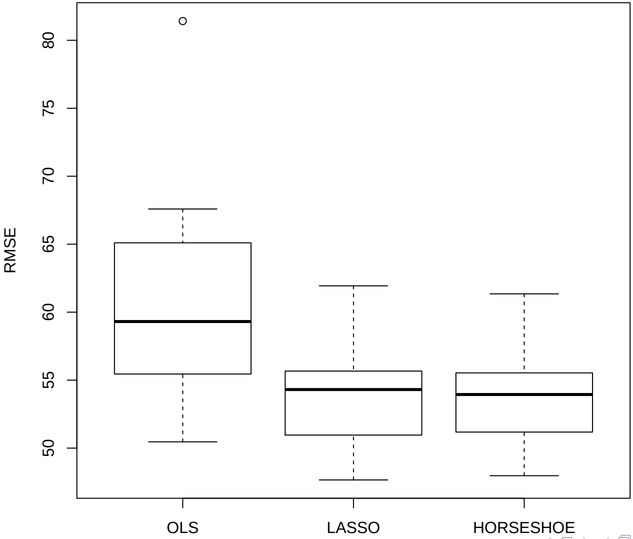
R packages

The `monomvn` package in R includes

- blasso
- bhs

See Diabetes.R code

Simulation Study with Diabetes Data



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Choice of prior? Properties? Fan & Li (JASA 2001) discuss Variable selection via nonconcave penalties and oracle properties

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Remember all models are wrong, but some may be useful!