Introduction to Linear Models

STA721 Linear Models Duke University
Wakefield Chapter 1 & 5

Merlise Clyde

August 26, 2014

Coordinates

- Instructor: Merlise Clyde
  214 Old Chemistry
  Office Hours Tues/Thur 4:20-5:20 or by appointment
- Teaching Assistant: Chris Glynn
- Course: Theory and Application of linear models from both a frequentist (classical) and Bayesian perspective

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- Introduce R programming as needed

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  - Causal interpretation
  - Testing of hypotheses
  - confirmatory or validation analyses
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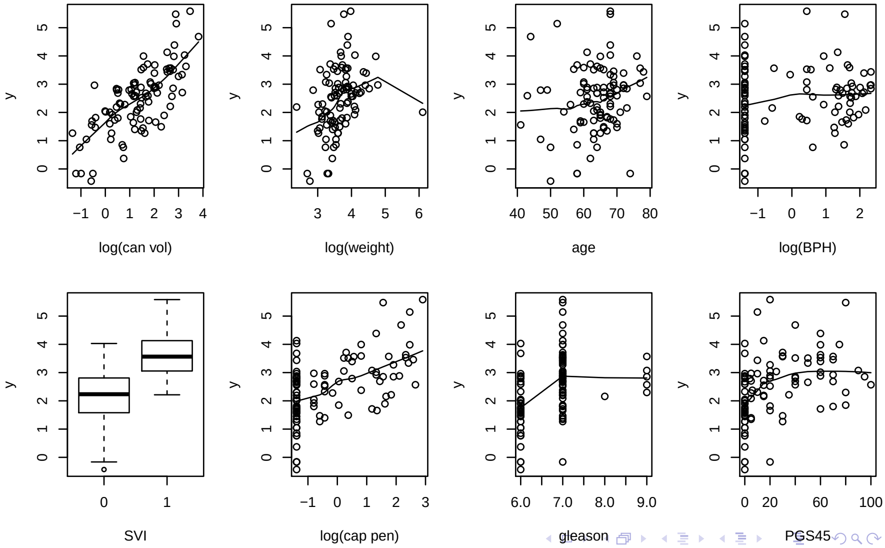
Prostate Example



Simple Linear Regression

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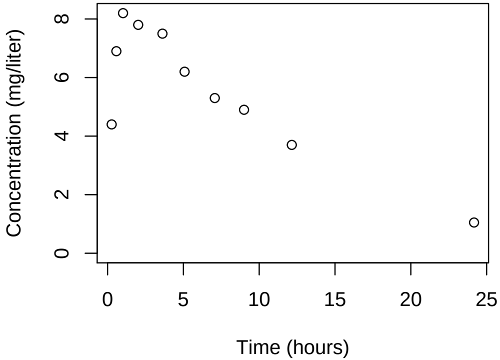
what should go into **X** and do we need all columns of **X** for inference about **Y**?

Nonlinear Models

Mean function may be an intrinsicaly nonlinear function of t

$$\mathsf{E}[Y_i] = f(t_i, \boldsymbol{\theta})$$



Quadratic Linear Regression

Taylor's Theorem:

$$f(t_i, \boldsymbol{\theta}) = f(t_0, \boldsymbol{\theta}) + (t_i - t_0) f'(t_0, \boldsymbol{\theta}) + (t_i - t_0)^2 \frac{f''(t_0, \boldsymbol{\theta})}{2} + R(t_i, \boldsymbol{\theta})$$

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Quadratic in x, but linear in β's

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Use Nonlinear Regression or other Nonparametric models

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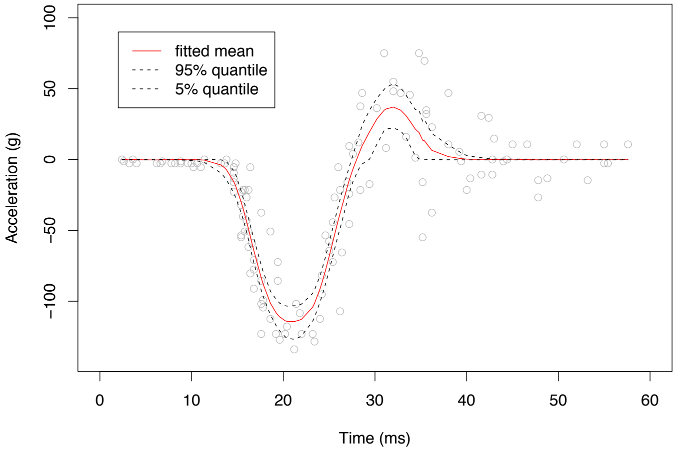
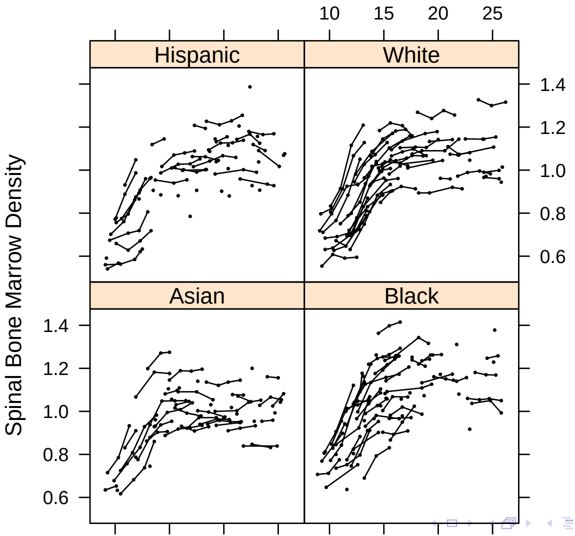
Linear in β given λ
Learn λ and J

Kernel Regression Example



Hierarchical Models - Spinal Bone Density



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All models are wrong, but some may be useful (George Box)

Ordinary Least Squares

Goal: Find the best fitting "line" or "hyper-plane" that minimizes

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Need Distribution Assumptions of Y (or ϵ) for testing and uncertainty measures $\Rightarrow$ Likelihood and Bayesian inference

Philosophy

- for many problems frequentist and Bayesian methods will give similar answers (more a matter of taste in interpretation)
- For small problems, Bayesian methods allow us to incorporate prior information which provides better calibrated answers
- for problems with complex designs and/or missing data Bayesian methods are often better easier to implement (do not need to rely on asymptotics)
- For problems involving hypothesis testing or model selection frequentists and Bayesian methods can be strikingly different.
- Frequentist methods often faster (particularly with "big data") so great for exploratory analysis and for building a "data-sense"
- Bayesian methods sit on top of Frequentist Likelihood

Important to understand advantages and problems of each perspective!