STA721

Homework 16

1. Show that if $\tau^2 \sim G(1/2, \lambda)$ and $\lambda \sim G(1/2, \psi)$ then $\tau \sim \mathcal{C}^+(0, \psi^{1/2})$. (i.e a student t distribution with 1 df supported on $\mathbb{R}^+$. (Find the density with normalizing constant).

2. Consider the model

$$\begin{aligned}
Y &= 1\alpha + X\beta + \epsilon \\
\epsilon_i \mid \lambda_i, \phi &\overset{\text{ind}}{\sim} N(0, 1/(\phi\lambda_i)) \\
\lambda_i &\overset{\text{iid}}{\sim} G(a/2, a/2) \\
\beta_j &\overset{\text{ind}}{\sim} N(0, 1/(\phi\gamma_j)) \\
\gamma_j &\overset{\text{ind}}{\sim} G(\delta/2, \delta/2) \\
p(\alpha, \phi) &\propto 1/\phi
\end{aligned}$$

where X has been centered and standardized.

(a) Show that marginal distributions $\epsilon_i \mid \phi$ are iid Student t with a degrees of freedom. Similarly show that $\beta_j \mid \phi$ are iid Student t with δ degrees of freedom.

(b) Derive the full conditional distributions for the blocks of parameters β, λ, γ, α and ϕ. These are all nice distributions (Normal, Gamma, Gamma, Normal, and Gamma, respectively, please specify distribution and hyperparameters)

(c) Modify the Gibbs Sampler code for JAGS for the stack loss data stackloss data using your choice of $\delta \leq a$. Provide plots of the posterior distributions for λ (side-by-side boxplots). What does this suggest about "outliers"? Construct credible intervals for the βs. How do these compare to the frequentist solutions or the model selection results from MC3.REG? How sensitive are the results to the choice of a and δ? (if you wish to also compare using BAS, download the latest version 1.0.9 from github:

```
library(devtools)
install_github("merliseclyde/BAS")
stack.out = cbind(stackloss, diag(n))
library(BAS)
BAS.stack = bas.lm(stack.loss ~ ., method="MCMC", prior="hyper-g-n",
                   a=3, modelprior=beta.binomial(1, 1),
                   data=stack.out, n.models=2^20)
```