

Work the following problems from Christensen (C) and Wakefield (W)

1. 1.11
2. 1.4 (see sketch from class notes)
3. 1.5.3
4. Suppose Σ is a real $p \times p$ positive semi-definite matrix. Then the Cholesky decomposition of $\Sigma = \mathbf{L}\mathbf{L}^T$ where \mathbf{L} is a lower triangular matrix with real, non-negative elements on the diagonal.

If you can generate standard normal random variates, $\mathbf{Z} = (z_1, \dots, z_p)^T$ with $z_i \stackrel{\text{iid}}{\sim} N(0, 1)$, then what is the distribution of $\mathbf{Y} = \mu + \mathbf{L}\mathbf{Z}$ for $\mu \in \mathbb{R}^p$? Explain why it does not matter for generating \mathbf{Y} that the Cholesky decomposition is not unique when Σ is not positive definite.