Morphological agreement in Minimalist Grammars

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Agreement: morphological manifestation of dependencies between words

Case study: Icelandic dative intervention

He walks.

There seems to have arrived a man.

 Agreement: morphological manifestation of dependencies between words

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Minimalist Grammars (MGs, Stabler 1997): formalization of Chomsky's Minimalist Program

Overview

 Agreement: morphological manifestation of dependencies between words

Case study: Icelandic dative intervention

He walks

There seems to have arrived a man.

- Minimalist Grammars (MGs, Stabler 1997): formalization of Chomsky's Minimalist Program
- Goal: extend MGs with morphological features and operations on them

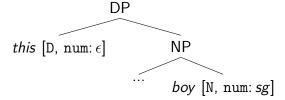
Agreement in MGs

Overview

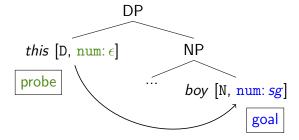
- Agreement in MGs
- 3 Case study: Icelandic dative intervention
- 4 Discussion

- Syntax is driven by feature checking/valuation
- Merge and Move:
 - operate on categorial features (T, V, D, N...)
 - build new structure
- Agree:
 - targets morphosyntactic features (num, per, case...)
 - creates dependencies within existing structure

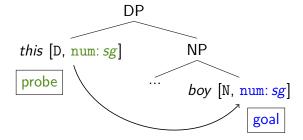
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A set of syntactic features:

```
Syn = base (categories)

\cup \{ = f \mid f \in base \} (selectors)

\cup \{ + f \mid f \in base \} (licensors)

\cup \{ -f \mid f \in base \} (licensees)
```

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Syn = base \qquad \qquad \text{(categories)}
\cup \{ = f \mid f \in base \} \qquad \text{(selectors)}
\cup \{ + f \mid f \in base \} \qquad \text{(licensors)}
\cup \{ -f \mid f \in base \} \qquad \text{(head movement selectors)}
\cup \{ *f \mid f \in base \} \qquad \text{("persistent" licensees)}
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A set of lexical items:

```
Lex \subset \Sigma^* \times Syn^*, where \Sigma is a set of pronounced segments
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A set of lexical items:

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• Two structure-building operations: *merge* and *move*

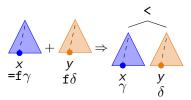
Case study: Icelandic dative intervention

merge:

Introduction

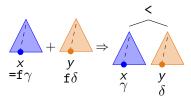
merge:

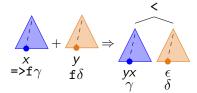
Introduction



merge:

Introduction

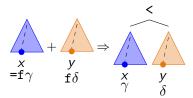


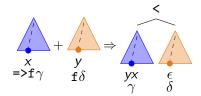


merge:

Introduction

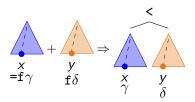
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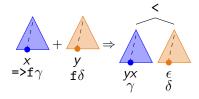




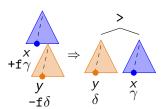
■ move:

merge:

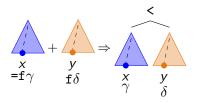


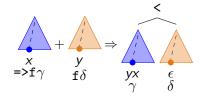


move:

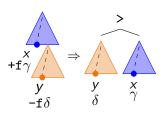


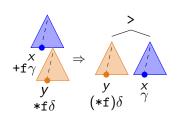
merge:





move:



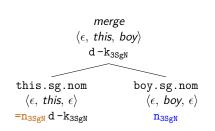


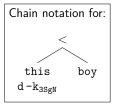
• Chain notation (Stabler 2001):

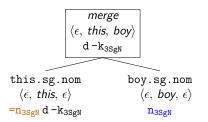
$$\langle {\sf Spec}, \; {\sf Head}, \; {\sf Comp} \rangle \; : \; {\sf Features}, \; \underbrace{{\sf Mover}_1, \; {\sf Mover}_2, \; \dots}_{{\sf Non-initial \; chains}}$$

Example grammar:

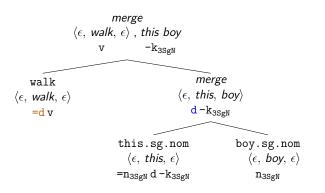
$$\begin{array}{lll} \texttt{this.sg.nom} & := \left<\epsilon, \; \textit{this}, \; \epsilon\right> \; :: \; = \texttt{n}_{3\texttt{SgN}} \; \texttt{d} \; - \texttt{k}_{3\texttt{SgN}} \\ \texttt{boy.sg.nom} & := \left<\epsilon, \; \textit{boy}, \; \epsilon\right> \; :: \; \texttt{n}_{3\texttt{SgN}} \\ \texttt{walk} & := \left<\epsilon, \; \textit{walk}, \; \epsilon\right> \; :: \; = \texttt{d} \; \texttt{v} \\ \texttt{prs.3sg} & := \left<\epsilon, \; -s, \; \epsilon\right> \; :: \; = \texttt{v} \; + \texttt{k}_{3\texttt{SgN}} \; \texttt{t} \\ \end{array}$$

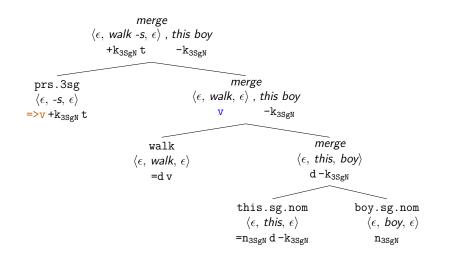


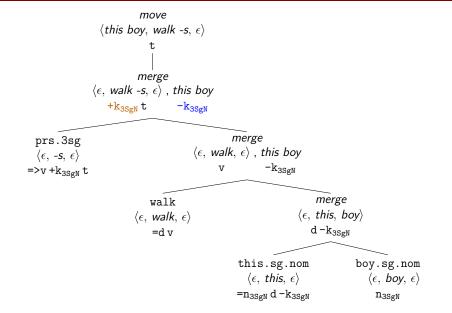




Introduction







Case study: Icelandic dative intervention

Overview

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- A Discussion

$$\texttt{boy.sg.nom} \; := \; \left\langle \epsilon, \; boy, \; \epsilon \right\rangle \; :: \; \texttt{n}_{\texttt{3SgN}}$$

Lexical items carry morphological feature bundles

$$\texttt{boy.sg} \; := \; \left\langle \epsilon, \; \left[\begin{smallmatrix} \mathsf{BOY} \\ \mathsf{num:sg} \\ \mathsf{per:3} \end{smallmatrix} \right], \; \epsilon \right\rangle \; :: \; \mathsf{n}_{\mathsf{3SgN}}$$

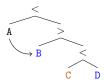
- Lexical items carry morphological **feature bundles**
- Heads exchange information across *merge* and *move* dependencies (Kobele 2012)

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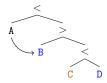
- Lexical items carry morphological **feature bundles**
- Heads exchange information across merge and move dependencies (Kobele 2012)
- Syntactic features annotated with channels
 - receiving channel: does it accept values from whatever checks it?
 - emitting channel: what values does it pass to whatever checks it?

$$\texttt{boy.sg} \; := \; \left\langle \epsilon, \; \left[\begin{smallmatrix} \texttt{BOY} \\ \texttt{num:}sg \\ \texttt{per:}3 \\ \texttt{case} : \epsilon \end{smallmatrix} \right], \; \epsilon \right\rangle \; :: \; \mathsf{n}_{\leftarrow}^{\left[\begin{smallmatrix} \texttt{num:}sg \\ \texttt{per:}3 \end{smallmatrix} \right] \rightarrow}$$

- Closest goal (Chomsky 2000)
 - overwriting values
 - last receiving channel wins

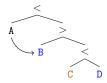


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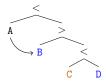
- No agreement in intermediate positions of movement (Bošković in press)
 - Relevant for starred licensees
 - Agreement just in case the licensee is deleted

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- No agreement in intermediate positions of movement (Bošković in press)
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- Long-distance agreement as a series of local exchanges

- Closest goal (Chomsky 2000)
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- No agreement in intermediate positions of movement (Bošković in press)
 - Relevant for starred licensees
 - Agreement just in case the licensee is deleted
- Long-distance agreement as a series of local exchanges
- Is this enough?

$$\begin{array}{c} \textit{merge} \\ \langle \epsilon, \left[\begin{smallmatrix} B \\ \text{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle, \left[\begin{smallmatrix} A \\ \text{m} : \epsilon \end{smallmatrix} \right] \\ b_{\leftarrow} - g_{\leftarrow} - h_{\leftarrow} \\ \\ \langle \epsilon, \left[\begin{smallmatrix} B \\ \text{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle \\ = \mathbf{a}^{\left[\begin{smallmatrix} \mathbf{m} : \epsilon \end{smallmatrix} \right]} \rightarrow b_{\leftarrow} - g_{\leftarrow} \\ \mathbf{a}_{\leftarrow} - h_{\leftarrow} \end{array}$$

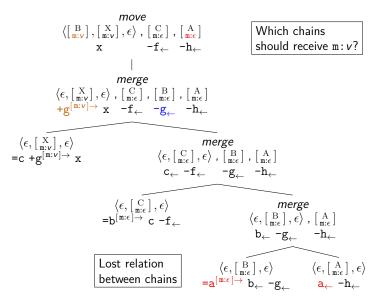
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$$\begin{array}{c} \textit{merge} \\ \langle \epsilon, \left[\begin{smallmatrix} \mathrm{C} \\ \mathrm{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle \text{ , } \left[\begin{smallmatrix} \mathrm{B} \\ \mathrm{m} : \epsilon \end{smallmatrix} \right], \left[\begin{smallmatrix} \mathrm{A} \\ \mathrm{m} : \epsilon \end{smallmatrix} \right] \\ c_{\leftarrow} - f_{\leftarrow} - g_{\leftarrow} - h_{\leftarrow} \\ \end{array} \\ \stackrel{\langle \epsilon, \left[\begin{smallmatrix} \mathrm{C} \\ \mathrm{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle}{>} c_{-} f_{\leftarrow} \\ \stackrel{\langle \epsilon, \left[\begin{smallmatrix} \mathrm{B} \\ \mathrm{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle}{>} c_{-} h_{\leftarrow} \\ \stackrel{\langle \epsilon, \left[\begin{smallmatrix} \mathrm{B} \\ \mathrm{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle}{>} c_{-} h_{\leftarrow} \\ \stackrel{\langle \epsilon, \left[\begin{smallmatrix} \mathrm{B} \\ \mathrm{m} 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$$\begin{array}{c} \textit{merge} \\ \langle \epsilon, \left[\begin{smallmatrix} \mathbf{C} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle \text{ , } \left[\begin{smallmatrix} \mathbf{B} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \left[\begin{smallmatrix} \mathbf{A} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right] \\ \mathbf{c}_{\leftarrow} - \mathbf{f}_{\leftarrow} - \mathbf{g}_{\leftarrow} - \mathbf{h}_{\leftarrow} \\ \end{array} \\ \stackrel{\langle \epsilon, \left[\begin{smallmatrix} \mathbf{C} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle}{\Rightarrow} \mathbf{c} - \mathbf{f}_{\leftarrow} \\ \begin{pmatrix} \epsilon, \left[\begin{smallmatrix} \mathbf{B} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle \\ \mathbf{b}_{\leftarrow} - \mathbf{g}_{\leftarrow} - \mathbf{h}_{\leftarrow} \\ \end{pmatrix} \\ \stackrel{\langle \epsilon, \left[\begin{smallmatrix} \mathbf{B} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle}{\Rightarrow} \mathbf{c} - \mathbf{h}_{\leftarrow} \\ \begin{pmatrix} \epsilon, \left[\begin{smallmatrix} \mathbf{B} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle \\ = \mathbf{a}^{\left[\mathbf{m} : \epsilon \end{smallmatrix} \right] \rightarrow} \mathbf{b}_{\leftarrow} - \mathbf{g}_{\leftarrow} \\ & \mathbf{a}_{\leftarrow} - \mathbf{h}_{\leftarrow} \\ \end{pmatrix}$$

$$\begin{array}{c} \textit{merge} \\ \langle \epsilon, \left[\begin{smallmatrix} \mathbf{X} \\ \mathbf{m} : \mathcal{V} \end{smallmatrix} \right], \epsilon \rangle \text{, } \left[\begin{smallmatrix} \mathbf{B} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \left[\begin{smallmatrix} \mathbf{A} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \left[\begin{smallmatrix} \mathbf{A} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right] \\ + \mathbf{g}^{\left[\mathbf{m} : \mathcal{V} \right] \rightarrow} \mathbf{x} - \mathbf{f}_{\leftarrow} - \mathbf{g}_{\leftarrow} - \mathbf{h}_{\leftarrow} \\ \\ \langle \epsilon, \left[\begin{smallmatrix} \mathbf{X} \\ \mathbf{m} : \mathcal{V} \end{smallmatrix} \right], \epsilon \rangle \\ = \mathbf{c} + \mathbf{g}^{\left[\mathbf{m} : \mathcal{V} \right] \rightarrow} \mathbf{x} \\ & \begin{array}{c} \textit{merge} \\ \langle \epsilon, \left[\begin{smallmatrix} \mathbf{C} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle , \left[\begin{smallmatrix} \mathbf{B} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \left[\begin{smallmatrix} \mathbf{A} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right] \\ \mathbf{c}_{\leftarrow} - \mathbf{f}_{\leftarrow} - \mathbf{g}_{\leftarrow} - \mathbf{h}_{\leftarrow} \\ \\ & \begin{array}{c} \langle \epsilon, \left[\begin{smallmatrix} \mathbf{B} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle \\ = \mathbf{b}^{\left[\mathbf{m} : \epsilon \right] \rightarrow} \mathbf{c} - \mathbf{f}_{\leftarrow} \\ \\ & \begin{array}{c} \langle \epsilon, \left[\begin{smallmatrix} \mathbf{B} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle \\ = \mathbf{a}^{\left[\mathbf{m} : \epsilon \right] \rightarrow} \mathbf{b}_{\leftarrow} - \mathbf{g}_{\leftarrow} & \mathbf{a}_{\leftarrow} - \mathbf{h}_{\leftarrow} \\ \\ & \begin{array}{c} \langle \epsilon, \left[\begin{smallmatrix} \mathbf{A} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \epsilon \rangle \\ = \mathbf{a}^{\left[\mathbf{m} : \epsilon \right] \rightarrow} \mathbf{b}_{\leftarrow} - \mathbf{g}_{\leftarrow} & \mathbf{a}_{\leftarrow} - \mathbf{h}_{\leftarrow} \\ \end{array}$$

```
move
                                                                  \langle \begin{bmatrix} B \\ m \cdot \nu \end{bmatrix}, \begin{bmatrix} X \\ m \cdot \nu \end{bmatrix}, \epsilon \rangle, \begin{bmatrix} C \\ m \cdot \epsilon \end{bmatrix}, \begin{bmatrix} A \\ m \cdot \epsilon \end{bmatrix}
                                                                                                                                           -f_{\leftarrow} -h_{\leftarrow}
                                                                                                                                        merge
                                                         \left<\epsilon, \left[\begin{smallmatrix}X\\ \text{m}:\nu\end{smallmatrix}\right], \epsilon\right> , \left[\begin{smallmatrix}C\\ \text{m}:\epsilon\end{smallmatrix}\right] , \left[\begin{smallmatrix}B\\ \text{m}:\epsilon\end{smallmatrix}\right] , \left[\begin{smallmatrix}A\\ \text{m}:\epsilon\end{smallmatrix}\right]
                                                          +g^{[m:v]} \rightarrow x -f_{\leftarrow} -g_{\leftarrow} -h_{\leftarrow}
         \langle \epsilon, \begin{bmatrix} X \\ m:v \end{bmatrix}, \epsilon \rangle
                                                                                                                                                                                                                                                        merge
                                                                                                                                                                                           \langle \epsilon, \begin{bmatrix} \mathrm{C} \\ \mathsf{m} \cdot \epsilon \end{bmatrix}, \epsilon \rangle , \begin{bmatrix} \mathrm{B} \\ \mathsf{m} \cdot \epsilon \end{bmatrix} , \begin{bmatrix} \mathrm{A} \\ \mathsf{m} \cdot \epsilon \end{bmatrix}
=c +g^{[m:v]} \rightarrow x
                                                                                                                                                                                                     c_{\leftarrow} - f_{\leftarrow} - g_{\leftarrow} - h_{\leftarrow}
                                                                                                                     \langle \epsilon, \begin{bmatrix} C \\ m:\epsilon \end{bmatrix}, \epsilon \rangle
=\mathbf{b}^{[m:\epsilon] \to} \mathbf{c} - \mathbf{f}_{\leftarrow}
                                                                                                                                                                                                                                                                                                                                                         merge
                                                                                                                                                                                                                                                                                                                    \langle \epsilon, \begin{bmatrix} B \\ m \cdot \epsilon \end{bmatrix}, \epsilon \rangle, \begin{bmatrix} A \\ m \cdot \epsilon \end{bmatrix}
                                                                                                                                                                                                                                                                                                                             b_{\leftarrow} -g_{\leftarrow} -h_{\leftarrow}
                                                                                                                                                                                                                                                         \begin{array}{ll} \left\langle \epsilon, \left[ \begin{smallmatrix} \mathbf{B} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \epsilon \right\rangle & \left\langle \epsilon, \left[ \begin{smallmatrix} \mathbf{A} \\ \mathbf{m} : \epsilon \end{smallmatrix} \right], \epsilon \right\rangle \\ = \mathbf{a}^{\left[ \mathbf{m} : \epsilon \right] \rightarrow} \mathbf{b}_{\leftarrow} - \mathbf{g}_{\leftarrow} & \mathbf{a}_{\leftarrow} - \mathbf{h}_{\leftarrow} \end{array}
```



Introduction

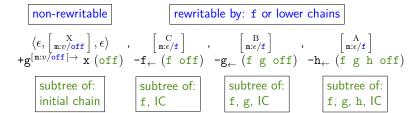
- Long-distance agreement as an unbroken sequence of local exchanges of values
- Where does this sequence end?
 - feature rewritability: the highest chain that can update the feature's value
- What path does it take?
 - chain lineages: hierarchical relations between chains

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```
non-rewritable
           rewritable by: f or lower chains
```

- Long-distance agreement as an unbroken sequence of local exchanges of values
- Where does this sequence end?
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- What path does it take?
 - chain lineages: hierarchical relations between chains



Example grammar revisited

$$\begin{array}{lll} \text{this} & := \left\langle \epsilon, \begin{bmatrix} \frac{\text{THIS}}{\text{num}:\epsilon/\text{on}} \\ \text{per}:\epsilon/\text{on} \\ \text{case}:\epsilon/\text{on} \end{bmatrix}, \; \epsilon \right\rangle \; :: \; = n \overset{[\text{case}:\epsilon/\text{on}] \to}{\leftarrow} \; d \; - k \overset{[\text{num}:\epsilon/\text{on}] \to}{\leftarrow} \; \text{(off)} \\ \\ \text{boy.sg} & := \left\langle \epsilon, \begin{bmatrix} \frac{\text{BOY}}{\text{num}:sg/\text{off}} \\ \text{per}:3/\text{off} \\ \text{case}:\epsilon/\text{on} \end{bmatrix}, \; \epsilon \right\rangle \; :: \; n \overset{[\text{num}:sg/\text{off}] \to}{\leftarrow} \; \text{(off)} \\ \\ \text{walk} & := \left\langle \epsilon, \begin{bmatrix} \frac{\text{BOY}}{\text{valk}} \\ \epsilon, \epsilon & \epsilon & \epsilon & \epsilon \\ \text{on} \end{bmatrix}, \; \epsilon \right\rangle \; :: \; = d \; v \; \text{(off)} \\ \\ \text{prs} & := \left\langle \epsilon, \begin{bmatrix} \frac{\text{PRS}}{\text{num}:\epsilon/\text{on}} \\ \text{per}:\epsilon/\text{on} \end{bmatrix}, \; \epsilon \right\rangle \; :: \; = v \; + k \overset{[\text{case}:nom/\text{off}] \to}{\leftarrow} \; t \; \text{(off)} \\ \\ \end{array}$$

Discussion

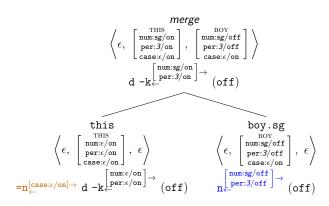
Example grammar revisited

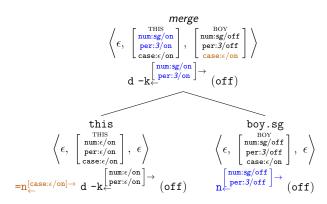
$$\begin{array}{lll} \text{this} & := \left\langle \epsilon, \begin{bmatrix} \frac{\text{THIS}}{\text{num}:\epsilon/\text{on}} \\ \frac{\text{per}:\epsilon/\text{on}}{\text{case}:\epsilon/\text{on}} \end{bmatrix}, \; \epsilon \right\rangle \; :: \; = n_{\leftarrow}^{[\text{case}:\epsilon/\text{on}] \to} \; d \; - k_{\leftarrow}^{\left[\frac{\text{num}:\epsilon/\text{on}}{\text{per}:\epsilon/\text{on}} \right] \to} \; (\text{off}) \\ \\ \text{boy.sg} & := \left\langle \epsilon, \begin{bmatrix} \frac{\text{BOY}}{\text{num}:\text{sg/off}} \\ \frac{\text{per}:3/\text{off}}{\text{case}:\epsilon/\text{on}} \end{bmatrix}, \; \epsilon \right\rangle \; :: \; n_{\leftarrow}^{\left[\frac{\text{num}:\text{sg/off}}{\text{per}:3/\text{off}} \right] \to} \; (\text{off}) \\ \\ \text{walk} & := \left\langle \epsilon, \begin{bmatrix} \frac{\text{PRS}}{\text{num}:\epsilon/\text{on}} \\ \frac{\text{per}:\epsilon/\text{on}}{\text{per}:\epsilon/\text{on}} \end{bmatrix}, \; \epsilon \right\rangle \; :: \; = v \; + k_{\leftarrow}^{\left[\frac{\text{case}:\text{nom/off}}{\text{off}} \right] \to} \; t \; (\text{off}) \\ \\ \\ \text{prs} & := \left\langle \epsilon, \begin{bmatrix} \frac{\text{PRS}}{\text{num}:\epsilon/\text{on}} \\ \frac{\text{per}:\epsilon/\text{on}}{\text{per}:\epsilon/\text{on}} \end{bmatrix}, \; \epsilon \right\rangle \; :: \; = v \; + k_{\leftarrow}^{\left[\frac{\text{case}:\text{nom/off}}{\text{off}} \right] \to} \; t \; (\text{off}) \\ \\ \end{array}$$

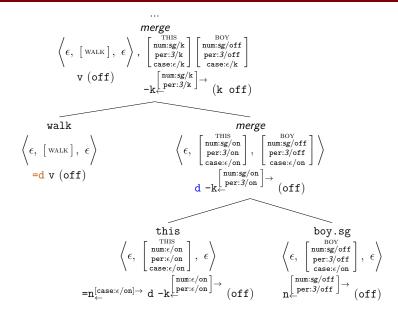
Discussion

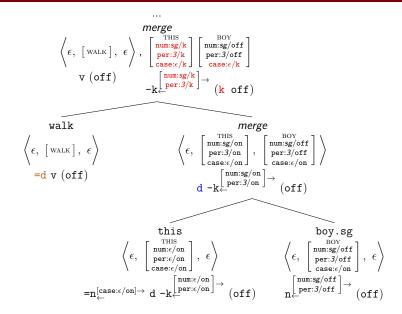
Example grammar revisited

$$\begin{array}{lll} \text{this} & := \left\langle \epsilon, \begin{bmatrix} \frac{\text{THIS}}{\text{num}:\epsilon/\text{on}} \\ \text{per}:\epsilon/\text{on} \\ \text{case}:\epsilon/\text{on} \end{bmatrix}, \; \epsilon \right\rangle \; :: \; = n \overset{[\text{case}:\epsilon/\text{on}] \to}{\leftarrow} \; d \; - k \overset{[\text{num}:\epsilon/\text{on}] \to}{\leftarrow} \\ \text{boy.sg} & := \left\langle \epsilon, \begin{bmatrix} \frac{\text{BOY}}{\text{num}:\text{sg/off}} \\ \text{per}:3/\text{off} \\ \text{case}:\epsilon/\text{on} \end{bmatrix}, \; \epsilon \right\rangle \; :: \; n \overset{[\text{num}:\text{sg/off}] \to}{\leftarrow} \\ \text{walk} & := \left\langle \epsilon, \begin{bmatrix} \text{WALK} \end{bmatrix}, \; \epsilon \right\rangle \; :: \; = d \; v \; (\text{off}) \\ \text{prs} & := \left\langle \epsilon, \begin{bmatrix} \frac{\text{PRS}}{\text{num}:\epsilon/\text{on}} \\ \text{per}:\epsilon/\text{on} \end{bmatrix}, \; \epsilon \right\rangle \; :: \; = v \; + k \overset{[\text{case}:\text{nom/off}] \to}{\leftarrow} \; t \; (\text{off}) \\ \end{array}$$



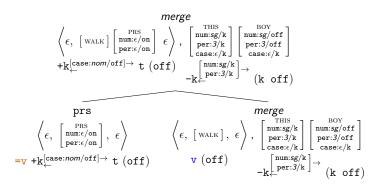


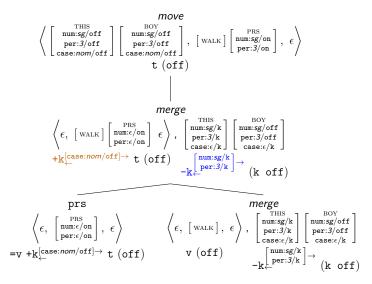


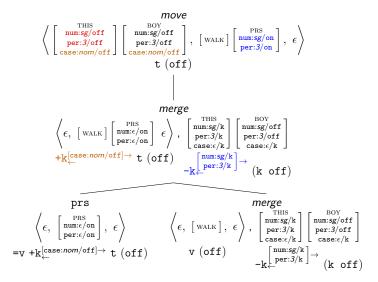


$$\begin{pmatrix} \epsilon, & [\text{WALK}], & \epsilon \end{pmatrix}, \begin{bmatrix} \text{merge} \\ \text{num:sg/k} \\ \text{per:}3/\text{k} \\ \text{case:}\epsilon/\text{k} \end{bmatrix} \begin{bmatrix} \text{num:sg/sf} \\ \text{num:sg/sf} \\ \text{case:}\epsilon/\text{k} \end{bmatrix}$$

$$v \text{ (off)} \qquad \begin{bmatrix} \text{num:sg/k} \\ \text{per:}3/\text{k} \end{bmatrix} \rightarrow \\ -\text{k} \leftarrow \begin{pmatrix} \text{num:sg/k} \\ \text{per:}3/\text{k} \end{pmatrix} \rightarrow \begin{pmatrix} \text{k off} \end{pmatrix}$$







Case study: Icelandic dative intervention

Overview

• Introduction

- Agreement in MGs
- 3 Case study: Icelandic dative intervention
- A Discussion

Icelandic verb agreement

Optional number agreement with the nominative DP:

```
Það finnst mörgum stúdentum tölvurnar
                                                  liótar.
EXPL find.SG many students.DAT computers.DEF.NOM ugly.NOM
Það finnast mörgum stúdentum tölvurnar
                                                  liótar.
EXPL find.PL many students.DAT computers.DEF.NOM ugly.NOM
'Many students find the computers ugly.'
```

Some dative experiencers disrupt agreement:

```
Það finnst fáum börnum tölvurnar
                                              liótar.
EXPL find.SG few children.DAT computers.DEF.NOM ugly.NOM
*Það finnast fáum börnum tölvurnar
                                               liótar.
```

EXPL find.PL few children.DAT computers.DEF.NOM ugly.NOM

^{&#}x27;Few children find the computers ugly.'

Analysis (Kučerová 2016)

 Agreement with the nominative if the dative DP undergoes Object Shift:

```
Pað ??finnst mörgum köttum fljótt mýsnar góðar. EXPL find.SG many cats.DEF.DAT quickly mice.DEF.NOM tasty Pað finnast mörgum köttum fljótt mýsnar góðar. EXPL find.PL many cats.DEF.DAT quickly mice.DEF.NOM tasty 'Many cats quickly find the mice tasty.'
```

Default agreement otherwise:

Pað finnst fljótt mörgum köttum mýsnar góðar.

EXPL find.SG quickly many cats.DEF.DAT mice.DEF.NOM tasty

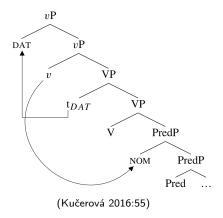
Pað ??finnast fljótt mörgum köttum mýsnar góðar.

EXPL find.PL quickly many cats.DEF.DAT mice.DEF.NOM tasty

'Many cats quickly find the mice tasty.'

Analysis (Kučerová 2016)

- Shiftable vs.
 non-shiftable DPs
- Shifted datives do not block v from probing the nominative
- Non-shiftable datives always block agreement



```
\texttt{many}{\sim} \quad := \left\langle \epsilon, \left[ \begin{smallmatrix} \text{MANY}{\text{num:} \rho l/\text{off}} \end{smallmatrix} \right], \epsilon \right\rangle \; :: \; \mathsf{d}^{[\text{num:} \rho l/\text{off}] \rightarrow} \; * \mathsf{k}^{[\text{num:} sg/\text{off}] \rightarrow}
                                                                                                                                                                                                                                                                                                       (off)
\mathtt{few}{\sim} \quad := \left\langle \epsilon, \left[ \begin{smallmatrix} \mathrm{FEW}{} \sim \\ \mathrm{num} : \mathit{pl}/\mathrm{off} \end{smallmatrix} \right], \epsilon \right\rangle \; :: \; \mathsf{d}^{[\mathrm{num} : \mathit{pl}/\mathrm{off}] \rightarrow} \; - \mathsf{k}^{[\mathrm{num} : \mathit{sg}/\mathrm{off}] \rightarrow} \right.
                                                                                                                                                                                                                                                                                                       (off)
                                           :=\left\langle \epsilon, \left[	ext{FIND}
ight], \epsilon 
ight
angle \, :: \, =\! \mathsf{sc}_{\leftarrow} = \mathsf{d} \,\, \mathsf{V}^{[\mathtt{num}:\epsilon/\mathtt{on}] 
ightarrow}
find
\mathtt{comp}{\sim} \quad := \left\langle \epsilon, \left[ \begin{smallmatrix} \mathtt{COMP}{\sim} \\ \mathtt{num}: \mathit{pl}/\mathtt{off} \end{smallmatrix} \right], \epsilon \right\rangle \; :: \; \mathsf{sc}^{[\mathtt{num}: \mathit{pl}/\mathtt{off}] \rightarrow} \quad (\mathtt{off})
                                      :=\left\langle \epsilon, \left[ \text{AgrO} \right], \epsilon \right
angle \; :: \; => V_{\leftarrow} \; + k_{\leftarrow} \; \mathsf{agrO}^{\left[ \mathsf{num} : \epsilon / \mathsf{on} \right] 	o }
Agr0
                                           :=\left\langle \epsilon, \left[ \mathbf{v} \right], \epsilon \right
angle :: = \mathsf{agr0}_{\leftarrow} \ \mathsf{v}^{[\mathtt{num}:\epsilon/\mathtt{on}] 
ightarrow} \quad (\mathtt{off})
v
                                          :=\left\langle \epsilon, \left[ \begin{smallmatrix} \mathbf{v}_{\mathtt{SHIFT}} \end{smallmatrix} \right], \epsilon 
ight
angle \; :: \; = \mathsf{f agr0}_{\leftarrow} \; + \mathsf{k} \; \mathsf{v}^{[\mathtt{num}:\epsilon/\mathtt{on}] 
ightarrow} \; \; (\mathtt{off})
v_{\text{shift}}
                                            :=\left\langle \epsilon,\left| \begin{smallmatrix} \mathrm{T} \\ \mathrm{num}:\epsilon/\mathrm{on} \end{smallmatrix} \right|,\epsilon \right\rangle ::=>\mathrm{v}_{\leftarrow}\;\mathsf{t} \quad (\mathsf{off})
Τ
```

$$\begin{array}{llll} \operatorname{many} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{MANY} \sim \\ \operatorname{num}: \rho l / \operatorname{off} \end{smallmatrix} \right], \epsilon \right\rangle \; :: \; \operatorname{d}^{[\operatorname{num}: \rho l / \operatorname{off}] \to} * \operatorname{k}^{[\operatorname{num}: sg / \operatorname{off}] \to} & (\operatorname{off}) \end{array}$$

$$few \sim & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{FEW} \sim \\ \operatorname{num}: \rho l / \operatorname{off} \end{smallmatrix} \right], \epsilon \right\rangle \; :: \; \operatorname{d}^{[\operatorname{num}: \rho l / \operatorname{off}] \to} - \operatorname{k}^{[\operatorname{num}: sg / \operatorname{off}] \to} & (\operatorname{off}) \end{array}$$

$$find & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{FIND} \right], \epsilon \right\rangle \; :: \; = \operatorname{sc}_{\leftarrow} = \operatorname{d} \; \operatorname{V}^{[\operatorname{num}: \epsilon / \operatorname{on}] \to} & (\operatorname{off}) \end{array}$$

$$comp \sim & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{COMP} \sim \\ \operatorname{num}: \rho l / \operatorname{off} \end{smallmatrix} \right], \epsilon \right\rangle \; :: \; \operatorname{sc}^{[\operatorname{num}: \rho l / \operatorname{off}] \to} & (\operatorname{off}) \end{array}$$

$$Agr0 & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{AGRO} \right], \epsilon \right\rangle \; :: \; = \operatorname{V}_{\leftarrow} + \operatorname{k}_{\leftarrow} \; \operatorname{agr0}^{[\operatorname{num}: \epsilon / \operatorname{on}] \to} & (\operatorname{off}) \end{array}$$

$$v & := \left\langle \epsilon, \left[v \right], \epsilon \right\rangle \; :: \; = \operatorname{agr0}_{\leftarrow} \; v^{[\operatorname{num}: \epsilon / \operatorname{on}] \to} & (\operatorname{off}) \end{array}$$

$$v_{\operatorname{shift}} & := \left\langle \epsilon, \left[\begin{smallmatrix} v \right], \epsilon \right\rangle \; :: \; = \operatorname{agr0}_{\leftarrow} + \operatorname{k} \; v^{[\operatorname{num}: \epsilon / \operatorname{on}] \to} & (\operatorname{off}) \end{array}$$

$$T & := \left\langle \epsilon, \left[\begin{smallmatrix} T \right], \epsilon \right\rangle \; :: \; = \operatorname{v}_{\leftarrow} \; \operatorname{t} & (\operatorname{off}) \end{array}$$

$$\begin{array}{llll} \operatorname{many} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{MANY} \\ \operatorname{num}: \rho l / \operatorname{off} \end{smallmatrix} \right], \epsilon \right\rangle & :: & \operatorname{d}^{[\operatorname{num}: \rho l / \operatorname{off}] \to} * \operatorname{k}^{[\operatorname{num}: sg / \operatorname{off}] \to} & (\operatorname{off}) \end{array} \\ \operatorname{few} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{FEW} \\ \operatorname{num}: \rho l / \operatorname{off} \end{smallmatrix} \right], \epsilon \right\rangle & :: & \operatorname{d}^{[\operatorname{num}: \rho l / \operatorname{off}] \to} - \operatorname{k}^{[\operatorname{num}: sg / \operatorname{off}] \to} & (\operatorname{off}) \end{array} \\ \operatorname{find} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{FIND} \\ \operatorname{num}: \rho l / \operatorname{off} \end{smallmatrix} \right], \epsilon \right\rangle & :: & \operatorname{sc} = \operatorname{d} V^{[\operatorname{num}: \epsilon / \operatorname{on}] \to} & (\operatorname{off}) \\ \operatorname{comp} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{COMP} \\ \operatorname{num}: \rho l / \operatorname{off} \end{smallmatrix} \right], \epsilon \right\rangle & :: & \operatorname{sc}^{[\operatorname{num}: \rho l / \operatorname{off}] \to} & (\operatorname{off}) \\ \operatorname{Agr0} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{AgrO} \\ \operatorname{s} \right], \epsilon \right\rangle & :: & \operatorname{sv} + \operatorname{k}_{\leftarrow} \operatorname{agr0}^{[\operatorname{num}: \epsilon / \operatorname{on}] \to} & (\operatorname{off}) \\ \operatorname{v} & := \left\langle \epsilon, \left[v \right], \epsilon \right\rangle & :: & \operatorname{sagr0}_{\leftarrow} v^{[\operatorname{num}: \epsilon / \operatorname{on}] \to} & (\operatorname{off}) \\ \operatorname{v}_{\operatorname{shift}} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{V}_{\operatorname{SHiff}} \\ \operatorname{num}: \epsilon / \operatorname{on} \end{array} \right], \epsilon \right\rangle & :: & \operatorname{sv}_{\leftarrow} \operatorname{t} & (\operatorname{off}) \\ \end{array}$$

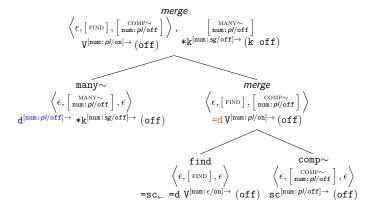
$$\begin{array}{llll} \operatorname{many} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{MANY} \\ \operatorname{num}: \rho l / \operatorname{off} \end{smallmatrix} \right], \epsilon \right\rangle & :: & \operatorname{d}^{[\operatorname{num}: \rho l / \operatorname{off}] \to} * \operatorname{k}^{[\operatorname{num}: sg / \operatorname{off}] \to} & (\operatorname{off}) \end{array} \\ \operatorname{few} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{FEW} \\ \operatorname{num}: \rho l / \operatorname{off} \end{smallmatrix} \right], \epsilon \right\rangle & :: & \operatorname{d}^{[\operatorname{num}: \rho l / \operatorname{off}] \to} - \operatorname{k}^{[\operatorname{num}: sg / \operatorname{off}] \to} & (\operatorname{off}) \end{array} \\ \operatorname{find} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{FIND} \\ \operatorname{num}: \rho l / \operatorname{off} \end{smallmatrix} \right], \epsilon \right\rangle & :: & \operatorname{sc} = \operatorname{d} \operatorname{V}^{[\operatorname{num}: \epsilon / \operatorname{on}] \to} & (\operatorname{off}) \\ \operatorname{comp} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{COMP} \\ \operatorname{num}: \rho l / \operatorname{off} \end{smallmatrix} \right], \epsilon \right\rangle & :: & \operatorname{sc}^{[\operatorname{num}: \rho l / \operatorname{off}] \to} & (\operatorname{off}) \\ \operatorname{Agr0} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{AGRO} \\ \operatorname{I} \right], \epsilon \right\rangle & :: & \operatorname{sv} - \operatorname{k}_{\leftarrow} & \operatorname{agr0}^{[\operatorname{num}: \epsilon / \operatorname{on}] \to} & (\operatorname{off}) \\ \operatorname{v} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{V} \right], \epsilon \right\rangle & :: & \operatorname{sagr0}_{\leftarrow} & \operatorname{v}^{[\operatorname{num}: \epsilon / \operatorname{on}] \to} & (\operatorname{off}) \\ \operatorname{T} & := \left\langle \epsilon, \left[\begin{smallmatrix} \operatorname{T} \\ \operatorname{num}: \epsilon / \operatorname{on} \right], \epsilon \right\rangle & :: & \operatorname{sv} - \operatorname{t} & (\operatorname{off}) \\ \end{array} \right.$$

Introduction

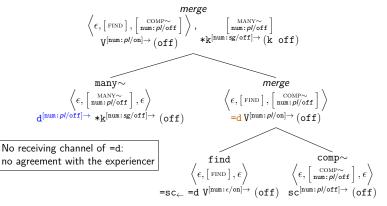
$\begin{array}{c} \textit{merge} \\ \left\langle \epsilon, \left[\begin{smallmatrix} \text{FIND} \end{smallmatrix}\right], \left[\begin{smallmatrix} \text{COMP} \sim \\ \text{num}: \textit{pl} / \text{off} \end{smallmatrix}\right] \right\rangle \\ = & \text{d} \, V^{\left[\text{num}: \textit{pl} / \text{on}\right] \rightarrow \left(\text{off}\right)} \\ \left\langle \epsilon, \left[\begin{smallmatrix} \text{FIND} \end{smallmatrix}\right], \epsilon \right\rangle & \left\langle \epsilon, \left[\begin{smallmatrix} \text{COMP} \sim \\ \text{num}: \textit{pl} / \text{off} \end{smallmatrix}\right], \epsilon \right\rangle \\ = & \text{sc}_{\leftarrow} = & \text{d} \, V^{\left[\text{num}: \epsilon / \text{on}\right] \rightarrow \left(\text{off}\right)} & \text{sc}^{\left[\text{num}: \textit{pl} / \text{off}\right] \rightarrow \left(\text{off}\right)} \end{array}$

Introduction

$\begin{array}{c} \textit{merge} \\ \left\langle \epsilon, \left\lceil \begin{smallmatrix} \text{FIND} \right\rceil, \left\lceil \begin{smallmatrix} \text{COMP} \sim \\ \text{num} : \textit{pl/off} \end{smallmatrix} \right\rceil \right\rangle \\ = & \text{d} \, V^{\left[\text{num} : \textit{pl/on}\right] \rightarrow} \left(\text{off} \right) \\ \\ \left\langle \epsilon, \left\lceil \begin{smallmatrix} \text{FIND} \right\rceil, \epsilon \right\rangle & \left\langle \epsilon, \left\lceil \begin{smallmatrix} \text{COMP} \sim \\ \text{num} : \textit{pl/off} \end{smallmatrix} \right\rceil, \epsilon \right\rangle \\ = & \text{sc.} = & \text{d} \, V^{\left[\text{num} : \epsilon / \text{on}\right] \rightarrow} \left(\text{off} \right) \\ \text{sc.} & \text{num} : \textit{pl/off} \right] \rightarrow \left(\text{off} \right) \end{array}$



Introduction



```
 \begin{cases} \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right], \left[ \substack{\text{COMP} \sim \\ \text{num}: pl/\text{off}} \right] \end{cases} \rangle, \left[ \substack{\text{num}: pl/\text{off} \\ \text{num}: pl/\text{off}} \right] \\ +k_{\leftarrow} \underbrace{\text{agrO}[\text{num}: pl/\text{on}] \rightarrow \left( \text{off} \right)}_{} *k^{\left[ \text{num}: \text{sg/off} \right] \rightarrow \left( k \text{ off} \right)} 
                                  =>V_{\leftarrow} +k_{\leftarrow} \operatorname{agr0}^{[\operatorname{num}:\epsilon/\operatorname{on}]\rightarrow} (\operatorname{off})
                                                                                                                                                                                                              \left<\epsilon, \left[\begin{smallmatrix} 	ext{FIND} \end{smallmatrix}\right], \left[\begin{smallmatrix} 	ext{COMP} \sim \ 	ext{num}: m{p} / 	ext{off} \end{smallmatrix}\right] \right>
                                                                                                                               \left\langle \epsilon, \left[ \begin{smallmatrix} \text{MANY} \sim \\ \text{num:} \ pl/\text{off} \end{smallmatrix} \right], \epsilon \right\rangle
                                                                                                   d^{[\text{num}:pl/off}] \rightarrow *k^{[\text{num}:sg/off}] \rightarrow (off)
                                                                                                                                                                                                                                                                     =dV^{[num:pl/on]} \rightarrow (off)
                                                                                                                                                                                                                                   \begin{cases} \text{find} & \text{comp -} \\ \left\langle \epsilon, \left[ \text{FIND} \right], \epsilon \right\rangle & \left\langle \epsilon, \left[ \frac{\text{comp -}}{\text{num:} \rho l / \text{off}} \right], \epsilon \right\rangle \end{cases} 
                                                                                                                                                                                                          =sc_{-}=dV^{[num:\epsilon/on]\rightarrow}(off)sc^{[num:pl/off]\rightarrow}(off)
```

Example derivation

```
 \begin{cases} \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right], \left[ \substack{\text{COMP} \sim \\ \text{num}: pl/\text{off}} \right] \end{cases} \rangle, \left[ \substack{\text{num}: pl/\text{off} \\ \text{num}: pl/\text{off}} \right] \\ +k_{\leftarrow} \underbrace{\text{agrO} \left[ \text{num}: pl/\text{on} \right] \rightarrow \left( \text{off} \right)}_{} *k^{\left[ \text{num}: \text{sg/off} \right] \rightarrow \left( k \text{ off} \right)} 
                                  =>V_{\leftarrow} +k_{\leftarrow} \operatorname{agr0}^{[\operatorname{num}:\epsilon/\operatorname{on}]\rightarrow} (\operatorname{off})
                                                                                                                                                                                                                \left<\epsilon, \left[\begin{smallmatrix} 	ext{FIND} \end{smallmatrix}\right], \left[\begin{smallmatrix} 	ext{COMP} \sim \ 	ext{num}: m{p} / 	ext{off} \end{smallmatrix}\right] \right>
                                                                                                                                \left\langle \epsilon, \left[ \begin{smallmatrix} \text{MANY} \sim \\ \text{num:} \ pl/\text{off} \end{smallmatrix} \right], \epsilon \right\rangle
                                                                                                    d^{[\text{num}:pl/off}] \rightarrow *k^{[\text{num}:sg/off}] \rightarrow (off)
                                                                                                                                                                                                                                                                        =dV^{[num:pl/on]} \rightarrow (off)
                                                                                                                                                                                                                                      \begin{cases} \text{find} & \text{comp -} \\ \left\langle \epsilon, \left[ \text{FIND} \right], \epsilon \right\rangle & \left\langle \epsilon, \left[ \frac{\text{comp -}}{\text{num:} \rho l / \text{off}} \right], \epsilon \right\rangle \end{cases} 
                                                                                                                                                                                                            =sc_{-}=dV^{[num:\epsilon/on]\rightarrow}(off)sc^{[num:pl/off]\rightarrow}(off)
```

```
 \begin{array}{c} \textit{move} \\ \left \langle \left[ \begin{smallmatrix} \text{MANY} \sim \\ \text{num:} \textit{pl/off} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \text{FIND} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \text{AGRO} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \text{COMP} \sim \\ \text{num:} \textit{pl/off} \end{smallmatrix} \right] \right \rangle \\ \text{agr0} \begin{bmatrix} \text{Inum:} \textit{sg/on} \end{smallmatrix} \rightarrow \left( \texttt{off} \right) \\ \left | \qquad \qquad \qquad \qquad \qquad \\ \textit{merge} \\ \left \langle \epsilon, \left[ \begin{smallmatrix} \text{FIND} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \text{AGRO} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \text{COMP} \sim \\ \text{num:} \textit{pl/off} \end{smallmatrix} \right] \right \rangle, \left[ \begin{smallmatrix} \text{MANY} \sim \\ \text{num:} \textit{pl/off} \end{smallmatrix} \right] \\ + \mathbf{k}_{\leftarrow} \text{agr0} \begin{bmatrix} \text{num:} \textit{pl/on} \end{smallmatrix} \rightarrow \left( \texttt{off} \right) \\ * \mathbf{k} \begin{bmatrix} \text{num:} \textit{sg/off} \end{smallmatrix} \right] \rightarrow \left( \mathbf{k} \text{ off} \right) \\ \end{array}
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Discussion

Default agreement: shiftable and non-shiftable DPs

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 \begin{array}{c} \textit{move} \\ \left \langle \begin{bmatrix} \text{\tiny MANY} \sim \\ \text{\tiny num:} \rho l / \text{\scriptsize off} \end{bmatrix}, \begin{bmatrix} \text{\tiny FIND} \end{bmatrix} \begin{bmatrix} \text{\tiny AGRO} \end{bmatrix}, \begin{bmatrix} \text{\tiny COMP} \sim \\ \text{\tiny num:} \rho l / \text{\scriptsize off} \end{bmatrix} \right \rangle \\ \text{\tiny agr0} \begin{bmatrix} \text{\tiny num:} s g / \text{\scriptsize on} \end{bmatrix} \rightarrow (\text{\tiny off}) \\ \\ \textit{\tiny merge} \\ \left \langle \epsilon, \begin{bmatrix} \text{\tiny FIND} \end{bmatrix} \begin{bmatrix} \text{\tiny AGRO} \end{bmatrix}, \begin{bmatrix} \text{\tiny COMP} \sim \\ \text{\tiny num:} \rho l / \text{\scriptsize off} \end{bmatrix} \right \rangle, \begin{bmatrix} \text{\tiny MANY} \sim \\ \text{\tiny num:} \rho l / \text{\scriptsize off} \end{bmatrix} \\ + \mathbf{k}_{\leftarrow} \text{\tiny agr0} \begin{bmatrix} \text{\tiny num:} \rho l / \text{\scriptsize on} \end{bmatrix} \rightarrow (\text{\tiny off}) \\ \\ *\mathbf{k}^{[\text{\tiny num:}} s g / \text{\tiny off}] \rightarrow (\mathbf{k} \text{\tiny off}) \\ \end{array}
```

Last channel wins: agreement via *move*

```
\left\langle \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right] \left[ \text{V} \right], \left[ \begin{array}{c} \text{MANY} \sim \\ \text{num:} \, pl/\text{off} \end{array} \right] \left[ \begin{array}{c} \text{COMP} \sim \\ \text{num:} \, pl/\text{off} \end{array} \right] \right\rangle
                                                                                                       v^{[num:sg/on]\rightarrow}(off)
 \left\langle \epsilon, \left[ \begin{smallmatrix} \mathbf{v} \end{smallmatrix} \right], \epsilon \right\rangle \qquad \qquad \left\langle \begin{bmatrix} \begin{smallmatrix} \mathbf{MANY} \sim \\ \mathbf{num}: \mathit{pl}/\mathsf{off} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \mathbf{FIND} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \mathbf{AGRO} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \mathbf{COMP} \sim \\ \mathbf{num}: \mathit{pl}/\mathsf{off} \end{smallmatrix} \right] \right\rangle 
                                                                                                                                                                                                                  agr0^{[num:sg/on]\rightarrow}(off)
                                                                                                             \left\langle \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right], \left[ \substack{\text{COMP} \sim \\ \text{num}: pl/\text{off}} \right] \right\rangle, \left[ \substack{\text{MANY} \sim \\ \text{num}: pl/\text{off}} \right] \\ +k_{\leftarrow} \operatorname{agr0} \left[ \substack{\text{num}: pl/\text{on} \rightarrow \\ \text{off}} \right] \wedge \left( \text{off} \right) \\ *k^{\left[ \text{num}: sg/\text{off} \right] \rightarrow} \left( k \text{ off} \right)
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\left\langle \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right] \left[ \text{V} \right], \left[ \begin{array}{c} \text{MANY} \sim \\ \text{num:} \, pl/\text{off} \end{array} \right] \left[ \begin{array}{c} \text{COMP} \sim \\ \text{num:} \, pl/\text{off} \end{array} \right] \right\rangle
                                                                                                       v^{[num:sg/on]\rightarrow}(off)
 \left\langle \epsilon, \left[ \begin{smallmatrix} \mathbf{v} \end{smallmatrix} \right], \epsilon \right\rangle \qquad \qquad \left\langle \begin{bmatrix} \begin{smallmatrix} \mathbf{MANY} \sim \\ \mathbf{num}: \mathit{pl}/\mathsf{off} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \mathbf{FIND} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \mathbf{AGRO} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \mathbf{COMP} \sim \\ \mathbf{num}: \mathit{pl}/\mathsf{off} \end{smallmatrix} \right] \right\rangle 
                                                                                                                                                                                                                agr0^{[num:sg/on]\rightarrow}(off)
                                                                                                            \left\langle \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right], \left[ \substack{\text{COMP} \sim \\ \text{num}: pl/\text{off}} \right] \right\rangle, \left[ \substack{\text{MANY} \sim \\ \text{num}: pl/\text{off}} \right] \\ +k_{\leftarrow} \operatorname{agr0} \left[ \substack{\text{num}: pl/\text{on} \rightarrow \\ \text{off}} \right] \wedge \left( \text{off} \right) \\ *k^{\left[ \text{num}: sg/\text{off} \right] \rightarrow} \left( k \text{ off} \right)
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Discussion

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\left\langle \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right] \left[ \text{V} \right] \left[ \begin{array}{c} \text{T} \\ \text{num:sg/on} \end{array} \right], \left[ \begin{array}{c} \text{MANY} \sim \\ \text{num:pl/off} \end{array} \right] \left[ \begin{array}{c} \text{COMP} \sim \\ \text{num:pl/off} \end{array} \right] \right\rangle
                                                                                                                                                             t(off)
  \left\langle \epsilon, \left[ \begin{smallmatrix} \mathbf{T} \\ \mathbf{num} : \epsilon / \mathbf{on} \end{smallmatrix} \right], \epsilon \right\rangle \qquad \left\langle \epsilon, \left[ \begin{smallmatrix} \mathbf{FIND} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \mathbf{AGRO} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \mathbf{V} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \mathbf{MANY} \sim \\ \mathbf{num} : \rho l / \mathbf{off} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \mathbf{COMP} \sim \\ \mathbf{num} : \rho l / \mathbf{off} \end{smallmatrix} \right] \right\rangle
                                                                                                                                                                                                                                                      v^{[num:sg/on]\rightarrow}(off)
                                                                                                                                                  \left\langle \epsilon, \left[ \begin{smallmatrix} \mathbf{v} \end{smallmatrix} \right], \epsilon \right\rangle \qquad \left\langle \left[ \begin{smallmatrix} \mathbf{MANY} \sim \\ \mathbf{num} : \mathit{pl}/\mathsf{off} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \mathbf{FIND} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \mathbf{AGRO} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \mathbf{COMP} \sim \\ \mathbf{num} : \mathit{pl}/\mathsf{off} \end{smallmatrix} \right] \right\rangle 
                                                                                             => agr0_{\sim} v^{[num:\epsilon/on]\rightarrow} (off)
                                                                                                                                                                                                                                                                                                                                                                 agr0^{[num:sg/on]\rightarrow}(off)
                                                                                                                                                                                                                                                            \left\langle \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right], \left[ \substack{\text{COMP} \sim \\ \text{num}: pl/\text{off}} \right] \right\rangle, \left[ \substack{\text{MANY} \sim \\ \text{num}: pl/\text{off}} \right] \\ +k_{\leftarrow} \operatorname{agr0} \left[ \substack{\text{num}: pl/\text{on} \rightarrow \\ \text{off}} \right] \wedge \left( \text{off} \right) \\ *k^{\left[ \text{num}: sg/\text{off} \right] \rightarrow} \left( k \text{ off} \right)
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\left\langle \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right] \left[ \text{V} \right] \left[ \begin{array}{c} \text{T} \\ \text{num: sg/on} \end{array} \right], \left[ \begin{array}{c} \text{MANY} \sim \\ \text{num: pl/off} \end{array} \right] \left[ \begin{array}{c} \text{COMP} \sim \\ \text{num: pl/off} \end{array} \right] \right\rangle
                                                                                                                                                              t(off)
  \left\langle \epsilon, \left[ \begin{smallmatrix} \mathbf{T} \\ \mathbf{num} : \epsilon / \mathbf{on} \end{smallmatrix} \right], \epsilon \right\rangle \qquad \left\langle \epsilon, \left[ \begin{smallmatrix} \mathbf{FIND} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \mathbf{AGRO} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \mathbf{V} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \mathbf{MANY} \sim \\ \mathbf{num} : \rho l / \mathbf{off} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \mathbf{COMP} \sim \\ \mathbf{num} : \rho l / \mathbf{off} \end{smallmatrix} \right] \right\rangle
                                                                                                                                                                                                                                                       v^{[num:sg/on]\rightarrow}(off)
                                                                                                                                                  \left\langle \epsilon, \left[ \begin{smallmatrix} \mathbf{v} \end{smallmatrix} \right], \epsilon \right\rangle \qquad \left\langle \left[ \begin{smallmatrix} \mathbf{MANY} \sim \\ \mathbf{num} : \mathit{pl}/\mathsf{off} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \mathbf{FIND} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \mathbf{AGRO} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \mathbf{COMP} \sim \\ \mathbf{num} : \mathit{pl}/\mathsf{off} \end{smallmatrix} \right] \right\rangle 
                                                                                             => agr0_{\sim} v^{[num:\epsilon/on]\rightarrow} (off)
                                                                                                                                                                                                                                                                                                                                                                  agr0^{[num:sg/on]\rightarrow}(off)
                                                                                                                                                                                                                                                            \left\langle \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right], \left[ \substack{\text{COMP} \sim \\ \text{num}: pl/\text{off}} \right] \right\rangle, \left[ \substack{\text{MANY} \sim \\ \text{num}: pl/\text{off}} \right] \\ +k_{\leftarrow} \operatorname{agr0} \left[ \substack{\text{num}: pl/\text{on} \rightarrow \\ \text{off}} \right] \wedge \left( \text{off} \right) \\ *k^{\left[ \text{num}: sg/\text{off} \right] \rightarrow} \left( k \text{ off} \right)
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Introduction

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 \begin{array}{c} \textit{move} \\ \left<\epsilon, \left\lceil \begin{smallmatrix} \text{FIND} \end{smallmatrix} \right\rceil \left\lceil \begin{smallmatrix} \text{AGRO} \end{smallmatrix} \right], \left\lceil \begin{smallmatrix} \text{COMP} \sim \\ \text{num}: \textit{pl} / \text{onf} \end{smallmatrix} \right] \right>, \quad \left\lceil \begin{smallmatrix} \text{MANY} \sim \\ \text{num}: \textit{pl} / \text{onf} \end{smallmatrix} \right] \\ = \text{agr0} \\ \boxed{\text{num}: \textit{pl} / \text{on} } \rightarrow \left( \text{off} \right) \\ \text{merge} \\ \left<\epsilon, \left\lceil \begin{smallmatrix} \text{FIND} \end{smallmatrix} \right] \left\lceil \begin{smallmatrix} \text{AGRO} \end{smallmatrix} \right], \left\lceil \begin{smallmatrix} \text{COMP} \sim \\ \text{num}: \textit{pl} / \text{off} \end{smallmatrix} \right] \right>, \quad \left\lceil \begin{smallmatrix} \text{MANY} \sim \\ \text{num}: \textit{pl} / \text{off} \end{smallmatrix} \right] \\ + \texttt{k}_{\leftarrow} \text{ agr0} \\ \boxed{\text{num}: \textit{pl} / \text{onf}} \rightarrow \left( \text{off} \right) \\ \end{array}
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Intermediate position:
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 \begin{array}{c} \textit{move} \\ \left\langle \epsilon, \left[ \text{\tiny FIND} \right] \left[ \text{\tiny AGRO} \right], \left[ \begin{array}{c} \text{\tiny COMP\sim} \\ \text{\tiny num:} pl/\text{\tiny off} \end{array} \right] \right\rangle, \quad \begin{bmatrix} \text{\tiny num:} pl/\text{\tiny off} \end{bmatrix} \\ \text{\tiny agr0} \\ \text{\tiny [num:} pl/\text{\tiny onl}) \rightarrow (\text{\tiny off}) \\ \text{\tiny merge} \\ \\ \left\langle \epsilon, \left[ \text{\tiny FIND} \right] \left[ \text{\tiny AGRO} \right], \left[ \begin{array}{c} \text{\tiny COMP\sim} \\ \text{\tiny num:} pl/\text{\tiny off} \end{array} \right] \right\rangle, \quad \begin{bmatrix} \text{\tiny MANY\sim} \\ \text{\tiny num:} pl/\text{\tiny off} \end{bmatrix} \\ +\mathbf{k}_{\leftarrow} \text{\tiny agr0} \\ \text{\tiny [num:} pl/\text{\tiny onl}) \rightarrow (\text{\tiny off}) \\ \end{array} \right. \\ *k_{\leftarrow} \text{\tiny [num:} pl/\text{\tiny onl}) \rightarrow (\text{\tiny off}) \\ \end{array}
```

```
 \begin{array}{c} \textit{merge} \\ & \left< \epsilon, \left\lceil \begin{smallmatrix} \text{FIND} \right\rceil \left\lceil A \text{GRO} \right\rceil \left\lceil \begin{smallmatrix} \text{V}_{\text{SHFT}} \right\rceil, \left\lceil \begin{smallmatrix} \text{COMP} \sim \\ \text{num}: \textit{pl/off} \end{array} \right] \right>, \quad \left\lceil \begin{smallmatrix} \text{MANY} \sim \\ \text{num}: \textit{pl/off} \right\rceil \right> \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \right> \\ & *k \left\lceil \begin{smallmatrix} \text{Num} : \textit{sg/off} \right\rceil \rightarrow \left( k \text{ off} \right) \\ & \left< \epsilon, \left\lceil \begin{smallmatrix} \text{V}_{\text{SHFT}} \right\rceil, \epsilon \right> \\ & \left< \epsilon, \left\lceil \begin{smallmatrix} \text{FIND} \right\rceil \left\lceil A \text{GRO} \right\rceil, \left\lceil \begin{smallmatrix} \text{COMP} \sim \\ \text{num}: \textit{pl/off} \right\rceil \right> \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{MANY} \sim \\ \text{num}: \textit{pl/off} \right\rceil \\ & +k_{\leftarrow} \text{ agr0} \left\lceil \begin{smallmatrix} \text{num}: \textit{pl/off} \right\rceil \right> \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left\lceil \begin{smallmatrix} \text{num} : \textit{pl/off} \right\rceil \\ & *k \left
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 \left\langle \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right] \left[ \text{V}_{\text{SHIFT}} \right], \left[ \substack{\text{COMP} \sim \\ \text{num}: \rho / / \text{off}} \right] \right\rangle, \left[ \substack{\text{MANY} \sim \\ \text{num}: \rho / / \text{off}} \right] \\ + k \, v^{\left[ \text{num}: \rho / / \text{on} \right] \rightarrow} \left( \text{off} \right) \\ * k^{\left[ \text{num}: sg / \text{off} \right] \rightarrow} \left( k \text{ off} \right) 
                   = agr0. +k v^{[\text{num}:\epsilon/\text{on}]} \rightarrow (\text{off})
```

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\left\langle \left[ \begin{smallmatrix} \text{MANY} \sim \\ \text{num:} \textit{pl}/\text{off} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \text{FIND} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \text{AGRO} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \text{V}_{\text{SHIFT}} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \text{COMP} \sim \\ \text{num:} \textit{pl}/\text{off} \end{smallmatrix} \right] \right\rangle
                                                                                                                                                                v^{[num:pl/on]\rightarrow}(off)
                                           \left\langle \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right] \left[ \text{V}_{\text{SHIFT}} \right], \left[ \substack{\text{COMP} \sim \\ \text{num}: \textit{pl}/\text{off}} \right] \right\rangle, \left[ \substack{\text{MANY} \sim \\ \text{num}: \textit{pl}/\text{off}} \right] \\ + k \, v^{\left[ \text{num}: \textit{pl}/\text{on} \right] \rightarrow} \left( \text{off} \right) \\ * k^{\left[ \text{num}: \textit{sg}/\text{off} \right] \rightarrow} \left( k \, \, \text{off} \right) 
 \begin{cases} \epsilon, \left[ \text{ FIND } \right] \left[ \text{ AGRO } \right], \left[ \substack{\text{COMP} \sim \\ \text{num:} \rho / \text{off }} \right] \end{cases} \rangle, \quad \begin{bmatrix} \text{MANY} \sim \\ \text{num:} \rho / \text{off } \end{bmatrix} \\ +k_{\leftarrow} \operatorname{agrO} \left[ \substack{\text{num:} \rho / \text{onl} \rightarrow \\ \text{off }} \right] \\ *k^{\left[ \text{num:} \text{sg/off } \right] \rightarrow} \left( k \text{ off } \right) \end{cases}
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Object Shift:
no receiving channel on +k; \left\langle \left[\begin{smallmatrix} \text{MANY} \sim \\ \text{num}: \rho l/\text{off} \end{smallmatrix}\right], \left[\begin{smallmatrix} \text{FIND} \end{smallmatrix}\right] \left[\begin{smallmatrix} \text{AGRO} \end{smallmatrix}\right] \left[\begin{smallmatrix} \text{V_{SHIFT}} \end{smallmatrix}\right], \left[\begin{smallmatrix} \text{COMP} \sim \\ \text{num}: \rho l/\text{off} \end{smallmatrix}\right] \right\rangle
no agreement
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 v^{[num:pl/on]} \rightarrow (off)
                                                                                                                                                                                                                                                                                                                                                                                                                                       \left\langle \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right] \left[ \text{V}_{\text{SHFT}} \right], \left[ \substack{\text{COMP} \sim \\ \text{num}: \rho / / \text{off}} \right] \right\rangle, \left[ \substack{\text{MANY} \sim \\ \text{num}: \rho / / \text{off}} \right] 
+ k v^{[\text{num}: \rho / / \text{on}] \rightarrow} (\text{off}) 
*k^{[\text{num}: \text{sg/off}] \rightarrow} (k \text{ off}) 
                                                                                                                                                                                                                                                                                                      \begin{array}{c} \text{``sill} \\ \left<\epsilon, \left[\,\text{Vsupt}\,\right], \epsilon\right> \\ => \text{agr0}\_+k \ v^{[\text{num}:\epsilon/\text{onl}] \rightarrow} \ (\text{off}) \end{array} \right. \\ \left<\epsilon, \left[\,\text{FIND}\,\right] \left[\,\text{AgrO}\,\right], \left[\,\text{CoMPeo}\\ \text{num}: \rho l/\text{onf}\,\right] \right> \\ => \text{agr0}\_+k \ v^{[\text{num}:\epsilon/\text{onl}] \rightarrow} \ (\text{off}) \end{array} \right. \\ \left<\epsilon, \left[\,\text{FIND}\,\right] \left[\,\text{AgrO}\,\right], \left[\,\text{CoMPeo}\\ \text{num}: \rho l/\text{off}\,\right] \right> \\ \left<\epsilon, \left[\,\text{Vsupt}\,\right], \left[\,\text{MANYe}\\ \text{num}: \rho l/\text{off}\,\right] \right> \\ \left<\epsilon, \left[\,\text{Supt}\,\right], \left[\,\text{CoMPeo}\\ \text{num}: \rho l/\text{off}\,\right] \right> \\ \left<\epsilon, \left[\,\text{Num}: \rho l/\text{off}\,\right] \right> \\ \left<\epsilon, \left[\,\text{Num}: \rho l/\text{off}\,\right] \right> \\ \left<\epsilon, \left[\,\text{Supt}\,\right], \left[\,\text{CoMPeo}\\ \text{num}: \rho l/\text{off}\,\right] \right> \\ \left<\epsilon, \left[\,\text{Num}: \rho l/\text{off}\,\right] \right> \\ \left<\epsilon, \left[\,\text{
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          \begin{cases} \epsilon, \left[ \text{ FIND } \right] \left[ \text{ AGRO } \right], \left[ \substack{\text{COMP} \sim \\ \text{num }: \rho l / \text{off }} \right] \end{cases} \rangle, \quad \begin{bmatrix} \text{num }: \rho l / \text{off } \right] \\ +k_{\leftarrow} \text{ agr0} \left[ \text{num }: \rho l / \text{onl} \right] + \left( \text{off} \right) \end{cases} \\ *k^{\left[ \text{num }: \text{sg} / \text{off } \right]} \rightarrow \left( k \text{ off } \right) \end{cases}
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\left\langle \epsilon, \left[ \text{ FIND } \right] \left[ \text{ AGRO } \right] \left[ \text{ V}_{\text{SHIFT }} \right] \left[ \text{ T}_{\text{num:} p//\text{on}} \right], \left[ \text{ MANY}_{\text{num:} pl/\text{off }} \right] \left[ \text{ COMP}_{\text{num:} pl/\text{off }} \right] \right\rangle
                                                                                                                                                                                                                t(off)
                                                                                                                                                                                                                               \left\langle \left[\begin{smallmatrix} \text{MANY} \sim \\ \text{num}: \textit{pl}/\text{off} \end{smallmatrix}\right], \left[\begin{smallmatrix} \text{FIND} \end{smallmatrix}\right] \left[\begin{smallmatrix} \text{AGRO} \end{smallmatrix}\right] \left[\begin{smallmatrix} \text{V}_{\text{SHIFT}} \end{smallmatrix}\right], \left[\begin{smallmatrix} \text{COMP} \sim \\ \text{num}: \textit{pl}/\text{off} \end{smallmatrix}\right] \right\rangle
\left\langle \epsilon, \begin{bmatrix} \mathbf{T} \\ \mathtt{num} : \epsilon / \mathtt{on} \end{bmatrix}, \epsilon \right\rangle
                                                                                                                                                                                                                                                                                                                                                         v^{[num:pl/on]} \rightarrow (off)
                                                                                                                                                                                               \left\langle \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right] \left[ \text{V}_{\text{SHIFT}} \right], \left[ \substack{\text{COMP} \sim \\ \text{num}: \rho | / \text{off}} \right] \right\rangle, \left[ \substack{\text{MANY} \sim \\ \text{num}: \rho | / \text{off}} \right] 
+k \, v^{[\text{num}: \rho | / \text{on}] \rightarrow} \left( \text{off} \right) 
*k^{[\text{num}: sg / \text{off}] \rightarrow} \left( k \text{ off} \right) 
                                                                                                                                                                                                     \begin{array}{c} \left\langle \epsilon, \left[ \mathbf{v}_{\mathsf{SHIFT}} \right], \epsilon \right\rangle & \left\langle \epsilon, \left[ \mathbf{FIND} \right] \left[ \mathbf{AGRO} \right], \left[ \mathbf{num} : \rho / \mathsf{off} \right] \right\rangle, & \left[ \mathbf{num} : \rho / \mathsf{off} \right] \\ + \mathbf{k} \ \mathbf{v}^{[\mathsf{num}} : \epsilon / \mathsf{on}] \rightarrow & \left( \mathsf{off} \right) & \mathsf{agrO}^{[\mathsf{num}} : \rho / \mathsf{on}] \rightarrow & \left( \mathsf{off} \right) & *\mathbf{k}^{[\mathsf{num}} : \mathsf{sg} / \mathsf{off}] \rightarrow & \left( \mathsf{k} \ \mathsf{off} \right) \\ \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                        \begin{cases} \epsilon, \left[ \text{ FIND } \right] \left[ \text{ AGRO } \right], \left[ \substack{\text{COMP} \sim \\ \text{num }: \rho l / \text{off }} \right] \end{cases} \rangle, \quad \begin{bmatrix} \text{num }: \rho l / \text{off } \right] \\ +k_{\leftarrow} \text{ agr0} \begin{bmatrix} \text{num }: \rho l / \text{onf } \right) \end{cases} \\ *k^{\left[ \text{num }: \text{sg/off } \right] } \cdot \left( \text{k off } \right) \end{cases}
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\left\langle \epsilon, \left[\text{FIND}\right] \left[\text{AGRO}\right] \left[\text{V}_{\text{SHIFT}}\right] \left[ \begin{array}{c} \text{T} \\ \text{num}: pl/\text{on} \end{array} \right], \left[ \begin{array}{c} \text{MANY} \sim \\ \text{num}: pl/\text{off} \end{array} \right] \left[ \begin{array}{c} \text{COMP} \sim \\ \text{num}: pl/\text{off} \end{array} \right] \right\rangle
                                                                                                                                                                                                                       t(off)
                                                                                                                                                                                                                                      \left\langle \left[ \begin{smallmatrix} \text{MANY} \sim \\ \text{num:} \textit{pl/off} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \text{FIND} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \text{AGRO} \end{smallmatrix} \right] \left[ \begin{smallmatrix} \text{V}_{\text{SHIFT}} \end{smallmatrix} \right], \left[ \begin{smallmatrix} \text{COMP} \sim \\ \text{num:} \textit{pl/off} \end{smallmatrix} \right] \right\rangle
\left\langle \epsilon, \begin{bmatrix} \mathbf{T} \\ \mathtt{num} : \epsilon / \mathtt{on} \end{bmatrix}, \epsilon \right\rangle
                                                                                                                                                                                                                                                                                                                                                                    v^{[num:pl/on]} \rightarrow (off)
                                                                                                                                                                                                     \left\langle \epsilon, \left[ \text{FIND} \right] \left[ \text{AGRO} \right] \left[ \text{V}_{\text{SHIFT}} \right], \left[ \substack{\text{COMP} \sim \\ \text{num}: \rho | / \text{off}} \right] \right\rangle, \left[ \substack{\text{MANY} \sim \\ \text{num}: \rho | / \text{off}} \right] 
+k \, v^{[\text{num}: \rho | / \text{on}] \rightarrow} \left( \text{off} \right) 
*k^{[\text{num}: sg / \text{off}] \rightarrow} \left( k \text{ off} \right) 
                                                                                                                                                                                                            \begin{array}{c} \left\langle \epsilon, \left[ \mathbf{v}_{\mathsf{SHIFT}} \right], \epsilon \right\rangle & \left\langle \epsilon, \left[ \mathbf{FIND} \right] \left[ \mathbf{AGRO} \right], \left[ \mathbf{num} : \rho / \mathsf{off} \right] \right\rangle, & \left[ \mathbf{num} : \rho / \mathsf{off} \right] \\ + \mathbf{k} \ \mathbf{v}^{[\mathsf{num}} : \epsilon / \mathsf{on}] \rightarrow & \left( \mathsf{off} \right) & \mathsf{agrO}^{[\mathsf{num}} : \rho / \mathsf{on}] \rightarrow & \left( \mathsf{off} \right) & *\mathbf{k}^{[\mathsf{num}} : \mathsf{sg} / \mathsf{off}] \rightarrow & \left( \mathsf{k} \ \mathsf{off} \right) \\ \end{array} 
                                                                                                                                                                                                                                                                                                                                                                                                    \begin{cases} \epsilon, \left[ \text{ FIND } \right] \left[ \text{ AGRO } \right], \left[ \substack{\text{COMP} \sim \\ \text{num }: \rho l / \text{off }} \right] \end{cases} \rangle, \quad \begin{bmatrix} \text{num }: \rho l / \text{off } \right] \\ +k_{\leftarrow} \text{ agr0} \begin{bmatrix} \text{num }: \rho l / \text{onf } \right) \end{cases} \\ *k^{\left[ \text{num }: \text{sg/off } \right] } \cdot \left( \text{k off } \right) \end{cases}
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Overview

Introduction

• Introduction

- Agreement in MGs
- 3 Case study: Icelandic dative intervention
- A Discussion

Discusssion

Discussion

Results:

- Modified MG formalism operating over bundles of morphological features
- Proof of concept: a straightforwardly expressed analysis of Icelandic dative intervention

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Ermolaeva & Edmiston 2017: Distributed Morphology over sequences of feature structures

Thank you for your attention!



$$\begin{array}{c} \textit{merge} \\ \langle \epsilon, \left[\begin{smallmatrix} B \\ m: \epsilon / on \end{smallmatrix} \right], \epsilon \rangle \text{ , } \left[\begin{smallmatrix} A \\ m: \epsilon / on \end{smallmatrix} \right] \\ b_{\leftarrow} - g_{\leftarrow} \text{ (off) } - h_{\leftarrow} \text{ (h off)} \\ \\ \langle \epsilon, \left[\begin{smallmatrix} B \\ m: \epsilon / on \end{smallmatrix} \right], \epsilon \rangle \\ =_{\mathbf{a}} \left[\begin{smallmatrix} A \\ m: \epsilon / on \end{smallmatrix} \right] \rightarrow b_{\leftarrow} - g_{\leftarrow} \text{ (off)} \\ a_{\leftarrow} - h_{\leftarrow} \text{ (off)} \end{array}$$

Case study: Icelandic dative intervention

$$\begin{array}{c} \textit{merge} \\ \langle \epsilon, \begin{bmatrix} B \\ m:\epsilon/on \end{bmatrix}, \epsilon \rangle \;, \quad \begin{bmatrix} A \\ m:\epsilon/on \end{bmatrix} \\ b_{\leftarrow} - g_{\leftarrow} \; (off) \quad -h_{\leftarrow} \; (\textbf{h} \; off) \\ \hline \langle \epsilon, \begin{bmatrix} B \\ m:\epsilon/on \end{bmatrix}, \epsilon \rangle & \langle \epsilon, \begin{bmatrix} A \\ m:\epsilon/on \end{bmatrix}, \epsilon \rangle \\ =_{\textbf{a}} \begin{bmatrix} m:\epsilon/on \end{bmatrix} \rightarrow b_{\leftarrow} - g_{\leftarrow} \; (off) & \textbf{a}_{\leftarrow} - h_{\leftarrow} \; (off) \end{array}$$

Case study: Icelandic dative intervention

Introduction

Appendix: long-distance agreement

$$\begin{array}{c} (\epsilon, \left[\begin{smallmatrix} \mathbf{C} \\ \mathbf{m} : \epsilon / \mathbf{on} \end{smallmatrix}\right], \epsilon) & \text{merge} \\ \mathbf{C}_{\leftarrow} - \mathbf{f}_{\leftarrow} \left(\mathbf{off} \right) & -\mathbf{g}_{\leftarrow} \left(\mathbf{g} \ \mathbf{off} \right) & -\mathbf{h}_{\leftarrow} \left(\mathbf{g} \ \mathbf{h} \ \mathbf{off} \right) \\ \mathbf{c}_{\leftarrow} - \mathbf{f}_{\leftarrow} \left(\mathbf{off} \right) & -\mathbf{g}_{\leftarrow} \left(\mathbf{g} \ \mathbf{off} \right) & -\mathbf{h}_{\leftarrow} \left(\mathbf{g} \ \mathbf{h} \ \mathbf{off} \right) \\ & \mathbf{merge} \\ \mathbf{e}_{\mathbf{b}} & \mathbf{merge} \\ \mathbf{e}_{\mathbf{b}} & \mathbf{e}_{\mathbf{c}} & -\mathbf{f}_{\leftarrow} \left(\mathbf{off} \right) \\ \mathbf{e}_{\leftarrow} - \mathbf{g}_{\leftarrow} \left(\mathbf{off} \right) & -\mathbf{h}_{\leftarrow} \left(\mathbf{h} \ \mathbf{off} \right) \\ \mathbf{e}_{\leftarrow} - \mathbf{g}_{\leftarrow} \left(\mathbf{off} \right) & -\mathbf{h}_{\leftarrow} \left(\mathbf{h} \ \mathbf{off} \right) \\ \mathbf{e}_{\mathbf{c}} & \mathbf{e}_{\mathbf{c}} & \mathbf{e}_{\mathbf{c}} & \mathbf{e}_{\mathbf{c}} \\ \mathbf{e}_{\mathbf{c}} & \mathbf{e}_{\mathbf{c}} & \mathbf{e}_{\mathbf{c}} \\ \mathbf{e}_{\mathbf{c}} & \mathbf{e}_{\mathbf{$$

Case study: Icelandic dative intervention

Introduction

$$\begin{array}{c} (\epsilon, \left[\begin{smallmatrix} \mathbf{C} \\ \mathbf{m} : \epsilon / \mathbf{on} \end{smallmatrix}\right], \epsilon) & \text{merge} \\ \mathbf{C}_{\leftarrow} - \mathbf{f}_{\leftarrow} \left(\mathbf{off} \right) & -\mathbf{g}_{\leftarrow} \left(\mathbf{g} \ \mathbf{off} \right) & -\mathbf{h}_{\leftarrow} \left(\mathbf{g} \ \mathbf{h} \ \mathbf{off} \right) \\ \mathbf{c}_{\leftarrow} - \mathbf{f}_{\leftarrow} \left(\mathbf{off} \right) & -\mathbf{g}_{\leftarrow} \left(\mathbf{g} \ \mathbf{off} \right) & -\mathbf{h}_{\leftarrow} \left(\mathbf{g} \ \mathbf{h} \ \mathbf{off} \right) \\ & \mathbf{merge} \\ \mathbf{e}_{\mathbf{b}} & \mathbf{merge} \\ \mathbf{e}_{\mathbf{b}} & \mathbf{e}_{\mathbf{c}} & -\mathbf{f}_{\leftarrow} \left(\mathbf{off} \right) \\ \mathbf{e}_{\leftarrow} - \mathbf{g}_{\leftarrow} \left(\mathbf{off} \right) & -\mathbf{h}_{\leftarrow} \left(\mathbf{h} \ \mathbf{off} \right) \\ \mathbf{e}_{\leftarrow} - \mathbf{g}_{\leftarrow} \left(\mathbf{off} \right) & -\mathbf{h}_{\leftarrow} \left(\mathbf{h} \ \mathbf{off} \right) \\ \mathbf{e}_{\mathbf{c}} & \mathbf{e}_{\mathbf{c}} & \mathbf{e}_{\mathbf{c}} & \mathbf{e}_{\mathbf{c}} \\ \mathbf{e}_{\mathbf{c}} & \mathbf{e}_{\mathbf{c}} & \mathbf{e}_{\mathbf{c}} \\ \mathbf{e}_{\mathbf{c}} & \mathbf{e}_{\mathbf{$$

$$\begin{array}{c} \textit{merge} \\ \langle \varepsilon, \left[\begin{smallmatrix} X \\ \mathtt{m:v/off} \end{smallmatrix} \right], \varepsilon \rangle &, \left[\begin{smallmatrix} C \\ \mathtt{m:e/f} \end{smallmatrix} \right] &, \left[\begin{smallmatrix} A \\ \mathtt{m:e/f} \end{smallmatrix} \right] \\ + g^{[\mathtt{m:v/off}]} \to x \ (off) & -f_{\leftarrow} \ (f \ off) & -g_{\leftarrow} \ (f \ g \ off) & -h_{\leftarrow} \ (f \ g \ h \ off) \\ \hline \\ \langle \varepsilon, \left[\begin{smallmatrix} X \\ \mathtt{m:v/off} \end{smallmatrix} \right], \varepsilon \rangle & \textit{merge} \\ = c & +g^{[\mathtt{m:v/off}]} \to x \ (off) & \langle \varepsilon, \left[\begin{smallmatrix} C \\ \mathtt{m:e/on} \end{smallmatrix} \right], \varepsilon \rangle &, \left[\begin{smallmatrix} A \\ \mathtt{m:e/on} \end{smallmatrix} \right] &, \left[\begin{smallmatrix} A \\ \mathtt{m:e/on} \end{smallmatrix} \right] \\ c_{\leftarrow} & -f_{\leftarrow} \ (off) & -g_{\leftarrow} \ (g \ off) & -h_{\leftarrow} \ (g \ h \ off) \\ \hline \\ \langle \varepsilon, \left[\begin{smallmatrix} A \\ \mathtt{m:e/on} \end{smallmatrix} \right], \varepsilon \rangle & \textit{merge} \\ = b^{[\mathtt{m:e/on}]} \to c & -f_{\leftarrow} \ (off) & \langle \varepsilon, \left[\begin{smallmatrix} A \\ \mathtt{m:e/on} \end{smallmatrix} \right], \varepsilon \rangle &, \left[\begin{smallmatrix} A \\ \mathtt{m:e/on} \end{smallmatrix} \right] \\ b_{\leftarrow} & -g_{\leftarrow} \ (off) & -h_{\leftarrow} \ (h \ off) \\ \hline \\ = a^{[\mathtt{m:e/on}]} \to b_{\leftarrow} & -g_{\leftarrow} \ (off) & a_{\leftarrow} & -h_{\leftarrow} \ (off) \\ \hline \end{array}$$