taking the equation and plugging in the hint:

$$\begin{aligned}
f(t+\Delta t, x) - f(t-\Delta t, x) &= -\frac{v\Delta t}{\Delta x}[f(t, x+\Delta x) - f(t, x-\Delta x)] \\
\varepsilon^{t+\Delta t} e^{ikx} - \varepsilon^{t-\Delta t} e^{ikx} &= -\frac{v\Delta t}{\Delta x}[\varepsilon^t e^{ik(x+\Delta x)} - \varepsilon^t e^{ik(x-\Delta x)}] \\
\varepsilon^t e^{ikx} \varepsilon^{-\Delta t}(\varepsilon^{2\Delta t} - 1) &= -\frac{v\Delta t}{\Delta x}\varepsilon^t e^{ikx}(e^{ik\Delta x} - e^{-ik\Delta x}) \\
\left(\varepsilon^{\Delta t}\right)^2 - 1 &= -\frac{v\Delta t}{\Delta x}\varepsilon^{\Delta t}(2i\sin(k\Delta x)) \\
0 &= \left(\varepsilon^{\Delta t}\right)^2 + 2i\frac{v\Delta t}{\Delta x}\sin(k\Delta x) - 1 \\
\implies \varepsilon^{\Delta t} &= -i\frac{v\Delta t}{\Delta x}\sin(k\Delta x) \pm \sqrt{1 - \left(\frac{v\Delta t}{\Delta x}\sin(k\Delta x)\right)^2}
\end{aligned}$$

which shows us that $v\Delta t \leq \Delta t$ for $|\varepsilon| \leq 1$. And more importantly there is no dissipation term which means that we do not expect ε to shrink as time goes on!