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J.P.Morgan

Designing robust trend-following system

Behind the scenes of trend-following

Trend-following has actively been on investors' radar for the last few decades. The J.P. Morgan primer on momentum strategies (Kolanovic and Wei, 2015) provides an extensive review of the momentum strategies. The current paper focuses on a concrete trend-following solution and analyzes its properties alongside the practical implementation.

More concretely, the goal of the current paper is threefold:

- First, a trend-following signal based on statistical theory is proposed and we analytically analyze its properties. We reconcile the theoretical results with stylized facts about trend-following investing – 'CTA smile', the link to straddles and the better performance of so-called 'slower' signals.
- Second, based on the theoretical results we propose a prototype trend-following solution that uses a unified approach across assets and diversifies across time-frames. Its performance versus benchmarks and diversification properties for long-only portfolios are highlighted within simulation examples.
- Third, we elaborate on the portfolio and risk management of the trend-following strategy. We illustrate how the risk-budgeting and the Hierarchical Risk Parity (HRP) approaches can be adapted to the trend-following framework. Various methods to manage the transaction costs aspects of the strategy have also been discussed. In particular, we show how to limit the downside in the case of trendless market and how to take into account the implications of the carry component in many futures and FX forwards.

Global Quantitative and Derivatives Strategy

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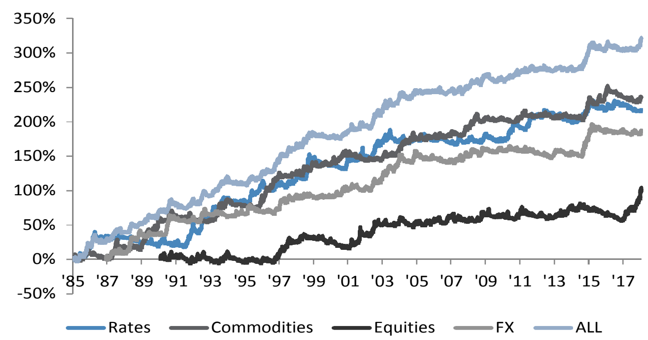
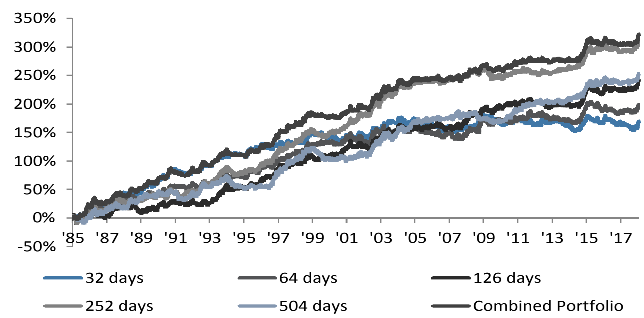
Cumulative Performance by Asset Class



Source: J.P. Morgan Quantitative and Derivatives Strategy

Cumulative performance of signals based on various lookback periods



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Performance Statistics by Asset Class

	Commodities	Equities	Rates	FX	Combination All Asset Classes
Annualized Return	6.82%	3.39%	6.23%	5.74%	9.27%
Annualized Volatility	9.47%	9.73%	9.39%	8.77%	9.04%
Sharpe	0.72	0.35	0.66	0.65	1.03
Max Drawdown	-20.93%	-23.45%	-21.38%	-16.37%	-13.60%

Source: J.P. Morgan Quantitative and Derivatives Strategy

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Introduction

Trend-following (also referred to in academic circles as time-series momentum[1]) has actively been on investors' radar for the last few decades. The longevity of the strategy and the appealing performance in the midst of the crisis of 2008 have helped to propel the assets managed by CTAs to more than $348bln[2]. The intuitive investment philosophy (well summarized by the 18th century British economist and trader David Ricardo as "Cut your losses, let your winners run") indisputably has its own merits as well.

The investor's interest has spurred a large amount of research into the reasons why the (time-series) momentum phenomenon arises and into attempts to design strategies that improve upon the benchmarks. The J.P. Morgan primer on momentum strategies (Kolanovic and Wei, 2015) provides an extensive review of the momentum strategies and proposes a framework for their design. The current paper focuses a concrete trend-following solution and analyzes its analytical properties alongside its practical implementation. The majority of the research on trend-following has been of an empirical nature and in our opinion there has been a relative lack of theoretical research linking the empirically observed characteristics of the strategy to theoretical results with a model framework. To some extent we try to fill this void with the current paper.

The goal of the current paper is threefold. First, we construct a trend-following signal that is rooted in statistical theory and analytically analyze its properties. We manage to reconcile the theoretical results with stylized facts about trend-following investing – the presence of a 'CTA smile' (see for example Hurst et. al. 2014) and the tendency of signals based on longer-term horizons (slow signals) to outperform (see Baltas and Kosowski 2013). Second, leveraging on the theoretical results we proposed a prototype trend-following solution that is diversified across time-frames and assets and uses a unified approach across assets. Third, we discuss the portfolio and risk management of the trend-following strategy. We illustrate how the risk-budgeting and the Hierarchical Risk Parity approaches can be adapted to the trend-following framework. Various approaches to manage the transaction costs aspects of the strategy have also been discussed.

We start by presenting a signal that is based on statistical hypothesis testing. We show that under certain conditions the trend-following signal is the also the delta of a straddle. Hence we make explicit the widely propagated link between trend-following and long straddle positions (see for example Fung and Hsieh 2011).

Subsequently, we analyze the profit drivers for the trend-following strategy based on the proposed signal. We show that the strategy (similarly to a straddle) is profitable whenever there are trends in either direction. Hence we demonstrate that the so-called "CTA smile" (see for example Hurst et. al. 2014) can be justified within a theoretical model as well. Furthermore, the strategy exhibits convexity. The absolute value of the Sharpe ratio of the underlying asset is of critical importance for the profitability of the strategy and the higher the number, the bigger the convexity embedded in the strategy. Furthermore, signals based on longer estimation periods possess ceteris paribus better profitability than signals based on shorter lookback periods.

Next, the time-series properties of the underlying asset are explicitly taken into account. We show that the autocorrelation is important only for the profitability of signals based on short lookback periods (typically less than a month). Naturally positive autocorrelation leads to profits while even small values of negative autocorrelation induce substantial losses. On the other hand the profitability of the signals based on longer lookback periods is unaffected by the time-series properties of the underlying.

The impact of transaction costs is also explicitly modelled. Results show that transaction costs are increasing with the bid-ask spread but decreasing with the volatility and the lookback period.

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[1] See Tobias, M., Ooi, Y. and Pedersen, L. "Time Series Momentum". Journal of Financial Economics 104 (2012): 228–250
[2] Estimate by BarclayHedge in the 3rd Quarter of 2017 (https://www.barclayhedge.com/research/indices/cta/Money_Under_Management.html).

In addition, the correlation between the P&L of the signals based on different lookback periods is derived and it is shown to depend on the ratio of the lookback periods. The theoretical values of the correlations are shown to closely match the empirical observations. It is demonstrated that averaging the signals across various timeframes is optimal if an appropriate correlation structure between P&Ls of signals is present. While averaging the signals among different lookback windows has been a common practice, certain conditions have to be present for its optimality.

Based on the theoretical results we propose a prototype trend-following solution that uses a unified methodology across asset and asset classes. The solution is diversified across various time-frameworks. The performance prototype trend-following solution is compared to benchmark indices under various fee structures scenarios. The diversification and hedging properties of trend-following with respect to long only portfolios are also demonstrated in simulations.

Recent innovations in portfolio management have been applied in the specific setting of the proposed trend-following algorithm. In particular, the inverse volatility approach is compared to a risk-budgeting approach. Furthermore, the Hierarchical Risk Parity approach (see the recent J.P. Morgan publications Cross Asset Portfolios of Tradable Risk Premia Indices and Post-Modern Portfolio Construction) has been also tailored to fit within the proposed trend-following framework.

Last but not the least we focus on cost control. We aim to incorporate short-signals that provide quicker reaction at inflection points in a cost-efficient way. We discuss the impact of 'carry' and show how our framework allows for incorporation of the carry component in the strategy design.

What statistics, trend-following and options have in common

From 101 statistical hypothesis testing to trend-following signals

A simple and intuitive measure of a trend is the average asset's return over a certain period. If it is positive, we can conclude the asset is trending upwards. Conversely, if it is negative, the asset is trending downwards. The more sizable the average return in absolute value, the higher our conviction for the presence of a trend.

Of course, every estimate entails some uncertainty which is typically linked to volatility and statistical theory comes in handy in quantifying that uncertainty. Let's denote the average return over period T at time t as $\overline{R_{t,T}}$ and the estimated volatility as $\widehat{\sigma_t}$. Under the standard assumption that R_t is i.i.d. $N(0,\sigma^2)$ it is well-known that $tstat_{t,T} = \frac{\sqrt{T}\,\overline{R_{t,T}}}{\widehat{\sigma_t}}$ has a Student's *t*-distribution with *T-1* degrees of freedom[3]. When the sample size increases, the t-distribution approaches the standard normal one (typically that happens for $T > 30$ as will be the case in most of our subsequent work).

We can easily construct statistical tests using the estimated t-statistic. Given our interest in the asset's trend-following behavior, we can for example use a one-sample test and test whether the average return μ is greater than zero when the estimate $\overline{R_{t,T}}$ turns out positive:

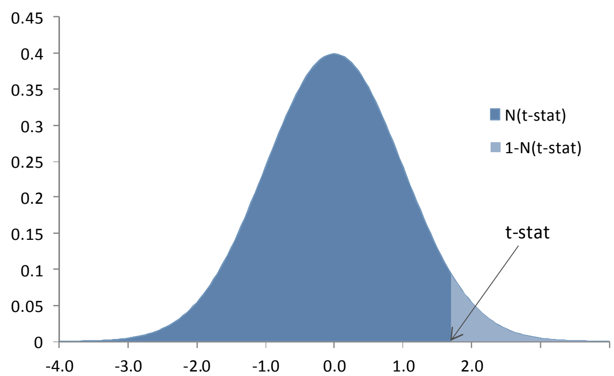
$$H0\colon \mu = 0 \text{ versus } H1\colon \mu > 0$$

The decision whether to accept or reject $H0$ at a certain confidence level is based on comparison of the calculated t-value to a critical value depending on the chosen confidence level. Hence, we will reject H0 when $1 - \mathrm{N}\left(tstat_{t,T}\right)$ is below the required confidence level, where N stands for the standard normal c.d.f. In general, the smaller $1 - \mathrm{N}\left(tstat_{t,T}\right)$ the higher is our confidence that $\mu > 0$. As $tstat_{t,T} > 0$, $\left(1 - N\left(tstat_{t,T}\right)\right)\epsilon[0,1/2]$. In case we want construct a trend-following signal

[3] The assumption is equivalent to Geometric Brownian Motion in continuous time as in the Black-Scholes world. Later we relax the assumption and assume an AR(1) process for the asset's returns. Note that in the case of an AR(1) process, the t-statistic becomes $\frac{\sqrt{T}\,\overline{R_{t,T}}}{\widehat{\sigma_t}} * \sqrt{\frac{1+\rho}{1-\rho}}$, where ρ is the autocorrelation coefficient (see van Belle (2002)). In the case of daily return data, the absolute value of the autocorrelation is small in magnitude and hence the value of the t-statistic is not significantly impacted.

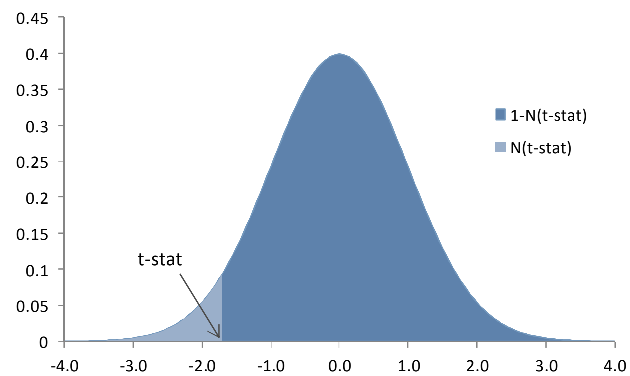
ranging from 0 to 1 (with higher signal denoting stronger trend-following behaviour), we can show that the linear combination $2 * N(tstat_{t,T}) - 1$ achieves that goal[4].

Figure 1: Testing for a positive mean



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 2: Testing for a negative mean



Source: J.P. Morgan Quantitative and Derivatives Strategy

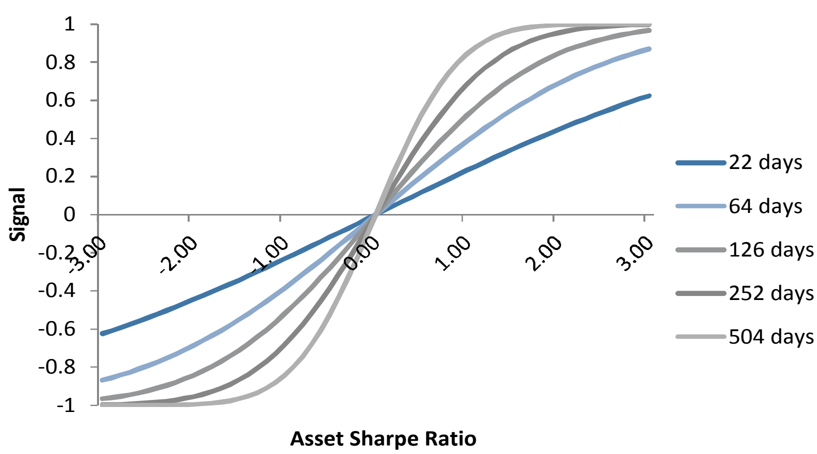
Similarly we consider the case when the estimated average return is negative. Then of interest is the hypothesis

$$H0: \mu = 0 \text{ versus } H1: \mu < 0$$

In this case, the smaller is $N(tstat_{t,T})$ the greater the confidence with which we can reject $H0$. As before we want to map $N(tstat_{t,T}) \epsilon \left[0, \frac{1}{2}\right]$ to a signal ranging from $[-1, 0]$. Again, the linear transformation that achieves this goal is $2 * N(tstat_{t,T}) - 1$.

In the end, we can construct a trend-following signal of the same form irrespective of the sign of the estimated average. Our trend-following signal rooted in statistical hypothesis testing will have the form: $2 * N(tstat_{t,T}) - 1$.

Figure 3: Signal values for various windows and asset's Sharpe ratios



Source: J.P. Morgan Quantitative and Derivatives Strategy

[4] That can be easily derived by solving a system of equations to find the linear combination that satisfies the desired mapping.

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What trend-following and options have in common

The P&L profile of trend-following has always been thought to resemble the P&L of a straddle. Trend-following benefits from sizable moves in either direction of the asset price and also tends to exhibit positive convexity. For example, Fung and Hsieh (2001) used lookback straddles to replicate the track record of actual trend-followers.

Below we make an explicit link between our trend-following signal and typical option strategies.

In the Black-Scholes world, the delta of a straddle is given by$2 * N(d1_t) - 1$. Let's assume that the strike of the option is set to the price T days ago and the maturity of the options is T. Under the assumption of zero interest rates, $d1_t = \frac{\ln\left(\frac{S_t}{S_{t-T}}\right)+\frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}$. Using the assumptions of the Geometric Brownian Motion and introducing $\varepsilon_t \sim N.I.D(0,1)$, we can write $d1_t = \frac{\sum_{s=t-T+1}^{t}\ln\left(\frac{S_s}{S_{s-1}}\right)+\frac{\sigma^2 T}{2}}{\sigma\sqrt{T}} = \frac{\sum_{s=t-T+1}^{t}(\mu+\sigma\varepsilon_s)}{\sigma\sqrt{T}} = \frac{\sum_{s=t-T+1}^{t} R_s}{\sigma\sqrt{T}} = \frac{\sqrt{T}\,\overline{R_{t,T}}}{\sigma}$. If we plug in an estimate of the volatility σ, we arrive at $d1_t = tstat_{t,T}$.

Hence, the delta of a straddle with appropriately chosen strike and maturity can also be viewed as a trend-following signal.

Below we compare the signal based on the T-statistic to some other commonly used trend-following signals. For example, the typical the Z-score measure ignores the uncertainty in mean estimation and hence it is not robust on theoretical grounds. It is also not acceptable from risk-management considerations as the signal (and hence the position) can become quite sizable (even with a negligible probability).

Table 1: Comparison Trend-Following Signals

	Construction Mechanism	Advantages/Drawbacks
Binary Signal	The signal is 1 when the average return over a particular lookback period is positive and -1 if it is negative.	• Intuitive long/short logic and easy calculation. • No consideration of the strength of the trend and its uncertainty.
Z-score Measure based	The signal is proportional to MA/EWMA return over a certain lookback period. In some cases an adjustment by volatility is made and a Z-score is arrived.	• Ignores the empirical fact that outsized returns are less probable. • The strength of the trend is taken into account but not its uncertainty.
Signal Based on the T-stat (delta-straddle signal)	The signal is equivalent to a statistical test whether the mean return of an asset is either positive or negative. It can also be interpreted as the delta of a straddle with specific input parameters.	• Theoretically robust as it takes into account the strength of the signal and its uncertainty. • Involves the c.d.f. of the standard normal distribution which makes theoretical calculations more evolved.

Source: J.P. Morgan Quantitative and Derivatives Strategy

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Profit drivers of 'delta-straddle' trend-following signals

The profit generation mechanism of trend-following has not been well comprehended beyond the general statement that 'trend-following is profitable when there are strong trends'. In the sections below we analyze the interactions between the Sharpe ratio of the asset and the Sharpe ratio of the trend-following and demonstrate analytically that trend-following exhibits a straddle-like P&L profile. We also derive expressions for the expected transaction costs and elaborate on the implications for the trade-off between having a reactive trend-following system and keeping a lid on the costs.

Gross P&L

In the appendix we derive the relationship between the gross P&L of the trend-following strategy and its lookback window, Sharpe ratio and autocorrelation properties. We deviate from the assumptions of the Black-Scholes world and assume an Autoregressive Process of order 1 (AR1) for the asset's returns.

Assume that returns follow an AR(1) model: $R_t = a + \rho R_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim N\left(0,\ \sigma_\varepsilon^2\right)$ and $|\rho| < 1$. It follows that $R_t \sim N\left(\frac{a}{1-\rho}, \frac{\sigma_\varepsilon^2}{1-\rho^2}\right) \sim N\left(\mu, \sigma^2\right)$. The expected gross P&L for a 'straddle' signal based on a lookback T is:

$$E(PL_{t,T}) = 2\frac{\mu}{\sigma}\Phi\left(\frac{\mu_{d1,T}}{\sqrt{1+\sigma_{d1,T}^2}}\right) - \frac{\mu}{\sigma} + 2\phi\frac{\sigma_{d1,T}}{\sqrt{1+\sigma_{d1,T}^2}} f\left(\frac{\mu_{d1,T}}{\sqrt{1+\sigma_{d1,T}^2}}\right)$$

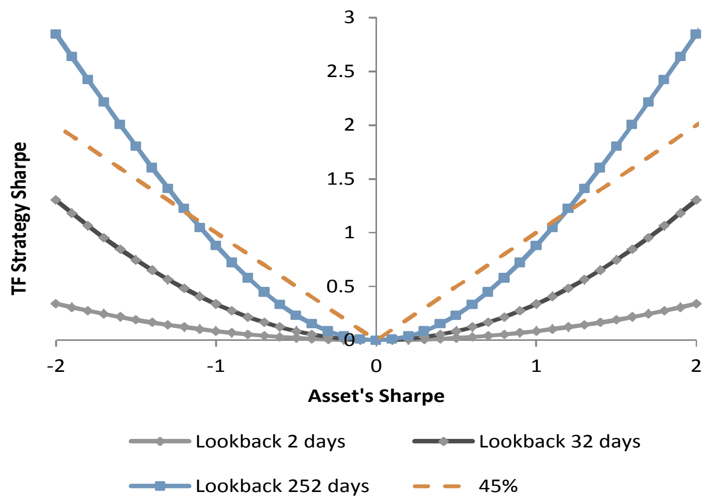
where $\mu_{d1,T}$, $\sigma_{d1,T}$ and ϕ are functions of μ, σ^2, ρ and T, Φ stands for the standard normal c.d.f and f for the standard normal p.d.f.

In case $\rho = 0$ (Black-Scholes assumption), it follows that $E(PL_{t+1,T}) = \frac{\mu}{\sigma}\left(2\Phi\left(\frac{\mu}{\sigma}\frac{\sqrt{T}}{\sqrt{2}}\right) - 1\right)$.

Similarly, if $\mu = 0$, we obtain that $E(PL_{t+1,T}) = \frac{2\rho\left(1-\rho^T\right)}{\sqrt{2\pi}\sqrt{2T(1-\rho)-2\rho\left(1-\rho^T\right)}}$.

In Figure 4 we have shown the profile of the gross P&L for various lookbacks and Sharpe ratios of the underlying asset for the case $\rho = 0$. We can notice that the P&L exhibits the typical straddle P&L payoff (once we abstract from costs). Both positive and negative drifts (positive and negative values of μ) generate positive profit. Furthermore, what is important for the profitability of the strategy is the Sharpe ratio of the asset (μ/σ). We can also notice the built-in convexity in the strategy. The Sharpe ratio of the trend-following increases faster than the increase in the absolute value of the Sharpe ratio of the underlying. When the lookback period is relatively large and the Sharpe ratio of the asset is sizable (above 1 in absolute value), the Sharpe ratio of the trend-following strategy exceeds the corresponding Sharpe ratio of the underlying. This is quite desirable, especially during periods of intense market sell-offs, when the Sharpe ratio of the underlying is sizably negative. A subtle implication of this result is that if the Sharpe ratio of an asset is stable and below 1, an investor might be better off holding the asset rather pursuing a trend-following strategy.

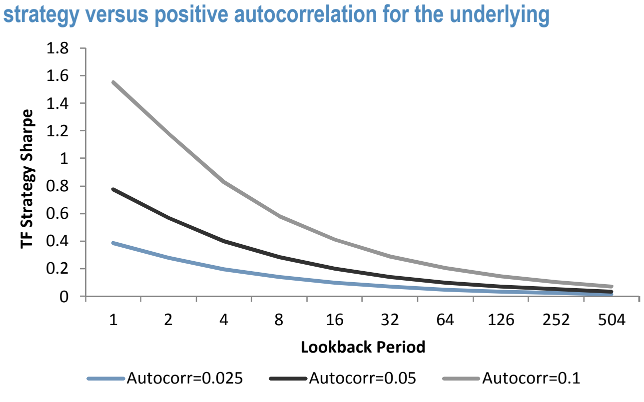
Figure 4: Sharpe ratio (based on gross P&L) of the trend-following strategy versus the Sharpe ratio of the underlying ($\rho = 0$)



Source: J.P. Morgan Quantitative and Derivatives Strategy

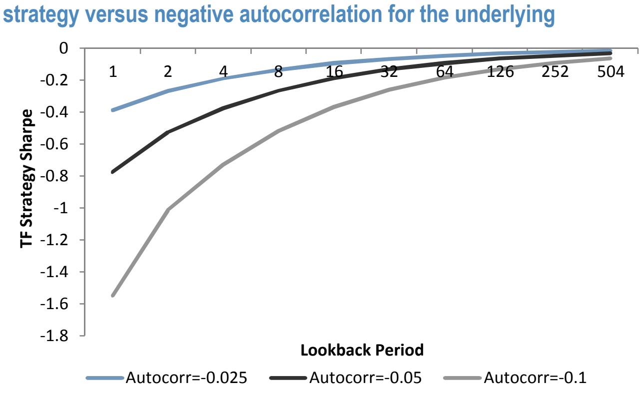
In Figures 5 and 6 we have plotted the Sharpe ratio of the trend-following strategy for various positive and negative values of the autocorrelation when there is no drift ($\mu = 0$). As expected positive autocorrelation leads to profits for the trend-following strategy while negative autocorrelation leads to losses. There are two important conclusions to be drawn from the results. First, the impact of autocorrelation is more pronounced for the profits generated by the signals based on short-term lookbacks. The P&L of the signals based on longer-term periods is expected to be expected to be immune to the impact of autocorrelation.[5] Hence, the signals based on the longer-term lookback periods will tend to be pure trend-following play for reasonable values of the autoregressive coefficients. Second, even small values of autocorrelation (note that we are discussing the autocorrelation of daily returns) can lead to a substantial positive or negative P&L when signals are based on short-term lookback periods. For example, when the autocorrelation coefficient is 0.1 a trend-following strategy based on a lookback period of 4 days is expected to produce a Sharpe ratio above 0.8.

Figure 5: Sharpe ratio (based on gross P&L) of the trend-following strategy versus positive autocorrelation for the underlying



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 6: Sharpe ratio (based on gross P&L) of the trend-following strategy versus negative autocorrelation for the underlying



Source: J.P. Morgan Quantitative and Derivatives Strategy

The results also have implication for mean-reversion strategy. A mean-reversion strategy can use a signal that is of opposite sign (profiting from on mean-reversion rather than on trend-following) and the P&L formula for the profitability of the

[5] For sufficiently large T and realistic values of ρ it follows $\rho^T \sim 0$. Hence, $E(PL_{t+1,T}) = \frac{2\rho(1-\rho^T)}{\sqrt{2\pi}\sqrt{2T(1-\rho)-2\rho(1-\rho^T)}} \sim \frac{2\rho}{\sqrt{2\pi}\sqrt{2T(1-\rho)-2\rho}} \sim 0$.

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mean-reversion will have the opposite sign of the trend-following one. Hence, mean-reversion will be profitable when autocorrelation is negative and will produce negative P&L when there are positive autocorrelation and strong trends. Similarly to trend-following there can be situations when mean-reversion will be profitable even if there are trends if the autocorrelation coefficient is sufficiently negative to offset the impact of trends. An interesting corollary concerns the periods over which the signals should be generated so that the mean-reversion strategy is profitable. Short-term periods are preferable as the impact of autocorrelation disappears when the signal is based on longer term periods. Such conclusions justify the common practice of designing mean-reversion strategies by focusing on the weekly frequency.

Furthermore, another important implication is that when short-term periods are considered trend-following signals should be better designed with a lookback period that is an even number while the mean-reversion signals should preferably make use of signals based on periods that are odd numbers[6]. For example, in the extreme case when autocorrelation is close to -1, a trend-following signal based on an even lookback period will be close to 0 (as we will have an alternating sequence of returns with an even length). That will be beneficial as the trend-following strategy will not take a position. Conversely, if the signal is based on a lookback period that is an odd number, the signal will be very close to 1 or -1 (depending on the sign of the last return) and the trend-following strategy will incur significant losses (as the next day return will be in the opposite direction of the current day one). Similarly, in the same setting a mean-reversion strategy using a signal based on an odd period will be profitable and it will not take positions if the signal is based on a period that is an even number.

Transaction Costs

For the implementation of every systematic strategy it is quite important to have a good understanding of the transaction costs involved in implementing the strategy. We distinguish between two types of transaction costs: running and execution. The running costs are linked to the size of the position and the execution costs are related to the change in the position.

Running Costs

Under the assumption that returns follow an AR(1) and the per unit running cost RC the expected running costs for a signal based on a lookback of T are:

$$E(RU_{t,T}) = \Bigg(2\Phi\left(\mu_{d1,T}/\sqrt{\sigma_{d1,T}^2+1}\right) + 2\Phi(-\mu_{d1,T}/\sigma_{d1,T}) - 4BvN(\mu_{d1,T}/\sqrt{\sigma_{d1,T}^2+1}, -\mu_{d1,T}/\sigma_{d1,T}; corr = -\sigma_{d1,T}/\sqrt{\sigma_{d1,T}^2+1})\Bigg) RC/\sigma$$

where $BvN(U,W;\rho)$ stands for the c.d.f of the standard bivariate normal distribution with correlation ρ evaluated at U and W and $\mu_{d1,T}$, $\sigma_{d1,T}$ and ϕ are functions of μ, σ^2, ρ and T and Φ stands for the standard normal c.d.f .

Under simplified assumptions that $\mu = 0$ and $\rho = 0$ (i.e. returns are a Gaussian noise), it follows that

$$E(RU_{t,T}) = -2\frac{\operatorname{asin}\left(-\frac{1}{\sqrt{2}}\right)}{\pi}\frac{RC}{\sigma} = \frac{1}{2} * \frac{RC}{\sigma}$$

All other things equal, the running costs are an increasing function of the ratio between the unit running cost and the volatility. In the extreme case when returns are a Gaussian white noise, the running costs are equal to half of that ratio. Furthermore, the graphs below show expected annualized running costs as a percentage of employed capital[7]. The running costs are increasing with the lookback period and the absolute value of the Sharpe Ratio of the underlying. Such results are intuitive as higher in magnitude Sharpe ratios generated larger signals and positions and for the same Sharpe ratio the

[6] When the autocorrelation is negative, ρ^T is positive when T is even and negative when T is odd.
[7] We assume that we target 10% annualized volatility and hence the employed capital is 10*annualized volatility.

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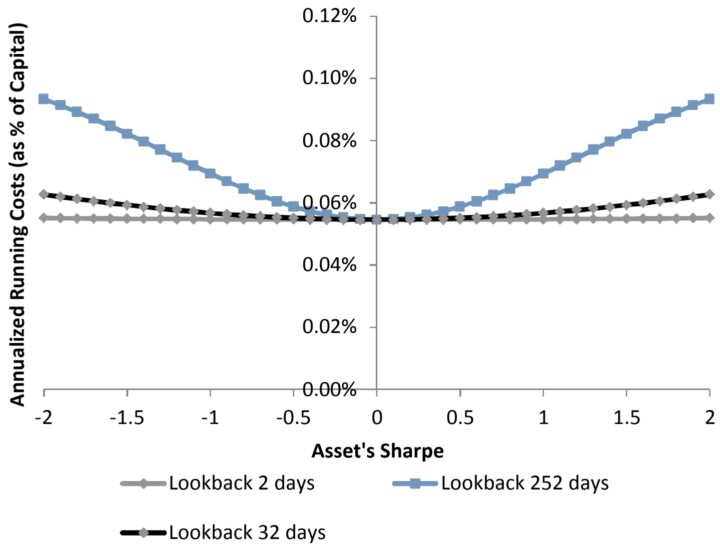
signals based on the longer term periods are bigger than the signals based on the shorter term periods. In general the magnitude of the running costs is small and rarely exceeds the running cost of the underlying asset.

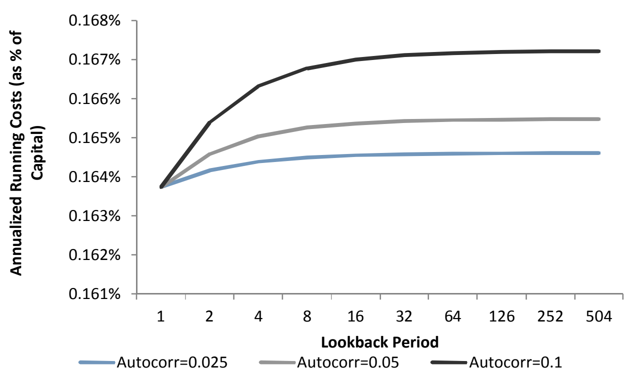
Figure 7: Annualized expected running costs (assuming unit running cost of 10bps per annum and daily volatility of 1%)



Source: J.P. Morgan Quantitative and Derivatives Strategy

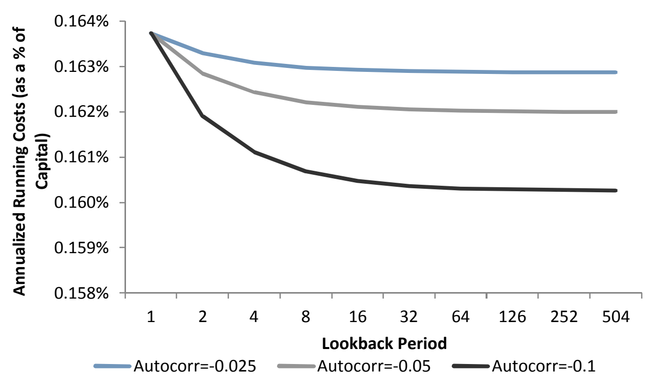
Note that in the case of pure autoregressive behavior (no drift in the autoregressive process) the running costs have a flat structure across various lookback periods and autocorrelation coefficients[8]. Given the P&L arguments presented in the previous section, the estimated running cost structure supports focusing on shorter term lookback periods when the strategies are designed to benefit from the autocorrelation properties. Note that the stronger the mean-reversion (the more negative is the autocorrelation), the lower the costs.

Figure 8: Annualized running costs as a % of capital for positive autocorrelation



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 9: Annualized running costs as a % of capital for negative autocorrelation



Source: J.P. Morgan Quantitative and Derivatives Strategy

[8] Note that the correlation coefficient in the bivariate normal distribution $corr = -\sigma_{d1,T} \big/ \sqrt{\sigma_{d1,T}^2 + 1}$ is decreasing in ρ and T when $\rho > 0$ and increasing in ρ and T when $\rho < 0$. Hence, running costs are increasing in ρ and T when $\rho > 0$ and decreasing in ρ and T when $\rho < 0$.

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Execution Costs

Under the assumption that returns follow an AR(1) and the per unit running cost EC the expected running costs for a signal based on a lookback of T are:

$$E(XC_{t,T}) = 4 * \left(\Phi\left(\frac{-\mu_{d1,T}}{\sqrt{\sigma^2_{d1,T}+1}} \right) - BvN\left(\frac{-\mu_{d1,T}}{\sqrt{\sigma^2_{d1,T}+1}}, \frac{-\mu_{d1,T}}{\sqrt{\sigma^2_{d1,T}+1}}; corr = 1 - \frac{\left(\frac{1-\rho^T}{T}\right)}{1+\sigma^2_{d1,T}} \right) \right) \frac{EC}{\sigma}$$

where $BvN(U, W; \rho)$ stands for the c.d.f of the standard bivariate normal distribution with correlation ρ evaluated at U and W and $\mu_{d1,T}$, $\sigma_{d1,T}$ and ϕ are functions of μ, σ^2, ρ and T and Φ stands for the standard normal c.d.f .

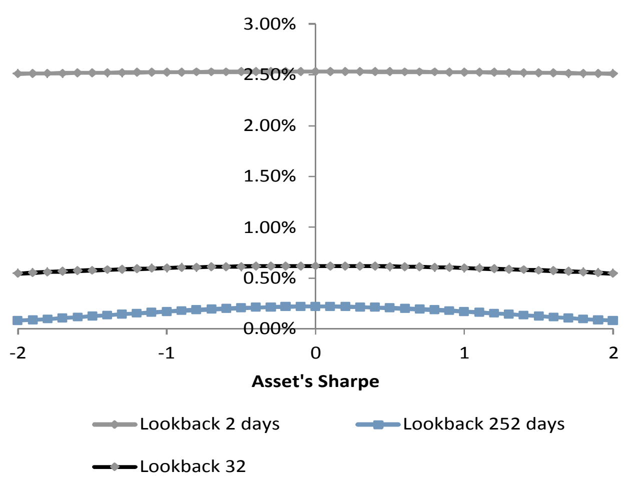
Under simplified assumptions that $\mu = 0$ and $\rho = 0$ (i.e. returns are a Gaussian noise), it follows that

$$E(XC_{t,T}) = \frac{2EC}{\pi\sigma} \operatorname{acos}(1 - 1/(2T))$$

Similarly to running costs, the ratio between the per unit execution cost and volatility is key. Under the assumption that returns are a Gaussian noise, the execution costs are a decreasing function of the lookback period.

In the case when no autocorrelation is present, the execution costs are also decreasing with the lookback period. The impact of the Sharpe ratio of the underlying is more pronounced for longer term lookbacks and the execution costs decrease with absolute value of the Sharpe ratio.

Figure 10: Annualized expected execution costs (assuming unit execution cost of 2bps and daily volatility of 1%)



Source: J.P. Morgan Quantitative and Derivatives Strategy

Note that in the case of pure autoregressive behavior (no drift in the autoregressive process) the execution costs are strongly dependent on the lookback period. The longer lookback periods produce result in substantially smaller execution fees. Furthermore, execution costs are decreasing in ρ when ρ > 0 and increasing in ρ when ρ < 0. The impact of the autocorrelation is much more muted in comparison to the period.

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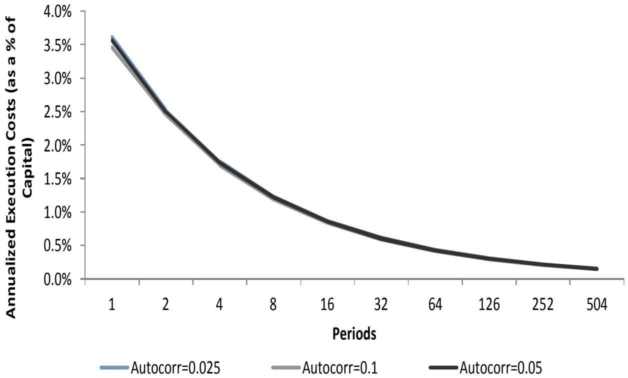
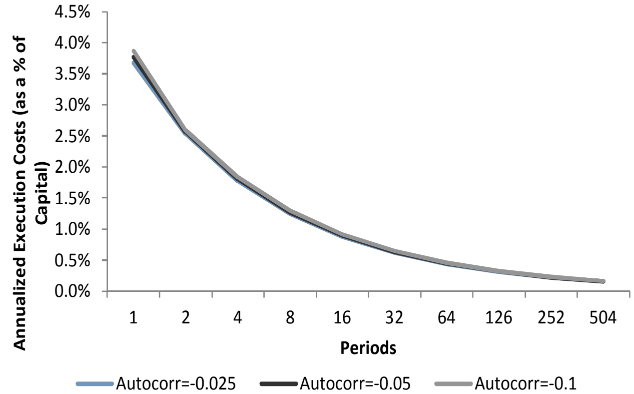
Figure 11: Annualized expected execution costs (assuming unit execution cost of 2bps and daily volatility of 1%)



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 12: Annualized expected execution costs (assuming unit execution cost of 2bps and daily volatility of 1%)



Source: J.P. Morgan Quantitative and Derivatives Strategy

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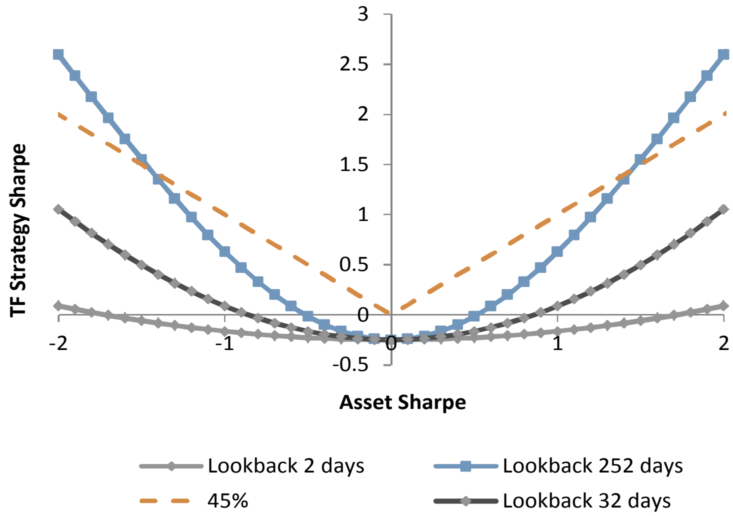
Net P&L

Knowing the analytical expressions for the expected P&L and transaction costs, we can naturally derive the net P&L. We start by considering the case when the autocorrelation coefficient is zero.

Figure 13: Sharpe ratio (after accounting for costs) of the trend-following strategy versus the Sharpe ratio of the underlying ($\rho = 0$)



Source: J.P. Morgan Quantitative and Derivatives Strategy

Due to the non-linear nature of the expressions for the expected P&L and transaction costs, it is difficult to derive the threshold Sharpe ratio of the underlying that renders the profitability of a signal based on a certain lookback period. Nevertheless, numerical results shed some interesting caveats for this relationship. In Figure 13 we have plotted the Sharpe ratio based on the net P&L of the trend-following strategy versus the Sharpe ratio of the underlying for various lookback periods. We use the transaction cost structure for S&P and assume a daily volatility of 1% (approximately 16% annualized). It is evident that signals based on short term lookbacks can only be profitable if the Sharpe ratio of the asset is quite sizable in either direction. For example, for a signal based on 2 days we need a Sharpe ratio above 2 and below -2 to assure the profitability of the strategy. For a signal based on 32 days, the Sharpe ratio should be above 1 or below -1. Even a signal based on a 1 year lookback period requires the absolute value of the Sharpe ratio to be bigger than 0.5 so that profitability is assured.

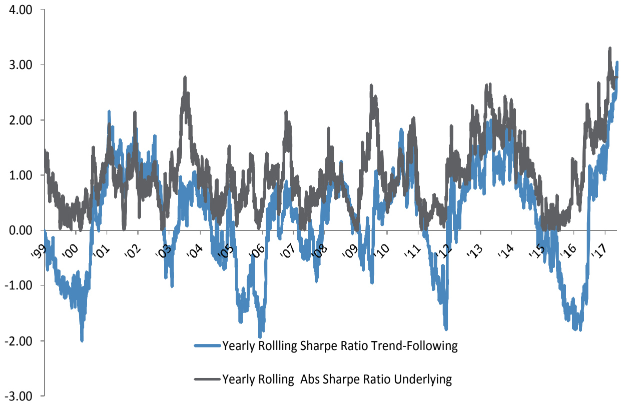
While such levels might seem a big hurdle at first sight, the table below shows that empirically such levels of absolute values of the Sharpe ratios are the rule rather than the exception. It is evident that empirically the absolute values of the Sharpe ratios have sufficient magnitude to render the trend-following strategy profitable. Hence, for the profitability of the trend-following strategy, of vital importance will be the persistence in the Sharpe ratio (return generation process respectively). The trends should last sufficiently long time so that they are captured by the signals.

Table 2: Average absolute value of the Sharpe ratio over various timeframes

Asset Class	Data Size (in Days)							
	4	8	16	32	64	126	252	504
Equities	8.3	5.0	3.3	2.3	1.6	1.2	0.9	0.7
FX	8.4	5.1	3.4	2.3	1.6	1.2	0.9	0.7
Commodities	8.2	5.0	3.4	2.4	1.7	1.2	0.9	0.6
Rates	8.5	5.1	3.5	2.5	1.8	1.4	1.1	0.9

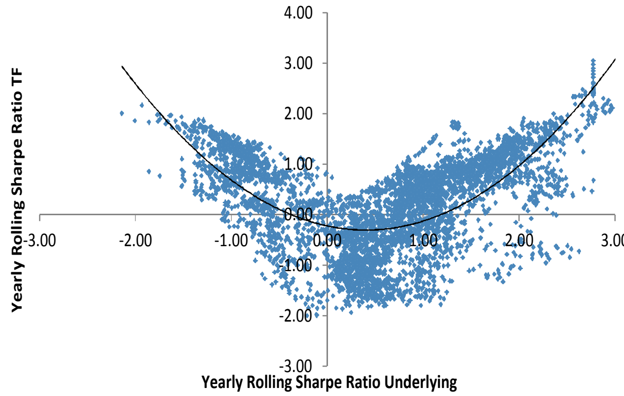
Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 14: Absolute Sharpe ratio of S&P500 versus the Sharpe ratio of the trend-following system (yearly horizon).



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 15: Scatter plot of the Sharpe ratio of the trend-following strategy versus the Sharpe ratio of S&P500 (yearly horizon).

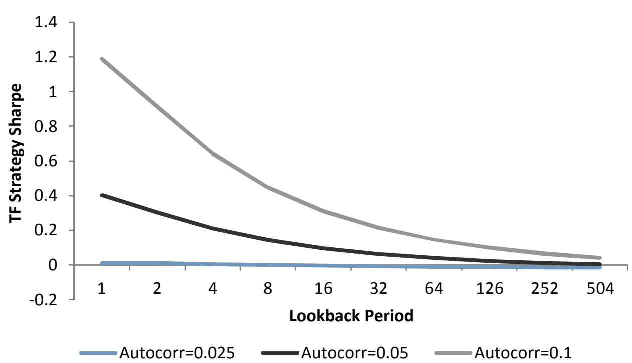


Source: J.P. Morgan Quantitative and Derivatives Strategy

Furthermore, we expect the Sharpe ratio of trend-following strategy to be below the absolute value of the Sharpe ratio of the asset. A sizable positive or negative Sharpe ratio of the underlying and long term lookback period are both necessary for the Sharpe ratio of the trend-following strategy to exceed the absolute value of the Sharpe of the underlying. For example, we need the Sharpe ratio of the underlying to be bigger in absolute value than 1.5 so that trend-following is more profitable than either holding or shorting the asset.

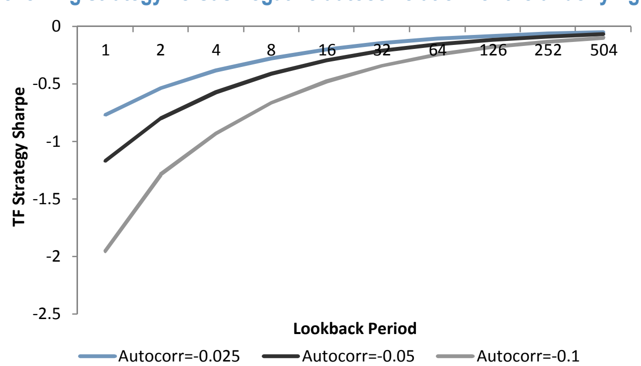
If the drift of the asset is stable (stays constant over a long period), it is much more profitable and cost-efficient to use signals based on longer lookback periods. For example, if we expect equities to exhibit a positive drift due to the embedded equity risk premia, it is preferable to use signals with longer lookback periods. The appeal of the shorter term lookback periods arises in two scenarios. Firstly, the duration of the trend might be smaller than a long lookback period. For example, if the trend changes direction every 6 months making use of a signal based on 1 year lookback will be detrimental. Secondly, during market reversals signals based on shorter lookback periods are more reactive and eventually mitigate the drawdowns of the slower trend-following systems.

Figure 16: Sharpe ratio (after accounting for costs)of the trend-following strategy versus positive autocorrelation for the underlying



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 17: Sharpe ratio (after accounting for costs) of the trend-following strategy versus negative autocorrelation for the underlying



Source: J.P. Morgan Quantitative and Derivatives Strategy

Benefiting from positive autocorrelations remains profitable after accounting for costs when short-term lookback are used. But negative autocorrelation of the same magnitude can lead to two times higher losses when quicker signals are employed. The profitability of signals based on long term periods remains relatively immune to the autocorrelation. The Sharpe ratio of the P&L generated by a signal based on a 1 year lookback is 0.06 when the autocorrelation is 0.1 and -0.13 when autocorrelation is -0.1.

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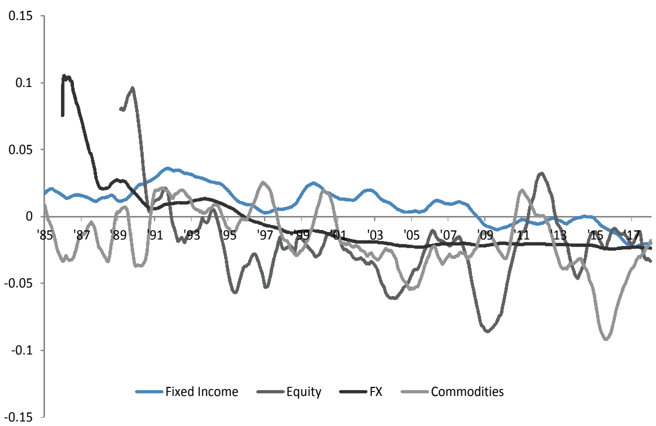
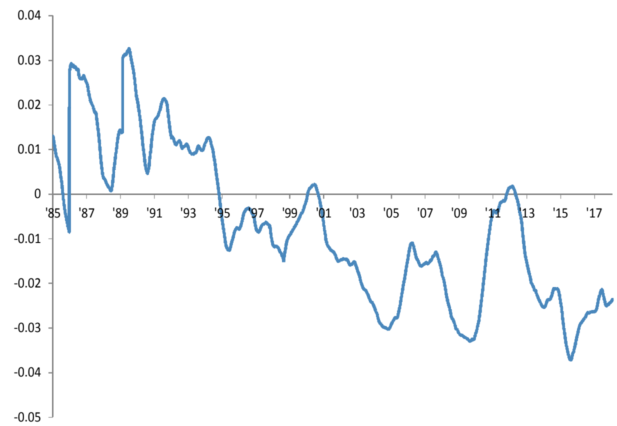
Below we have shown the average autocorrelation coefficients through time for various asset classes and the average value across all asset classes. While in the beginning of our sample most of the autocorrelation values were positive they have gradually turned negative. As we witness later, this dynamic will be quite important for the trend-following strategies that make use of short-term lookback windows.

Figure 18: Average autocorrelation coefficient per asset class (data window=1 year)



Source: J.P. Morgan Quantitative and Derivatives Strategy

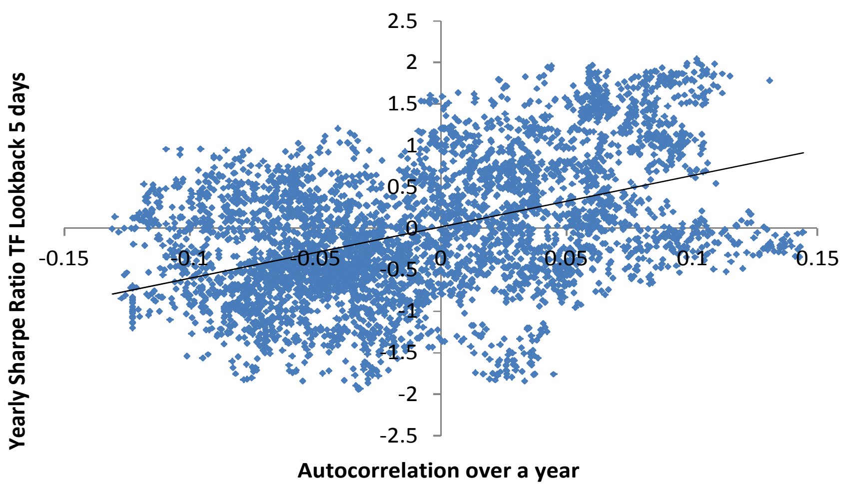
Figure 19: Average autocorrelation coefficient across all asset classes (data window=1 year)



Source: J.P. Morgan Quantitative and Derivatives Strategy

As an empirical example, let us consider a trend-following strategy applied to the EUR/USD exchange rate that uses a signal based on a 5 day lookback period. The impact of the autocorrelation is quite evident with negative values of autocorrelation resulting on average in negative Sharpe ratio and positive values of autocorrelation producing on average a positive Sharpe ratio.

Figure 20: Sharpe ratio of the trend-following strategy (lookback period) applied to EURUSD versus the autocorrelation coefficient of the underlying



Source: J.P. Morgan Quantitative and Derivatives Strategy

Lookback period selection

In empirical work the problem of data window selection is often encountered and there are opposite forces in play. On the one hand, a long enough estimation sample is needed to produce reliable estimates. On the other hand, too long a window masks the recent developments.

A similar problem arises when a systematic strategy is being designed. In trend-following there is virtually an infinite choice of lookback windows. The investment logic and experience dictates that the signals based on short-term lookback windows are more reactive but will typically entail higher costs. The signals based on longer-term lookback periods are inherently more stable but their performance disappoints at inflection points.

One option to select the lookback periods is the empirical route and the choice is often based on backtested performance. As mentioned by Baltas and Kosowski (2013) '...12-month horizon generates the largest Sharpe ratio for trend-following strategies across each asset class'. Asness et al. (2013) use the past 12-month cumulative raw return on the asset skipping the most recent month's return and mention that this is a 'common measure'. Moskowitz et al. (2012) also focused on the 12 month time-series momentum strategy with a 1 month holding period. Lempérière et al. (2015) make use of a 5 month lookback period. The common feature of all those studies is that, although various lookback periods have been backtested, a single one was selected.

Another option is to combine several lookback periods aiming to achieve diversification. Hurst et al. (2017) use an equally weighted combination of 1-month, 3-month and 12-month time series momentum strategies. Baz et al. (2015) construct an aggregated signal based on 3 EWMA Crossovers.

In the following we discuss the optimal way to select the lookback periods by explicitly taking into account the properties of the signal and more concretely the correlation between the P&L generated by signals based on various lookback periods. The relevant derivations can be found in the Appendix.

The correlation between the P&L streams generated by 'delta-straddle' type signals based on lookback periods T_1 and T_2 ($T_1 < T_2$) is given by the formula below:

$$\rho = 6 * \operatorname{asin}\left(0.5 * \sqrt{\frac{T_1}{T_2}}\right) / \pi$$

The main caveat of the result is that it is the ratio between the lookback periods $\frac{T_1}{T_2}$ is important rather than their difference $(\mathrm{T}_1 - \mathrm{T}_2)$. For example, if we plug in $\frac{\mathrm{T}_2}{\mathrm{T}_1} = 2$ then ρ=0.69.

In Tables 4 and 5 we have shown theoretical and empirical correlations for selected lookback periods[9]. Though the theoretical correlation has been derived under simplifying assumption, the deviations between the theoretical and the empirical values are negligible. The biggest average absolute deviation stands at 0.04.

Knowing the correlation matrix we can estimate Equal Risk Contribution (ERC) weights. The optimal weights are quite close to equal. This is an interesting result as it shows that averaging the signals (when the lookback periods are selected with particular ratio) is close to an optimal solution

Table 3: ERC weights

Period	1	2	4	8	16	32	64	126	252	504
Weight	0.120	0.103	0.095	0.092	0.090	0.090	0.092	0.095	0.103	0.120

Source: J.P. Morgan Quantitative and Derivatives Strategy

[9] Note that the theoretical correlation matrix is a Toeplitz matrix.

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Table 4: Theoretical correlation matrix for various lookback periods

Periods	1	2	4	8	16	32	64	126	252	504
1	1	0.69	0.48	0.34	0.24	0.17	0.12	0.09	0.06	0.04
2		1.00	0.69	0.48	0.34	0.24	0.17	0.12	0.09	0.06
4			1.00	0.69	0.48	0.34	0.24	0.17	0.12	0.09
8				1.00	0.69	0.48	0.34	0.24	0.17	0.12
16					1.00	0.69	0.48	0.34	0.24	0.17
32						1.00	0.69	0.49	0.34	0.24
64							1.00	0.70	0.49	0.34
126								1.00	0.69	0.48
252									1.00	0.69
504										1.00

Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 5: Empirical correlation matrix for various lookback periods

Periods	1	2	4	8	16	32	64	126	252	504
1	1	0.70	0.49	0.35	0.25	0.17	0.12	0.07	0.03	-0.01
2		1.00	0.69	0.50	0.35	0.25	0.18	0.10	0.05	0.01
4			1.00	0.70	0.50	0.36	0.25	0.15	0.09	0.03
8				1.00	0.71	0.51	0.37	0.25	0.17	0.09
16					1.00	0.72	0.53	0.38	0.27	0.16
32						1.00	0.74	0.55	0.41	0.26
64							1.00	0.75	0.56	0.38
126								1.00	0.74	0.53
252									1.00	0.74
504										1.00
Average absolute deviation from theoretical values	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.04	0.04

Source: J.P. Morgan Quantitative and Derivatives Strategy

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Prototype Trend-Following Solution

Data Universe and Transaction Costs

Our data universe covers liquid futures across equities, currencies, commodities and fixed income. More details can be found in the Appendix. We allocate equally across the asset classes. Furthermore, every asset receives a risk weight (allocation) that reflects its liquidity relative to the rest of the assets in the same asset class. For futures we use data on the daily volumes from the relevant exchanges while for currencies we make use of the BIS (Bank for International Settlements) transactions volume data.

As a robustness check we have analyzed performance of the performance with alternative datasets and we have not found substantial performance differences. The first alternative dataset includes the same asset universe but the weights are equally distributed among the assets within the same asset class. The second alternative dataset consists of a larger universe that also makes use of risk weights based on liquidity. More details can be found in the subsequent sections.

A conservative cost structure has been assumed in the backtest simulations. The table below outlines the average execution and running costs per asset class in various periods. The adjustments for the various periods have been done in accordance to Jones (2012). Before 1993 we assume that transaction costs were on average 4 times higher than the current levels and 1.5 times higher on average between 1993-2002[10].

Table 6: Average transaction costs per asset class in different subperiods

Period	Equities		FX		Commodities		Fixed Income	
	Bid-Ask Spread (bps)	Annual Replication Cost (bps)	Bid-Ask Spread (bps)	Annual Replication Cost (bps)	Bid-Ask Spread (bps)	Annual Replication Cost (bps)	Bid-Ask Spread (bps)	Annual Replication Cost (bps)
Before 1993	19.8	44.0	18.7	32.0	17.2	39.7	18.3	33.6
1993-2002	7.4	16.5	7.0	12.0	6.4	14.9	6.9	12.6
Since 2003	5.0	11.0	4.7	8.0	4.3	9.9	4.6	8.4

Source: J.P. Morgan Quantitative and Derivatives Strategy

Benchmark Trend-Following Solution

Our benchmark solution is based on an aggregate signal that averages signals based on 32 days, 64 days, 126 days, 252 days and 504 days. The lookback periods are selected so that the diversification and correlation benefits are optimal. The identical ratio of proportionality among the consecutive lookback periods guarantees that averaging the signals based on various lookback periods is justified by the implications of the earlier results.

Standard portfolio and risk management techniques are employed in our benchmark solution. The position in every asset is proportional to the signal and the risk weight inversely proportional to its volatility[11]. The aggregate portfolio is dynamically risk-managed on an expanding window basis and an average annualized volatility of 10% is targeted.

The proposed solution also includes a floor on the adjustment of the position. If the absolute value of the adjustment in the position is below the floor, the position will not be adjusted. The floor corresponds to a change in the signal of 0.25.[12] Such a transaction cost mitigating approach has become a standard cost management technique. Later in the paper we demonstrate the results for various values of the floor parameter.

[10] The transaction costs are 6 times higher at the beginning of our data sample in 1985 and gradually decrease to 2 times higher at the end of 1992. From 1993 to 2003 the transaction costs move from 2 times the current levels to the current levels.

[11] We make use of volatility forecasts based on EWMA approach. The smoothing parameter used is 0.94 (half-life of approx. 11 days).

[12] As the positon at every point is time is $Signal(t)/vol(t)$, the floor is set to $0.25/vol(t)$.

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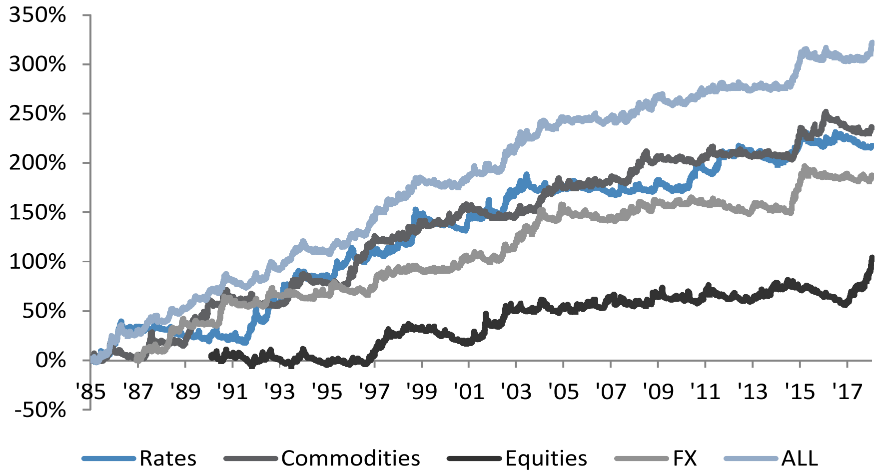
Backtested Performance

Below the cumulative performance of the benchmark approach in various asset classes as well as the performance of the combined portfolio are shown[13]. Commodities have historically had the most appealing trend-following track-record (commodities are also the asset class upon which the CTA industry originated). The asset class that has been historically been the most challenging for the trend-following approach is equities.

Figure 21: Cumulative Performance by Asset Class



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 7: Performance Statistics by Asset Class

	Commodities	Equities	Rates	FX	Combination All Asset Classes
Annualized Return	6.82%	3.39%	6.23%	5.74%	9.27%
Annualized Volatility	9.47%	9.73%	9.39%	8.77%	9.04%
Sharpe	0.72	0.35	0.66	0.65	1.03
Max Drawdown	-20.93%	-23.45%	-21.38%	-16.37%	-13.60%

Source: J.P. Morgan Quantitative and Derivatives Strategy

There are strong diversification benefits from aggregating among asset classes. The Sharpe ratio of the combined portfolio is more than 40% bigger than the Sharpe ratio of the best performing asset class – commodities. The drawdown of the combined portfolio is also well-controlled and stands at less than 1.5 times the annualized volatility. The main reason for the substantial diversification benefit is the extremely low average correlation between the trend-following strategies in different asset classes (it stands at 0.07).

Table 8: Correlation matrix among the P&L in various asset classes

	Equities	FX	Commodity	Rates
Equities	1.00			
FX	0.05	1.00		
Commodity	0.03	0.14	1.00	
Rates	0.09	0.10	0.04	1.00

Source: J.P. Morgan Quantitative and Derivatives Strategy

[13] The return calculations do included the interest earned on the capital invested and do not account for management and performance fees.

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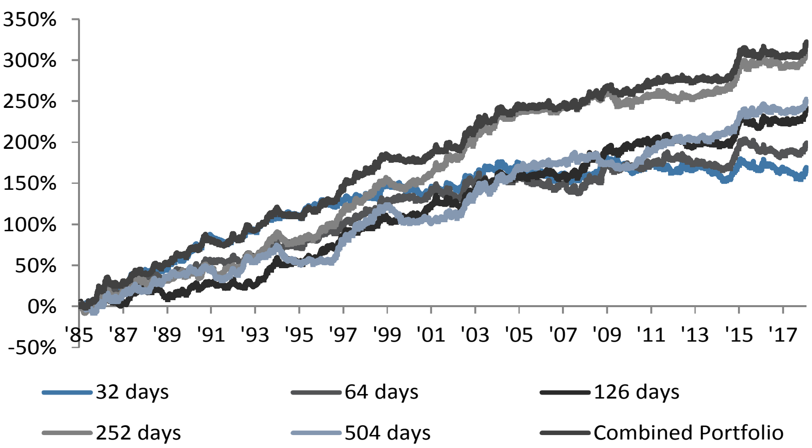
The diversification benefits among the various lookback windows have already been discussed within our theoretical framework and the backtested results below are in line with our earlier conclusions. While the combined portfolio does not substantially improve upon the Sharpe ratio of the best performing lookback period (1 year), the drawdown measure is improved by more than 5%. The empirical results are also in accordance with theoretical results that suggest that longer term lookback periods can potentially do better as the threshold value of Sharpe ratio to assure profitability is lower and the overall expected transaction costs are lower. The additional (and somewhat disguised) pre-requisite for appealing performance is to have sufficient stability in the trends so that it can be captured by the signals based on longer term lookback windows.

Figure 22: Cumulative performance of signals based on various lookback periods



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 9: Performance statistics for signals based on various lookback periods

	32 days	64 days	126 days	252 days	504 days	Combined Portfolio
Annualized Return	4.67%	5.55%	6.70%	8.87%	7.28%	9.27%
Annualized Volatility	9.23%	9.23%	9.28%	9.18%	9.15%	9.04%
Sharpe	0.51	0.60	0.72	0.97	0.80	1.03
Max Drawdown	-26.67%	-23.80%	-17.77%	-19.05%	-22.10%	-13.60%

Source: J.P. Morgan Quantitative and Derivatives Strategy

In the Appendix we have presented a detailed table of the monthly returns of the strategy. Below we present the histogram of the monthly returns together with a few performance statistics. On average the strategy would have returned 85bp per month. It is interesting to note the asymmetric distribution and in particular the positive skewness of the strategy. While the worst monthly return has been -6.38%, the maximum return would have been 9.63%.

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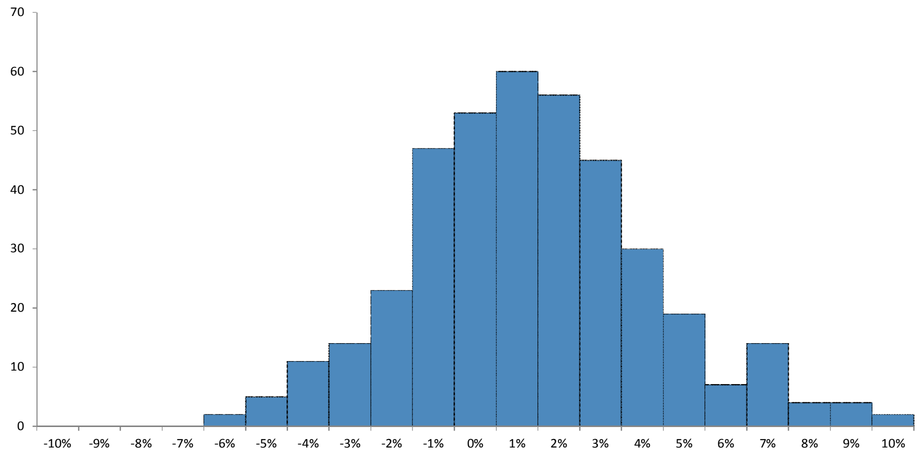
Figure 23: Histogram of the monthly returns



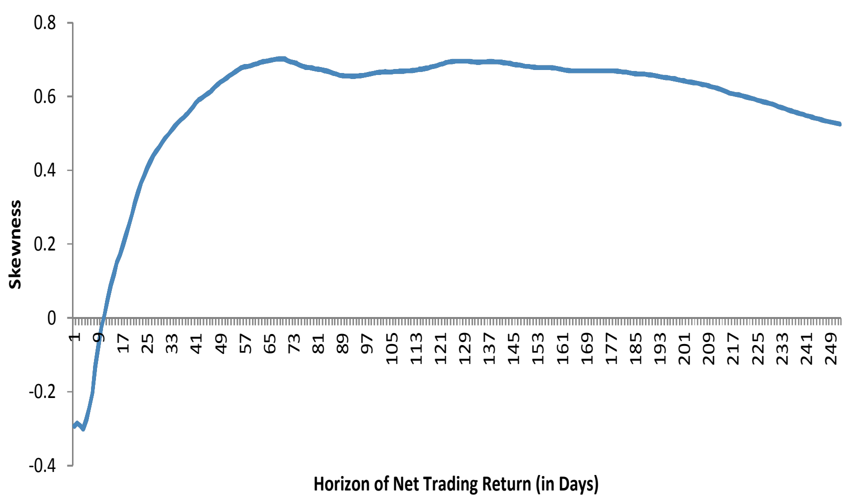
Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 10: Performance statistics for the monthly strategy returns

Average Monthly Return	Volatility of the Average Monthly Return	Min	Max	Skewness	Kurtosis
0.85%	2.88%	-6.38%	9.63%	0.33	0.34

Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 24: Skewness of the net return aggregated over different horizons



Source: J.P. Morgan Quantitative and Derivatives Strategy

The positive skewness of the trend-following strategies is often cited as one of its main merits. It is well-known that many of the typical risk-premia strategies exhibit a negative skewness (for example short volatility strategies, carry strategies in various asset classes etc)[14]. Martin and Bana (2012) have analyzed within a theoretical framework the skewness of various non-linear strategies (the delta-straddle signal is a particular case of the non-linear strategies discussed). They show that

[14] For example, Lempérière et.al. (2014) find a strong relationship between the negative skewness of the strategy and the expected return. Trend-following is a notable exception having both positive expected return and positive skewness. Hence, Lempérière et al. (2014) consider trend-following more as a market anomaly rather than a risk-premia strategy.

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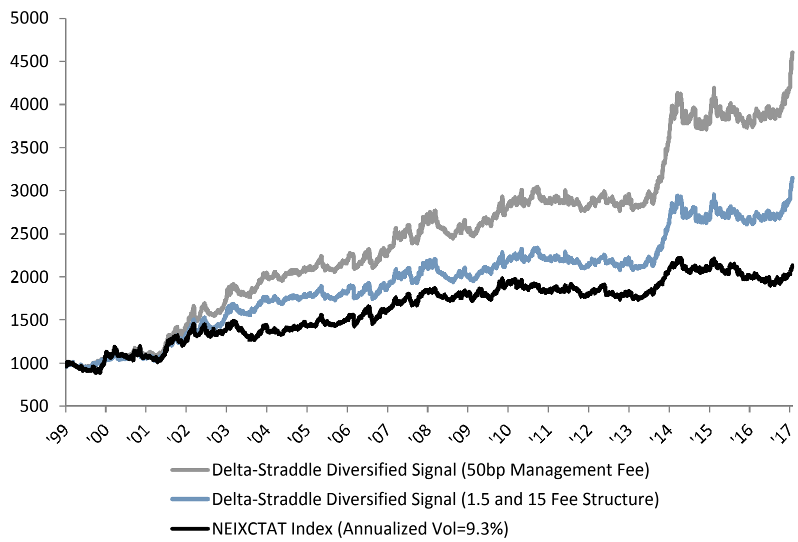
even when there are no-trends, the cumulated over a certain period trading return of the trend-following strategy exhibits skewness. Martin and Bana (2012) also demonstrate that the skewness peaks at a certain period over which the returns are cumulated. Below we have plotted the skewness of the returns of our benchmark solution cumulated over various periods and we can indeed notice such a result. The presence of transaction costs and negative autocorrelation in reality can lead to negative skewness over very short-horizons.

Our benchmark solution also manages to capture the bulk of the returns of the big trend-following funds as represented by the NEIXCTAT Index[15]. We have simulated the performance of our prototype solutions under various fees assumptions. First, we have assumed an average fee structure of 1.5% management fee and 15% performance fee. The fees imposed start at the typical 2 and 20 fee structure at the beginning of the period considered and finish at 1% management fee and 10% performance fee. The assumed fee structure reflects the gradual pressure on the fees in the recent years. The second fee scenario assumes 50bp flat management fee and no performance fee. The correlation between the benchmark solution under the different fee scenarios and the NEIXCTAT Index stands above 80%. The prototype solution fares well in comparison to the trend-following benchmark with a Sharpe ratio that is bigger with more than 60% even in the aggressive fee scenario and better controlled drawdown.

Figure 25: Compounded performance of the benchmark solution under different fee scenarios



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 11: Comparative performance statistics

	Benchmark Solution 1.5 and 15 Fee Structure	Benchmark Solution 50bp Management Fee	NEIXCTAT Index
Annualized Return	6.56%	8.61%	4.48%
Annualized Volatility	9.32%	9.43%	9.32%
Sharpe	0.70	0.91	0.48
Max Drawdown	-12.26%	-11.86%	-15.64%

Source: J.P. Morgan Quantitative and Derivatives Strategy

As a robustness check we have also checked the performance of the 'delta-straddle' type trend-following signal versus the Z-score signal and the binary signal. The delta-straddle signal outperforms but nevertheless all the track records share similar characteristics. The average correlation between the three approaches is 0.97. As pointed out in Levine and Pedersen (2015) there is equivalence among many of the commonly used trend-following signals once appropriate adjustments for the

[15] The NEIXCTAT Index (also known as SG Trend Index) is designed to track the 10 largest (by AUM) trend following CTAs. The index is equally weighted, and rebalanced and reconstituted annually.

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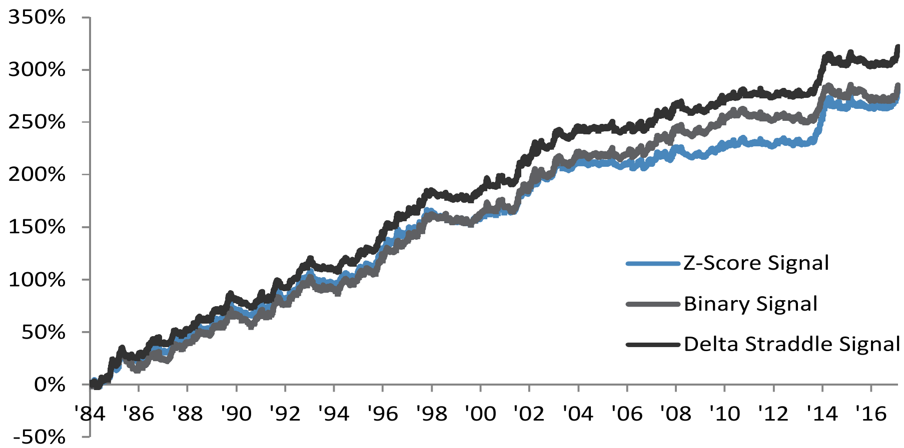
lookback periods have been made. In line with our discussion earlier having a robust signal from a theoretical point of view is quite important but not less important is the choice (or the mixture) of the lookback windows.

Figure 26: Cumulative performance various trend-following signals



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 12: Performance statistics for various lookback windows

	Z-Score Signal	Binary Signal	Delta Straddle Signal
Annualized Return	8.16%	8.25%	9.27%
Annualized Volatility	8.98%	9.12%	9.04%
Sharpe	0.91	0.9	1.03
Max Drawdown	-14.00%	-17.70%	-13.60%

Source: J.P. Morgan Quantitative and Derivatives Strategy

Diversification properties of trend-following strategies

In addition to the attractive feature of positive skewness that the trend-following strategies possess, trend-following strategies bring substantial diversification benefits for the long-only portfolios. As we have already shown in the theoretical sections, trend-following strategies exhibit convexity and when the move on the downside is sizable enough the return of the trend-following strategy will more than compensate the loss in the underlying. It has also been well-known that the magnitude of the sell-offs is typically quite sizable and therefore the offset with the trend-following strategies is quite appealing.

To verify this hypothesis empirically we have constructed portfolios that consist of long positions in the underlyings from our asset universe. The portfolios are well targeted to have an annualized volatility of 10% and utilize the same risk weights for the individual assets as in our benchmark solution. We have also constructed combined portfolios that invest 50% in the long-only portfolio and 50% in the trend-following system. The diversification benefits are quite evident in all asset classes except for fixed income. In fixed income, the directionality of the market has led to a lot of overlap between the positions of the trend-following system and those of the long-only portfolio.

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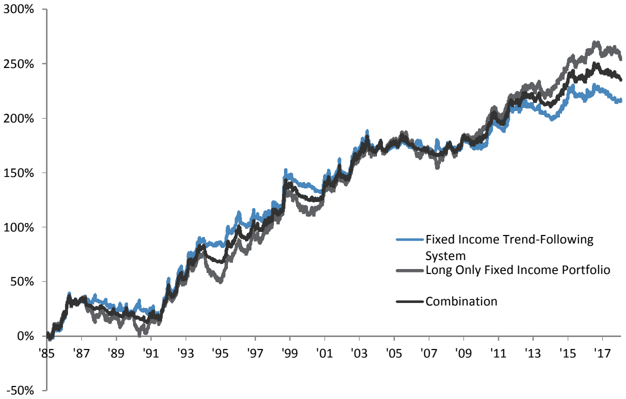
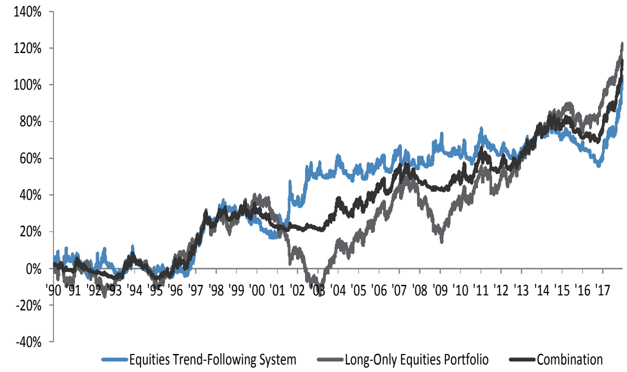
Figure 27: Cumulative returns for a long only fixed income portfolio, a fixed-income trend-following system and their combination



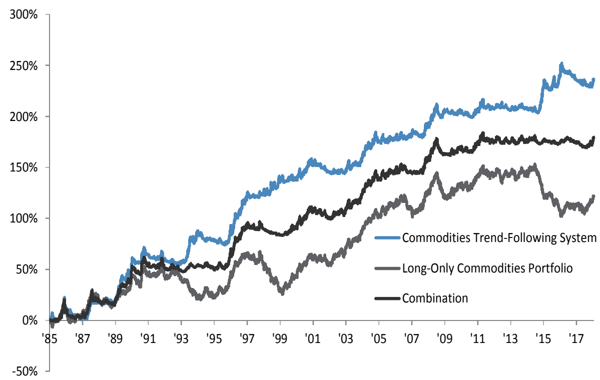
Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 28: Cumulative returns for a long only equities portfolio, equities trend-following system and their combination



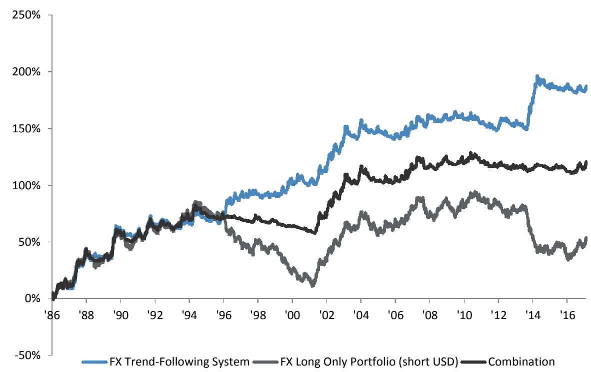
Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 29: Cumulative returns for a long only commodities portfolio, commodities trend-following system and their combination



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 30: Cumulative returns for a long only FX portfolio, FX trend-following system and their combination



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 13: Performance statistics for fixed income and equities

	Fixed Income			Equities		
	Trend-Following System	Long-Only Portfolio	Combination	Trend-Following System	Long-Only Portfolio	Combination
Return	6.23%	7.23%	6.78%	3.39%	4.20%	3.86%
Vol	9.39%	9.60%	8.47%	9.73%	9.82%	7.56%
Sharpe	0.66	0.75	0.80	0.35	0.43	0.51
Max DD	-21.38%	-31.34%	-22.52%	-23.45%	-42.90%	-14.44%

Source: J.P. Morgan Quantitative and Derivatives Strategy

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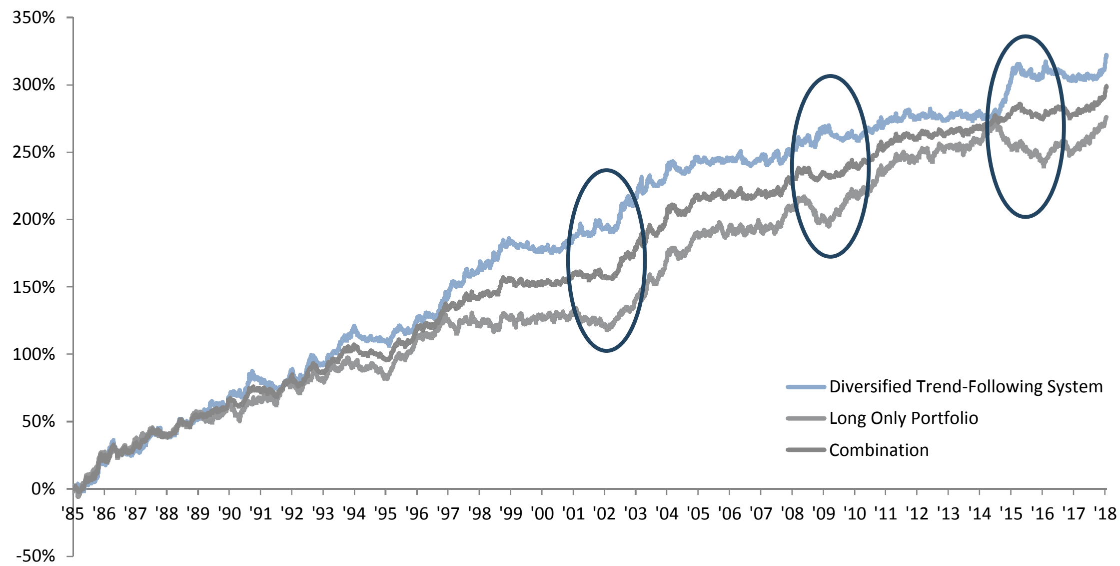
Table 14: Performance statistics for commodities and FX

	Commodities			FX		
	Trend-Following System	Long-Only Portfolio	Combination	Trend-Following System	Long-Only Portfolio	Combination
Return	6.82%	3.47%	5.14%	5.74%	1.66%	3.70%
Vol	9.47%	10.08%	7.71%	8.77%	9.32%	6.92%
Sharpe	0.72	0.34	0.67	0.65	0.18	0.53
Max DD	-20.93%	-40.62%	-17.55%	-16.37%	-53.88%	-22.29%

Source: J.P. Morgan Quantitative and Derivatives Strategy

The diversification benefits stand out when we are looking at the multi-asset portfolios as well. The Sharpe ratio of the combined portfolio is even greater than the trend-following one and the drawdown is decreased by more than 60%.

Figure 31: Cumulative returns for a long only portfolio across asset classes, a diversified trend-following system and their combination



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 15: Performance statistics

	All Asset Classes		
	Trend-Following System	Long-Only Portfolio	Combination
Annualized Return	9.27%	7.86%	8.61%
Annualized Volatility	9.04%	9.22%	7.31%
Sharpe	1.03	0.85	1.18
Maximum Drawdown	-13.60%	-29.15%	-11.20%

Source: J.P. Morgan Quantitative and Derivatives Strategy

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Portfolio management of the trend-following portfolio

Risk budgeting

So far when we have been constructing the portfolios we have ignored the impact of correlations among assets. The allocation algorithm in the benchmark portfolio – allocating proportionally to the product of the signal-to-vol ratio and the risk weight – would have been optimal if assets were perfectly correlated[16]. Below we aim additionally to incorporate the correlation structure when we construct our trend-following portfolio.

In line with our earlier investment philosophy the risk contribution of an asset is set to be proportional to the absolute value of the signal and the risk weight. In the optimization procedure we also constrain that we have long positions in the assets with positive signals and short positions in the assets with the negative signals. As shown by Bai et. al. (2016) such an optimization problem is well-defined and has a unique solution. A similar approach has been adopted by Baltas (2015) though he does not make use of signals of different scales, equal allocation to asset classes and asset-wise risk weights.

Similarly to the benchmark case the overall portfolio volatility is again targeted to be on average 10%. Note that depending on the strengths of the signals at a certain point in time the volatility of portfolio can be below or above the targeted value[17]. Our covariance matrix is estimated from the sample correlation matrix with an estimation period of 252 days and the forecasted volatilities.

A potential danger with the risk-budgeting approach when short-positions are allowed is taking too much leverage. There is a lot of uncertainty in the estimation the covariance matrix. It is often possible that the signals of two assets point in opposite directions when the assets' returns are positively correlated or that the signals point in the same direction and the assets' returns are negative correlated. In such situations a potential estimation error in the correlation matrix can be propagated as excessive leverage can be taken by the algorithm due to an ill-envisaged offset.

To mitigate the danger of such situations we have introduced an additional parameter that limits the size of the offset allowed by the algorithm. In situations where the direction of the signals does not match what the correlation coefficient implies, we compare the absolute value of the correlation coefficient to a cap parameter. If the absolute value of the correlation coefficient is above the cap parameter, we set the correlation to the cap parameter and adjusting for the original sign of the correlation.

[16] See Bruder and Roncalli (2012).

[17] At every time *t* the target annualized portfolio volatility is equal to $(10\% * \sum_{i=1}^{n} abs(S_{it}) * RiskWeight_i) * t / \sum_{s=1}^{t} \sum_{i=1}^{n} abs(S_{is}) * RiskWeight_i)$, where S_{is} stands for signal for asset *i* and time *s*.

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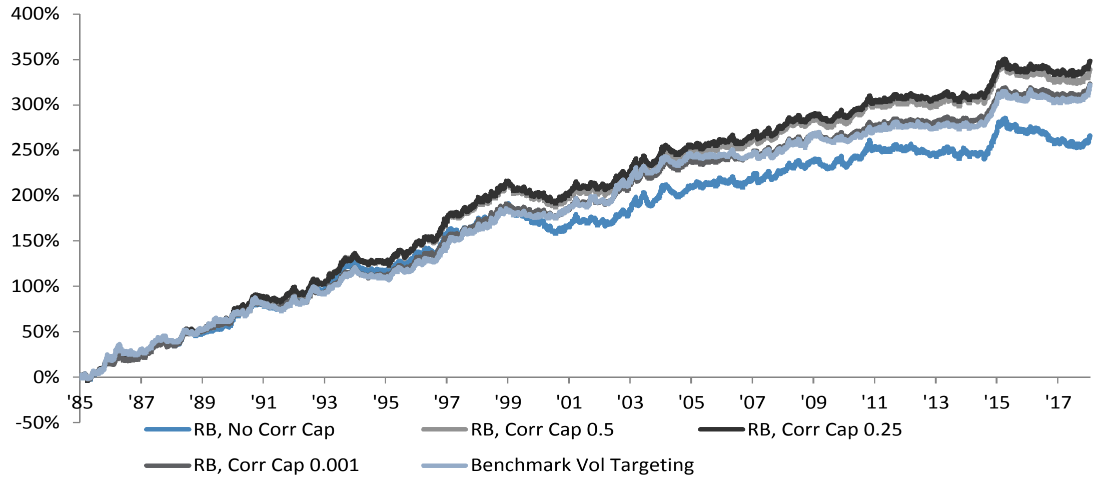
Figure 32: Cumulative returns for the risk-budgeting approach for various values of the correlation floor parameter



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 16: Performance statistics for various values of the correlation floor parameter

	RB, No Corr Cap	RB, Corr Cap 0.5	RB, Corr Cap 0.25	RB, Corr Cap 0.001	Benchmark Vol Targeting
Annualized Return	7.66%	9.30%	10.03%	9.76%	9.27%
Annualized Volatility	10.24%	8.07%	9.28%	9.83%	9.04%
Sharpe	0.75	1.15	1.08	0.99	1.03
Maximum Drawdown	-29.2%	-14.3%	-21.6%	-23.3%	-13.60%

Source: J.P. Morgan Quantitative and Derivatives Strategy

Note that the optimal choice lies between the extreme values. No cap allows for an excessive offset and leverage that can easily disappoint if the signal predictions turn wrong. Extreme capping might be penalizing not recognizing the diversification benefits.

It is prudent to note that the performance impact of using a risk budgeting approach depends on a mixture of factors. From a portfolio management point of view the risk will more efficiently distributed if the correlation among assets differs significantly[18]. The assets that have lower correlations to the remaining ones will receive higher allocations in comparison to the benchmark case. Therefore, to some extent the performance impact will be dependent on the relative trend-following performance of the assets with low correlations versus those with high correlation. Furthermore, the historical correlations should be a good representation of the future ones for the risk budgeting approach to lead to a more efficient capital allocation.

A Hierarchical Risk Budgeting approach

Lopez de Prado (2017) introduced 'Hierarchical Risk Parity' (HRP) as a way to mitigate some of problems with portfolio optimization problems that involve inversion of the covariance matrix. Lau et. al. (2017) applied the approach to the area of risk-premia investing in the J.P. Morgan paper "Cross Asset Portfolios of Tradable Risk Premia Indices". J.P. Morgan paper "Post-Modern Portfolio Construction" by Hlavaty and Smith (2017) allows for inclusion of an 'alpha' component.

In our application we take on board the HRP and adopt it to the trend-following framework. Along the way we have also made some modifications which tackle some issues encountered.

[18] As noted by Bruder and Roncalli (2012) if the correlations among the assets are the same the optimal Risk Budgeting solution is close to the solution that assumes perfect correlation among assets. When all assets are perfectly correlated the allocation is proportional to the risk-budget and inversely proportional to the volatility. In our case that corresponds exactly to the approach taken in our benchmark solution.

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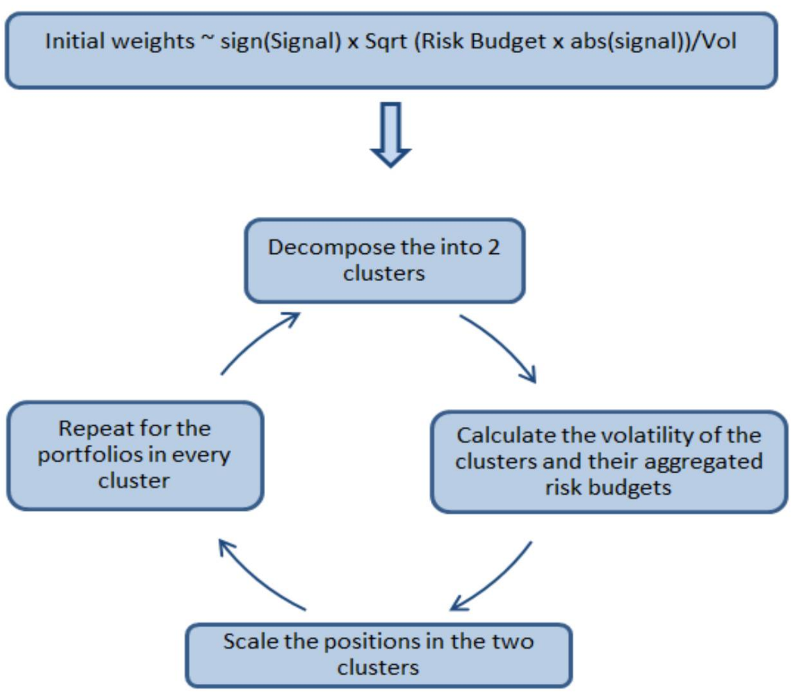
As within the standard approach we consecutively divide the universe into two clusters. But we use correlation based clustering to determine the two clusters. In our approach the number of assets does not have to be the same in the two clusters. Our experience with the original approach has been that highly-correlated assets might end up in different clusters (we have in particular experienced that for equities) and such a problem affects the quality of capital allocation which within HRP rests on the assumption that the two clusters have a low correlation between themselves.

Furthermore, as the trend-following system can take both long and short positions we need an adjustment in correlation matrix to reflect this fact. If we have a short signal in a certain asset we inverse the row and the column corresponding to this asset in the correlation matrix when we determine the clusters.

The starting point of our allocation is similar to the risk budgeting framework. The algorithm should allocate bigger positions to the assets with larger signals and bigger risk weights. Under a simplified assumption that the assets' returns are uncorrelated we assign initial weights that have the sign of the signal, proportional to the square root of the absolute value of the product of the signal and the risk weight and inversely proportional to the volatility[19]. Hence, in our case we can refer to the algorithm as Hierarchical Risk Budgeting (HRB) approach.

Figure 33:Flow-diagram of the Hierarchical Risk Budgeting approach



Source: J.P. Morgan Quantitative and Derivatives Strategy

Once we obtain the clusters we rescale the positions within the clusters taking into aggregated for the cluster risk-budget (which is the sum of the risk-budget for the assets in the cluster) and the volatility of the clusters. The whole algorithm runs via a recursive function until we have 2 assets left in a cluster[20].

Note that in the case of the HRB the danger of overleveraging in the case when the signals and the correlations of two assets point in different directions is less pronounced. If we have discrepancy in the direction of the signals and the correlations it

[19] As shown by Bruder and Roncalli (2012) if the assets are not correlated the optimal weight of asset i is $w_i = \frac{b_i/\sigma_i}{\sum_{j=1}^{n} b_j/\sigma_j}$, where b_i is the risk budget for asset i and σ_i is its volatility. In our case $b_i \sim abs(S_i) * RiskWeight_i$

[20] Please contact us if you need additional details with code implementation.

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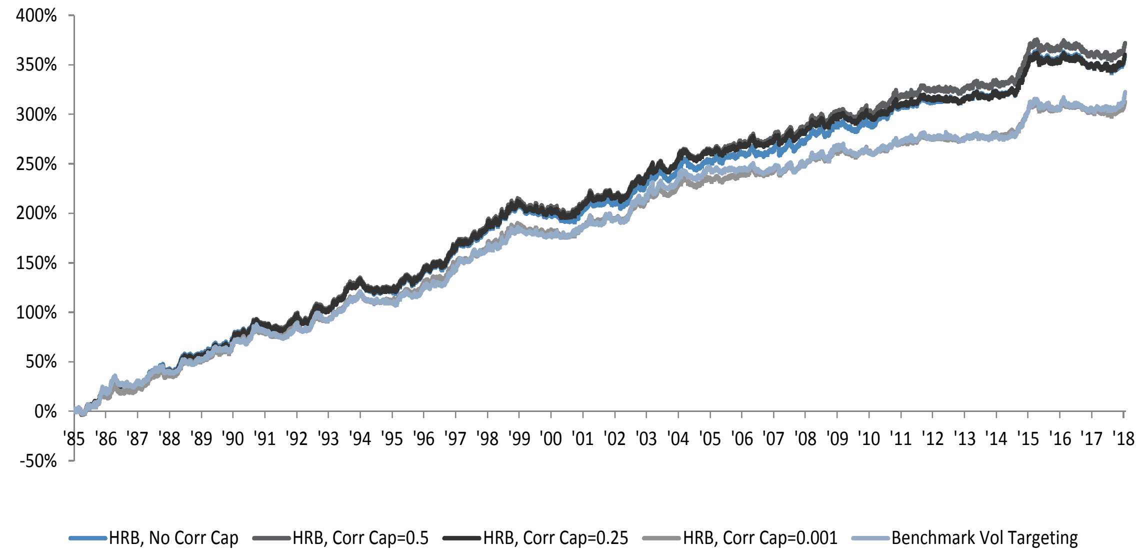
is quite likely that the assets will end up in different clusters during the initial split. Hence, the allocation algorithm will not take into account the offset as it will allocated directly to the clusters.

Such a conjecture is confirmed by the results shown below. We have experimented with various values for the correlation cap and there has not been a substantial improvement in the performance statistics. As in the case of risk-budgeting the Sharpe ratio has been increased but at the cost of a slightly higher drawdown.

Figure 34: Cumulative returns for various values of the correlation cap parameter



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 17: Performance statistics for various values of the cap parameter for Hierarchical Risk Budgeting approach

	HRB, No Corr Cap	HRB, Corr Cap=0.5	HRB, Corr Cap=0.25	HRB, Corr Cap=0.001	Benchmark Vol Targeting
Annualized Return	10.35%	10.70%	10.36%	8.99%	9.27%
Annualized Volatility	9.71%	9.53%	9.16%	7.94%	9.04%
Sharpe	1.07	1.12	1.13	1.13	1.03
Maximum Drawdown	-20.26%	-18.96%	-16.98%	-13.96%	-13.60%

Source: J.P. Morgan Quantitative and Derivatives Strategy

Controlling costs

A simple way to reduce turnover costs

As we have already mentioned our benchmark prototype solution includes a simple mechanism for control of costs. In simple terms, a trade is undertaken only if its size is sufficiently large. We compare the change in the position to the ratio of a cap parameter and the asset's volatility[21]. The values of the cap reflect the magnitude of the changes in the signal – for example 0.1, 0.2 etc. Below we have presented the simulation results for various values of the cap parameter (up to a maximum value of 0.25). It is obvious that even such a simple rule can improve performance with higher values of the cap parameter typically commanding a better Sharpe. We have used the value of 0.25 as our final choice as it improves the Sharpe ratio by around 10%. We do not think it is prudent to consider values beyond 0.25 as the agility of the system will start to be impacted.

[21] At every point in time, the absolute value of change in positon should be greater than $cap/vol(t)$.

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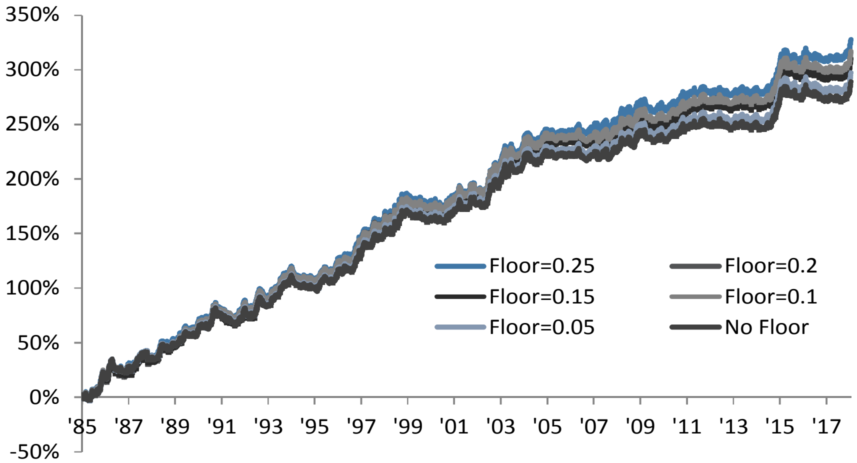
Figure 35: Cumulative returns for various values of the floor parameter



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 18: Performance statistics for various values of the floor parameter

	Floor=0.25	Floor=0.2	Floor=0.15	Floor=0.1	Floor=0.05	No Floor
Annualized Return	9.27%	9.11%	8.92%	9.12%	8.56%	8.32%
Annualized Volatility	9.04%	9.07%	9.06%	9.07%	9.05%	9.04%
Sharpe	1.03	1.00	0.99	1.01	0.95	0.92
Maximum Drawdown	-13.60%	-13.45%	-14.17%	-13.54%	-14.00%	-14.22%

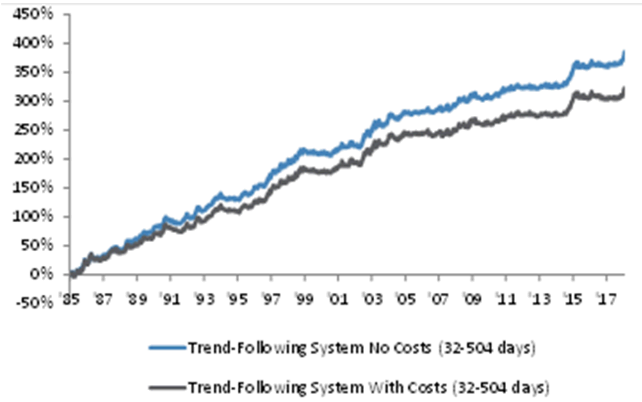
Source: J.P. Morgan Quantitative and Derivatives Strategy

Limiting costs in the absence of trends

In the theoretical parts we have shown that costs play much bigger role for the profitability of the trend-following strategies based on short-term lookback periods than for those based on long-term lookback periods. Below we have shown an empirical illustration of those results.

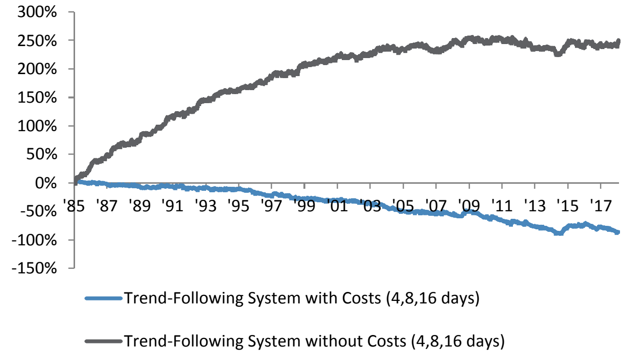
The performance of our benchmark trend-following system has just been marginally improved when no costs are assumed. In contrast the performance of a trend-following system based 4, 8 and 16 days lookbacks moves from attractive positive performance when no costs are accounted for to a negative performance when costs are taken into account. Our goal will be to control for the impact of costs and simultaneously trigger signals based on short term lookback periods when volatility spikes.

Figure 36: Cumulative returns of the benchmark solution with and without costs



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 37: Cumulative returns of a trend-following system based on 4,8 and 16 days with and without costs



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 19: Performance statistics for various versions of the trend-following strategy with and without costs

	Trend-Following System No Costs (32-504 days)	Trend-Following System with Costs (32-504 days)	Trend-Following System No Costs (4,8,16 days)	Trend-Following System with Costs (4,8,16 days)
Annualized Return	11.00%	9.27%	7.85%	7.85%
Annualized Volatility	9.10%	9.04%	9.08%	9.06%
Sharpe	1.21	1.03	0.86	-0.36
Maximum Drawdown	-11.60%	-13.60%	-28.53%	-89.17%

Source: J.P. Morgan Quantitative and Derivatives Strategy

The most unfavorable environment for the performance of the trend-following strategy is the lack of trends. As we have already discussed in the previous sections the problem becomes even more acute if volatility is low and the signals are generated with short-term lookback periods. Below we have plotted the expected transaction costs per year for various asset classes using as inputs the average transaction costs for the asset class (both execution and running fees) and the average asset class volatility for the whole sample as well as the volatility in 2017[22]. It is evident that signals based on short-term signals can lead to excessive losses in the absence of trends and the problem has been accentuated with the drop in volatility in 2017. In terms of costs FX and Equities seem to be the most expensive asset classes given the volatility structure that was in place in 2017.

[22] The figures have been calculated under the assumption that the targeted annualized volatility on the invested period is 10%.

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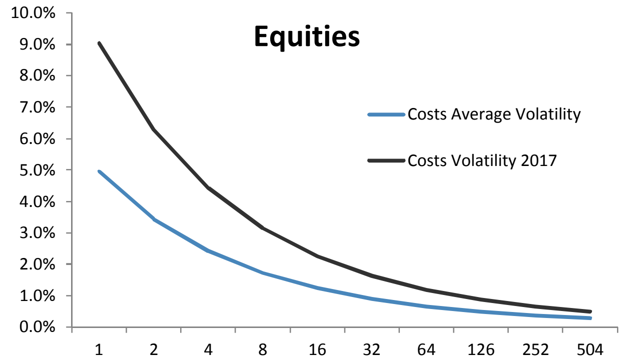
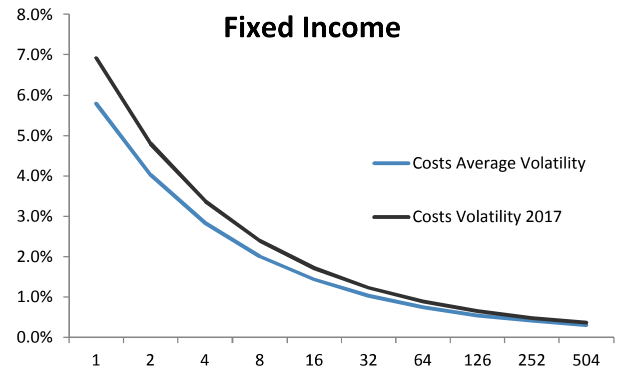
Figure 38: Expected transaction costs for equities as a percentage of capital in the absence of trends



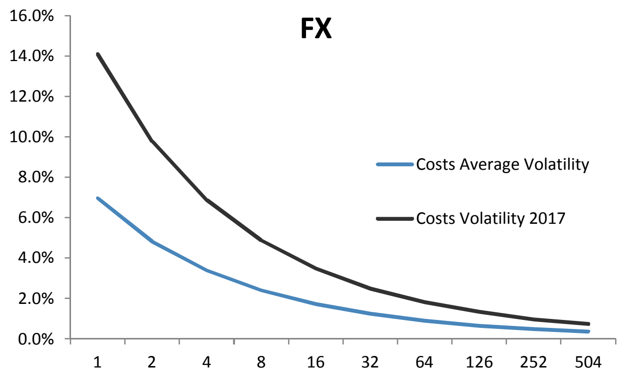
Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 39: Expected transaction costs for fixed income as a percentage of capital in the absence of trends



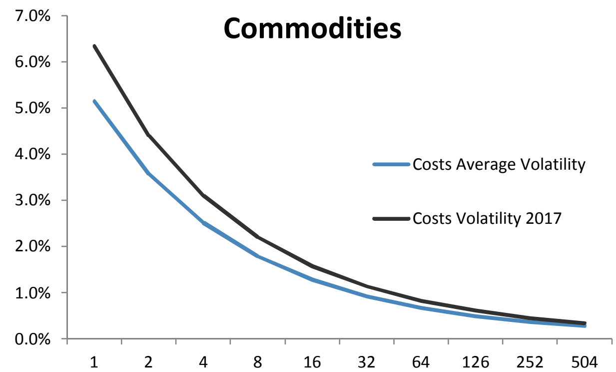
Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 40: Expected transaction costs for FX as a percentage of capital in the absence of trends



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 41: Expected transaction costs for commodities as a percentage of capital in the absence of trends



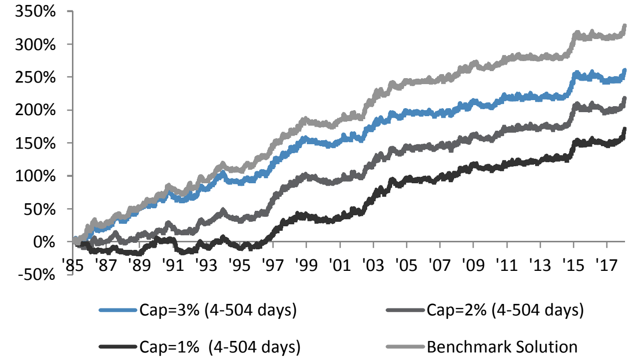
Source: J.P. Morgan Quantitative and Derivatives Strategy

We make use of our theoretical framework to limit the downside in the case of trendless. We expand our range of lookback periods to incorporate additionally 4, 8 and 16 days. Subsequently, we impose a cap on the costs calculated under the assumption that the market is trendless. In such way if the costs calculated for a given lookback period exceed the cap the signal based on that lookback period will not be taken into account when the aggregated signal is calculated.

We have experimented with various caps ranging from 1% to 3% and we have considered the performance from 1985 as well as from 2003 (when the current transaction structure comes in place).

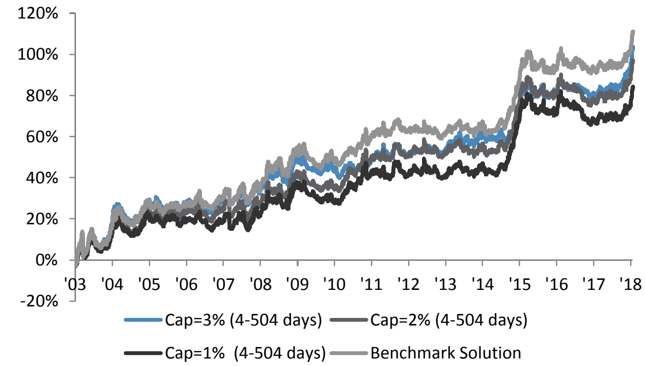
If we consider the track record since 1985 we can see that the trend-following systems that include short-term lookback windows and impose caps under-perform in the very first years. As we have substantially adjusted upwards the transaction costs in the past a cap approach can often switch off some signals. As the cap mechanism is often triggered some performance can be forgone when there are strong trends. In general the capping mechanism comes at a cost. It limits the losses in trendless markets but if the markets turn out to be strongly trending, potential profits will not materialize.

Figure 42: Cumulative returns for a trend-following system that includes short-term lookback periods since 1985



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 43: Cumulative returns for a trend-following system that includes short-term lookback periods since 2003



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 20: Performance statistics for a trend-following system that includes short-term lookback periods and caps costs

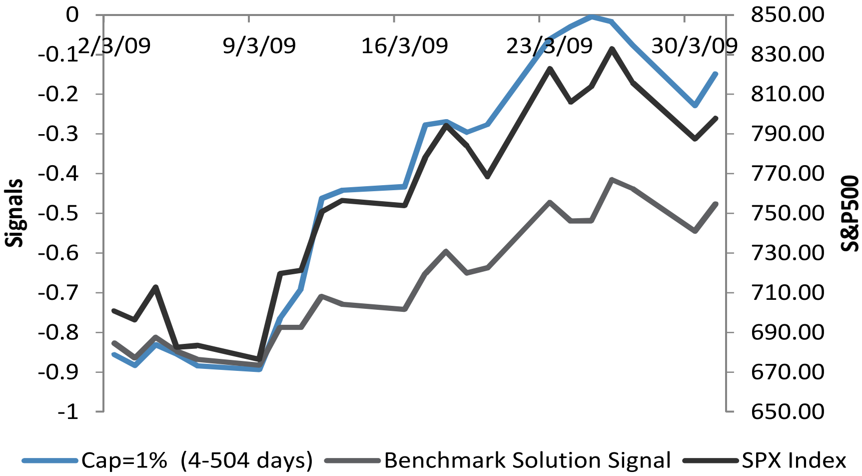
	Since 1985				Since 2003			
	Cap=1% (4-504 days)	Cap=2% (4-504 days)	Cap=3% (4-504 days)	Benchmark Solution	Cap=1% (4-504 days)	Cap=2% (4-504 days)	Cap=3% (4-504 days)	Benchmark Solution
Annualized Return	5.00%	6.27%	7.50%	9.27%	6.64%	6.23%	5.40%	7.12%
Annualized Volatility	8.37%	8.58%	8.74%	9.04%	9.04%	8.99%	8.96%	9.34%
Sharpe	0.60	0.73	0.86	1.03	0.73	0.69	0.60	0.76
Maximum Drawdown	-21.04%	-17.64%	-17.33%	-13.60%	-10.71%	-14.08%	-15.44%	-11.85%

Source: J.P. Morgan Quantitative and Derivatives Strategy

Looking at the period since 2003 when the current transaction cost structure comes in place we can see that trend-following systems that employ short-term lookback periods and control costs can generate performance similar to the benchmark solution. The 1% cap solution even has a more attractive drawdown measure (even after accounting for costs) though it might be considered excessively stringent.

Such a result is quite appealing as having a wide range of lookback windows leads to better diversification and a swifter reaction by the trend-following system at inflection points. Below we illustrate how a system with cap of 1% would have would have been much more reactive than the benchmark solution during the reversal in the S&P500 bearish trend in March'09.

Figure 44: Comparison between the S&P signals of the benchmark solution and Cap=1% (4-504 days) system during the reversal in March'09



Source: J.P. Morgan Quantitative and Derivatives Strategy

Taking carry into consideration

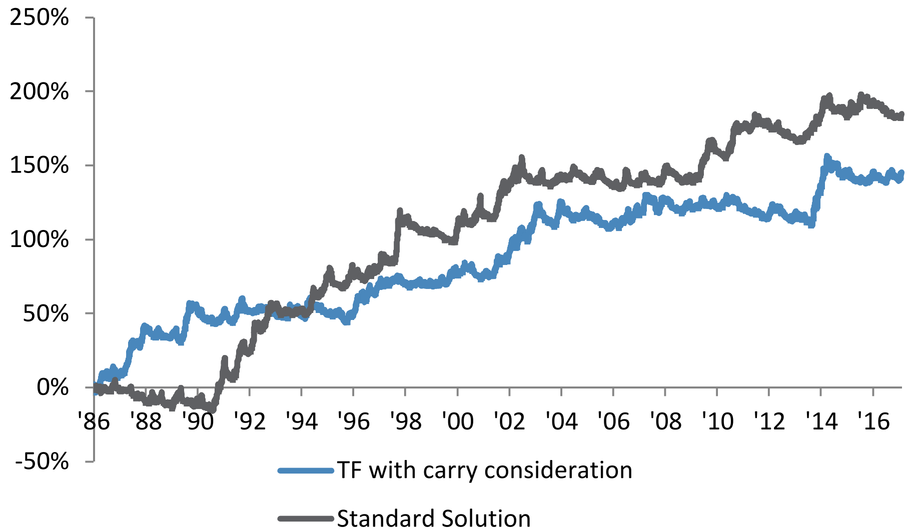
Often the trend-following strategies benefit from the carry present in the underlying futures or FX forwards. For example, the fixed income trend-following strategies have benefited substantially from keeping a long futures position and profiting on slide in the yield curve. Similarly in FX many of the high-yielding currencies have tended to appreciate. While trend-following in high carry assets on the long side is to some extent straightforward, having a short position in high carry assets might is challenging as the trend-following gain might not offset the loss due to negative carry. For example, the potential reversal in the bullish fixed income trend might pose challenges in front the trend-following systems.

Our framework is well-suited to take into account carry within trend-following. It can consider the carry as an additional component in the expected P&L calculation and hence link the strength of the signal, the current trend and the size of the carry within a unified framework.

Below we show an example of taking into account the carry component in FX. At every point in time, for every lookback period we calculate the expected P&L taking into account the carry and compare it to the expected costs. We use estimates of the asset drift, its volatility and assume that the autocorrelation is zero. If the expected net P&L is positive, we take into account the signal based on the respected lookback. In such a framework we will typically not go short assets with high carry if the signals are small.

The chart below compares the dynamic approach that takes into account carry to the standard approach. It is interesting that on occasions the two strategies can de-correlate (the average correlation is just 0.12).

Figure 45: Cumulative returns for a trend-following system in FX with and without carry considerations



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 21: Performance characteristics for a trend-following system in FX with and without carry considerations

	TF with carry consideration	Standard Solution
Return	4.47%	5.64%
Vol	8.76%	9.31%
Sharpe	0.51	0.61
Max DD	-20.25%	-20.28%

Source: J.P. Morgan Quantitative and Derivatives Strategy

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Appendix

Data Universe

	Bloomberg Ticker	Name of the Asset	Risk Weight	Asset Class Weight
Equities	**GX1 Index**	DAX Index	2.24%	25%
	VG1 Index	DJ Euro Stoxx 50	3.53%	
	Z 1 Index	FTSE100 Index	1.00%	
	ES1 Index	S&P 500 Index	12.06%	
	FTJGUSSE Index	Russell 2000 EMini	0.69%	
	NQ1 Index	Nasdaq 100 E-Mini	1.32%	
	NI1 Index	Nikkei 225 Index	1.08%	
	TP1 Index	OSE Japan Topix Index	0.88%	
	KM1 Index	KOSPI 200 Index	0.94%	
	HI1 Index	Hang Seng Index	1.26%	
Currencies	**EURUSDCR Index**	EUR Total Return	9.46%	25%
	GBPUSDCR Index	GBP Total Return	3.87%	
	SEKUSDCR Index	SEK Total Return	0.67%	
	CADUSDCR Index	CAD Total Return	1.54%	
	JPYUSDCR Index	JPY Total Return	6.75%	
	AUDUSDCR Index	AUD Total Return	2.07%	
	NZDUSDCR Index	NZD Total Return	0.63%	
Commodities	**CO1 Comdty**	Brent Crude	4.64%	25%
	CL1 Comdty	WTI Crude	9.75%	
	HO1 Comdty	Heating Oil	1.06%	
	XB1 Comdty	Gasoline	0.96%	
	NG1 Comdty	Natural Gas	1.77%	
	GC1 Comdty	Gold	2.89%	
	SI1 Comdty	Silver	0.60%	
	HG1 Comdty	Comex Copper	0.65%	
	C 1 Comdty	Corn	0.60%	
	S 1 Comdty	Soybean	2.09%	
Fixed Income	**ED4 Comdty**	Eurodollar	2.52%	25%
	TU1 Comdty	US Treasury Note 2Y	1.38%	
	FV1 Comdty	US Treasury Note 5Y	2.06%	
	TY1 Comdty	US Treasury Note 10Y	3.83%	
	US1 Comdty	US Treasury Long Bond	0.85%	
	DU1 Comdty	Euro Schatz (2y)	1.58%	
	OE1 Comdty	Euro Bobl (5y)	2.96%	
	RX1 Comdty	Euro Bund (10y)	5.04%	
	OAT1 Comdty	French Govt. Bonds (10y)	0.82%	
	IK1 Comdty	Italian Govt. Bonds (10y)	0.65%	
	G 1 Comdty	Long Gilt (10y)	0.60%	
	JB1 Comdty	Japanese Gov't Bond (10y)	1.47%	
	YM1 Comdty	Australian Gov't Bond (3y)	0.56%	
	XM1 Comdty	Australian Gov't Bond (10y)	0.66%	

Source: J.P. Morgan Quantitative and Derivatives Strategy

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Monthly Return Series

Table 22: Monthly returns of the benchmark trend-following strategy

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
1985	0.98%	2.30%	-4.55%	0.67%	6.77%	0.03%	-0.37%	2.97%	1.77%	8.72%	5.32%	-2.12%	**24.04%**
1986	-2.39%	8.93%	6.30%	-3.17%	-4.55%	1.63%	-1.67%	2.77%	-2.83%	0.34%	0.56%	2.47%	**7.79%**
1987	0.78%	0.35%	2.54%	1.49%	3.21%	2.99%	1.72%	-1.66%	2.60%	-4.18%	1.15%	0.80%	**12.18%**
1988	-1.16%	0.57%	1.44%	4.12%	8.00%	-2.16%	-1.75%	1.70%	-1.92%	2.30%	3.57%	-1.62%	**13.32%**
1989	0.77%	1.74%	3.57%	0.67%	5.40%	-0.35%	-0.46%	-1.08%	0.76%	-0.57%	0.78%	4.87%	**17.05%**
1990	3.85%	0.45%	-0.16%	2.69%	-5.34%	3.74%	4.04%	4.90%	6.66%	-5.43%	-0.88%	-0.03%	**14.57%**
1991	-1.67%	-1.60%	-0.15%	-0.76%	-0.71%	-1.30%	-1.45%	2.50%	2.80%	2.04%	1.93%	6.08%	**7.63%**
1992	-6.38%	-0.45%	0.50%	0.07%	1.85%	5.55%	6.03%	2.07%	-0.60%	-3.86%	-1.31%	0.99%	**3.88%**
1993	1.02%	4.98%	-0.03%	3.36%	1.00%	1.49%	5.17%	3.60%	-2.01%	3.82%	1.85%	4.35%	**32.31%**
1994	-3.63%	-3.09%	-0.87%	-1.02%	-1.41%	1.06%	-1.14%	-0.26%	0.23%	0.22%	-1.36%	-0.91%	**-11.62%**
1995	-0.03%	0.50%	6.09%	1.13%	3.26%	-0.72%	-1.06%	-1.81%	0.40%	1.32%	2.61%	6.05%	**18.86%**
1996	2.67%	-3.06%	2.05%	3.00%	-0.91%	1.91%	-3.42%	2.21%	5.91%	4.72%	7.10%	-0.59%	**23.17%**
1997	5.94%	1.14%	1.36%	1.10%	-1.44%	2.32%	9.63%	-5.66%	2.27%	-0.14%	0.96%	2.24%	**20.72%**
1998	1.35%	0.04%	2.82%	-2.72%	3.85%	1.30%	0.45%	8.85%	1.98%	-1.62%	2.03%	0.45%	**19.95%**
1999	0.03%	-1.51%	-2.03%	1.96%	-1.84%	0.33%	-1.73%	0.98%	-0.17%	-1.78%	1.15%	2.75%	**-1.99%**
2000	-0.83%	0.77%	-0.84%	-1.52%	-0.07%	-0.32%	-1.17%	4.59%	-0.89%	2.35%	2.76%	0.77%	**5.51%**
2001	1.47%	2.10%	4.90%	-5.16%	0.23%	-0.99%	1.28%	3.09%	6.26%	1.26%	-6.00%	0.22%	**8.23%**
2002	0.61%	-0.66%	-2.39%	0.98%	1.58%	7.84%	6.04%	2.72%	6.57%	-4.52%	-2.26%	7.69%	**25.90%**
2003	4.63%	4.95%	-4.11%	0.48%	6.56%	-1.19%	-3.57%	-0.26%	1.07%	1.18%	2.01%	6.58%	**19.12%**
2004	2.03%	4.38%	-0.71%	-4.95%	-0.62%	-1.17%	-0.45%	1.88%	2.69%	3.15%	3.15%	0.81%	**10.27%**
2005	-2.54%	0.90%	0.17%	-2.86%	0.65%	1.10%	1.24%	-0.10%	-0.21%	-2.15%	1.93%	-0.53%	**-2.49%**
2006	1.91%	-2.13%	2.56%	2.29%	-2.47%	-1.96%	-1.91%	-0.46%	0.31%	1.86%	1.47%	1.21%	**2.51%**
2007	2.24%	-4.26%	-0.91%	2.42%	3.48%	0.55%	-2.87%	-3.30%	4.28%	4.00%	-1.70%	0.61%	**4.13%**
2008	3.71%	4.71%	1.37%	-1.39%	1.77%	3.26%	-3.46%	-2.50%	-0.20%	6.14%	3.91%	2.49%	**21.13%**
2009	-0.03%	1.17%	-2.70%	-2.15%	-0.16%	-2.00%	-0.72%	0.15%	2.18%	-0.84%	4.56%	-5.09%	**-5.81%**
2010	-1.31%	1.12%	1.63%	1.96%	1.14%	1.48%	-1.34%	3.83%	0.39%	2.51%	-3.98%	3.96%	**11.69%**
2011	0.27%	3.07%	-1.62%	4.85%	-3.36%	-1.53%	3.99%	1.84%	0.66%	-3.82%	0.23%	0.74%	**5.03%**
2012	0.95%	0.62%	0.33%	-0.74%	3.34%	-4.68%	2.06%	-1.05%	0.13%	-2.43%	0.42%	1.60%	**0.28%**
2013	3.58%	-1.39%	2.01%	1.49%	-1.28%	-1.31%	-0.04%	-1.41%	-0.61%	1.78%	2.60%	1.00%	**6.42%**
2014	-4.31%	0.82%	-0.35%	0.56%	1.37%	2.84%	-3.46%	4.23%	5.94%	2.07%	8.65%	4.04%	**23.95%**
2015	9.62%	-1.13%	3.38%	-4.93%	1.09%	-2.75%	3.11%	-4.98%	2.06%	-2.15%	2.45%	-1.59%	**3.29%**
2016	6.16%	2.09%	-3.58%	-1.44%	-0.85%	3.46%	0.22%	-2.38%	-0.05%	-1.65%	-0.91%	0.77%	**1.46%**
2017	-1.58%	3.98%	-1.03%	-0.40%	0.37%	-0.95%	1.30%	0.02%	-0.87%	3.42%	3.27%	1.26%	**8.93%**

Source: J.P. Morgan

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Correlation between the P&L of Two Trend-Following Signals

Let's assume that the asset's returns R_t are i.i.d. variables from a normal distribution $N(0,\sigma^2)$.[23] Under such assumptions $d1_{t,T} = \frac{\ln\left(\frac{S_t}{S_{t-T}}\right)+\sigma^2 T/2}{\sigma\sqrt{T}} = \frac{\sum_{s=t-T+1}^{t} R_s}{\sigma\sqrt{T}} = \frac{\sqrt{T}\,\overline{R_{t,T}}}{\sigma}$, where $d1_{t,T}$ is the Black-Scholes $d1$ statistics calculated at time t, for an option of maturity T and strike S_{t-T} and $\overline{R_{t,T}}$ is the average asset return from $t-T+1$ to t. Note that $d1_{t,T} \sim N(0,1)$.

If we consider two lookback periods T_1 and T_2 ($T_1 < T_2$), it follows that $Correlation\left(d1_{t,T_1}, d1_{t,T_2}\right) = E\left(d1_{t,T_1} d1_{t,T_2}\right) = \sqrt{T_1/T_2}$.

Let $S_{t,T}$ denote the trend-following signal at time t based on lookback period T and $PL_{t+1,T}$ is the P&L at time *t+1* based on signal $S_{t,T}$. The position is proportional to the signal and inversely proportional to the volatility and hence $PL_{t+1,T} = R_{t+1} S_{t,T}/\sigma$. Furthermore, let Φ denote the c.d.f. of the standard normal distribution.

Now $E(PL_{t+1,T}) = 0$ and $Var(PL_{t+1,T}) = E((PL_{t+1,T})^2) = E(E_t((PL_{t+1,T})^2))=$

$= E((S_{t,T})^2) = E\left((2\Phi(d1_{t,T})+1)^2\right) = 4E((\Phi(d1_{t,T})^2) - 4E(\Phi(d1_{t,T}))+1$.

If $x \sim N(0,1)$ we can show that $E((\Phi(x))^2) = \int_{-\infty}^{+\infty} (\Phi(x))^2 f(x)dx = 1/3$.[24] Hence, $Var(PL_{t+1,T}) = \frac{1}{3}$.

It follows that:

$$\rho = correlation\left(PL_{t+1,T_1}, PL_{t+1,T_2}\right) = \frac{E\left(\left(2\Phi(d1_{t,T_1})-1\right)\left(2\Phi(d1_{t,T_2})-1\right)\right)}{\sqrt{Var\left(PL_{t+1,T_1}\right)Var\left(PL_{t+1,T_2}\right)}} = 12\left(\left(\Phi(d1_{t,T_1})\Phi(d1_{t,T_2})\right)\right) - 3$$

$d1_{t,T_1}$ and $d1_{t,T_2}$ have a bivariate normal distribution with mean vector $\bar{\mu}_1 = [0,0]$ and covariance matrix $\Sigma_1 = \begin{bmatrix} 1 & \sqrt{T_1/T_2} \\ \sqrt{T_1/T_2} & 1 \end{bmatrix}$. Using Lemma 1 in Harmann (2017), we can show that $E\left(\left(\Phi(d1_{t,T_1})\Phi(d1_{t,T_2})\right)\right) = P(x < 0, y < 0)$ where x and y have a bivariate normal distribution with mean vector $\bar{\mu}_2 = [0,0]$ and covariance matrix $\Sigma_2 = \begin{bmatrix} 2 & \sqrt{T_1/T_2} \\ \sqrt{T_1/T_2} & 2 \end{bmatrix}$.[25]

We can make use of the properties of the bivariate normal distribution and further conclude that $E\left(\left(\Phi(d1_{t,T_1})\Phi(d1_{t,T_2})\right)\right) = P(x<0,y<0) = P\left(\frac{x}{\sqrt{2}} < 0, \frac{y}{\sqrt{2}} < 0\right) = \frac{1}{4} + \frac{\operatorname{asin}(0.5*\sqrt{\frac{T_1}{T_2}})}{2\pi}$.[26] After simplification $\rho = 6\operatorname{asin}(0.5*\sqrt{\frac{T_1}{T_2}})/\pi$.

Note that the correlation is solely dependent on the ratio between the periods T_1 and T_2. If we target particular value of ρ, it follows that $T_1/T_2 = 4(\sin\left(\frac{\rho\pi}{6}\right))^2$.

---

[23] At first sight the assumption of a Gaussian white noise might seem restrictive. In practice the returns processes over different timeframes may differ. In extreme cases we can even have trends in opposite direction. Hence, we consider the assumption a good compromise. Later we show that it is also a realistic one as the theoretical and empirical correlation matrices are quite close.

[24] The result follows from integration by parts. Later we derive the same result as a particular case for the variance of P&L under AR(1) process.

[25] Alternatively, we can use the conditional distribution of $d1_{t,T_2}$ given $d1_{t,T_1}$ integrate out $d1_{t,T_2}$.

[26] The result can be found in Stuart and Ord (1998) and Rose and Smith (2002), p. 231.

The correlations can be considered relatively high. For example, for $\frac{T_1}{T_2} = 0.5$ it follows that ρ=0.69.

Expected (Gross) P&L when the Asset's Return Follows an AR(1) Process

Let's assume that $R_t = a + \rho R_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. It follows that $R_t \sim N\left(\frac{a}{1-\rho}, \frac{\sigma_\varepsilon^2}{1-\rho^2}\right) \sim N(\mu, \sigma^2)$.

We know that $PL_{t+1,T} = \frac{R_{t+1} S_{t,T}}{\sigma} = R_{t+1}\left(2\Phi(d1_{t,T}) - 1\right)/\sigma$.

For the subsequent derivations we need to find the correlation between R_{t+1}/σ and $d1_{t,T}$: $Correlation(\frac{R_{t+1}}{\sigma}, d1_{t,T}) = Cov\left(\frac{R_{t+1}}{\sigma}, d1_{t,T}\right)/\sqrt{Var(d1_{t,T})}$.

Proceeding further[27]:

$$Cov\left(\frac{R_{t+1}}{\sigma}, d1_{t,T}\right) = \frac{\sum_{s=t-T+1}^{t} Cov(R_{t+1} R_s)}{\sigma^2 \sqrt{T}} = \frac{1}{\sqrt{T}}(\rho + \rho^2 + \cdots + \rho^T) = \frac{\rho(1-\rho^T)}{\sqrt{T}(1-\rho)}.$$

The derivation of $Var(d1_{t,T})$ requires more algebraic operations:

$$Var(d1_{t,T}) = Var\left(\frac{\sum_{s=t-T+1}^{t} R_s}{\sigma\sqrt{T}}\right) = \frac{1}{T}(T + 2(T-1)\rho + 2(T-2)\rho^2 + \cdots. + 2\rho^{T-1})$$

After simplifications involving geometric series $Var(d1_{t,T}) = \frac{T(1-\rho^2) - 2\rho(1-\rho^T)}{T(1-\rho)^2}$.

Hence, $Correlation(\frac{R_{t+1}}{\sigma}, d1_{t,T}) = \frac{\rho(1-\rho^T)}{\sqrt{T(1-\rho^2) - 2\rho(1-\rho^T)}}$.

Now let's denote $X = \frac{R_{t+1}}{\sigma} \sim N(\mu/\sigma, 1)$ and $Y = d1_{t,T} \sim N(\mu_{d1,T}, \sigma_{d1,T}^2)$ with $\mu_{d1,T} = \frac{\sqrt{T}\mu}{\sigma}$ and $\sigma_{d1,T}^2 = Var(d1_{t,T})$. Note that X and Y are jointly bivariate normal with correlation $\phi = \frac{\rho(1-\rho^T)}{\sqrt{T(1-\rho^2) - 2\rho(1-\rho^T)}}$. Hence, $X|Y \sim N\left(\frac{\mu}{\sigma} + \frac{\phi(Y - \mu_{d1,T})}{\sigma_{d1,T}}, 1 - \phi^2\right)$ and $Y|X \sim N\left(\mu_{d1,T} + \sigma_{d1,T}\phi(X - \frac{\mu}{\sigma}), (1-\phi^2)\sigma_{d1,T}^2\right)$.

It follows that

$$E(PL_{t+1,T}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(2\Phi(y) - 1) f(x,y)\,dxdy =$$
$$= 2\int_{-\infty}^{\infty} \Phi(y) f(y) \int_{-\infty}^{\infty} x f(x|y)\,dxdy - \int_{-\infty}^{\infty} x f(x) \int_{-\infty}^{\infty} f(y|x)\,dydx$$

Let $X^* \sim N(0,1)$ and $Y^* \sim N(0,1)$. Making use of the conditional distributions of $X|Y$ and $Y|X$ and formulas 10010.8 and 10011.3 in Owen (1980) and with f denoting the c.d.f. of the standard normal distribution, we obtain:

[27] Note that in the following it is assumed that the volatility σ^2 of the AR(1) process is a known quantity. In an AR(1) the estimate of the volatility is asymptotically normal, $\hat{\sigma}^2 \sim N(\sigma^2, \frac{2\sigma^4(1+\rho^2)}{T(1-\rho^2)})$ (see Crack, T. and Ledoit, O. (2010)). For financial daily data $abs(\rho)$ is sufficiently small and the sample size dominates the error of the estimate. For example, if we assume $\rho = 0$, the standard error of the estimate will be less than 10% of the true value when T=252 days (1 year).

$$E(PL_{t+1,T}) = 2\int_{-\infty}^{\infty}\Phi(y)f(y)\left(\frac{\mu}{\sigma}+\frac{\phi(y-\mu_{d1,T})}{\sigma_{d1,T}}\right)dy - \int_{-\infty}^{\infty}xf(x)dx =$$

$$= 2\left(\frac{\mu}{\sigma}-\phi\frac{\mu_{d1,T}}{\sigma_{d1,T}}\right)\int_{-\infty}^{\infty}\Phi\left(\sigma_{d1,T}y^* + \mu_{d1,T}\right)f(y^*)dy^*$$

$$+2\frac{\phi}{\sigma_{d1,T}}\int_{-\infty}^{\infty}\Phi\left(\sigma_{d1,T}y^*+\mu_{d1,T}\right)f(y^*)(\sigma_{d1,T}y^*+\mu_{d1,T})\,dy^* - \frac{\mu}{\sigma} =$$

$$= 2\frac{\mu}{\sigma}\Phi\left(\frac{\mu_{d1,T}}{\sqrt{1+\sigma_{d1,T}^2}}\right) - \frac{\mu}{\sigma} + 2\phi\frac{\sigma_{d1,T}}{\sqrt{1+\sigma_{d1,T}^2}}f\left(\frac{\mu_{d1,T}}{\sqrt{1+\sigma_{d1,T}^2}}\right)$$

In case $\rho = 0$, it follows that $E(PL_{t+1,T}) = \frac{\mu}{\sigma}\left(2\Phi\left(\frac{\mu}{\sigma}\frac{\sqrt{T}}{\sqrt{2}}\right)-1\right)$.

Similarly, if $\mu = 0$, we obtain $E(PL_{t+1,T}) = \frac{2\rho(1-\rho^T)}{\sqrt{2\pi}\sqrt{2T(1-\rho)-2\rho(1-\rho^T)}}$.

Given that we will be using estimates of parameters of the AR(1) process, the uncertainty embedded in the estimates based on shorter periods is greater. Below we make use the Delta Theorem to approximate the volatility in our estimate of the expected P&L. For simplicity we will assume that the uncertainty arises only due to estimate $\hat{\mu}$ of the mean μ.[28] Let's assume that $f(\mu) = E(PL_{t+1,T})$. In an AR(1) process, $\sqrt{T}(\hat{\mu}-\mu) \sim N(0, \sigma^2\,(1+\rho)/(1-\rho))$.[29] From the Delta Theorem it follows that $\sqrt{T}\left(f(\hat{\mu}) - f(\mu)\right) \sim N(0, (f(\mu)')^2\sigma^2\,(1+\rho)/(1-\rho))$.

The derivative of the expected P&L with respect to μ can be derived straightforwardly as:

$$(f(\mu)') = \frac{2}{\sigma} * \Phi\left(\frac{\mu_{d1,T}}{\sqrt{1+\sigma_{d1,T}^2}}\right) + 2\frac{\mu}{\sigma}f\left(\frac{\mu_{d1,T}}{\sqrt{1+\sigma_{d1,T}^2}}\right)\frac{\frac{\sqrt{T}}{\sigma}}{\sqrt{1+\sigma_{d1,T}^2}} - \frac{1}{\sigma} - 2\phi\frac{\sigma_{d1,T}}{\sqrt{1+\sigma_{d1,T}^2}}f\left(\frac{\mu_{d1,T}}{\sqrt{1+\sigma_{d1,T}^2}}\right)\frac{\mu_{d1,T}}{1+\sigma_{d1,T}^2}\frac{\sqrt{T}}{\sigma}$$

Expected Transaction Costs when the Asset's Return is an AR(1) Process

Expected Running Costs

The running costs are proportional to the absolute nominal value of the position that we hold every day. If $RU_{t,T}$ denotes the running costs at time *t* for a signal based on a lookback of *T* days and RC stands for the per unit running cost then $RU_{t,T} = Abs\left(S_{t,T}\right) * RC/\sigma$. Subsequently,

$$E\left(RU_{t,T}\right) = \left(P(S_{t,T} > 0)E(S_{t,T}\middle|S_{t,T} > 0) + P(S_{t,T} < 0)E(-S_{t,T}\middle|S_{t,T} < 0)\right) * \frac{RC}{\sigma}$$

Introducing the standard normal variable $Z \sim N(0,1)$, we obtain:

[28] An alternative (at the cost of complexity) is to use the variance-covariance matrix of the estimates of the autoregressive process $(\hat{\mu}, \hat{\rho}, \hat{\sigma}^2)$ that can be obtained from Maximum Likelihood estimation and apply the Delta theorem accordingly.

[29] See Crack, T and Ledoit, O. (2004), "Central Limit Theorems When Data Are Dependent: Addressing the Pedagogical Gaps", Working Paper. Available at SSRN: https://ssrn.com/abstract=587562.

$$
\begin{aligned}
&P(S_{t,T}>0)E(S_{t,T}|S_{t,T}>0) = P(d1_{t,T}>0)E(2\Phi(d1_{t,T})-1|d1_{t,T}>0) = \\
&= 2P(d1_{t,T}>0)E(\Phi(d1_{t,T})|d1_{t,T}>0) - P(d1_{t,T}>0) = \\
&= 2P(Z<d1_{t,T}, d1_{t,T}>0) - \left(1-\Phi(-\mu_{d1,T}/\sigma_{d1,T})\right)
\end{aligned}
$$

We have shown in the previous section that if returns follow an AR(1) process $d1_{t,T} \sim N(\mu_{d1,T}, \sigma^2_{d1,T}) = N\left(\frac{\sqrt{T}\,\mu}{\sigma}, \frac{T(1-\rho^2)-2\rho(1-\rho^T)}{T(1-\rho)^2}\right)$. It follows that $X = Z - d1_{t,T} \sim N(-\mu_{d1,T}, \sigma^2_{d1,T}+1)$. Hence, $Cov(X, d1_{t,T}) = Cov\left((Z-d1_{t,T}), d1_{t,T}\right) = -Var(d1_{t,T})$ and $Corr(X, d1_{t,T}) = -\sigma_{d1,T}/\sqrt{\sigma^2_{d1,T}+1}$.

It follows that:

$$
\begin{aligned}
&P(Z<d1_{t,T}, d1_{t,T}>0) = P(Z<d1_{t,T}) - P(Z<d1_{t,T}, d1_{t,T}<0) = \\
&= P(X<0) - P(X<0, d1_{t,T}<0) = \\
&= \Phi\left(\frac{\mu_{d1,T}}{\sqrt{\sigma^2_{d1,T}+1}}\right) - BvN\left(\frac{\mu_{d1,T}}{\sqrt{\sigma^2_{d1,T}+1}}, -\frac{\mu_{d1,T}}{\sigma_{d1,T}}; corr = -\sigma_{d1,T}/\sqrt{\sigma^2_{d1,T}+1}\right)
\end{aligned}
$$

where $BvN(U, W; \rho)$ stands for the c.d.f of the standard bivariate normal distribution with correlation ρ evaluated at U and W.

Similarly,

$$
\begin{aligned}
&P(S_{t,T}<0)E(-S_{t,T}|S_{t,T}<0) = -P(d1_{t,T}<0)E(2\Phi(d1_{t,T})-1|d1_{t,T}<0) = \\
&= -2\,BvN\left(\frac{\mu_{d1,T}}{\sqrt{\sigma^2_{d1,T}+1}}, -\frac{\mu_{d1,T}}{\sigma_{d1,T}}; corr = -\sigma_{d1,T}/\sqrt{\sigma^2_{d1,T}+1}\right) + \Phi\left(-\frac{\mu_{d1,T}}{\sigma_{d1,T}}\right)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&E(RU_{t,T}) = (2\Phi\left(\mu_{d1,T}/\sqrt{\sigma^2_{d1,T}+1}\right) + 2\Phi(-\mu_{d1,T}/\sigma_{d1,T}) - \\
&4 * BvN(\mu_{d1,T}/\sqrt{\sigma^2_{d1,T}+1}, -\mu_{d1,T}/\sigma_{d1,T}; corr = -\sigma_{d1,T}/\sqrt{\sigma^2_{d1,T}+1}) - 1)RC/\sigma
\end{aligned}
$$

Under simplified assumptions that $\mu = 0$ and $\rho = 0$ (i.e. returns are a Gaussian noise), it follows that

$$E(RU_{t,T}) = -2\frac{\operatorname{asin}\left(-\frac{1}{\sqrt{2}}\right)}{\pi}\frac{RC}{\sigma} = \frac{1}{2} * \frac{RC}{\sigma}.$$

Note that in this case the expected running costs are independent of the lookback period. For example, if assume 10bp running fee per year and a volatility of 10%, the expected running costs are 0.3% per year.

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Expected Execution Costs

The execution costs are linked to the absolute value of the change in nominal position. If $XC_{t,T}$ denotes the execution costs at time t for a signal based on a lookback period of T and EC is the per unit execution cost then $XC_{t,T} = Abs\left(S_{t,T} - S_{t-1,T}\right) * EC/\sigma$.

Let start by analyzing the case when $S_{t,T} > S_{t-1,T}$. We are interested in the expression below:

$$E(S_{t,T} - S_{t-1,T} | S_{t,T} > S_{t-1,T}) P\left(S_{t,T} > S_{t-1,T}\right) =$$

$$= 2E(\Phi(d1_{t,T}) - \Phi(d1_{t-1,T}) | d1_{t,T} > d1_{t-1,T}) P\left(d1_{t,T} > d1_{t-1,T}\right) =$$

$$= 2P(d1_{t-1,T} < Z < d1_{t,T}) = 2P(d1_{t-1,T} - Z < 0, d1_{t,T} - Z > 0),$$

where $Z \sim N(0,1)$.

Let's assume that returns follow an AR(1) process, i.e. $R_t = a + \rho R_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. It follows that $R_t \sim N\left(\frac{a}{1-\rho}, \frac{\sigma_\varepsilon^2}{1-\rho^2}\right) \sim N(\mu, \sigma^2)$. Previously we have shown that $d1_{t,T} \sim N\left(\mu_{d1,T}, \sigma_{d1,T}^2\right) = N\left(\frac{\sqrt{T}\,\mu}{\sigma}, \frac{T(1-\rho^2) - 2\rho(1-\rho^T)}{T(1-\rho)^2}\right)$.

Let's denote $X = d1_{t,T} - Z$ and $Y = d1_{t-1,T} - Z$. It follows that $X \sim N\left(\mu_{d1,T}, \sigma_{d1,T}^2 + 1\right)$ and $Y \sim N\left(\mu_{d1,T}, \sigma_{d1,T}^2 + 1\right)$.

Subsequently,
$Cov\left(d1_{t,T}, d1_{t-1,T}\right) = Cov\left(\left(\frac{R_t - R_{t-T}}{\sigma\sqrt{T}} + d1_{t-1,T}\right), d1_{t-1,T}\right) = \frac{1}{\sigma\sqrt{T}} E\left(R_t d1_{t-1,T}\right) - \frac{1}{\sigma\sqrt{T}} E(R_{t-T} d1_{t-1,T}) + \sigma_{d1,T}^2$.

Proceeding further $\frac{1}{\sigma\sqrt{T}} E\left(R_t, d1_{t-1,T}\right) = \frac{\sum_{s=t-T}^{t-1} E(R_t R_s)}{\sigma^2 T} = \frac{(\rho + \rho^2 + \dots + \rho^T)\sigma^2 + T\mu^2}{\sigma^2 T} = \frac{\rho(1-\rho^T)}{T(1-\rho)} + \frac{\mu^2}{\sigma^2}$. Similarly, $\frac{1}{\sigma\sqrt{T}} E\left(R_{t-T}, d1_{t-1,T}\right) = \frac{\sum_{s=t-T}^{t-1} E(R_{t-T} R_s)}{\sigma^2 T} + \frac{\mu^2}{\sigma^2} = \frac{(1 + \rho^2 + \dots + \rho^{T-1})\sigma^2 + T\mu^2}{\sigma^2 T} = \frac{(1-\rho^T)}{T(1-\rho)} + \frac{\mu^2}{\sigma^2}$. It follows that $Cov\left(d1_{t,T}, d1_{t-1,T}\right) = -\frac{1-\rho^T}{T} + \sigma_{d1,T}^2$. Hence, $Cov(X,Y) = Cov\left(d1_{t,T}, d1_{t-1,T}\right) + 1$ and $Corr(X,Y) = 1 - \left(\frac{1-\rho^T}{T}\right)/(\sigma_{d1,T}^2 + 1)$.

We can proceed similarly for the case when the position is decreasing:

$$E(S_{t-1,T} - S_{t,T} | S_{t,T} < S_{t-1,T}) P\left(S_{t,T} < S_{t-1,T}\right) = 2P(X < 0, Y > 0)$$

Subsequently, making use of the bivariate normal distribution:

$$E(XC_{t,T}) = 2 * \left(P(Y < 0, X > 0) + P(X < 0, Y > 0)\right) * \frac{EC}{\sigma} =$$

$$= 4 * \left(\Phi\left(\frac{-\mu_{d1,T}}{\sqrt{\sigma_{d1,T}^2 + 1}}\right) - BvN\left(\frac{-\mu_{d1,T}}{\sqrt{\sigma_{d1,T}^2 + 1}}, \frac{-\mu_{d1,T}}{\sqrt{\sigma_{d1,T}^2 + 1}}; corr = 1 - \frac{\left(\frac{1-\rho^T}{T}\right)}{1 + \sigma_{d1,T}^2}\right)\right) \frac{EC}{\sigma}$$

In the special case when returns are a Gaussian white noise, it follows that $Corr\left(d1_{t,T}, d1_{t-1,T}\right) = 1 - 1/(2T)$. Using the properties of the bivariate normal distribution, in this case we obtain:

$E(XC_{t,T}) = 4 * (0.25 - asin(1 - 1/(2T))/\,(2\pi)\,) EC/\sigma = \frac{2EC}{\pi\sigma} \text{acos}(1 - 1/(2T))$

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P&L Volatility under AR(1) Return Dynamics

The derivation of the P&L volatility under the general assumption of an AR(1) return process is quite evolved. We prefer to evaluate numerically $E\left(PL_{t,T}^2\right)$ when needed.

It is straightforward to calculate the P&L volatility when return process is a Gaussian white noise with a drift ($\rho = 0$). Let's again use the notations $X = \frac{R_{t+1}}{\sigma} \sim N(\mu/\sigma, 1)$ and $Y = d1_{t,T} \sim N(\mu_{d1,T}, \sigma_{d1,T}^2)$. It follows that

$$E\left(PL_{t+1,T}^2\right) = E(x^2(2\Phi(y)-1)^2) = E(x^2)E(4\big(\Phi(y)\big)^2 - 4\big(\Phi(y)\big) + 1)$$

$$E(x^2) = 1 + (\mu/\sigma)^2$$

Let $Y^* \sim N(0,1)$ and making use of formulas 10010.8 and 20010.3 in Owen (1980):

$$E\left(\left(\Phi(Y)\right)^2\right) = \int_{-\infty}^{\infty}\left(\Phi(y)\right)^2 f(y)dy = \int_{-\infty}^{\infty}\left(\Phi(\mu_{d1,T} + \sigma_{d1,T}y^*)\right)^2 f(y^*)dy^* = \, = BvN(\mu_{d1,T}/\sqrt{1+\sigma_{d1,T}^2},$$
$$\mu_{d1,T}/\sqrt{1+\sigma_{d1,T}^2}\; ; corr = \sigma_{d1,T}^2/(1+\sigma_{d1,T}^2))$$

$$E\left(\Phi(Y)\right) = \Phi\left(\mu_{d1,T}/\sqrt{1+\sigma_{d1,T}^2}\right)$$

If returns are a Gaussian white noise ($\mu = 0$ and $\rho = 0$), the P&L volatility is independent of the lookback period.[30]

$$Var\left(PL_{t,T}\right) = E\left(PL_{t,T}^2\right) = 4BvN\left(0,0; corr = \frac{1}{2}\right) - 4\Phi(0) + 1 = \frac{2\,\mathrm{asin}\left(\frac{1}{2}\right)}{\pi} = 1/3$$

[30] The result was also shown in the section 'Correlation between the P&L of Two Trend-Following Signals'.

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