

This assignment is marked out of 100. Marks for each part are indicated below. This assignment is to be completed individually. Present your results as two brief reports, one for each of the following questions.

QUESTION 0.1. Consider the following boundary value problem:

$$\begin{aligned} -y'''(x) + y''(x) - 4xy'(x) + (8x + 3)y(x) &= x^2, \\ y(-1) &= -10, \\ y(0.5) &= 1, \\ y(1.5) &= -3. \end{aligned}$$

- Using shooting method, 4th order Runge-Kutta, with $h = 0.01$, solve the BVP, explain your approach, plot your solution, and provide *Python* code. (20 Marks) *To help tutors with marking, include the output of the guess $y'(-1) = 1$, $y''(-1) = 1$. Try half a dozen or so trial and error guesses for the boundary conditions.*
- Investigate using the Lehmer-Shur algorithm to solve this BVP with shooting method. (5 Marks) *The marks for this are for your attempt at getting initial conditions to converge. There are several ways to do this including generalised bisection.*
- Solve the BVP using difference method with $h = 0.1$, explain your approach, plot your solution, and provide *Python* code. (30 Marks) *Expect to get more unknowns than equations, depending on which derivative approximations you use. Decide what to do about this.*
- Solve the following boundary value problem (10 Marks) using difference method

$$\begin{aligned} y''(x) + (\cos(y(x)) + 2)^2 - 9x^3y(x)^2 &= 0, \\ y(1) &= 1, \\ y(1.8) &= 3. \end{aligned}$$

Nonlinear systems of equations may have multiple solutions...

QUESTION 0.2. Use Newton's method to find two whole numbers x, y satisfying the system of equations

$$\begin{aligned} (1) \quad x^3 + 3x^2y - 2xy^2 - 7y^3 &= -604894015496000, \\ (2) \quad -15x^2 - 57xy - 67y^2 &= -26864190700. \end{aligned}$$

- Provide your solution, a detailed discussion of how the method works, and your *Python* code. (20 Marks) *Find complex solutions. I miss-typed the system of equations. There is no need to rewrite code to accommodate complex numbers. Just make you guesses complex.*
- Experimentally, how many solutions does this system have in \mathbb{R}^2 ? (5 Marks) *Try different initial guesses to decide experimentally how many solutions there might be in \mathbb{C}^2 .*
- How many solutions does this system have in \mathbb{R}^2 theoretically by writing $y = f(x)$ in Eqn. (2)? (5 Marks) *Think of quadratic formula for Eqn. (2).*
- Write *Maxima* code to solve the system for two whole numbers x, y by brute force search. How long did it take to find a solution using this method? (5 Marks) *Look in Maxima's help file to work out how to use 'if', 'then', 'for', 'and'. Try looking for solutions to $x^2 - 5y^2 = 4$ over \mathbb{Z}^2 , there are a few, by trying out $(1, 1)$, $(1, 2)$, $(1, 3), \dots$, $(2, 1)$, $(2, 2)$, $(2, 3), \dots$. Then look for simultaneous solutions to the system $x^2 - 5y^2 = 4$ and $x^2 - 13y^2 = 4$. Once it works, replace these equations with Eqn.s (1) and (2). Maybe don't press shift enter though!*