Homework 4, Math 181A Winter 2023

Due by Saturday noon, February 4 (pacific time).

Relevant section in textbook by Larsen and Marx: 5.4, 5.5.

Relevant lecture notes: Lecture 8, Lecture 9 and Lecture 10.

Problem 1: Larsen and Marx question 5.4.19.

Problem 2: Larsen and Marx question 5.4.20. (Add: first show that the two estimators in the problem are unbiased.)

Problem 3: Larsen and Marx question 5.4.22.

Problem 4: Suppose $X_1, \ldots, X_n$ are i.i.d. random variables with $\mathbb{E}[X_i] = \theta$ and $\mathrm{Var}(X_i) = \sigma^2$. Let $\hat{\theta}_1 = \bar{X}$. Suppose $\hat{\theta}_2$ is another estimator based on $X_1, \ldots, X_n$, which is known to be unbiased for θ. Assume that $\mathrm{Var}(\hat{\theta}_2) = \mathrm{Var}(\hat{\theta}_1)/2$ and $\mathrm{Cov}(\hat{\theta}_1, \hat{\theta}_2) = \mathrm{Var}(\hat{\theta}_1)/3$. Consider the estimator $\hat{\theta} = c\hat{\theta}_1 + (1-c)\hat{\theta}_2$, where $c \in [0, 1]$.

(a) Calculate the bias, variance, and mean squared error of $\hat{\theta}$. Hint: You shall use the fact $\mathrm{Cov}(aX, bY) = ab\ \mathrm{Cov}(X, Y)$ for any random variables X, Y and real numbers a, b.

(b) For what value of c is the mean squared error of $\hat{\theta}$ the smallest?

Problem 5: Larsen and Marx question 5.5.1.

Problem 6: Larsen and Marx question 5.5.2.