Homework 7, Math 181A Winter 2023

Due by Saturday noon, March 4 (pacific time).
Relevant section in textbook by Larsen and Marx: 6.3, 6.4.
Relevant lecture notes: Lecture 16, Lecture 17 and Lecture 18

Problem 1: Larsen and Marx question 6.3.9 (use exact binomial test). Change subproblem (b) to "Compute the power of the test when $p = 0.65$."

Problem 2: We are going to perform an exact binomial test for the population proportion. Let $H_0 : p = 0.4$ and we get a sample of size 10. The test statistic is $X = X_1 + \ldots + X_{10}$. Below is the table of probability mass function for Binomial(10, 0.4).

k	0	1	2	3	4	5	6	7	8	9	10
$P(X = k)$	0.006	0.040	0.121	0.215	0.251	0.201	0.111	0.042	0.011	0.002	0.000

(a) If $H_1 : p \neq 0.4$, what is the critical region for the test statistic X so that the significance level α is closest to but does not exceed 0.1?
(b) If $H_1 : p > 0.4$, what is the critical region for the test statistic X so that the significance level α is closest to but does not exceed 0.1?

Problem 3: Larsen and Marx question 6.4.3. You should first do question 6.2.2 with $z_{\alpha/2} = 1.88$ for $\alpha = 0.06$.

Problem 4: Larsen and Marx question 6.4.7.

Problem 5: Larsen and Marx question 6.4.18.

Problem 6: Larsen and Marx question 6.4.20.

R Simulation: Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Test $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$. Define the effect size $\delta = \frac{\mu - \mu_0}{\sigma}$, the number of standard deviations the true mean is away from the tested one.
(a) If it is known that $\sigma^2 = 1$, given $\alpha = 0.05$, derive a formula which shows how the power

depends on the effect size δ and sample size n. You can use the notation $\Phi(x)$ to indicate the cumulative distribution function of the standard normal distribution.

(b) Plot δ (x-axis with range $[-2, 2]$) versus power (y-axis) for $n = 10$ and 40 in one figure. Comment on how the power changes with δ and n from the plot.