Instructions

- **Final Part A receipt.** This **Final Part A** is available for download from **Gradescope** at **3:00 pm** Pacific Time on Wednesday **June 8**. In the unlikely event that you cannot download the exam at that time you should email both Prof. Chow at <bechow@ucsd.edu> and TA Zhiyuan Jiang <z5jiang@ucsd.edu> to receive an emailed copy of the exam. If you are taking the final asynchronously, you will be able to download the final at your given start time.
- **Exam format.** There are **4 questions**, worth 10 points each. Total: **40 points**. Time: Your exam will not be counted late if uploaded by **4:15 pm**.
- **Format for your solutions.** You should hand write your solutions neatly and darkly on your own blank or lined 11”×8.5” paper or on your iPad/tablet in an organized way. Do not type solutions.
- **Allowed resources.** The resources you may use are: Hubbard and Hubbard’s book, any materials on the canvas site, piazza discussion, and the google drive, all for this course only. Students cannot communicate with each other or any people by any means. E.g., no use of Discord, texting, messaging, etc. is allowed. You may not search the internet outside the above resources. You may not use any calculator or mathematical software except for addition, multiplication, subtraction, and division.
- **Submission.** Please make sure your files are legible before submitting and carefully follow the **Gradescope** instructions to match every question to the appropriate pages of your submission.
- **Deadline.** The deadline to upload your Final Part A solutions to **Gradescope** is 4:15 pm (Pacific Time) on June 8. Submissions after 4:15 pm will be deducted 1/2 point per minute late, with no Final Part A’s accepted after 4:30 pm. Email your exam to both <bechow@ucsd.edu> and <z5jiang@ucsd.edu> if you feel you cannot upload to gradescope in time to timestamp your submission; after that, upload to Gradescope as soon as possible.

1. (10 points) Let $S = \left\{(u,v) \in \mathbb{R}^2 \,\middle|\, 0 \le u \le \sin(v),\ 0 \le v \le \pi/4\right\}$. Define the map $\Phi : S \to \mathbb{R}^2$ by $\Phi \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u^2 + e^v \\ \sin(v) \end{pmatrix}$. You may assume WITHOUT proof that Φ satisfies all of conditions of the change of variables formula such as being one-to-one. Compute the integral
$$\int_{\Phi(S)} y \, dx \, dy.$$

2. (10 points) Let $U = \left\{(u_1,u_2) \in \mathbb{R}^2 \,\middle|\, 0 \le u_2 \le u_1,\ 0 \le u_1 \le 1\right\}$. Compute the area (2-dimensional volume) of the parametrized surface $M = \gamma(U) \subset \mathbb{R}^3$, where
$$\gamma \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 2u_1 \\ -2\sqrt{3}u_1 \\ e^{-u_2} \end{pmatrix}. \tag{1}$$

3. (10 points) Define a vector field on $\mathbb{R}^3$ by
$$\vec{F} = \begin{bmatrix} x^2 + f(y,z) \\ y^2 + g(x,z) \\ z^2 + h(x,y) \end{bmatrix},$$
where f, g, h are each smooth functions of two variables. Let $W = [0,a] \times [0,b] \times [0,c]$ be a solid rectangular region in $\mathbb{R}^3$, where $a, b, c > 0$. Compute the surface integral $\int_{\partial W} \Phi_{\vec{F}}$, where ∂W has its usual orientation (associated to the outward pointing normal).

4. (10 points) Define
$$M = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \,\middle|\, \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,\ z \ge 0 \right\},$$
where $a, b, c > 0$, which is a compact surface with boundary. Orient M by the outward pointing normal $\vec{N}$ (which has positive z-component when $z > 0$). Parametrize the boundary ∂M by the map
$$\gamma(t) = (a\sin(t), b\cos(t), 0), \quad 0 \le t \le 2\pi.$$
Is the map γ orientation-preserving for the orientation on ∂M induced by the orientation on M?