

# Homework 1, Math 181A Winter 2023

Due by Saturday noon, January 14 (pacific time).

Relevant section in textbook by Larsen and Marx: 3.2-3.7, 3.9, 3.11, 4.2-4.4.

Relevant lecture notes: Lecture 1 and Lecture 2.

**Problem 1:** Larsen and Marx question 3.6.6.

**Problem 2:** Larsen and Marx question 3.9.20.

**Problem 3:** Larsen and Marx question 4.3.33.

**Problem 4:** Larsen and Marx question 4.3.34.

**Problem 5:** Apply Chebyshev's inequality to prove the Weak Law of Large Numbers for the sample mean of i.i.d. random variables with a finite variance.

## R simulation:

Let  $X_1, \dots, X_n$  be i.i.d. random variables from a uniform distribution on  $[0, 2]$ . Generate and plot 10 paths of sample means from  $n = 1$  to  $n = 40$  in one figure for each case. Describe what you can observe from each plot and relate to the theory in this specific context to empirically check the Law of Large Numbers.

(a) When  $n$  is large,

$$\frac{X_1 + \dots + X_n}{n} \text{ converges to } E[X_i].$$

(b) When  $n$  is large,

$$\frac{X_1^2 + \dots + X_n^2}{n} \text{ converges to } E[X_i^2].$$