Homework due Friday, October 27, at 11:00 pm Pacific Time.

A. Let (X, d) be a metric space. For any two nonempty subsets $A, B \subset X$ define

$$d(A, B) = \inf\{d(x, y) : x \in A, y \in B\}.$$

Note that if $A \cap B \neq \emptyset$, then $d(A, B) = 0$. Prove or provide a counter example to the following statements.

(1) If A and B are two disjoint subsets of X, then $d(A, B) > 0$.
(2) If A and B are two separated subsets of X, then $d(A, B) > 0$. (Two subsets are called separated if $A \cap \bar{B} = \emptyset$ and $B \cap \bar{A} = \emptyset$)
(3) If A and B are two disjoint open subsets of X, then $d(A, B) > 0$.
(4) If A and B are two disjoint closed subsets of X, then $d(A, B) > 0$.

B.

(1) Let (X, d) be a metric space. Prove that the closed neighborhood

$$\overline{N}_r(x) = \{y \in X : d(x, y) \leq r\}$$

is a closed set.
(2) Let $(\mathbb{R}^n, d)$ with the standard metric. Prove that the closure of the open neighborhood $N_r(x)$ is the closed neighborhood.
(3) Is it true in general that the closure of the open neighborhood $N_r(x)$ is the closed neighborhood? If true, prove it and if false, give a counter example?

C. Let (X, d) be a metric space. Prove or disprove: $A \subset X$ is dense in X if and only if $A \cap O \neq \emptyset$ for all non-empty open subsets $O \subset X$.

D. Rudin, Chapter 2 (page 43), problems # 9, 22.

The following problems are for your practice, and will not be graded.

(1) Show that every closed subset of $\mathbb{R}^n$ is an intersection of countably many open sets.
(2) Let (X, d) be a metric space. If A and B are two disjoint compact subsets of X, then $d(A, B) > 0$.