MATH 170B HOMEWORK 2

Interpolation

§1: Find the polynomial of least degree that interpolates this set of data:

x	3	7	1	2
y	10	146	2	1

§2: Prove that if g interpolates the function f at $x_0, \ldots, x_{n-1}$ and if h interpolates f at $x_1, \ldots, x_n$, then the function

$$g(x) + \frac{x_0 - x}{x_n - x_0}[g(x) - h(x)]$$

interpolates f at $x_0, \ldots, x_n$. Notice that g and h need not be polynomials.

§3: Let p be a polynomial of degree $\leq n-1$ that interpolates the function $f(x) = \sinh x$ at any set of n nodes in the interval $[-1, 1]$, subject only to the condition that one of the nodes is 0. Prove that the error obeys this inequality on $[-1, 1]$:

$$|p(x) - f(x)| \leq \frac{2^n}{n!}|f(x)|$$

§4: Find the Lagrange and Newton forms of the interpolating polynomial for these data:

x	-2	0	1
$f(x)$	0	1	-1

Write both polynomials in the form $a+bx+cx^2$ to verify that they are identical as functions.

§5: Using the functions L_i defined as below:

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j} \qquad \text{for } 0 \leq i \leq n$$

and based on nodes $x_0, \ldots, x_n$, show that for any f,

$$\sum_{i=0}^{n} f(x_i)L_i(x) = \sum_{i=0}^{n} f[x_0, \ldots, x_i] \prod_{j=0}^{i-1}(x - x_j).$$

§6: (Continuation) Prove this formula:

$$f[x_0, \ldots, x_n] = \sum_{i=0}^{n} f(x_i) \prod_{\substack{j=0 \\ j \neq i}}^{n} (x_i - x_j)^{-1}$$

§7: Use divided differences to write the Newton interpolating polynomial for these data:

x	4	2	0	3
$f(x)$	63	11	7	28