MATH 170C HOMEWORK 4

(1) (§8.6, 3) Write the third-order ordinary differential equation

$$\begin{cases} x''' + 2x'' - x' - 2x = e^t \\ x(8) = 3, \qquad x'(8) = 2, \qquad x''(8) = 1 \end{cases}$$

as an autonomous (time-independent) system of first-order equations.

(2) (§8.7, 8) Illustrate Theorem 2 with the boundary-value problem

$$\begin{cases} x'' = t + x^2 - 3x' \\ x(3) = \alpha, \qquad x(7) = \beta \end{cases}$$

Show that it is equivalent to

$$\begin{cases} x'' = 48 + 64t + 16x^2 - 12x' \\ x(0) = \alpha, \qquad x(1) = \beta \end{cases}$$

(3) (§8.8, 4) Show how the shooting method can be used to solve a two-point boundary-value problem of the following type, in which the constants α, β, and c_{ij} are all given:

$$\begin{cases} x'' = u(t) + v(t)x + w(t)x' \\ c_{11}x(a) + c_{12}x'(a) = \alpha \\ c_{21}x(b) + c_{22}x'(b) = \beta \end{cases}$$

Hint: Let x_1 solve the inhomogeneous differential equation with initial conditions $x_1(a)$, $x_1'(a)$ specified in such a way that $c_{11}x_1(a) + c_{12}x_1'(a) = \alpha$. Let x_2 solve the homogeneous differential equation with initial conditions $x_2(a) = -c_{12}$ and $x_2'(a) = c_{11}$. Consider $x_1 + \lambda x_2$.