**Homework due Friday, November 3, at 11:00 pm Pacific Time.**

A. Let $(X, d)$ denote a set $X$ equipped with discrete metric. Show that $A \subset X$ is compact if and only if it is finite.

B. Rudin, Chapter 2 (page 43), problems #12, 23, 24, 25, 26.

The following problems are for your practice, and will not be graded.

(1) Let $(X, d)$ be a metric space and let $K \subset X$ be a compact set. Let $x \in K^c$. Define $d(x, K)$ as follows

$$d(x, K) = \inf\{d(x, y) : y \in K\}.$$

Prove: there exists some $z \in K$ so that $d(x, K) = d(x, z)$.

(2) Let $\{x_1, x_2, \ldots\}$ be a countable subset of $\mathbb{R}$. Assume that there are finitely many points $y_1, \ldots, y_m \in \mathbb{R}$ so that the following holds.

$$\forall \epsilon > 0,\ \exists N \text{ such that } \forall n > N,\ \exists 1 \le \ell \le m,\ |x_n - y_\ell| < \epsilon.$$

Set $K = \{x_1, x_2, \ldots\} \cup \{y_1, \ldots, y_m\}$.

(a) Prove that $K$ is compact.

(b) Using the definition of a compact set, prove that $K$ is compact. (In part (b) you are supposed to use the definition and not the Heine-Borel theorem.)

(3) Let $(X_1, d_1)$ and $(X_2, d_2)$ be two compact metric spaces. Let $X = X_1 \times X_2$ and define

$$d\big((x_1, x_2), (y_1, y_2)\big) = \max\{d_1(x_1, y_1), d_2(x_2, y_2)\}.$$

Prove that $(X, d)$ is a compact metric space.