Math 20D HW5

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Section 4.7

Problem 3 The functions are continuous for $t \in (0, \infty)$, so a unique solution exists.
Problem 10 The associated characteristic equation is

$$r^2 + r - 6 = (r+3)(r-2) \implies r = -3, 2$$

The general solution is

$$y = C_1 t^{-3} + C_2 t^2$$

Problem 11 The associated characteristic equation is

$$r^2 + 4r + 4 = 0 = (r+2)^2 \implies r = -2$$

The general solution is

$$y = C_1 t^{-2} + C_2 t^{-2} \ln t$$

Problem 16 The associated characteristic equation is

$$r^2 - 4r + 6 \implies r = \frac{4 \pm \sqrt{-8}}{2} = 2 \pm \sqrt{2} i$$

The general solution is

$$y = C_1 t^2 \cos(\sqrt{2} \ln t) + C_2 t^2 \sin(\sqrt{2} \ln t)$$

Problem 25

1. True
2. False

Problem 26

1. No
2. No
3. Yes
4. The Wronskian is

$$t^3(3t|t|) - 3t^2|t^3| = 3t^4|t| - 3t^2|t^3| = 3t^4|t| - 3t^4|t| = 0$$

Problem 28

1. No
2. No
3. Yes
4. The Wronskian is

$$t^2(4|t|) - 2t(2t|t|) = 4t^2|t| - 4t^2|t| = 0$$

Section 9.1

Problem 3

$$\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} 7 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Problem 4

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}' = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 1 \\ \sqrt{\pi} & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Problem 12 The equation is rewritten as

$$\begin{bmatrix} x'' \\ x' \\ y'' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 0 & 1 & -2 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & -3 & -1 \end{bmatrix} \begin{bmatrix} x' \\ x \\ y' \\ y \end{bmatrix}$$

Substituting in

$$x_1 = x', \quad x_2 = x, \quad x_3 = y', \quad x_4 = y$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 0 & 1 & -2 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & -3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Section 9.3

Problem 9

$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ -1 & 4 & 0 & 1 \end{bmatrix} \implies \begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & \frac{9}{2} & \frac{1}{2} & 1 \end{bmatrix}$$
$$\implies \begin{bmatrix} 2 & 0 & \frac{8}{9} & -\frac{2}{9} \\ 0 & \frac{9}{2} & \frac{1}{2} & 1 \end{bmatrix}$$
$$\implies \begin{bmatrix} 1 & 0 & \frac{4}{9} & -\frac{1}{9} \\ 0 & 1 & \frac{1}{9} & \frac{2}{9} \end{bmatrix}$$

This aligns with the formula

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$$

Problem 17 Using the equation for the inverse of a 2×2,

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{3e^5}\begin{bmatrix} 4e^{4t} & -e^{4t} \\ -e^{t} & e^{t} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 4e^{-t} & -e^{-t} \\ -e^{-4t} & e^{-4t} \end{bmatrix}$$

Problem 18 Using the equation for the inverse of a 2×2,

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{-2\sin^2 2t - 2\cos^2 2t}\begin{bmatrix} -2\sin 2t & -\cos 2t \\ -2\cos 2t & \sin 2t \end{bmatrix} = \begin{bmatrix} \sin 2t & \frac{1}{2}\cos 2t \\ \cos 2t & -\frac{1}{2}\sin 2t \end{bmatrix}$$

Problem 22

$$ad - bc = 24 - 24 = 0$$

Problem 27

$$\det(A - rI) = (1-r)(4-r) + 2 = 6 - 5r + r^2 = (r-2)(r-3) \implies r = 2, 3$$

Problem 28

$$\det(A - rI) = (3-r)(4-r) - 6 = 6 - 7r + r^2 = (r-1)(r-6) \implies r = 1, 6$$

Problem 31

$$x' = \begin{bmatrix} 3e^{3t} \\ 6e^{3t} \\ -3e^{3t} \end{bmatrix}$$

Problem 35

$$x' = \begin{bmatrix} 3e^{3t} \\ 6e^{3t} \end{bmatrix}$$

$$Ax = \begin{bmatrix} e^{3t} + 2e^{3t} \\ -2e^{3t} + 8e^{3t} \end{bmatrix} = \begin{bmatrix} 3e^{3t} \\ 6e^{3t} \end{bmatrix}$$

Section 9.4

Problem 2

$$\begin{bmatrix} r(t) \\ \theta(t) \end{bmatrix}' = \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} r(t) \\ \theta(t) \end{bmatrix} + \begin{bmatrix} t^2 \\ 1 \end{bmatrix}$$

Problem 6 Let $x_1 = x$ and $x_2 = x'$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ t^2 \end{bmatrix}$$

Problem 9

$$x_1' = 5x_1 + 2e^{-2t}$$

$$x_2' = -2x_1 + 4x_1 - 3e^{-2t}$$

Problem 14 We have the scalar valued functions,

$$c_1 te^{-t} + c_2 e^{-t} = 0$$

$$c_1 e^{-t} + c_2 e^{-t} = 0$$

Subtracting the second row from the first row yields that $c_1(t-1) = 0$. Thus $c_1 = 0$ and $c_2 = 0$ and the functions are independent.

Problem 15 The second function is -3 times the first function. Thus the functions are dependent.

Problem 16 At the point $t = \frac{\pi}{2}$, the Wronskian is -1. Therefore the Wronskian is nonzero everywhere and the functions are independent.

Problem 21 They do not form a fundamental solution set since the functions are dependent. The second function is -2 times the first one.

Problem 22 At $t = 0$, the Wronskian is -5 so the function is independent. There are also 2 solutions, so the functions form a fundamental set. The matrix is

$$\begin{bmatrix} 3e^{-t} & e^{4t} \\ 2e{-}t & -e^{4t} \end{bmatrix}$$

The general solution is

$$c_1 \begin{bmatrix} 3e^{-t} \\ 2e{-}t \end{bmatrix} + c_2 \begin{bmatrix} e^{4t} \\ -e4t \end{bmatrix}$$