Math 181B: Homework 2

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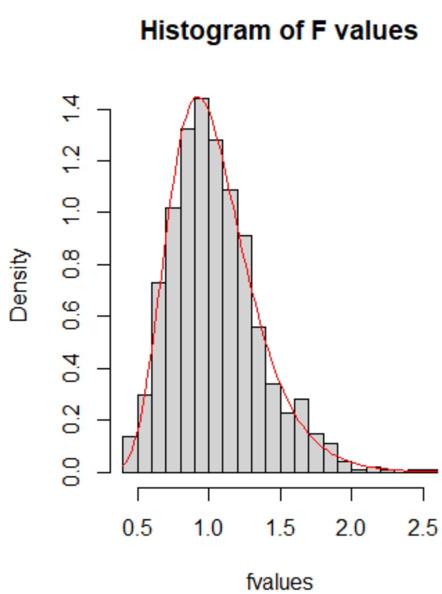
Exercise 1

1.
```
getf = function() {
        sampleX = rnorm(50, mean=1, sd=2)
        sampleY = rnorm(50, mean=0, sd=2)
        varX = var(sampleX)
        varY = var(sampleY)
        f = varY/varX
        return(f)
}

fvalues = rep(0, 1000)

for (i in 1:1000) {
        fvalues[i] = getf()
}

hist(fvalues, breaks=30, prob=TRUE, main = "Histogram of F values")
curve(df(x, 49, 49), add=TRUE, col="red")
```



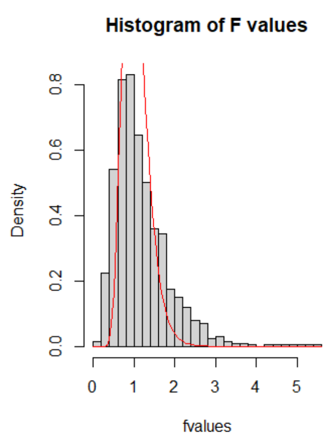
2.
```
rejections = rep(0, 1000)

for (i in 1:1000) {
        fvalue = getf()
        if (fvalue <= qf(0.005,49,49) | fvalue >= qf(0.995,49,49)) {
        rejections[i] = 1;
        }
}

sum(rejections)/1000
# > sum(rejections)/1000
# [1] 0.008
```

We can see that the proportion of rejections is 0.008, which is approximately $\alpha = 0.01$.

3.
```
getf = function() {
  sampleX = rexp(50, rate=10)
  sampleY = rexp(50, rate=10)
  varX = var(sampleX)
  varY = var(sampleY)
  f = varY/varX
  return(f)
}
...
# > sum(rejections)/1000
# [1] 0.175
```



The histogram no longer fits the f-distribution and the rejection rate has increased to 0.175.

Exercise 2

1. We have the null hypothesis $H_0 : \sigma_X = \sigma_Y$ and the alternative hypothesis $H_1 : \sigma_X \neq \sigma_Y$. We assume independence and normality of both samples.

```
# Import files
setwd("C:/Users/merri/Documents/MATH-31H/MATH 181B/Homework 2")
regular = unlist(read.csv("regular.csv"))
fast = unlist(read.csv("fast.csv"))

# Calculate F value
varX = var(regular)
varY = var(fast)
#H0 assumes varX = varY, and so f is just varY/varX
f = varY/varX # f = 0.18

# Do rejection check by using quantiles for alpha=0.05
# Two sided since H1 says varX != varY
if (f <= qf(0.025, 9, 9) | f >= qf(0.975, 9, 9)) {
  print("Reject, variances are not equal")
} else {
  print("Fail to reject, variances can be assumed to be equal")
}
# qf(0.025,9,9) = 0.248 and qf(0.975, 9, 9) = 4.03
# Printed "Reject, variances are not equal"
# Therefore we reject the null hypothesis and cannot assume the variances ar
```

We ended up rejecting the null hypothesis since $f = 0.18 < 0.248 = f_{0.025,9,9}$, meaning that we cannot assume that the variances are equal.

2. Since we rejected the null hypothesis, we know that a ratio of 1, which is what is assumed by H0, should not be in the CI.

3. We have that $\mu_X = \mu_Y$ and $H1 : \mu_X > \mu_Y$. We assume normality from the law of large numbers and independence of the two samples.

```
# Do a HT on H0: mu_X = mu_Y and H1: mu_X > mu_Y.
# We cannot assume that sigma_x = sigma_y,
# so we use Welch's approximation for this calculation.

# Calculate mean and std
Xbar = mean(regular)
Ybar = mean(fast)
Sx = sd(regular)
Sy = sd(fast)
n = length(regular)
m = length(fast)
```

```
# The test statistic is
Tv = (Xbar - Ybar)/sqrt(Sx^2/n + Sy^2/m) # Tv= 0.0726

# v degrees of freedom
v = round((Sx^2/n + Sy^2/m)^2/(Sx^4/n^2/(n-1) + Sy^4/m^2/(m-1)))

# Find P(t_12 > 0.0726)
pt(Tv, v, lower=F)
# Yields value of 0.4716732
```

Since $p = 0.4716732 > 0.03$, we fail to reject the null hypothesis and we cannot say that the fast glue drys faster that the regular glue.

4.
```
var.test(regular, fast)\$p.value
# 0.01754406 so reject null hypothesis

var.test(regular, fast)\$conf.int
# 1.381679 22.395125 so 1 is not in the interval

t.test(regular, fast, alternative = "greater")\$p.value
# p value of 0.4716667, so fail to reject
```

Exercise 3

The mean of the confidence interval is -0.08. We have that

$$\frac{23}{50} - \frac{y}{50} = -0.08$$

Therefore $y = 27$, meaning Kate cleaned the stove right after cooking 27 times.