Math 20D HW4

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Section 4.6

Problem 3 The root of the auxillary equation is

$$r^2 - 2r + 1 \implies r = 1$$

From this,

$$y_p = v_1(t)e^t + v_2(t)te^t$$

The denominator of the integrals is

$$1(e^t e^t(t+1) - e^t te^t)) = e^{2t}$$

Taking the integrals,

$$v_1 = \int -(t^{-1}e^{-1})te^t\,dt = \int -1\,dt = -t$$

$$v_2 = \int (t^{-1}e^{-1})e^t\,dt = \int t^{-1}\,dt = \ln t$$

Therefore the general solution is

$$y = C_1 e^t + C_2 te^t + \ln(t)te^t$$

Problem 6 The homogeneous equation has general solution

$$C_1 \cos 3t + C_2 \sin 3t$$

The denominator of the integrals is

$$1(3\cos^2 t + 3\sin^2 t) = 3$$

Taking the integrals,

$$v_1 = \int -\frac{\sec^2(3t)\sin 3t}{3}\,dt = -\frac{1}{9}\sec 3t$$

$$v_2 = \int \frac{\sec^2(3t)\cos 3t}{3}\,dt = \frac{1}{9}\ln|\sec 3t + \tan 3t|$$

Therefore the general solution is

$$y = C_1 \cos 3t + C_2 \sin 3t - \frac{1}{9} + \frac{1}{9}\sin 3t \ln|\sec 3t + \tan 3t|$$

Problem 9 The auxillary roots are

$$r^2 - 1 \implies r = -1, 1$$

Therefore the particular solution has equation

$$y_p = (A_1 t + A_0) + B_0$$

The only constants that solve the equation are

$$y_p = -2t - 4$$

Using variation of parameters,

$$y_p = v_1(t)e^{-t} + v_2(t)e^t$$

The denominator is

$$1(1+1) = 2$$

Taking the integrals,

$$v_1 = \int -(t+2)(e^t)\,dt = -e^t(t+1)$$

$$v_2 = \int (t+2)e^{-t}\,dt = -e^{-t}(t+3)$$

Therefore,

$$y_p = -t - 1 - t - 3 = -2t - 4$$

Using undetermined coefficients is a lot quicker.

Problem 16 We can use the method of undetermined coefficients. The roots of the auxillary equation are

$$r^2 + 5r + 6 = (r+3)(r+2) \implies r = -3, -2$$

Therefore the particular solution and its derivatives have form

$$y_p = A_2 t^2 + A_1 t + A_0$$

$$y_p' = 2A_2 t + A_1$$

$$y_p'' = 2A_2$$

Solving for the constants and adding the homogenous solution gives

$$y = 3t^2 - 5t + \frac{19}{6} + c_1 e^{-2t} + c_2 e^{-3t}$$

Problem 18 We can use the method of variation of parameters. The auxillary equation has roots

$$r^2 - 6r + 9 = (r-3)^2 \implies r = 3$$

The denominator has form

$$1(e^{3t}(3te^{3t} + e^{3t}) - (3e^{3t})(te^{3t})) = e^{6t}(3t+1) - 3te^{6t} = e^{6t}$$

Taking the integrals

$$v_1 = \int -\frac{t^{-3}e^{3t}te^{3t}}{e^{6t}}\,dt = \int -\frac{1}{t^2}\,dt = t^{-1}$$

$$v_2 = \int \frac{t^{-3}e^{3t}e^{3t}}{e^{6t}}\,dt = \int t^{-3}\,dt = -\frac{1}{2t^2}$$

Thus the general solution is

$$y = \frac{e^{3t}}{2t} + c_1 e^{3t} + c_2 te^{3t}$$

Review Chapter 2

Problem 1 The equation is separable

$$(y-1)e^{-y}\,dy = e^x\,dx \implies -ye^{-y} = e^x + C$$

Problem 2 The equation is linear

$$\mu = e^{\int P(x)\,dx} = e^{-4x}$$

Multiplying both sides by μ yields

$$\frac{d}{dx}[e^{-4x}y] = 32x^2e^{-4x} \implies e^{-4x}y = e^{-4x}(8x^2+4x+1)+C \implies y = 8x^2+4x+1+Ce^{4x}$$

Problem 16 The equation is linear

$$\mu = e^{\int P(x)\,dx} = e^{-\ln(\cos x)} = \sec x$$

Multiplying both sides by μ yields

$$\frac{d}{dx}[\sec(x)y] = -\tan x \implies \sec(x)y = \ln(\cos x)+C \implies y = \cos(x)\ln(\cos x)+C\cos x$$

Problem 23 The equation is exact since both derivatives equal 1. Integrating M yields

$$F(x,y) = yx - \frac{1}{2}x^2 + g(y)$$

Taking the derivative relative to y yields

$$x+y = x+g'(y) \implies g'(y) = y \implies g(y) = \frac{1}{2}y^2$$

Therefore the solution is

$$yx - \frac{1}{2}x^2 + \frac{1}{2}y^2 = C$$

Problem 28 The equation can be rewritten as the exact equation

$$(-x+y+1)\,dx + (x+y+5)\,dy = 0$$

Integrating M yields

$$F(x,y) = -\frac{1}{2}x^2 + xy + x + g(y)$$

Taking the derivative relative to y yields

$$x+y+5 = x+g'(y) \implies g'(y) = y+5 \implies g(y) = \frac{1}{2}y^2+5y$$

Therefore the solution is

$$-\frac{1}{2}x^2 + xy + x + \frac{1}{2}y^2 + 5y = C$$

Problem 34 The equation is linear. The integrating factor is

$$\mu = e^{\int P(x)\,dx} = e^{-2\ln x} = x^{-2}$$

Multiplying both sides by the integratin factor yields

$$\frac{d}{dx}[x^{-2}y] = \cos xy = x^2\sin x + Cx^2$$

Plugging in the initial value...

$$2 = 0 + C\pi^2 \implies C = \frac{2}{pi^2}$$

The final solution is

$$y = x^2\sin x + \frac{2}{\pi^2}x^2$$