MAT140B WINTER 2024: PROBLEM SET 6

Due: S 02/25/2023, by midnight

Directions: You can collaborate, but must write up the solutions independently and in a good handwriting. **Consulting solutions to problem sets of previous semesters or internet solutions is not allowed.**

Problem 1. Recall that $C([0,1])$ is a complete metric space with metric $d(f,g) = \sup_{x\in[0,1]} |f(x)-g(x)|$. For any $0 < \alpha \leq 1$ and $M \geq 0$ define

$$C^{\alpha}_{M}([0,1]) = \{f : [0,1] \to \mathbb{R} \Big| |f(x) - f(y)| \leq M|x-y|^{\alpha} \ \ \forall\ x, y \in [0,1]\}.$$

Note that $C^{\alpha}_{M}([0,1]) \subset C([0,1])$. Prove that bounded subsets of $C^{\alpha}_{M}([0,1])$ are compact in $C([0,1])$.

Problem 2. Chapter 7: 20, 25 (This problem shows one reason why Arzela-Ascoli theorem is useful).

Reading. Read the sections on Exponential and Trigonometric functions in Chapter 8 of Rudin, and use their properties freely in the next exercise. We will justify them in class.

Problem 3. Chapter 8: 1, 4, 5

Problem 4. Chapter 8: 6 (only part (a)), 7