MATH 170B HOMEWORK 3

Hermite Interpolation and Splines

§1: Determine whether this is a quadratic spline function:

$$f(x) = \begin{cases} x & x \in (-\infty, 1] \\ -\frac{1}{2}(2-x)^2 + \frac{3}{2} & x \in [1, 2] \\ \frac{3}{2} & x \in [2, \infty) \end{cases}$$

§2: Determine all the values of a, b, c, d, e, for which the following function is a cubic spline:

$$f(x) = \begin{cases} a(x-2)^2 + b(x-1)^3 & x \in (-\infty, 1] \\ c(x-2)^2 & x \in [1, 3] \\ d(x-2)^2 + e(x-3)^3 & x \in [3, \infty) \end{cases}$$

§3: Using the development of cubic splines as a model, derive the appropriate equations and algorithm to provide a quadratic spline interpolant to data (t_i, y_i) for $0 \leq i \leq n$, where $t_0 < t_1 < \cdots < t_n$. If Q is the spline interpolant, then the numbers $z_i = Q'(t_i)$ are well-defined. Find the equations governing $z_0, z_1, \ldots, z_n$. You should discover that one of the z points can be arbitrary, say $z_0 = 0$.

§4: [1] 6.6

References

[1] Süli, Endre, and David F. Mayers. An introduction to numerical analysis. Cambridge university press, 2003.