Math 181A: Homework 6

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Problem 1: 6.3.3.

The null hypothesis is that $p = 0.65$. The alternative hypothesis is that $p < 0.65$. The estimated proportion is $\frac{72}{120} = 0.6$. The test statistic is

$$z = \frac{0.6 - 0.65}{\sqrt{\frac{0.65(1-0.65)}{120}}} = -1.15.$$

Assuming that H_0 is true, the p value is

$$P(Z \leq -1.15) = 0.12507 > 0.05.$$

Therefore, we cannot reject the null hypothesis.

Problem 2: 6.3.4.

The critical region for $\alpha = 0.14$ is $[1.08, \infty)$. We have that

$$z = \frac{\hat{p} - 0.45}{\sqrt{\frac{0.45(1-0.45)}{200}}} \geq 1.08 \implies \hat{p} \geq 0.45 + 1.08 \cdot \sqrt{\frac{0.45(1-0.45)}{200}} = 0.49.$$

For a sample size of 200, there must be at least 98 successes to reject the null hypothesis.

Problem 3: 6.3.6.

The null hypothesis is that $p = 1/12$ The alternative hypothesis is that $p < 1/12$. The test statistic is

$$z = \frac{16/348 - 1/12}{\sqrt{\frac{1/12(1-1/12)}{348}}} = -2.52$$

The p value is

$$P(Z \leq -2.52) = 0.00587 < 0.05.$$

Since the p-value is less than 0.05, we can reject the null hypothesis.

Problem 4: 6.2.1.

1. The critical region for $\alpha = 0.08$ is $(-\infty, -1.4]$. Reject the null hypothesis if
$$\frac{\bar{y} - 120}{18/\sqrt{25}} < -1.4.$$
We should reject the null hypothesis since
$$\frac{114.2 - 120}{18/\sqrt{25}} = -1.61 < -1.4.$$

2. The critical region for $\alpha = 0.01$ is $(-\infty, -2.576] \cup [2.576, \infty)$. Reject the null hypothesis if
$$\left|\frac{\bar{y} - 42.9}{3.2/\sqrt{16}}\right| > 2.576.$$
We should reject the null hypothesis since
$$\left|\frac{45.1 - 42.9}{3.2/\sqrt{16}}\right| = 2.75 > 2.576.$$

3. The critical region for $\alpha = 0.13$ is $[2.22, \infty)$. Reject the null hypothesis if
$$\frac{\bar{y} - 14.2}{4.1/\sqrt{9}} > 1.12.$$
We should reject the null hypothesis since
$$\frac{15.8 - 14.2}{4.1/\sqrt{9}} = 1.17 > 1.12.$$

Problem 5: 6.2.6.

The probability that the mean lands in the $(29.9, 30.1)$ is

$$P(\frac{29.9-30}{6/\sqrt{16}} \leq Z \leq \frac{30.1-30}{6/\sqrt{16}}) = P(Z \leq 0.07) - P(Z \leq -0.07) = 0.52790 - 0.47210 = 0.0558.$$

The test has a significance level of 0.056, but the interval is a poor choice for the critical region because it rejects the null hypothesis for values that are closest with $\mu = 30$. For $\alpha = 0.056$, the critical region is $(-\infty, -1.91] \cup [1.91, \infty)$ Thus, the null hypothesis should be rejected if $\bar{y}$ falls in the range

$$(-\infty, -1.91 \cdot \frac{6}{\sqrt{16}}] \cup [1.91 \cdot \frac{6}{\sqrt{16}}, \infty) = (-\infty, 27.1] \cup [32.9, \infty)$$

Problem 6: 6.2.8.

1. The p value is
$$P(Z \leq -1.61) = 0.05370 < 0.08.$$
Thus, the p value agrees with my decision to reject the null hypothesis.

2. The p value is
$$2 * P(Z \geq 2.75) = 0.0060 < 0.01.$$
Thus, the p value agrees with my decision to reject the null hypothesis.

3. The p value is
$$P(Z \geq 1.17) = 0.121 < 0.13.$$
Thus, the p value agrees with my decision to reject the null hypothesis.