Instructions

- **Midterm receipt.** This **Midterm 1** is available for download from **Gradescope** at **2:55 pm** Pacific Time on Friday **April 22**. In the unlikely event that you cannot download the exam at that time you should email both Prof. Chow at <bechow@ucsd.edu> and TA Zhiyuan Jiang <z5jiang@ucsd.edu> to receive an emailed copy of the exam. If you are taking the midterm asynchronously, you will be able to download the midterm at your given start time.

- **Exam format.** There are **4 questions**, worth 10 points each. Total: **40 points**. Time: Although the official time for the exam is 3:00 pm to 3:50 pm, you may start up to 5 minutes early (upon download) and you have a 10 minute grace period to upload your exam to Gradescope, so your exam will not be counted late if uploaded by **4:00 pm**.

- **Format for your solutions.** You should hand write your solutions neatly and darkly on your own blank or lined 11"×8.5" paper or on your iPad/tablet in an organized way. Do not type solutions.

- **Allowed resources.** The resources you may use are: Hubbard and Hubbard's book, any materials on the canvas site, piazza discussion, and the google drive, all for this course only. Students cannot communicate with each other or any people by any means. E.g., no use of Discord, texting, messaging, etc. is allowed. You may not search the internet outside the above resources. You may not use any calculator or mathematical software except for addition, multiplication, subtraction, and division.

- **Submission.** Please make sure your files are legible before submitting and carefully follow the **Gradescope** instructions to match every question to the appropriate pages of your submission.

- **Deadline.** The deadline to upload your midterm solution to **Gradescope** is 4:00 pm (Pacific Time) on April (= 50 minutes plus 10-minute grace period). Submissions after 4:00 pm will be deducted 1/2 point per minute late, with no midterms accepted after 4:15 pm. Email your exam to both <bechow@ucsd.edu> and <z5jiang@ucsd.edu> if you feel you cannot upload to gradescope in time to timestamp your submission; after that, upload to Gradescope as soon as possible.

1. (10 points) Compute the determinant of the following 3×3 matrix:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ x & y & z \end{bmatrix} \tag{Matrix}$$

x, y, and z are real numbers.

Use your answer to find the volume of the parallelopiped spanned by the three vectors

$$\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}. \tag{Vectors}$$

2. (10 points) Evaluate the following iterated integral by reversing the order of integration:

$$\int_0^b \left(\int_{x^3}^{b^3} y^{5/3} \, e^{-y^3} \, dy \right) dx. \tag{Double}$$

3. (10 points) Let $f : \mathbb{R} \to \mathbb{R}$ be a function with the property that

$$|f(x) - f(y)| \leq 3\,|x - y| \quad \text{for all}\, x, y \in \mathbb{R}. \tag{Lip}$$

Prove that f is integrable on the interval $[0, 1)$ by directly estimating the difference $U_N(f) - L_N(f)$ of the N-th upper and lower sums over $[0, 1)$.

If you prefer, this is the same as estimating this difference for the function $\mathbf{1}_{[0,1)} f : \mathbb{R} \to \mathbb{R}$.

4. (10 points) Let S be the first quadrant of the unit circle. That is, S is defined by the inequalities $x^2 + y^2 \leq 1$, $x \geq 0$, and $y \geq 0$. Define the map $\Phi : S \to \mathbb{R}^2$ by $\Phi \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u^3 - 3uv^2 \\ 3u^2 v - v^3 \end{pmatrix}$. You may assume WITHOUT proof that Φ satisfies all of conditions of the change of variables formula such as being one-to-one. Compute the integral

$$\int_{\Phi(S)} dx \, dy. \tag{Area}$$

Hint: The relevant determinant calculation yields a quartic polynomial in u and v which is a nice "squared" expression. Also, after using the change of variables formula, evaluate the resulting integral by using polar coordinates.