Math 31BH: Assignment 9

Due 03/13 at 23:59

Merrick Qiu

Consider the function $f\colon \mathbb{R}^5 \to \mathbb{R}^2$ defined by

$$f(x_1, x_2, y_1, y_2, y_3) = (f_1(x_1, x_2, y_1, y_2, y_3), f_2(x_1, x_2, y_1, y_2, y_3)),$$

where

$$\begin{aligned} f_1(x_1, x_2, y_1, y_2, y_3) &= 2e^{x_1} + x_2 y_1 - 4y_2 + 3 \\ f_2(x_1, x_2, y_1, y_2, y_3) &= x_2 \cos x_1 - 6x_1 + 2y_1 - y_3. \end{aligned}$$

Let $\mathbf{u} = (0, 1, 3, 2, 7)$, and put $\mathbf{a} = (0, 1)$ and $\mathbf{b} = (3, 2, 7)$.

1. Evaluate $f(\mathbf{u})$.

   Solution:

   $$f_1(0, 1, 3, 2, 7) = 2e^0 + 1(3) - 4(2) + 3 = 2 + 3 - 8 + 3 = 0$$

   $$f_2(0, 1, 3, 2, 7) = 1\cos(0) - 6(0) + 2(3) - 7 = 1 - 0 + 6 - 7 = 0$$

   Therefore, $f(u) = (0, 0)$.

2. Find the matrix $J_f(\mathbf{u})$ of the linear transformation $f'(\mathbf{u}, \cdot)\colon \mathbb{R}^5 \to \mathbb{R}^2$ with respect to the standard bases.

   Solution: The Jacobian for f is defined by

   $$J_f = \begin{bmatrix} \frac{\partial}{\partial x_1} f_1 & \frac{\partial}{\partial x_2} f_1 & \frac{\partial}{\partial y_1} f_1 & \frac{\partial}{\partial y_2} f_1 & \frac{\partial}{\partial y_3} f_1 \\ \frac{\partial}{\partial x_1} f_2 & \frac{\partial}{\partial x_2} f_2 & \frac{\partial}{\partial y_1} f_2 & \frac{\partial}{\partial y_2} f_2 & \frac{\partial}{\partial y_3} f_2 \end{bmatrix} = \begin{bmatrix} 2e^{x_1} & y_1 & x_2 & -4 & 0 \\ -x_2 \sin x_1 - 6 & \cos x_1 & 2 & 0 & -1 \end{bmatrix}$$

   Plugging this in for u gets

   $$J_f(u) = \begin{bmatrix} 2 & 3 & 1 & -4 & 0 \\ -6 & 1 & 2 & 0 & -1 \end{bmatrix}$$

3. Let A be the 2×2 matrix obtained by taking the first two columns of $J_f(\mathbf{u})$. Show that A is invertible.

   Solution: A is

   $$A = \begin{bmatrix} 2 & 3 \\ -6 & 1 \end{bmatrix}$$

A is invertible if its determinant is nonzero.

$$\det A = 2 - (-18) = 20 \neq 0$$

Therefore, A is invertible.

4. Prove that there exists a continuously differentiable function g defined on an open set $D \subseteq \mathbb{R}^3$ containing $\mathbf{b}$ and mapping to $\mathbb{R}^2$ such that $g(\mathbf{b}) = \mathbf{a}$ and $f(g(\mathbf{v}), \mathbf{v}) = (0, 0)$ for all $\mathbf{v} \in D$.

   Solution: Since the matrix A is invertible, the implicit function theorem says that for some open set $D \subseteq \mathbb{R}^3$ with $b \in D$, there exists a function $g : D \to \mathbb{R}^2$ with $g(b) = a$, that g is continuously differentiable, and that $f(g(v), v) = (0, 0)$ for all $v \in D$.

5. Calculate the matrix of $g'(\mathbf{b}, \cdot)\colon \mathbb{R}^3 \to \mathbb{R}^2$ with respect to the standard bases.

   Solution: The inverse of a 2×2 matrix can be found by switching the diagonal elements, negating the other elements, and dividing by the original determinant. Therefore,

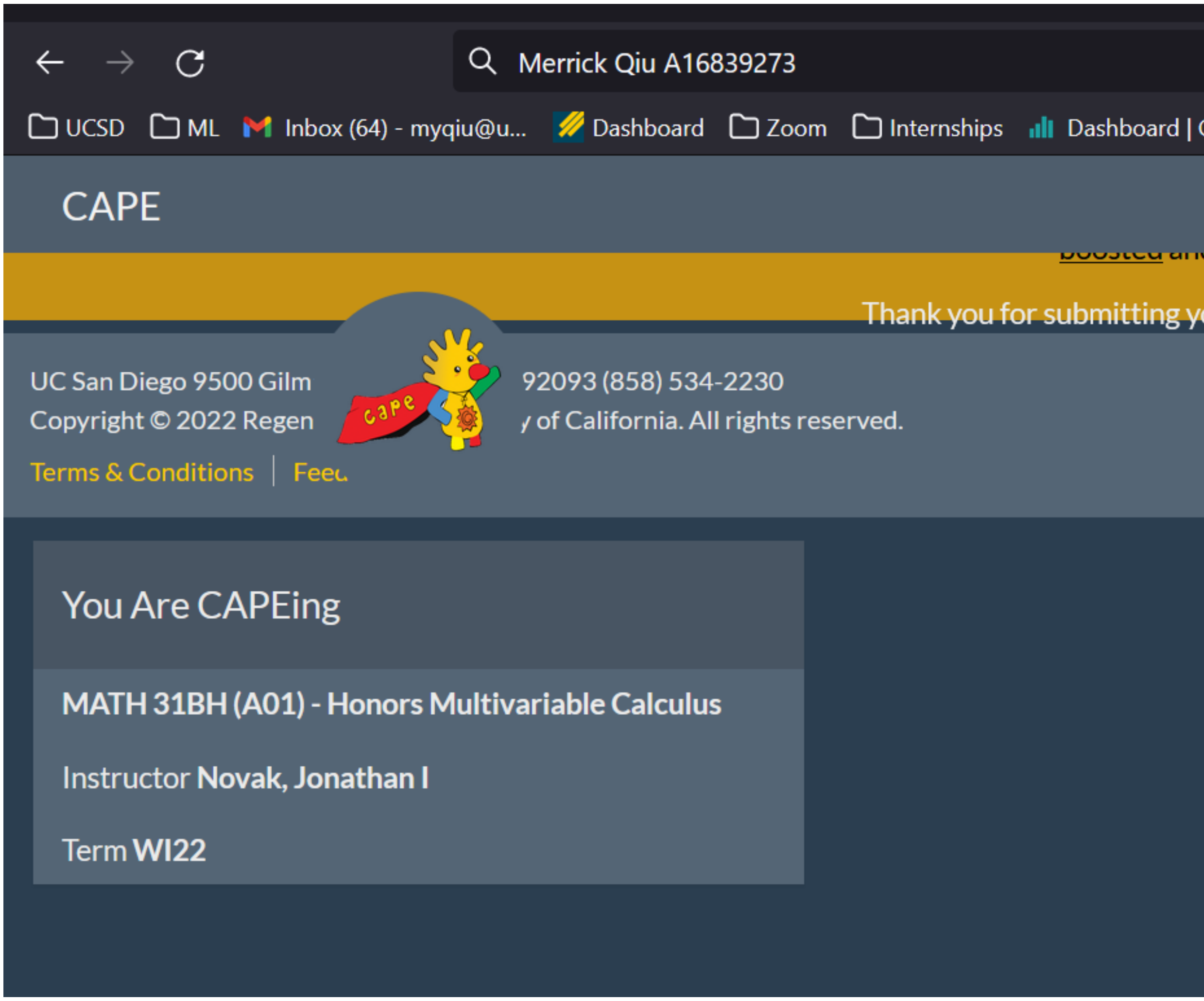
$$A^{-1} = \frac{1}{20} \begin{bmatrix} 1 & -3 \\ 6 & 2 \end{bmatrix}$$

   Using the implicit function theorem, the matrix ends up being

$$[g'(b)] = -A^{-1}B = -\frac{1}{20} \begin{bmatrix} 1 & -3 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 1 & -4 & 0 \\ 2 & 0 & -1 \end{bmatrix} = \frac{1}{20} \begin{bmatrix} 5 & 4 & -3 \\ -10 & 24 & 2 \end{bmatrix}$$

   where B is the matrix consisting of the last 3 columns of the Jacobian matrix.

Here is my capes confirmation:


Merrick Qiu A16839273
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Instructor Novak, Jonathan I
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