Math 170C: Homework 4

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Problem 1

Letting $x_0 = t$, $x_1 = x$, $x_2 = x'$, and $x_3 = x''$ yields

$$\begin{cases} x_0' = 1 \\ x_1' = x_2 \\ x_2' = x_3 \\ x_3' = e^{x_0} - 2x_3 + x_2 + 2x_1 \end{cases}$$

with initial condition $X = (8, 3, 2, 1)^T$

Problem 2

We can transform the first problem to the second with the change of variables $t = 3 + 4s$, $y(s) = x(3 + 4s)$, $y'(s) = 4x'(3 + 4s)$, $y''(s) = 16x''(3 + 4s)$

$$
\begin{aligned}
x(3) = \alpha &\implies y(0) = \alpha \\
x(7) = \beta &\implies y(1) = \beta \\
x'' = t + x^2 - 3x' &\implies y'' = 16((3 + 4s) + y^2 - 3(y'/4)) \\
&\qquad\;\;\, = 48 + 64s + 16y^2 - 12y'.
\end{aligned}
$$

Thus, theorem 2 holds for this problem.

Problem 3

Suppose we find a solution x_1 with initial conditions $x_1(a)$ and $x_1'(a)$ such that $c_{11}x_1(a) + c_{12}x_1'(a) = \alpha$. Then consider x_2 such that $x_2(a) = -c_{12}$ and $x_2'(a) = c_{11}$. Consider the solution $x_1 + \lambda x_2$. This satisfies the initial condition at a since

$$\begin{aligned}
c_{11}(x_1(a) + \lambda x_2(a)) + c_{12}(x_1'(a) + \lambda x_2'(a)) &= (c_{11}x_1(a) + c_{12}x_1'(a)) + (-\lambda c_{11}c_{12} + \lambda c_{12}c_{11}) \\
&= \alpha + 0 \\
&= \alpha
\end{aligned}$$

We then want to select λ such that

$$c_{21}(x_1(b) + \lambda x_2(b)) + c_{22}(x_1'(b) + \lambda x_2'(b)) = \beta$$

Solving for λ yields

$$\lambda = \frac{\beta - c_{21}x_1(b) - c_{22}x_1'(b)}{x_2(b) + x_2'(b)}.$$