# Math 31BH: Assignment 4

Due 01/30 at 23:59

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1. Verify that the following sets $C \subseteq \mathbb{R}^2$ are curves by giving an explicit paramterization, i.e. for each give a continuously differentiable function defined on a subset of $\mathbb{R}$ and taking values in $\mathbb{R}^2$ whose image is $C$.

   (a) The parabola $C = \{(x, y) \in \mathbb{R}^2 : y + 1 = (x - 2)^2\}$.

   (b) The circle $C = \{(x, y) \in \mathbb{R}^2 : (x - 1)^2 + (y - 2)^2 = 4\}$.

   (c) The ellipse $C = \{(x, y) \in \mathbb{R}^2 : 4x^2 + y^2 = 1\}$.

   (d) The ellipse $C = \{(x, y) \in \mathbb{R}^2 : 4x^2 + y^2 = 1\}$.

   (e) The set $C = \{(x, y) \in \mathbb{R}^2 : x = |y|\}$.

   **Solution:**

   (a) $f(t) = (t, (t - 2)^2 - 1)$
   (b) $f(t) = (2\cos(t) + 1, 2\sin(t) + 2)$
   (c) $f(t) = (\frac{\cos(t)}{2}, \sin(t))$
   (d) Ditto
   (e) $f(t) = (t^2, t\|t\|)$

2. For each of the curves $C$ in the previous problem, find the equation of the tangent line at each point on $C$ where it exists, and specify those points at which it does not.

   **Solution:**

   (a) $f'(t) = (1, 2t - 4)$ so
   $y - ((t - 2)^2 - 1) = (2t - 4)(x - t)$
   (b) $f'(t) = (-2\sin(t), 2\cos(t))$ so
   $y - (2\sin(t) + 2) = -\cot(t)(x - (2\cos(t) + 1))$ with
   $x = 3$ at $(3, 0)$ and $x = -1$ at $(-1, 0)$
   (c) $f'(t) = (-\frac{1}{2}\sin(t), \cos(t))$ so
   $y - \sin(t) = \frac{\cot(t)}{2}(x - \frac{\cos(t)}{2})$ with
   $x = \frac{1}{2}$ at $(\frac{1}{2}, 0)$ and $x = -\frac{1}{2}$ at $(-\frac{1}{2}, 0)$
   (d) Ditto
   (e) $f'(t) = (2t, 2\|t\|)$ so
   $y = x$ when $t > 0$ and $y = -x$ when $t < 0$ with no tangent line at $t = 0$