# HW 4 - due 11/04 at 11:59 pm.

Math 181B, Spring 22, Rava

Follow closely the 'Hw guide' under Files in the folder 'Course Contents' on how to write, scan and submit your homework.
On any problem involving R, you should include your code and output as part of your answer. You may take a screenshot of the code/output, or write it by hand.
Be careful with notation, remember to define the parameters and the random variables you intend to use.

## 1 Exercise 1

[3 points] Use the fact that, under the simple linear model, $\frac{(n-2)S^2}{\sigma^2} \sim \chi^2_{n-2}$ to prove that $\left[\frac{(n-2)S^2}{\chi^2_{1-\alpha,n-2}}, \infty\right]$ is a $100(1-\alpha)\%$ upper one-sided CI for $\sigma^2$, i.e. prove that $P\left(\sigma^2 \geq \frac{(n-2)S^2}{\chi^2_{1-\alpha,n-2}}\right) = 1-\alpha$.

## 2 Exercise 2

We want to use linear regression to predict professors' salary, in dollars, from the number of years spent in their current rank. We use a dataset that collects data on 52 professors.

a) [4 points] Complete the following summary:

| | Estimate | Std. Error | t value | $\Pr(>\|t\|)$ |
|---|---|---|---|---|
| Intercept | 18166.1 | 1003.7 | | |
| years | 752.8 | | | $7.34e{-}09$ |

Make sure to spell out your calculations.

b) [4 points] Do we have enough evidence to conclude that professors' salary increases on average when the number of years spent in their current rank increases? Perform a HT with significance level 0.03. Remember to define hypotheses, write down the value of the test statistic, the degrees of freedom, the p-value and write a meaningful conclusion.

## 3 Exercise 3

You can find on Canvas the dataset 'cars.csv'. The data was extracted from the 1974 Motor Trend US magazine, and comprises fuel consumption and 10 aspects of automobile design and performance for 32

automobiles. You want to use the dataset to predict fuel consumption (column 'mpg', units: miles/gallon) from gross horsepower (column 'hp', units: hp).

a) [1 point] Import the dataset in R and perform linear regression. What is the fitted regression line?

b) [2 points] What is the value of $R^2$? Interpret the value of $R^2$ in this contest.

c) [1 point] What does the value of $R^2$ tell you about the appropriateness of linear regression for predicting cars' fuel consumption from their gross horsepower?

d) [3 points] Let the car's gross horsepower be three standard deviations above the mean. How many standard deviations above or below the mean the predicted fuel consumption of this car would be according to this model?

e) [2 points] Use R to find a 90% CI for the average fuel consumption of all the cars with gross horsepower equal to 150 hp. Write a meaningful sentence to interpret the CI.

f) [2 points] Use R to find a 90% PI for the fuel consumption of a car with gross horsepower equal to 150 hp. Write a meaningful sentence to interpret the CI.