Mathematics 100A Homework 2
Due: Tuesday October 15 2024

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Chapter and problem numbers, if any, refer to *Algebra*, second edition, by Michael Artin. Please feel free to reach out to me or the TAs if you have any questions.

1. (Chapter 2, Problem 4.1) Let a and b be elements of a group G. Suppose that a has order 7, and $a^3b = ba^3$. Prove that $ab = ba$.

2. (Chapter 2, Problem 4.3) Let a and b be elements of a group G. Prove that ab and ba have the same order.

3. (Chapter 2, Problem 4.4) Suppose G is group that contains no proper subgroup. Prove G is finite and has order 1 or order p where p is prime.

4. (Chapter 2, Problem 4.10) Suppose G is a group, and $a, b \in G$ have finite order.

   (a) Suppose G is abelian. Prove ab has finite order.

   (b) Show by example that if G is not abelian, then ab need not have finite order.

5. (Chapter 2, Problem 5.1) Let $\varphi : G \to G'$ be a surjective group homomorphism.

   (a) A group G is said to be **cyclic** if $G = \langle x \rangle$ is generated by single element $x \in G$. Prove that if G is cyclic, then G' is cyclic.

   (b) Prove that if G is abelian, then G' is abelian.