Problem 6.10.1

$$
\begin{aligned}
\operatorname{vol}_3 U &= \int_{\partial U} \frac{1}{3}(z\,dx \wedge dy + y\,dz \wedge dx + x\,dy \wedge dz) \\
&= \int_U \frac{1}{3}(\,dx \wedge dy \wedge dz + y\,dz \wedge dx \wedge dy + \,dy \wedge dz\,dx) \\
&= \int_U dx \wedge dy \wedge dz
\end{aligned}
$$

Problem 6.10.2

$$
\begin{aligned}
\int_{C \cup D} x\,dy \wedge dz + y\,dz \wedge \,dx + z\,dx \wedge dy &= \int_w 3\,dx \wedge dy \wedge dz \\
&= 3\operatorname{vol}_3 W \\
&= 3(\frac{1}{3}a\pi a^2) \\
&= \pi a^3
\end{aligned}
$$

We can ignore the D since it is a flat disk.

Problem 6.10.3

$$
\begin{aligned}
\int_U x_1\,dx_2 \wedge dx_3 \wedge dx_4 &= \left(\int_{\partial U_{1,2,3,4}} + \int_{\partial U_{2,3,4}} + \int_{\partial U_{1,3,4}} + \int_{\partial U_{1,2,4}} + \int_{\partial U_{1,2,3}}\right) dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \\
&= \int_{\partial U_{1,2,3,4}} dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \\
&= \int_0^a \int_0^{a-x_4} \int_0^{a-x_4-x_3} \int_0^{a-x_4-x_3-x_2} dx_1 \wedge \,dx_2 \wedge \,d_3 \wedge \,d_4 x
\end{aligned}
$$