Math 100A: Homework 1

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Problem 1

Since $G \subset S$ and the law of composition is associative on S, the same law of composition is associative on G. Since $1 \in S$ and 1 is its own inverse, $1 \in G$ so G has an identity. From the definition of G, each element in G has an inverse. If $x \in G$ and $y \in G$ then $xy \in G$ since $xy \in S$ and xy has inverse $y^{-1}x^{-1}$, so G is closed.

Problem 2

Matrix multiplication for $\mathrm{SL}_2(\mathbf{Z})$ is associative since we can expand the matrix multiplications below to show they are equal.

$$\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix}\right) \begin{bmatrix} i & j \\ k & l \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \left(\begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix}\right)$$

We have the identity element

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Since each matrix has nonzero determinant so each matrix in $\mathrm{SL}_2(\mathbf{Z})$. is invertible, and the inverse of a matrix with determinant 1 also has determinant 1. The product of two matricies with determinant 1 also has determinant 1 so $\mathrm{SL}_2(\mathbf{Z})$ is closed under multiplication.

Problem 3

We can define $\rho(x) = e^x$. Thus $\rho(x + y) = e^{x+y} = e^x e^y = \rho(x)\rho(y)$.