Mathematics 100A Homework 1
Due: Tuesday October 8 2024

Instructions: Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Chapter and problem numbers, if any, refer to *Algebra*, second edition, by Michael Artin. Please feel free to reach out to me or the TAs if you have any questions.

1. (Chapter 2, problem 2.2) Let S be a set with an associative law of composition and with an identity element. Let $G = \{x \in S : x \text{ has an inverse}\}$. Prove that G is a group with the law of composition from S.

2. Let $\mathrm{SL}_2(\mathbf{Z}) = \{\gamma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) : a, b, c, d \in \mathbf{Z} \text{ and } \det(\gamma) = 1\}$. Prove that multiplication of matrices makes $\mathrm{SL}_2(\mathbf{Z})$ a group. **Note**: Be sure to explain why $\mathrm{SL}_2(\mathbf{Z})$ is closed under taking inverses.

3. A group homomorphism $\rho : G_1 \to G_2$ is said to be *trivial* if $\rho(g) = 1$ for all $g \in G_1$. The homomorphism is said to be non-trivial otherwise. Find a non-trivial group homomorphism $\rho : \mathbf{R} \to \mathbf{R}^\times$. Here $\mathbf{R}$ means the group of real numbers under addition, and $\mathbf{R}^\times$ means the group of nonzero real numbers under multplication.