Math 181A: Homework 3

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Problem 1: 5.3.2 and 5.3.10

1. Taking the average of all the subjects yields that $\bar{x} = 0.77$. The critical value for 95% is $Z_{\alpha/2} = 1.96$. The margin of error is $ME = 1.96 \cdot \frac{0.09}{\sqrt{19}} = 0.04$. We have that the confidence interval is $(0.73, 0.81)$. Since 0.80 is in the confidence interval, it is likely that the detergent does not cause respiratory illness.

2. Modeling his performance as a Bernoulli distribution, he has standard deviation $\sigma = \sqrt{0.356 \cdot (1 - 0.356)} = 0.479$. Thus he has confidence interval

$$(0.356 - 1.96 \cdot \frac{0.479}{\sqrt{540}}, 0.356 + 1.96 \cdot \frac{0.479}{\sqrt{540}}) = (0.316, 0.396)$$

Problem 2: 5.3.26

Assuming the upper bound of $\hat{p} = 0.4$ gives

$$n \geq \frac{2.58^2(0.4)(0.6)}{0.05^2} = 639.01.$$

Thus $n = 640$ is a lower-bound.

Problem 3

The margin of error of the first sample is

$$ME = 1.96 \cdot \frac{0.45 \cdot 0.55}{n} = \frac{0.485}{n}$$

The margin of error of the second sample is

$$ME_{new} = 1.645 \cdot \frac{0.48 \cdot 0.52}{n_{new}} = \frac{0.411}{n_{new}}$$

Setting ME_{new} to be a third of ME yields

$$\begin{aligned} ME_{new} = \frac{1}{3} ME &\implies \frac{0.411}{n_{new}} = \frac{0.162}{n} \\ &\implies n_{new} = 2.54n. \end{aligned}$$

Problem 4: 5.4.6

In order for $Y_{min} = y$, one of the $\binom{n}{1}$ variables must have value y with density $\frac{1}{\theta}$ and the other $n-1$ variables must have value $\geq y$ with probability $\frac{\theta-y}{\theta}$. Therefore, $f_{Y_{min}}(y) = n\frac{1}{\theta}\left(\frac{\theta-y}{\theta}\right)^{n-1}$. Finding the expected value of Y_{min} yields

$$\begin{aligned}
\int_0^\theta n\frac{1}{\theta}\left(\frac{\theta-y}{\theta}\right)^{n-1} dy &= \frac{n}{\theta}\int_0^\theta \left(\frac{\theta-y}{\theta}\right)^{n-1} dy \\
&= \frac{n}{\theta}\left[-\frac{\theta}{n(n+1)}(ny+\theta)\left(\frac{\theta-y}{\theta}\right)^{n}\right]_0^\theta \\
&= -\frac{1}{n+1}(0-\theta) \\
&= \frac{\theta}{n+1}.
\end{aligned}$$

Therefore an unbiased estimator would be $\hat{\theta} = (n+1)Y_{min}$.

Problem 5: 5.4.9

The expected value of Y is

$$\begin{aligned}
E[Y] &= \int_0^{\frac{1}{\theta}} 2y^2\theta^2 \\
&= \left[\frac{2}{3}y^3\theta^2\right]_0^{\frac{1}{\theta}} \\
&= \frac{2}{3}\frac{1}{\theta}
\end{aligned}$$

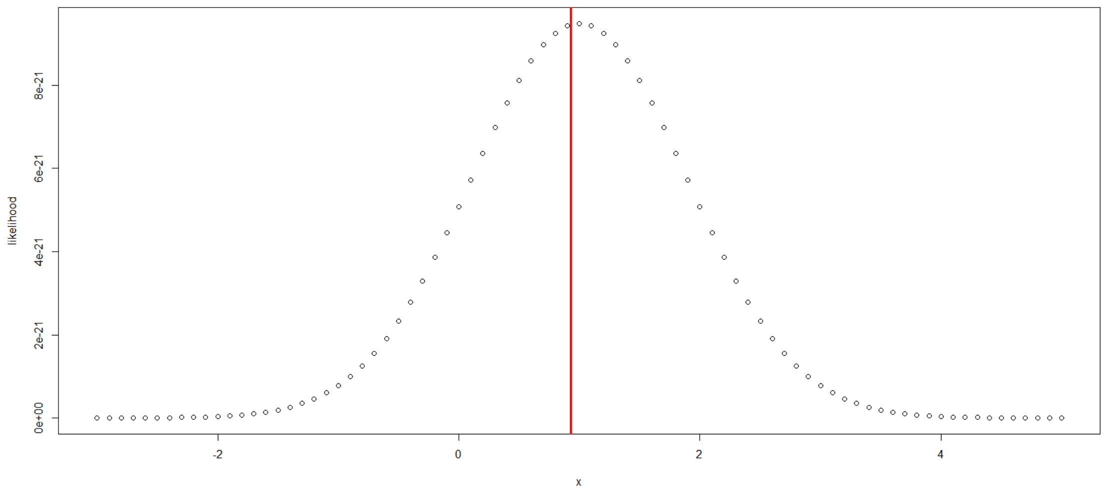
We have that

$$\begin{aligned}
E[c(Y_1 + 2Y_2)] &= 3cE[Y] \\
&= 2c\frac{1}{\theta}
\end{aligned}$$

Therefore, $c = \frac{1}{2}$.

Problem 6: R Simulation

1. Here is the plot of the likelihood function, with the red line representing the sample mean.



2. Here is the plot of the log likelihood function, with the red line representing the sample mean.