

Fall 2024 Math 120A Homework 6
Due Wednesday, 11:59pm, Nov. 13th, PT.

Submit this homework through Gradescope. If you are not familiar with Gradescope, check the website for an instruction on how to use Gradescope.

Solve the following problems. Please show your work.

1. Let $u(x, y) = y^2 + 2y - x^2$.
 - (a) Prove u is harmonic on \mathbb{R}^2 .
 - (b) Find all harmonic conjugates v of u on \mathbb{R}^2 .
 - (c) Let v be your answer in (b). Find $f(z)$ such that $f = u + iv$. Express the function f in terms of z .
2. Find $\log(1 - i)$ and $\text{Log}(1 - i)$.
3. Find $(-i)^{(-i)}$ and P.V. $(-i)^{(-i)}$.
4. Find $i^{(1+i)}$ and P.V. $i^{(1+i)}$.
5. Use the definition of $\sin z$ and $\cos z$ to prove $\cos z = \sin(z + \frac{\pi}{2})$ and $\sin^2 z + \cos^2 z = 1$.

Academic Integrity for the homework: For the homework, you are allowed to discuss the homework problems with your classmates, Soumya and also me. But you need to write your solution down all by yourself. Simply copying the solution from other sources will be regarded as academic violation.

Note: Depending on the assigned workload for our grader, possibly only a subset of the questions will be graded. But you are supposed to finish all the above problems.