MATH 170B HOMEWORK 4

Numerical Intergration

§1: Derive the Newton-Cotes formula for $\int_0^1 f(x)dx$ based on nodes 0, $\frac{1}{3}$, $\frac{2}{3}$, and 1.

§2: Verify that the following formula is exact for polynomials of degree ≤ 4.

$$\int_0^1 f(x)dx \approx \frac{1}{90}\left[7f(0) + 32f\left(\frac{1}{4}\right) + 12f\left(\frac{1}{2}\right) + 32f\left(\frac{3}{4}\right) + 7f(1)\right].$$

(continuation) From the formula above, obtain a formula for $\int_a^b f(x)dx$ that is exact for all polynomials of degree 4. (Apply a suitable change of variables to the integral and the quadrature formula)

§3: Find the formula

$$\int_0^1 f(x)dx \approx A_0 f(0) + A_1 f(1)$$

that is exact for all functions of the form $f(x) = ae^x + b\cos(\pi x/2)$.
(Write down conditions for the quadrature formula to be exact for $f_1(x) = e^x$ and $f_2(x) = \cos(\pi x/2)$ and solve for A_0 and A_1.)

§4: Use the Lagrange interpolation polynomial to derive the formula of the form

$$\int_0^1 f(x)dx \approx Af\left(\frac{1}{3}\right) + Bf\left(\frac{2}{3}\right).$$

Transform this formula to one for integration over $[a, b]$.