MATH 170C ASSIGNMENT 3

(1) (§8.5, 1) Discuss these multistep methods in light of Theorem 1 (p. 558), on multistep method stability and consistency:
   (a) $x_n - x_{n-2} = 2hf_{n-1}$
   (b) $x_n - x_{n-2} = h\left[\frac{7}{3}f_{n-1} - \frac{2}{3}f_{n-2} + \frac{1}{3}f_{n-3}\right]$
   (c) $x_n - x_{n-1} = h\left[\frac{3}{8}f_n + \frac{19}{24}f_{n-1} - \frac{5}{24}f_{n-2} + \frac{1}{24}f_{n-3}\right]$

(2) (§8.5, 3) Show that every multistep method in which $p(z) = z^k - z^{k-1}$ (such as the Adams methods) and $\sum_{i=0}^{k} b_i = 1$ is stable, consistent, and convergent.

(3) Write a MATLAB routine to solve an initial-value problem $x' = f(t,x)$ with $x(t_0) = x_0$ on an interval $a \le t \le b$ using the fourth-order Runge–Kutta method with stepsize h. This function should be written so that it can be called in MATLAB by typing:

```
[x,t] = RK4(@f,x0,a,b,h)
```

   (a) Consider the following initial-value problem,
   $$x' = \lambda x + \cos t - \lambda \sin t, \qquad x(0) = 0$$
   Compare your numerical solution (from RK4) to the exact solution on the interval $[0,5]$ for different values of $\lambda = 5, -5, -10$, and stepsize $h = 0.01$. What effect does λ have on the numerical accuracy?

(4) Write a MATLAB routine to solve an initial-value problem $x' = f(t,x)$ with $x(t_0) = x_0$ on an interval $a \le t \le b$ using the fourth-order Adams-Moulton method with stepsize h. This function should be written so that it can be called in MATLAB by typing:

```
[x,t] = AM4(@f,x0,a,b,h, TOL, MaxIters)
```

Use the RK4 method you implemented earlier to obtain the starting values, and use a fixed point iteration to solve the nonlinear equation.
   (a) Consider the following initial-value problem,
   $$x' = -2tx^2, \qquad x(0) = 1$$
   Compute the solution on the interval $[0,1]$ with stepsize $h = 0.25$ and compare your results with the exact solution $x(t) = 1/(1+t^2)$.