Math 181A: Homework 7

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Problem 1: 6.3.9.

The level of significance is

$$\begin{aligned} P(k \leq 3) &= \binom{7}{0}(0.75)^0(0.25)^7 + \binom{7}{1}(0.75)^1(0.25)^6 + \binom{7}{2}(0.75)^2(0.25)^5 + \binom{7}{3}(0.75)^3(0.25)^4 \\ &= 0.0706 \end{aligned}$$

When $p = 0.65$,

$$\begin{aligned} P(k \leq 3) &= \binom{7}{0}(0.65)^0(0.35)^7 + \binom{7}{1}(0.65)^1(0.35)^6 + \binom{7}{2}(0.65)^2(0.35)^5 + \binom{7}{3}(0.65)^3(0.35)^4 \\ &= 0.1998 \end{aligned}$$

Problem 2: Population Exact Binomial Test

1. We have that $\alpha/2 = 0.05$ From the lefthand side, $0.006 + 0.040 = 0.046 < 0.05$. From the righthand side, $0.000 + 0.002 + 0.011 = 0.013 < 0.05$. The critical region is $k \leq 1$ or $k \geq 8$.

2. From the righthand side, $0.000+0.002+0.011+0.042 = 0.055$. The critical region is $k \geq 7$.

Problem 3: 6.4.3.

6.2.2. The test statistic is $z = \frac{\bar{y}-95}{15/\sqrt{22}}$. The critical region for $\alpha/2 = 0.03$ is $z \leq -1.88$ or $z \geq 1.88$. Thus a value of $\bar{y} \leq 88.99$ or $\bar{y} \geq 101.01$ would cause H_0 to be rejected.

6.4.3. The power is

$$\begin{aligned}
P(\bar{y} \leq 88.99 \,|\, \mu = 90) + P(\bar{y} \geq 101.01 \,|\, \mu = 90) &= P(\frac{\bar{y}-90}{15/\sqrt{22}} \leq \frac{88.99-90}{15/\sqrt{22}}) + P(\frac{\bar{y}-90}{15/\sqrt{22}} \geq \frac{101.01-90}{15/\sqrt{22}}) \\
&= P(Z \leq -0.32) + P(Z \geq 3.44) \\
&= 0.3745 + 0.0003 \\
&= 0.3748
\end{aligned}$$

Problem 4: 6.4.7

The test statistic is $z = \frac{\bar{y}-200}{15/\sqrt{n}}$ The critical region for $\alpha = 0.10$ is $Z \leq -1.28$. H_0 is rejected when $\bar{y} \leq 200 - \frac{19.2}{\sqrt{n}}$. The power is

$$
\begin{aligned}
P(\bar{y} \leq 200 - \frac{19.2}{\sqrt{n}} \,|\, \mu = 197) &= P(\frac{\bar{y}-197}{15/\sqrt{n}} \leq \frac{200 - \frac{19.2}{\sqrt{n}} - 197}{15/\sqrt{n}}) \\
&= P(Z^* \leq \frac{3\sqrt{n} - 19.2}{15}) = 0.75
\end{aligned}
$$

This means that $\frac{3\sqrt{n}-19.2}{15} \geq 0.67$ which implies that $n \geq 95$.

Problem 5: 6.4.18

1.

$$
\begin{aligned}
P(k \le 2 \mid \lambda = 6) &= \frac{e^{-6}6^0}{0!} + \frac{e^{-6}6^1}{1!} + \frac{e^{-6}6^2}{2!} \\
&= 0.062
\end{aligned}
$$

2.

$$
\begin{aligned}
P(k > 2 \mid \lambda = 4) &= 1 - P(k \le 2) \\
&= 1 - \frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!} \\
&= 1 - 0.238 \\
&= 0.762
\end{aligned}
$$

Problem 6: 6.4.20

We have that

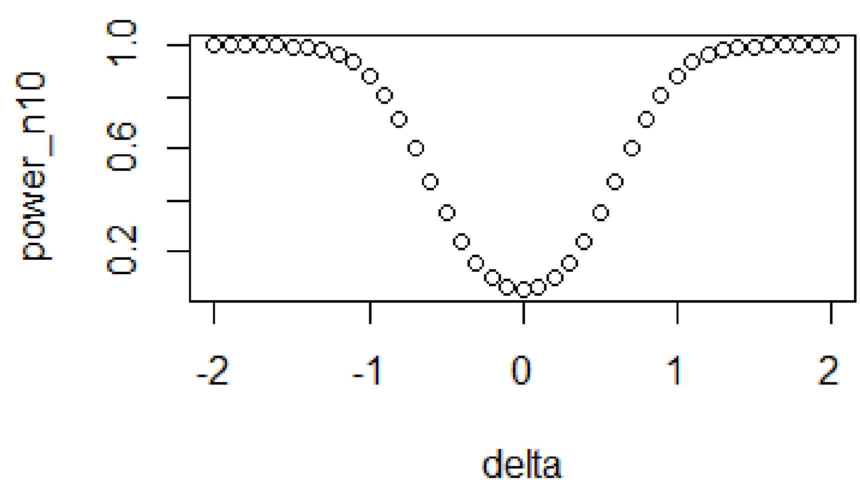
$$\beta = P(y \leq \ln 10) = 1 - e^{-\lambda \ln 10} = 1 - 10^{-\lambda}$$

R Problem

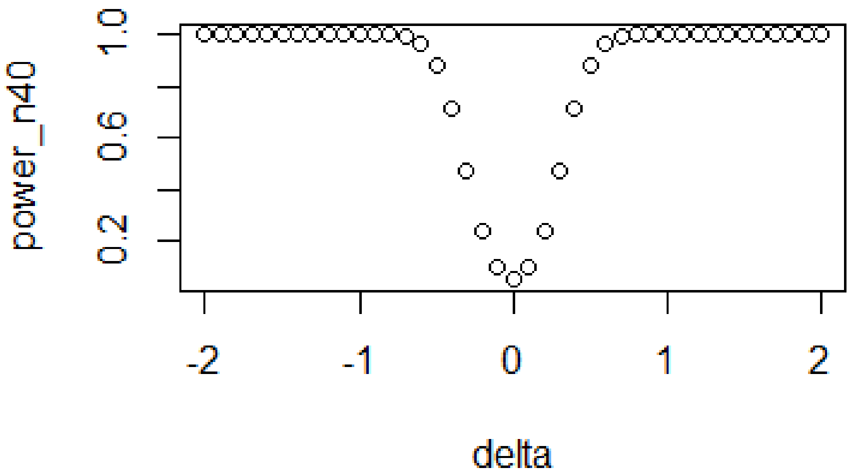
1. The test statistic is $z = \sqrt{n}(\bar{x} - \mu_0)$. The critical region for $\alpha = 0.05$ is $\bar{x} < \mu_0 - \frac{1.96}{\sqrt{n}}$ or $\bar{x} > \mu_0 + \frac{1.96}{\sqrt{n}}$ The power is

$$
\begin{aligned}
&P(\sqrt{n}(\bar{x} - \mu) < \sqrt{n}(\mu_0 - \frac{1.96}{\sqrt{n}} - \mu)) + P(\sqrt{n}(\bar{x} - \mu) > \sqrt{n}(\mu_0 + \frac{1.96}{\sqrt{n}} - \mu)) \\
=&P(Z^* < \sqrt{n}(\mu_0 - \frac{1.96}{\sqrt{n}} - \mu)) + P(Z^* > \sqrt{n}(\mu_0 + \frac{1.96}{\sqrt{n}} - \mu)) \\
=&\Phi(\sqrt{n}(\mu_0 - \frac{1.96}{\sqrt{n}} - \mu)) + \Phi(-\sqrt{n}(\mu_0 + \frac{1.96}{\sqrt{n}} - \mu)) \\
=&\Phi(\sqrt{n}(-\frac{1.96}{\sqrt{n}} - \delta)) + \Phi(-\sqrt{n}(\frac{1.96}{\sqrt{n}} - \delta)) \\
=&\Phi(-1.96 - \delta\sqrt{n}) + \Phi(-1.96 + \delta\sqrt{n})
\end{aligned}
$$

2. Here is a graph of $n = 10$



Here is a graph of $n = 40$



We can see that the greater the absolute value of δ and the greater the value of n, the greater the power is