MATH 170B HOMEWORK 5

Numerical Differentiation and Optimization

§1: Derive the following two formulas for approximating the third derivative. Find their error terms. Which formula is more accurate?

$$f'''(x) \approx \frac{1}{h^3}[f(x+3h) - 3f(x+2h) + 3f(x+h) - f(x)]$$
$$f'''(x) \approx \frac{1}{2h^3}[f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)]$$

§2: Using Taylor series, derive the error term for the approximation

$$f'(x) \approx \frac{1}{2h}[-3f(x) + 4f(x+h) - f(x+2h)]$$

§3: Let a be a given n-vector and $A_{n\times n}$ be given. Compute the gradient and Hessian of

$$f(x) = a^T x$$

§4: Show that the one-dimensional minimizer of a strongly convex function along the ray $x_k + \alpha p_k$ is given by

$$\alpha_k = -\frac{\nabla f_k^T p_k}{p_k^T Q p_k}$$

where it is given that a strongly convex quadratic function has the form

$$f(x) = \frac{1}{2}x^T Q x + b^T x, \qquad Q > 0$$

§5: Prove that

$$||Bx|| \geq ||x||/||B^{-1}||$$

for any non-singular matrix B. Use this fact to establish

$$\cos\theta_k = \frac{-\nabla f_k^T p_k}{||\nabla f_k||||p_k||} \geq \frac{1}{M}$$

for some constant M.