MATH 140B WINTER 2024: PROBLEM SET 1

Due: S 21/01/2024, by 11:59pm

Directions: You can collaborate, but must write up the solutions independently and in a good handwriting. **Consulting solutions to problem sets of previous semesters or internet solutions is not allowed.**

Problem 1. Prove that every Lipschitz continuous function $f : A \to \mathbb{R}$ is also uniformly continuous on A. Do the same for Hölder continuous functions.

Problem 2. Suppose $f : \mathbb{R} \to \mathbb{R}$ and that there exists some $\alpha > 1$ so that $|f(x) - f(y)| \leq |x - y|^\alpha$ for all $x, y \in \mathbb{R}$. Show that $f(x) = f(0)$ for all $x \in \mathbb{R}$. This is why it doesn't make sense to define a concept of Hölder continuity for $\alpha > 1$.

Problem 3. Assume that $f : [0, 1] \to \mathbb{R}$ is twice differentiable and that for all $x \in [0, 1]$, $f(x)f''(x) \geq 0$ and $f(0) = 0$. Show that f is monotonic. Give an example to show that $f(0) = 0$ is necessary for the conclusion to hold.

Problem 4. Chapter 5: 2, 3, 4, 5, 6