

## Mathematics 100A Homework 3

**Due: Tuesday October 22 2024**

**Instructions:** Please write clearly and fully explain your solutions. It is OK to work with others to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. Chapter and problem numbers, if any, refer to *Algebra*, second edition, by Michael Artin. Please feel free to reach out to me or the TAs if you have any questions.

1. (Chapter 2, Problem 8.3) Suppose  $p$  is a prime number, and  $G$  is a group with order  $p^n$  for a positive integer  $n$ . Prove that  $G$  contains an element of order  $p$ . **Hint:** Induct on  $n$ .
2. (Chapter 2, Problem 8.5) Suppose  $G$  is a finite group, and that  $G$  contains an element of order 10 and an element of order 6. Prove that  $|G|$  is a multiple of 30.
3. (Chapter 2, Problem 8.6) Suppose  $\varphi : G \rightarrow G'$  is a group homomorphism,  $|G| = 18$  and  $|G'| = 15$ . Assume that  $\varphi$  is not the trivial homomorphism. What is the order of  $\ker(\varphi)$ ?
4. (Chapter 2, Problem 8.10) Suppose  $G$  is a group, and  $N$  is a subgroup of  $G$  of index 2. Prove that  $N$  is normal.