Math 170C: Homework 3

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Problem 1

(a) $(a_0, a_1, a_2) = (-1, 0, 1)$ and $(b_0, b_1, b_2) = (0, 2, 0)$

$$p(z) = z^2 - 1 = (z-1)(z+1) \qquad q(z) = 2z$$

The method is stable since the roots of p are in the unit disk and simple. The method is consistent since $p(1) = 0$ and $p'(1) = 2 = q(1)$.

(b) $(a_0, a_1, a_2, a_3) = (0, -1, 0, 1)$ and $(b_0, b_1, b_2, b_3) = (\frac{1}{3}, -\frac{2}{3}, \frac{7}{3}, 0)$

$$p(z) = z^3 - z = z(z-1)(z+1) \qquad q(z) = \frac{7}{3}z^2 - \frac{2}{3}z + \frac{1}{3}$$

The method is stable since the roots of p are in the unit disk and simple. The method is consistent since $p(1) = 0$ and $p'(1) = 2 = q(1)$.

(c) $(a_0, a_1, a_2, a_3) = (0, 0, -1, 1)$ and $(b_0, b_1, b_2, b_3) = (\frac{1}{24}, -\frac{5}{24}, \frac{19}{24}, \frac{3}{8})$

$$p(z) = z^3 - z^2 = z^2(z-1) \qquad q(z) = \frac{3}{8}z^3 + \frac{19}{24}z^2 - \frac{5}{24}z + \frac{1}{24}$$

The method is stable since the roots of p are in the unit disk and 1 is a simple root. The method is consistent $p(1) = 0$ and $p'(1) = 1 = q(1)$.

Problem 2

A multistep method with $p(z) = z^k - z^{k-1} = z^k(z-1)$ is stable since the roots are in the unit disk and 1 is a simple root. The method is also consistent since

$$p(1) = 0$$

$$p'(1) = k - (k-1) = 1 = \sum_{i=0}^{k} b_i = q(1).$$

Since the method is stable and consistent, the method is also convergent.

Problem 3

The RK4 code is as follows

```
function [x,t] = RK4(f,x0,a,b,h)
    m = round((b-a)/h)+1;
    t = linspace(a,b,m);
    x = zeros(1,m);
    t(1) = a;
    x(1) = x0;
    for i = 2:m
        t_n = t(i-1);
        x_n = x(i-1);

        k1 = f(t_n, x_n);
        k2 = f(t_n+h/2, x_n+h*k1/2);
        k3 = f(t_n+h/2, x_n+h*k2/2);
        k4 = f(t_n+h, x_n+h*k3);
        x(i) = x_n + h/6*(k1 + 2*k2 + 2*k3 + k4);
    end
end
```

The exact solution is $x = \sin(t)$, and the numerical accuracy is much greater for negative values of λ.

```
>> t_real = linspace(0,5, 501);
>> x_real = sin(t_real);
>> [x_5,t_5] = RK4(@(t, x)(5*x + cos(t) - 5*sin(t)),0,0,5, 0.01);
>> [x_neg5,t_neg5] = RK4(@(t, x)(-5*x + cos(t) + 5*sin(t)),0,0,5, 0.01);
>> [x_neg10,t_neg10] = RK4(@(t, x)(-10*x + cos(t) + 10*sin(t)),0,0,5, 0.01);
>> x_5(501) - x_real(501)

ans =

22.6730

>> x_neg5(501) - x_real(501)

ans =

2.5755e-09

>> x_neg10(501) - x_real(501)

ans =

1.0622e-08
```

Problem 4

The code is as follows

```
function [x,t] = AM4(f,x0,a,b,h,TOL,MaxIters)
[x,t] = RK4(f,x0,a,b,h);
m = size(x,2);
for i = 4:m
    % Fixed point method
    for j = 1:MaxIters
        k1 = 9/24*f(t(i), x(i));
        k2 = 19/24*f(t(i-1), x(i-1));
        k3 = -5/24*f(t(i-2), x(i-2));
        k4 = 1/24*f(t(i-3), x(i-3));

        prev_x = x(i);
        x(i) = x(i-1) + h*(k1+k2+k3+k4);
        if x(i) - prev_x < TOL
            break
        end
    end
end
end
```

The code is accurate up to the provided TOL.

```
>> t_real = linspace(0,1, 5);
>> x_real = 1./(1+t_real.^2)

x_real =

1.0000    0.9412    0.8000    0.6400    0.5000

>> [x,t] = AM4(@(t, x)(-2*t*x*x),1,0,1, 0.25, 0.0001, 1000);
>> x

x =

1.0000    0.9412    0.7999    0.6388    0.4995
```