MATH 170C HOMEWORK 7

(1) (§9.1, 2) Show that the function

$$u(x,t) = \sum_{n=1}^{N} c_n \exp(-n^2\pi^2 t)\sin(n\pi x)$$

solves the heat conduction problem $u_x x = u_t$ with boundary conditions

$$\begin{cases} u(x,0) = \sum_{n=1}^{N} c_n \sin(n\pi x) \\ u(0,t) = u(1,t) = 0 \end{cases}$$

(2) (§9.3, 1) Let ∇^2 denote the Laplacian operator: $\nabla^2 u = u_{xx} + u_{yy}$. Prove that a problem of the form

$$\begin{cases} \nabla^2 u = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

can be solved by the following three steps: **i.** Find g such that $\nabla^2 g = f$. **ii.** Solve the Dirichlet problem in Ω, using $-g$ for boundary values. **iii.** Add g to the function obtained in step **ii** to find u.

(3) (§9.4, 1) Prove that if w is an analytic function of z (with $w = u + iv$ and $z = x + iy$), then u and v are harmonic. Hint: Use the Cauchy-Riemman equations, $u_x = v_y$, $u_y = -v_x$, which are necessary and sufficient conditions for a function to be analytic.