Math 170A: Homework 2

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Question 1

The matrix does not have an LU decomposition because $A_{1,1} = 0$. The operations on A are as follows.

$$\begin{bmatrix} 0 & 1 & 1 & 2 \\ 2 & 0 & 0 & 0 \\ 1 & 1 & 1 & 3 \\ 1 & 0 & 2 & 4 \end{bmatrix} \implies \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 1 & 1 & 1 & 3 \\ 1 & 0 & 2 & 4 \end{bmatrix} \implies \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 2 & 4 \end{bmatrix} \implies$$
$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 2 & 4 \end{bmatrix} \implies \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 4 \end{bmatrix} \implies \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The operations on L are as follows.

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \implies \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \implies \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \end{bmatrix} \implies$$
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \end{bmatrix} \implies \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \end{bmatrix} \implies \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \end{bmatrix}$$

We swapped R_1 with R_2 and R_3 with R_4 so the permutation matrix is

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

We add the identity to L to get

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.5 & 0 & 1 & 0 \\ 0.5 & 1 & 0 & 1 \end{bmatrix}$$

U is the resulting matrix after the the row operations

$$U = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 2

$$\begin{aligned}
r_{11} &= \sqrt{a_{11}} = 1 \\
r_{12} &= \frac{a_{12}}{r_{11}} = -2 \\
r_{13} &= \frac{a_{13}}{r_{11}} = 0 \\
r_{22} &= \sqrt{a_{22} - r_{12}^2} = 3 \\
r_{23} &= \frac{a_{23} - r_{12}r_{13}}{r_{22}} = 2 \\
r_{33} &= \sqrt{a_{33} - r_{13}^2 - r_{23}^2} = 1
\end{aligned}$$

$$R = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Since we were able to perform all the math for the Cholesky-factorization and since the matrix is symmetric, we know that A is positive-definite.

The first and second row of B are not independent, so B has an eigenvalue of 0 and it does not have a Cholesky factorization.

Question 3

B is symmetric since

$$\begin{aligned} B^T &= (X^T A X)^T \\ &= X^T A^T (X^T)^T \\ &= X^T A X \\ &= B \end{aligned}$$

Let $y \in \mathbb{R}^n$ and define $x = Xy$. B is positive definite since

$$\begin{aligned} y^T B y &= y^T (X^T A X) y \\ &= (Xy)^T A (Xy) \\ &= x^T A x \\ &> 0 \end{aligned}$$

Question 4

We can see that AP^T is equivalent to tranposing A so that the columns are now rows, permuting the rows, and then transposing the resulting matrix back such that the columns are permuted.

$$AP^T = ((AP^T)^T)^T = (PA^T)^T$$

In row i of P, the 1 is in column $p(i)$. Since P^T permutes the columns in P, it will send the $p(i)$th column to the ith column. This will result in the 1 being in position A_{ii} for all i, which is the identity matrix.

Question 5

The Backsub function used is the same as listed in the problem.

```
function x = ge_pp_solve(A,b)
n = size(A,1);

if (size(A,2) ~= n) || (size(b,1) ~= n) || (size(b,2)
    ~= 1)
    error('cannot solve this system')
end

for i=1:n
    % Swap rows
    [pivot, i_star] = max(abs(A(i:n,i)));
    i_star = i_star + i - 1;
    if pivot == 0
        error('cannot do GE')
    end

    if i_star ~= i
        % Swap for A
        tempA = A(i, :);
        A(i,:) = A(i_star, :);
        A(i_star,:) = tempA;
        % Swap for B
        tempB = b(i);
        b(i) = b(i_star);
        b(i_star) = tempB;
    end

    % Row Operation
    for j=(i+1):n
        l = A(j,i)/A(i,i);
        A(j,i) = 0;
        for k = (i+1):n
            A(j,k) = A(j,k) - l*A(i,k);
        end
        b(j) = b(j) - l*b(i);
    end
end

U = A;

x = BackSub(U,b);
end
```