# Math 170C: Homework 7

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## Problem 1

The solution is

$$u(x,t) = \sum_{n=1}^{N} c_n e^{-n^2\pi^2 t} \sin(n\pi x)$$

The solution has the correct initial time condition since

$$u(x,0) = \sum_{n=1}^{N} c_n e^0 \sin(n\pi x) = \sum_{n=1}^{N} c_n \sin(n\pi x).$$

The solution has the correct initial distance conditions since

$$u(0,t) = \sum_{n=1}^{N} c_n e^{-n^2\pi^2 t} \sin(0) = 0$$

$$u(1,t) = \sum_{n=1}^{N} c_n e^{-n^2\pi^2 t} \sin(n\pi) = 0$$

It satisfies the differential equation since

$$\begin{aligned}
u_{xx} &= \frac{\partial}{\partial x} n\pi \sum_{n=1}^{N} c_n e^{-n^2\pi^2 t} \cos(n\pi x) \\
&= -n^2\pi^2 \sum_{n=1}^{N} c_n e^{-n^2\pi^2 t} \sin(n\pi x) \\
&= u_t
\end{aligned}$$

## Problem 2

First we find $g$ such that $\nabla^2 g = f$ in $\Omega$. Then we solve the dirichlet problem in $\Omega$, using $-g$ for the boundary values. Call this solution $v$ so that

$$\begin{cases} \nabla^2 v = 0 & \text{in } \Omega \\ v = -g & \text{on } \partial\Omega \end{cases}$$

Thus, $u = v + g$ will equal 0 on the boundary and it will equal $\nabla^2 u = f$ in $\Omega$, which solves the problem.

## Problem 3

By the Cauchy-Riemann equations we have that $u_x = v_y$ and $u_y = -v_x$. We need to prove that $u$ and $v$ are harmonic, meaning that $\nabla^2 u = 0$ and $\nabla^2 v = 0$. These directly follow from the Cauchy-Riemann equations since

$$\begin{aligned}\nabla^2 u &= u_{xx} + u_{yy} \\ &= v_{yx} - v_{xy} \\ &= 0\end{aligned}$$

$$\begin{aligned}\nabla^2 v &= v_{xx} + v_{yy} \\ &= -u_{yx} + u_{xy} \\ &= 0\end{aligned}$$