Math 120A: Homework 9

Merrick Qiu

1. Page 159: Problem 1(a)(b)

$$f(z) = \frac{z^2}{z+3}$$

Since this function is the quotient of two polynomials, it is only not analytic at $z = -3$ which is outside of the region bound by the contour. Therefore the function is analytic everywhere interior to and on C so the integral is zero by the Cauchy-Goursat theorem.

$$f(z) = ze^{-z}$$

Since z is analytic and e^{-z} is analytic, their product is analytic everywhere. Therefore the integral is zero by the Cauchy-Goursat theorem.

2. Page 159: Problem 2(a)

$$f(z) = \frac{1}{3z^2 + 1}$$

Since this function is the quotient of two polynomials, it is analytic everywhere except when $3z^2 + 1 = 0$, where $z = \pm\frac{1}{\sqrt{3}}i$, which is inside of the square and outside of the region bound by the circle and the square. Therefore by the corollary, the two integrals are equal.

3. Page 170: Problem 1(a)(b)

Since e^{-z} is analytic, by the cauchy integral formula

$$\int_C \frac{e^{-z}\,dz}{z-(\pi i/2)} = 2\pi i e^{-\pi i/2} = 2\pi$$

Since $\frac{\cos z}{z^2+8}$ is analytic interior to C,

$$\int_C \frac{\cos z}{z(z^2+8)}\,dz = 2\pi i \frac{\cos 0}{0^2+8} = \frac{2\pi i}{8}$$

4. Page 170: Problem 2

Since $\frac{1}{z+2i}$ is analytic inside the circle,

$$\int_C \frac{1}{(z-2i)(z+2i)}\,dz = 2\pi i \frac{1}{2i+2i} = \frac{\pi}{2}$$

By the cauchy extension,

$$\int_C \frac{1}{(z-2i)^2(z+2i)^2}\,dz = \frac{2\pi i}{1!}\left(-\frac{2}{(2i+2i)^3}\right) = \frac{\pi}{16}$$

5. Page 170: Problem 3

Since $2s^2 - s - 2$ is analytic everywhere,

$$g(2) = \int_C \frac{2s^2 - s - 2}{s - 2}\, ds = 2\pi i(2(2)^2 - 2 - 2) = 8\pi i$$

If $|z| > 3$ the cauchy-goursat theorem says that $g(z) = 0$.

6. Page 170: Problem 4

If z is inside then by the cauchy extension

$$\frac{d^2}{ds^2}s^3 + 2s = \frac{d}{ds}3s^2 + 2 = 6s$$

$$\begin{aligned}\int_C \frac{s^3 + 2s}{(s-z)^3}\, ds &= \frac{2\pi i}{2!}\,(6z) \\ &= 6\pi i z\end{aligned}$$

If z is outside then the cauchy-goursat theorem says that $g(z) = 0$.