Instructions

- **Midterm receipt.** This **Midterm 2** is available for download from **Gradescope** at **2:55 pm** Pacific Time on Friday **May 20**. In the unlikely event that you cannot download the exam at that time you should email both Prof. Chow at <bechow@ucsd.edu> and TA Zhiyuan Jiang <z5jiang@ucsd.edu> to receive an emailed copy of the exam. If you are taking the midterm asynchronously, you will be able to download the midterm at your given start time.

- **Exam format.** There are **4 questions**, worth 10 points each. Total: **40 points**. Time: Although the official time for the exam is 3:00 pm to 3:50 pm, you may start up to 5 minutes early (upon download) and you have a 10 minute grace period to upload your exam to Gradescope, so your exam will not be counted late if uploaded by **4:00 pm**.

- **Format for your solutions.** You should hand write your solutions neatly and darkly on your own blank or lined 11"×8.5" paper or on your iPad/tablet in an organized way. Do not type solutions.

- **Allowed resources.** The resources you may use are: Hubbard and Hubbard's book, any materials on the canvas site, piazza discussion, and the google drive, all for this course only. Students cannot communicate with each other or any people by any means. E.g., no use of Discord, texting, messaging, etc. is allowed. You may not search the internet outside the above resources. You may not use any calculator or mathematical software except for addition, multiplication, subtraction, and division.

- **Submission.** Please make sure your files are legible before submitting and carefully follow the **Gradescope** instructions to match every question to the appropriate pages of your submission.

- **Deadline.** The deadline to upload your midterm solution to **Gradescope** is 4:00 pm (Pacific Time) on May 20 (= 50 minutes plus 10-minute grace period). Submissions after 4:00 pm will be deducted 1/2 point per minute late, with no midterms accepted after 4:15 pm. Email your exam to both <bechow@ucsd.edu> and <z5jiang@ucsd.edu> if you feel you cannot upload to gradescope in time to timestamp your submission; after that, upload to Gradescope as soon as possible.

1. (10 points) Consider the ellipsoid $M = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; x_1^2 + \frac{x_2^2}{4} + \frac{x_3^2}{9} = 1 \right\}$. Consider the point $\mathbf{x} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$ on the surface M, and the basis of tangent vectors at $\mathbf{x}$ defined by $\vec{v}_1 = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$. Consider the transverse vector $\vec{t} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$. Is the basis $\vec{v}_1, \vec{v}_2$ direct (a.k.a. positively oriented) for the orientation defined by $\vec{t}$?

2. (10 points) Let $U = \left\{ (u_1, u_2) \in \mathbb{R}^2 \,\middle|\, u_1^2 + u_2^2 \leq 1 \right\}$. Compute the area (2-dimensional volume) of the parametrized surface $M = \gamma(U) \subset \mathbb{R}^3$, where

$$\gamma \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_2 \\ -u_1 \\ u_1 u_2 \end{pmatrix}. \tag{1}$$

   Hint: After you set up the integral, you can then use the standard polar coordinates integration formula.

3. (10 points) Let $D = \left\{ (u_1, u_2) \in \mathbb{R}^2 \,\middle|\, 0 \leq u_1 \leq \sin(u_2),\ 0 \leq u_2 \leq \frac{\pi}{2} \right\}$. Define the map $\Phi : D \to \mathbb{R}^2$ by $\Phi \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_1 \sin(u_2) \\ u_1 \cos(u_2) \end{pmatrix}$. Use the change of variables formula to compute the integral $\int_{\Phi(D)} x_2 \, |dx_1 dx_2|$.

4. (10 points) Let $U = \left\{ (u_1, u_2) \in \mathbb{R}^2 \,\middle|\, 0 \leq u_1 \leq 1,\ 0 \leq u_2 \leq 1 \right\}$. Define the parametrization $\gamma : U \to \mathbb{R}^3$ by $\gamma \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_1^2 \\ u_2^2 \\ u_1 + u_2 \end{pmatrix}$ of the surface $M = \gamma(U) \subset \mathbb{R}^3$. Define the 2-form $\varphi = x_3 \, dx_1 \wedge dx_2$. Compute $\int_M \varphi$.