MATH 170C ASSIGNMENT 2

(1) Derive the three-stage Runge–Kutta method that corresponds to the collocation points $c_1 = \frac{1}{4}$, $c_2 = \frac{1}{2}$, $c_3 = \frac{3}{4}$, and determine its order of accuracy.

(2) (§8.4, 4) Use the method of undetermined coefficients to derive the **fourth-order Adams-Bashforth formula**

$$x_{n+1} = x_n + \frac{h}{24}[55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}]$$

(3) (§8.4, 5) Derive the **fourth-order Adams-Moulton formula**

$$x_{n+1} = x_n + \frac{h}{24}[9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2}]$$

(4) (§8.4, 6) Prove that is every element of Π_m (polynomials of degree $\leq m$) is correctly integrated by the formula

$$\int_0^1 f(x)dx \approx \sum_{i=-n}^{n} A_i f(i)$$

then the same is true of the formula

$$\int_{t_0}^{t_0+h} f(x)dx \approx h \sum_{i=-n}^{n} A_i f(t_0 + ih)$$

Hint: consider the change of variables formula for integrals.

(5) (§8.4, 12) The formula

$$x_{n+1} = (1 - A)x_n + Ax_{n-1} + \frac{h}{12}\left[(5 - A)x'_{n+1} + 8(1 + A)x'_n + (5A - 1)x'_{n-1}\right]$$

is known to be exact for all polynomials of degree m or less for all A. Determine A so that it will be exact for all polynomials of degree $m + 1$. Find A and m.