MATH140C SPRING 2024: PROBLEM SET 1

Directions: You can collaborate, but must write up the solutions independently and in a good handwriting. **Consulting solutions to problem sets of previous semesters or internet solutions is not allowed.**

Problem 1. Let $E = \{\mathbf{v}_1 \dots, \mathbf{v}_r\}$ be a linearly independent subset of a vector space X and $\mathbf{y} \in X$.

(1) Prove that $\mathbf{y} \in \text{span}(E)$ if and only if $E \cup \{\mathbf{y}\}$ is linearly dependent.

(2) Prove that if $E \cup \{\mathbf{y}\}$ is linearly dependent, then $\text{span}(E) = \text{span}(E \cup \{\mathbf{y}\})$.

Problem 2. Rudin: 9.1, 9.2, 9.3, 9.4, 9.5