Math 120A: Homework 7

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Problem 1

$$e^{z_0 t} = e^{x_0 t} e^{i y_0 t} = e^{x_0 t} \cos y_0 t + i e^{x_0 t} \sin y_0 t$$

$$\begin{aligned}
(e^{z_0 t})' &= (e^{x_0 t} \cos y_0 t)' + i(e^{x_0 t} \sin y_0 t)' \\
&= (-y_0 e^{x_0 t} \sin y_0 t + x_0 e^{x_0 t} \cos y_0 t) + i(y_0 e^{x_0 t} \cos y_0 t + x_0 e^{x_0 t} \sin y_0 t) \\
&= (x_0 + i y_0)(e^{x_0 t} \cos y_0 t + i e^{x_0 t} \sin y_0 t) \\
&= z_0 e^{z_0 t}
\end{aligned}$$

Problem 2

$$(1-2it)^2 = 1 - 4it - 4t^2 = (1-4t^2) - i(4t)$$

$$\begin{aligned}\int_0^1 1 - 4t^2\,dt - i\int_0^1 4t\,dt &= \left[t - \frac{4}{3}t^3\right]_0^1 - i\left[2t^2\right]_0^1 \\ &= -\frac{1}{3} - 2i\end{aligned}$$

Problem 3

Note that

$$\frac{1}{5i}e^{5it} = e^{5it}$$

Therefore,

$$\int_0^{\frac{\pi}{10}} e^{5it}\,dt = \left[\frac{1}{5i}e^{5it}\right]_0^{\frac{\pi}{10}} = \frac{1}{5} - \frac{1}{5i} = \frac{1}{5} + i\frac{1}{5}$$

Problem 4

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\begin{aligned}
\int_C \frac{z-2}{z}\,dz &= \int_0^\pi \frac{2e^{it}-2}{2e^{it}} 2ie^{it}\,dt \\
&= \int_0^\pi i(2e^{it}-2)\,dt \\
&= \left[2e^{it} - 2it\right]_0^\pi \\
&= (-2-2\pi i) - (2) \\
&= -4 - 2\pi i
\end{aligned}
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