Math 20D HW3

Merrick Qiu

April 16, 2022

Section4.3

Problem 3 The roots are

$$r^2 - 6r + 10 \implies r = \frac{6 \pm \sqrt{36-40}}{2} \implies r = 3 \pm i$$

The general solution is

$$y = c_1 e^{3t}\cos t + c_2 e^{3t}\sin t$$

Problem 6 The roots are

$$r^2 - 4r + 7 \implies r = \frac{4 \pm \sqrt{16-28}}{2} \implies r = 2 \pm \sqrt{3}i$$

The general solution is

$$y = c_1 e^{2t}\cos(\sqrt{3}t) + c_2 e^{2t}\sin(\sqrt{3}t)$$

Problem 17 The roots are

$$r^2 - r + 7 = 0 \implies r = \frac{1 \pm \sqrt{1-28}}{2} \implies r = \frac{1}{2} \pm \frac{3\sqrt{3}}{2}$$

The general solution is

$$y = c_1 e^{\frac{t}{2}}\cos(\frac{3\sqrt{3}t}{2}) + c_2 e^{\frac{t}{2}}\sin(\frac{3\sqrt{3}t}{2})$$

Problem 24 The roots are

$$r^2 + 9 = 0 \implies r = 3i$$

The general solution is

$$y = c_1 \cos 3t + c_2 \sin 3t$$

At the initial conditions,

$$1 = c_1$$

$$1 = 3c_2$$

The final solution is

$$y = \cos 3t + \frac{1}{3}\sin 3t$$

Problem 25 The roots are

$$r^2 - 2r + 2 = 0 \implies r = \frac{2 \pm \sqrt{4-8}}{2} \implies r = 1 \pm i$$

The general solution is

$$y = c_1 e^t \cos t + c_2 e^t \sin t$$

At the initial conditions,

$$e^\pi = -c_1 e^\pi$$

$$0 = c_1 e^\pi - c_2 e^\pi$$

The final solution is

$$y = e^t \sin t - e^t \cos t$$

Problem 26 The roots are

$$r^2 - 2r + 1 = 0 \implies r = \frac{2 \pm \sqrt{4-4}}{2} \implies r = 1$$

The general solution is

$$y = c_1 e^t + c_2 t e^t$$

At the initial conditions,

$$1 = c_1$$

$$-2 = c_1 + c_2$$

The final solution is

$$y = e^t - 3te^t$$

Problem 28 The roots are

$$r^2 + br + 4 = 0 \implies r = \frac{-b \pm \sqrt{b^2 - 16}}{2}$$

For $b = 5$,

$$r_5 = -\frac{5}{2} \pm \frac{3}{2} \implies r_5 = -4, -1$$

The general solution is

$$y_5 = c_1 e^{-4t} + c_2 e^{-t}$$

At the initial conditions,

$$1 = c_1 + c_2$$

$$0 = -4c_1 - c_2$$

The final solution is

$$y_5 = -\frac{1}{3} e^{-4t} + \frac{4}{3} e^{-t}$$

For $b = 4$,

$$r_4 = -2$$

The general solution is

$$y_4 = c_1 e^{-2t} + c_2 t e^{-2t}$$

At the initial conditions,

$$1 = c_1$$

$$0 = -2c_1 + c_2$$

The final solution is

$$y_4 = e^{-2t} + 2te^{-2t}$$

For $b = 2$,

$$r_2 = -1 \pm \sqrt{3} i$$

The general solution is

$$y_2 = c_1 e^{-t} \cos \sqrt{3} t + c_2 e^{-t} \sin \sqrt{3} t$$

At the initial conditions,

$$1 = c_1$$

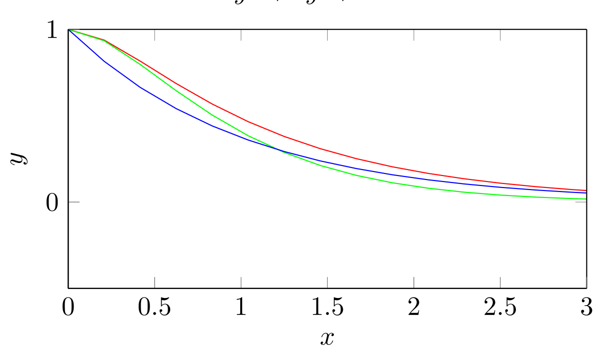
$$0 = -c_1 + \sqrt{3} c_2$$

The final solution is

$$y_2 = e^{-t} \cos \sqrt{3} t + \frac{1}{\sqrt{3}} e^{-t} \sin \sqrt{3} t$$

Graphing the equations yields (not quite sure why $b = 2$ graph isn't working),

$y'' + by' + 4 = 0$



Section 4-4

Problem 4 Method works since $3^t = e^(\ln 3t)$
Problem 11 The roots are

$$r^2 + r = 0 \implies r = -1, 0$$

Since $r = \ln 2$ is not a root, the general form is

$$y = A_0 2^x$$

Plugging this into the original equation yields

$$\ln(2)^2 A_0 2^x + A_0 2^x = 2^x \implies A_0 = \frac{1}{\ln(2)^2 + 1}$$

The particular solution is

$$y = \frac{2^x}{\ln(2)^2 + 1}$$

Problem 14 The roots are

$$2r^2 + 1 = 0 \implies r = \frac{1}{\sqrt{2}} i$$

Since $r = 2$ is not a root, the general form is

$$y = A_0 e^{2t}$$

Plugging this into the original equation yields

$$8A_0 e^{2t} + A_0 e^{2t} = 9e^{2t} \implies A_0 = 1$$

The particular solution is

$$y = e^{2t}$$

Problem 18 The roots are

$$r^2 - 2r + 1 = 0 \implies r = 1$$

Since $r = 1$ is a double root, the general form is

$$y = A_0 t^2 e^t$$

Plugging this into the original equation yields

$$A_0((2e^t + 2te^t + 2te^t + t^2 e^t) - 2(2te^t + t^2 e^t) + (t^2 e^t)) = 8e^t \implies A_0(2e^t) = 8e^t \implies A_0 = 4$$

The particular solution is

$$y = 4t^2 e^t$$

Problem 25 The roots are

$$r^2 + 2r + 4 = 0 \implies r = \frac{-2 \pm \sqrt{4 - 16}}{2} \implies r = -1 \pm \sqrt{3} i$$

$2 + 3i$ is not a root, so the general solution is

$$y = e^{2t}(A \cos 3t + B \sin 3t)$$

Finding the derivatives yields

$$\begin{aligned} y' &= 2e^{2t}(A \cos 3t + B \sin 3t) + e^{2t}(-3A \sin 3t + 3B \cos 3t) \\ &= e^{2t}((2A + 3B) \cos 3t + (-3A + 2B) \sin 3t) \end{aligned}$$

$$\begin{aligned} y'' &= 2e^{2t}((2A + 3B) \cos 3t + (-3A + 2B) \sin 3t) + e^{2t}((-6A - 9B) \sin 3t + (-9A + 6B) \cos 3t) \\ &= e^{2t}((-5A + 12B) \cos 3t + (-12A - 5B) \sin 3t) \end{aligned}$$

Plugging these into the original equation yields,

$$(3A + 18B)\cos 3t + (-18A + 3B)\sin 3t = 111\cos 3t$$

Therefore, the particular solution is

$$y = e^{2t}(\cos 3t + 6\sin 3t)$$

Problem 28 The auxillary equation has the double root $r = 3$, so $s = 2$.

$$y = t^2(A_6t^6 + A_5t^5 + A_4t^4 + A_3t^3 + A_2t^2 + A_1t + A_0)e^{3t}$$

Problem 31 The auxillary equation has roots $r = -1 \pm i$, so $s = 1$

$$y = t(A_3t^3 + A_2t^2 + A_1t + A_0)e^{-t}\cos t + t(B_3t^3 + B_2t^2 + B_1t + B_0)e^{-t}\sin t$$

Problem 32 The auxillary equation has roots $r = -3, 4$ so $s = 1$

$$y = t(A_3t^3 + A_2t^2 + A_1t + A_0)e^{-3t}$$

Section 4-5

Problem 1

1. $5\cos t$
2. $\cos t - e^{2t}$
3. $4\cos + 6e^{2t}$

Problem 4 The roots are

$$r^2 + r = 0 \implies r = -1, 0$$

The general solution is therefore

$$y = t + C_1 e^{-t} + C_2$$

Problem 18 The roots are

$$r^2 - 1 = 0 \implies r = -1, 1$$

The particular solution has form

$$y_p(t) = A_1 t + A_0$$

Solving for the coefficients yields

$$y_p(t) = 11t - 1$$

The general solution is therefore

$$y(t) = 11t - 1 + C_1 e^{-t} + C_2 e^t$$

Problem 23 The equation is linear and the integrating factor is

$$\mu = e^{-x}$$

Multiplying both sides by the integrating factor yields

$$\frac{d}{dx} e^{-x} y = e^{-x} \implies e^{-x} y = -e^{-x} \implies y = -1$$

The equation has root $r = 1$, so the general form is

$$y = Ce^t - 1$$

At $y(0) = 0$,

$$y = e^t - 1$$

Problem 24 Integrating twice yields

$$y' = 3t^2 + C_1$$

$$y = t^3 + C_1 t + C_2$$

At the initial values $C_1 = -1$ and $C_2 = 3$,

$$y = t^3 - t + 3$$

Problem 25 The roots are

$$r^2 + 1 = 0 \implies r = \pm i$$

The homogenous solution is therefore

$$y_h = C_1 \cos x + C_2 \sin x$$

The particular solution is in the form

$$y_p = A_0 e^{-x}$$

Solving gives $A_0 = 1$. The general solution has form

$$y = e^{-x} + C_1 \cos x + C_2 \sin x$$

Solving for constants yields

$$y = e^{-x} - \cos x + \sin x$$

Problem 28 The roots are

$$r^2 + r - 12 = 0 \implies (r+4)(r-3) = 0 \implies r = -4, r = 3$$

The homogenous solution is

$$y_h = C_1 e^{-4t} + C_2 3t$$

For $r = 1, 2$ are not roots, so the particular solution has form

$$y_p = Ae^t + Be^{2t} + \frac{1}{12}$$

Solving for the constants gives $A = -\frac{1}{10}$ and $B = -\frac{1}{3}$ The general solution has form

$$y = -\frac{1}{10}e^t - \frac{1}{6}e^{2t} + \frac{1}{12} + C_1 e^{-4t} + C_2 3t$$

Plugging in the initial values gives

$$1 = -\frac{4}{15} + C_1 + C_2$$

$$3 = -\frac{13}{30} - 4C_1 + 3C_2$$

Therefore the final solution is

$$y = -\frac{1}{10}e^t - \frac{1}{6}e^{2t} + \frac{1}{12} + \frac{1}{60}e^{-4t} + \frac{7}{6}3t$$