Gear System Optimization

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ME 423 Numeric Optimization
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Fall 2020

Background:

While carrying out my undergraduate capstone project at Saint Martin's University it became necessary to design a gear reduction. Once I had the reduction designed I researched the cost to implement the system into the greater design and found that the cost was more than prohibitive. The forces involved demanded the use of steel gears, and steel gears are difficult to source and expensive to purchase in small quantities. With that realization it became apparent that the ability to produce prototype-grade gearing with additive manufacturing methods would be useful.

Purpose:

The purpose of this project was to design a Matlab program which would optimize a gearing system for 3D printing.

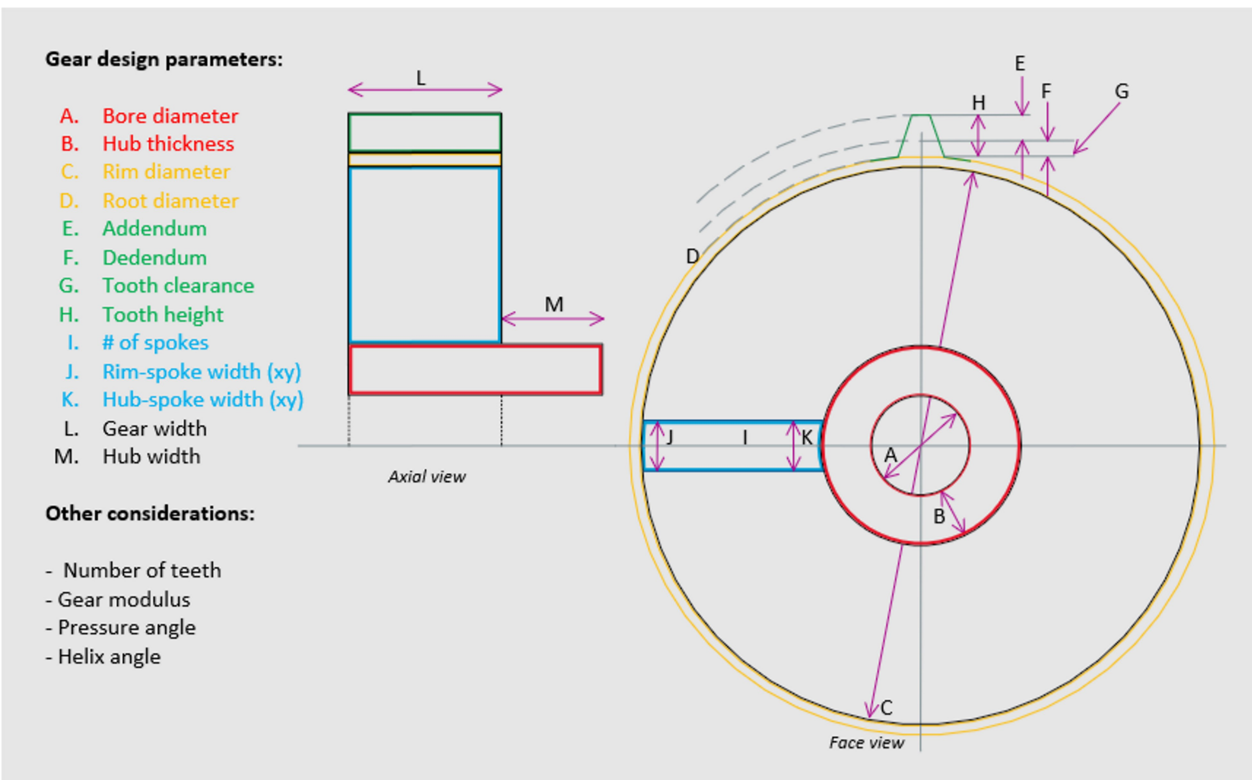
Objective:

To begin this project, the cost of printing was simplified down to the volume of the part. There are, of course, more complicated ideas behind the cost of 3D printing. For example the overall length and width of a part as well as the supporting structures required for some printing methods both can add to the cost aside from the actual material used for the useful part of the print. However, both of these attributes are yet dependent on the part's volume and so, by reducing the volume, the cost of these are also reduced.

Optimize the gear's functionality and cost by minimizing its volume.

Method:

A general gear design was made that includes four basic components: the hub, the spokes, the rim, and the gear teeth. Below is a graphic which shows this design.



The program will generate dimensions for each of these parameters, some of which are constrained and some which are not. By constraining some parameters with equations pertaining to size and stress, an optimal volume gear can be generated. Following is a detailed explanation of the objective, constraint and analysis functions used in this project.

Objective Equation:

$$Volume_{total} = V_{hub} + V_{spokes} + V_{rim} + V_{teeth}$$

Constraining Equations:

Gear teeth:

$$\sigma_{contact} \leq \sigma_{contact-allowable}$$
$$\sigma_{bending} \leq \sigma_{yield}$$

Other stress:

$$\sigma_{hub} \leq \sigma_{yield}$$
$$\sigma_{rim} \leq \sigma_{yield}$$
$$\sigma_{spoke-proximal} \leq \sigma_{yield}$$
$$\sigma_{spoke-distal} \leq \sigma_{yield}$$

Dimensional:

$$Ø_{bore} \leq Ø_{hub}$$
$$Ø_{hub} \leq Ø_{rim}$$
$$(Ø_{rim} + 1\text{ mm}) \leq Ø_{root}$$
$$\text{Width}_{spoke} \leq \left(\frac{Ø_{hub}}{2} \times \sin(60) \times 2\right)$$
$$3 \leq N_{spokes}$$
$$2 \times \text{thick}_{tooth} \times N_{teeth} = \pi \times Ø_{root}$$

Boundaries (common):

$0.75 \leq$ modulus ≤ 2.5
0.209 (rad) $\leq$ presure angle $\leq$ 4.19 (rad)
0.0 (rad) $\leq$ helix angle $\leq$ 0.785 (rad)
0.003 (m) $\leq$ face width $\leq$ 0.010 (m)
0.003 (m) $\leq$ hub width $\leq$ 0.010 (m)
0.003 (m) $\leq$ Ø bore $\leq$ 0.012 (m)
$7 \leq$ smallest # teeth ≤ 15

Boundaries (unique):

0.009 (m) $\leq$ Ø hub $\leq$ 0.100 (m)
0.012 (m) $\leq$ Ø rim $\leq$ 0.100 (m)
0.014 (m) $\leq$ Ø root $\leq$ 0.100 (m)
0.001 (m) $\leq$ spoke width $\leq$ 0.030 (m)
0.002 (m) $\leq$ tooth thickness $\leq$ 0.010 (m)

There is a difference laid out between the common and unique variables in the design of this program. This is based on the nature of a gear system. While the dimensions of the hub diameter and the rim or root diameter must vary from gear size to gear size, design variables like the gear modulus, and the governing dimensions of the teeth of the gear must be common throughout each of the gears for them to properly interface with each other. This leads into the design of the program.

User Interface Flow:

1) The user inputs:
   a) desired gear ratio
   b) desired number of gear/pinion sets (one set is two units)
   c) initial torque and velocity the gearing will be driven with
   d) name of the output file

2) The user specifies:
   a) print material properties
   b) gear loading factors (k)

3) The user adjusts:
   a) printing boundary conditions - as needed for size of gears (Unique Boundaries)
   b) gear commonalities (Common Boundaries)

4) The user runs the program.

5) The gear parameters of each gear/pinion are exported to a .xls file in the program's directory.

Program Flow:

1) User entered data is compiled for later distribution
2) The function GearCommons is called
   a) input: boundary conditions
      pertinent user entered data: q
   b) output: optimized common parameters: zopt
      volume of gears: fopt

   c) method: the function fmincon is called using the following functions:

      $objfun$ A:
      - calls for analysis using the function $DimOpt$
        input: optimized parameters: z
        user entered data: q
        boundary conditions
        Output: total_volume
      - dictates the objective function based on volume

      $nonlcon\ A$
      - no constraints applicable

3) The function DimOpt is called again to recalculate the unique parameters of each gear based on the optimized common parameters
   a) input: the optimized common parameters: zopt
      user entered information: q
      boundary conditions
   b) output: volume data
      optimized unique parameters
      dimension data
      gear system data

4) The data is exported to a .xls file in the program's directory

5) Explanation of the function DimOpt
   a) input: commonly optimized parameters: z
      user entered data: q
      boundary conditions
   b) output: volume
      dim_opt - the optimized unique dimensions: x
      dim_out - the optimized other dimensions
      the gear system information: y

   c) method:
      1) the function GearTrainCalc is called
         input: desired gear system data: w (gear ratio, number of sets, smallest number of teeth)
         output: a gear system meeting the imposed requirements: y
      2) the function fmincon is called using the following functions:

         $objfun\ B$
         - calls for analysis using the function Gear_Analysis
           input: the uniquely optimized parameters: x
           the gear train data: y
           user entered data: k
           output: the stress, volume and dimensional calculations: data_out
         - dictates the objective function based on volume

         $nonlcon\ B$
         - calls for analysis using the function Gear_Analysis
           input: the uniquely optimized parameters: x
           the gear train data: y
           user entered data: k
           output: the stress, volume and dimensional calculations: data_out
           the dimensions of the unique parameters: dim_out
         - dictates the constraint equations for the unique parameters

Results:

The results were that the program successfully ran and produced dimensions that make sense. Whether the output models would pass an FEA analysis is another question.

Using steel as the material with a desired gear ratio of 40:1 and a total of three sets of gear/pinions, the following data and corresponding models were generated.

Gear #	1	
Bore diam.	0.007919	(m)
Hub OD	0.016613	(m)
Rim diam.	0.137493	(m)
Root diam.	0.159637	(m)
Spoke width	0.007936	(m)
Tooth thickness	0.005554	(m)
Addendum	0.001504	(m)
Dedendum	0.001504	(m)
Tooth height	0.003007	(m)
Gear modulus	1.63	
Pressure angle	0.3140	(radians)
Helix angle	0.3925	(radians)
Face width	0.021128	(m)
Hub width	0.007779	(m)
Tooth count	46	
Spoke count	3	
Gear volume	1.02E-04	m^3

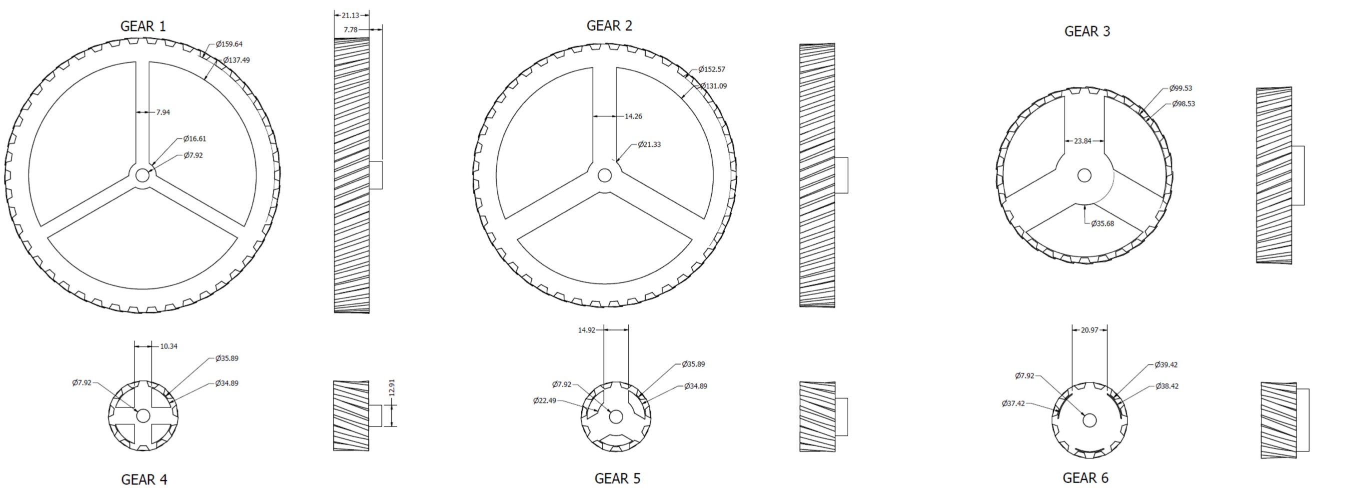
Gear #	2	
Bore diam.	0.007919	(m)
Hub OD	0.021329	(m)
Rim diam.	0.131086	(m)
Root diam.	0.152566	(m)
Spoke width	0.018471	(m)
Tooth thickness	0.005554	(m)
Addendum	0.001504	(m)
Dedendum	0.001504	(m)
Tooth height	0.003007	(m)
Gear modulus	1.63	
Pressure angle	0.3140	(radians)
Helix angle	0.3925	(radians)
Face width	0.021128	(m)
Hub width	0.007779	(m)
Tooth count	44	
Spoke count	3	
Gear volume	1.02E-04	m^3

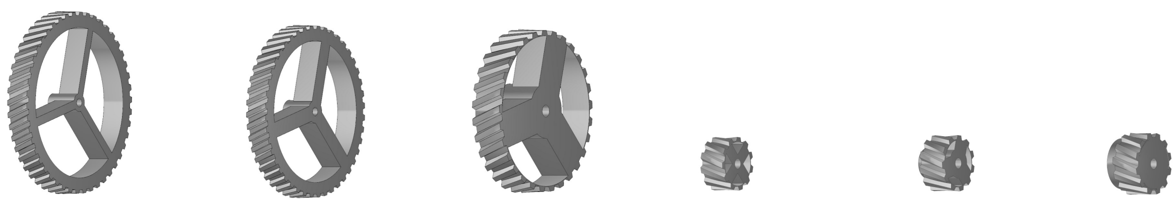
Gear #	3	
Bore diam.	0.007919	(m)
Hub OD	0.035681	(m)
Rim diam.	0.098529	(m)
Root diam.	0.099529	(m)
Spoke width	0.023842	(m)
Tooth thickness	0.005554	(m)
Addendum	0.001504	(m)
Dedendum	0.001504	(m)
Tooth height	0.003007	(m)
Gear modulus	1.63	
Pressure angle	0.3140	(radians)
Helix angle	0.3925	(radians)
Face width	0.021128	(m)
Hub width	0.007779	(m)
Tooth count	29	
Spoke count	3	
Gear volume	1.02E-04	m^3

Gear #	4	
Bore diam.	0.007919	(m)
Hub OD	0.012907	(m)
Rim diam.	0.034886	(m)
Root diam.	0.035886	(m)
Spoke width	0.010341	(m)
Tooth thickness	0.005554	(m)
Addendum	0.001504	(m)
Dedendum	0.001504	(m)
Tooth height	0.003007	(m)
Gear modulus	1.63	
Pressure angle	0.3140	(radians)
Helix angle	0.3925	(radians)
Face width	0.021128	(m)
Hub width	0.007779	(m)
Tooth count	11	
Spoke count	4	
Gear volume	1.02E-04	m^3

Gear #	5	
Bore diam.	0.007919	(m)
Hub OD	0.022493	(m)
Rim diam.	0.034886	(m)
Root diam.	0.035886	(m)
Spoke width	0.014916	(m)
Tooth thickness	0.005554	(m)
Addendum	0.001504	(m)
Dedendum	0.001504	(m)
Tooth height	0.003007	(m)
Gear modulus	1.63	
Pressure angle	0.3140	(radians)
Helix angle	0.3925	(radians)
Face width	0.021128	(m)
Hub width	0.007779	(m)
Tooth count	11	
Spoke count	3	
Gear volume	1.02E-04	m^3

Gear #	6	
Bore diam.	0.007919	(m)
Hub OD	0.037422	(m)
Rim diam.	0.038422	(m)
Root diam.	0.039422	(m)
Spoke width	0.020965	(m)
Tooth thickness	0.005554	(m)
Addendum	0.001504	(m)
Dedendum	0.001504	(m)
Tooth height	0.003007	(m)
Gear modulus	1.63	
Pressure angle	0.3140	(radians)
Helix angle	0.3925	(radians)
Face width	0.021128	(m)
Hub width	0.007779	(m)
Tooth count	12	
Spoke count	3	
Gear volume	1.02E-04	m^3





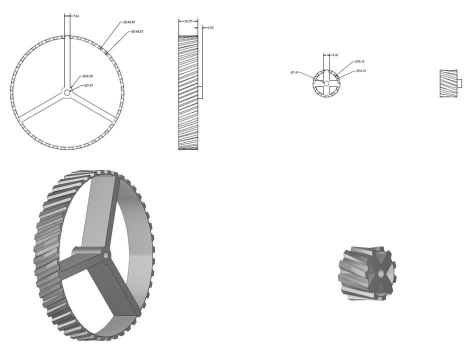
Gear 1 Gear 2 Gear 3 Gear 4 Gear 5 Gear 6

Using the materials steel and nylon 12 with a desired gear ratio of 4:1 and a total of one set of gear/pinions, the following data and corresponding models were generated so that effect of changing the material specifications in the algorithm might be seen. On the left is the steel set and the right is the nylon set of results.

Steel

Gear #	1	
Bore diam.	0.007465	(m)
Hub OD	0.016503	(m)
Rim diam.	0.148870	(m)
Root diam.	0.149870	(m)
Spoke width	0.007610	(m)
Tooth thickness	0.005460	(m)
Addendum	0.001536	(m)
Dedendum	0.001536	(m)
Tooth height	0.003072	(m)
Gear modulus	1.66	
Pressure angle	0.3140	(radians)
Helix angle	0.3912	(radians)
Face width	0.026365	(m)
Hub width	0.006526	(m)
Tooth count	44	
Spoke count	3	
Gear volume	9.24E-05	m^3

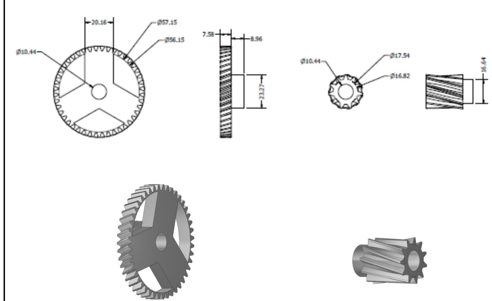
Gear #	2	
Bore diam.	0.007465	(m)
Hub OD	0.012973	(m)
Rim diam.	0.034164	(m)
Root diam.	0.035164	(m)
Spoke width	0.009448	(m)
Tooth thickness	0.005460	(m)
Addendum	0.001536	(m)
Dedendum	0.001536	(m)
Tooth height	0.003072	(m)
Gear modulus	1.66	
Pressure angle	0.3140	(radians)
Helix angle	0.3912	(radians)
Face width	0.026365	(m)
Hub width	0.006526	(m)
Tooth count	11	
Spoke count	4	
Gear volume	9.24E-05	m^3



Nylon12

Gear #	1	
Bore diam.	0.010442	(m)
Hub OD	0.023273	(m)
Rim diam.	0.056147	(m)
Root diam.	0.057147	(m)
Spoke width	0.020155	(m)
Tooth thickness	0.002155	(m)
Addendum	0.001606	(m)
Dedendum	0.001606	(m)
Tooth height	0.003212	(m)
Gear modulus	1.74	
Pressure angle	0.3140	(radians)
Helix angle	0.3865	(radians)
Face width	0.007582	(m)
Hub width	0.008963	(m)
Tooth count	44	
Spoke count	3	
Gear volume	2.15E-05	m^3

Gear #	2	
Bore diam.	0.010442	(m)
Hub OD	0.016641	(m)
Rim diam.	0.016822	(m)
Root diam.	0.017538	(m)
Spoke width	0.019606	(m)
Tooth thickness	0.002155	(m)
Addendum	0.001606	(m)
Dedendum	0.001606	(m)
Tooth height	0.003212	(m)
Gear modulus	1.74	
Pressure angle	0.3140	(radians)
Helix angle	0.3865	(radians)
Face width	0.007582	(m)
Hub width	0.008963	(m)
Tooth count	11	
Spoke count	3	
Gear volume	2.15E-05	m^3



Conclusions:

The program was written to minimize the volume of gear systems with regards to a very specific gear design. Based on the data collected, the program may have worked. When the dimensions are plugged into a CAD system, they make geometric sense. However, I haven't had to time to verify the stress calculations with an FEA program, so it is not yet possible to tell if the program successfully met the stress constraints. If the models do not hold up, it is likely due to a flaw in the analysis calculations used to generate the parametric relations. As a note, the analysis equations as well as the CAD design were simplified. A more robust program would include complete stress transformations as well as chamfers and radii in the gear's design to minimize stress concentrations.