COMP 5320/6320
Design and Analysis of Computer Networks
Homework Assignment 1
Due on Tuesday, Oct. 4, on Canvas

Instruction: Every student should finish the following questions independently. Please give justification for the results (i.e., show the calculation process and the corresponding diagram). Submit a scan of your answer sheet as a pdf file to Canvas.

1. Calculate the total time required to transfer a 1000-KB file in the following cases, assuming an RTT of 50 ms, a packet size of 1 KB data, and an initial 2×RTT of "handshaking" before data is sent. We define the total time of transferring the file as the time elapsed from the starting of initial handshaking to the instant when the last bit arrives at the receiver.

(a) The bandwidth is 1.5 Mbps, and data packets can be sent continuously.

Total time = handshake time + transmit time + prop. delay

handshake time = $2 \times RTT = 2 \cdot (0.050 \text{ sec}) = 0.1 \text{ sec}$

transmit time = $1000 \text{ KB} / 1.5 \text{ Mbps}$

$= \dfrac{1000(2^{10})(8) \text{ bits}}{1.5 \text{ Mbps}}$

$= \dfrac{8{,}192{,}000 \text{ b}}{1{,}500{,}000 \text{ bps}} = 5.461 \text{ sec}$

propagation delay = $RTT/2 = 0.050 \text{ sec}/2 = 0.025 \text{ sec}$

Total time = $0.1 \text{ sec} + 5.461 \text{ sec} + 0.025 \text{ sec}$

$= \boxed{5.586 \text{ sec}}$

(b) The bandwidth is 1.5 Mbps, but after we finish sending each data packet we must wait one RTT before sending the next.

Total time = $5.586 \text{ sec} + 999 \cdot RTT$

$= 5.586 \text{ sec} + 999 \cdot 0.050 \text{ sec}$

$= 5.586 \text{ sec} + 49.95 \text{ sec} = \boxed{55.536 \text{ sec}}$

(c) The bandwidth is "infinite," meaning that we take transmit time to be zero, and up to 20 packets can be sent per RTT.

Total time = $2 RTT + 49 RTT + RTT/2$

$= 2(0.050) + 49(0.050) + 0.050/2$

$= \boxed{2.575 \text{ sec}}$

(d) The bandwidth is infinite, and during the first RTT we can send one packet (2^{1-1}), during the second RTT we can send two packets (2^{2-1}), during the third we can send four (2^{3-1}), and so on.

$$2^{1-1} + 2^{2-1} + 2^{3-1} + \ldots + 2^{n-1} + 2^{n}$$

$$= 1 + 2 + 4 + \ldots + 2^{n-1} + 2^{n} = 2^{n+1} - 1$$

when $n = 9$, the equation equals 1023 packets

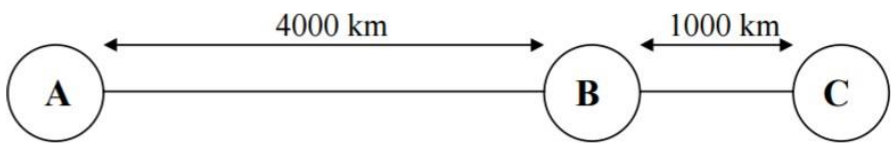
$$\text{Total time} = 2\,RTT + 9\,RTT + RTT/2$$

$$= 2(0.050) + 9(0.050) + 0.050/2$$

$$= \boxed{0.575 \text{ sec}}$$

2. In the figure below, all frames are generated at node A and sent to node C through node B. Determine the minimum transmission rate required between nodes B and C so that the buffers of node B are not flooded, based on the following conditions:

- The data rate between A and B is 100 kilobits/s.
- The propagation delay is 5 μs/km for both lines.
- There are full-duplex lines between the nodes.
- All data frames are 1000 bits long; ACK frames are separate frames of negligible length.
- Between A and B, a sliding-window flow control with a window size of 3 is used.
- Between B and C, stop-and-wait flow control is used.
- There are no errors.



$$Prop_{ab} = \frac{5\,\mu s}{\cancel{km}} \cdot 4000\,\cancel{km} = 20000\,\mu s = 0.02\,s$$

$$Trans_{ab} = \frac{1000 \text{ bits}}{100 \text{ kbits/s}} = \frac{1000 \text{ bits}}{100000 \text{ bits/s}} = 0.01\,s$$

$$Time_{ab} = Trans_{ab} + 2(Prop_{ab})$$
$$= 0.01 \text{ sec} + 2(0.02 \text{ sec}) = 0.05 \text{ sec}$$

$$Prop_{bc} = \frac{5\,\mu s}{\cancel{km}} \cdot 1000\,\cancel{km} = 5000\,\mu s = 0.005\,s$$

$$Trans_{bc} = \frac{1000 \text{ bits}}{x \text{ bits/s}}$$

$$Time_{bc} = 2(0.005 \text{ sec}) + \frac{1000 \text{ bits}}{x \text{ bits/sec}}$$

$$= 0.01 \text{ sec} + \frac{1000 \text{ bits}}{x \text{ bits/sec}} \quad \text{(for 1 frame)}$$

$$3Time_{bc} = 0.03 \text{ sec} + \frac{3000 \text{ bits}}{x \text{ bits/sec}} \quad \text{(for 3 frames)}$$

$$0.05 \text{ sec} \le 0.03 \text{ sec} + \frac{3000 \text{ bits}}{x \text{ bits/sec}}$$

$$0.02 \text{ sec} \le \frac{3000 \text{ bits}}{x \text{ bits/sec}} \Rightarrow \frac{x \text{ bits}}{\text{sec}} = 150000$$

$$= \boxed{150 \text{ kilobits/sec}}$$

3. For each of the following sets of codewords, please give the appropriate (n,k,d) designation where n is number of bits in each codeword, k is the number of message bits transmitted by each code word and d is the minimum Hamming distance between codewords. What is the coding rate (k/n), error detection capability, and error correction capability for each coding scheme?

A. {111, 100, 001, 010}

$n = 3,\ k = 2,\ d = 2$

code rate $= k/n = 2/3$ $\Rightarrow$ $\boxed{(3, 2, 2) \quad cr = 2/3}$

error detection:

$2 = x + 1 \Rightarrow x = 1$

$\boxed{\text{can detect 1 corrupt bit}}$

error correction:

$2 = 2x + 1 \Rightarrow x = 1/2$

$\boxed{\text{can correct } 1/2 \approx \emptyset \text{ bit errors}}$

B. {00000, 01111, 10100, 11011}

$n = 5,\ k = 2,\ d = 2$

code rate $= k/n = 2/5$ $\Rightarrow$ $\boxed{(5, 2, 2) \quad cr = 2/5}$

error detection:

$2 = x + 1 \Rightarrow x = 1$

$\boxed{\text{can detect 1 corrupt bit}}$

error correction:

$2 = 2x + 1 \Rightarrow x = 1/2$

$\boxed{\text{can correct } 1/2 \approx \emptyset \text{ bit errors}}$

4. In an error-correction code, an important constraint that the coding scheme must satisfy is that the number of added check bits should be sufficient to identify unique error patterns (note that no error is also a valid error pattern). Now suppose that we decide to use a (n, 20, 3) error correction code to transmit 20-bit messages. What's the minimum value of n that will allow the code to correct single bit errors? Show the reason and the calculation for full credits.

Based on the constraint specified above: "the number of added check bits should be sufficient to identify unique error patterns," the following equation can be given:

$$2^{n-k} \geq n + 1$$

$n + 1$ indicates that there are at least $n + 1$ unique error patterns (the "+1" accounts for the error pattern with no errors). $n - k$ bits can represent 2^{n-k} states. This number of states must therefore be at least $n + 1$.

So using this equation we can get the minimum n value:

$$2^{n-20} \geq n + 1$$

$$2^{25-20} \geq 25 + 1$$

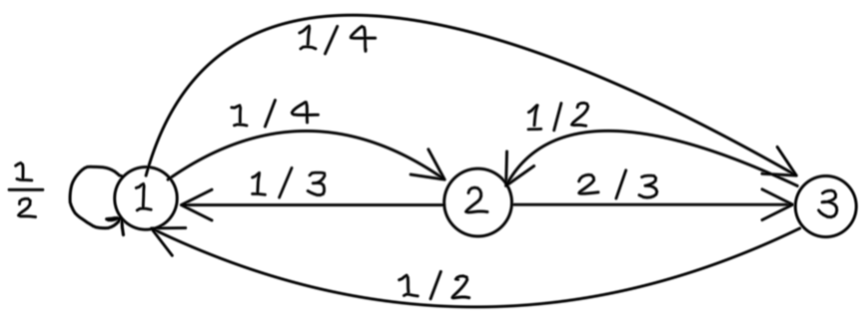
$$32 \geq 26$$

$$\boxed{n = 25}$$

5. Consider the Markov chain with three states, S={1, 2, 3}, that has the following transition matrix

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}.$$

(a) Draw the state transition diagram for this chain.



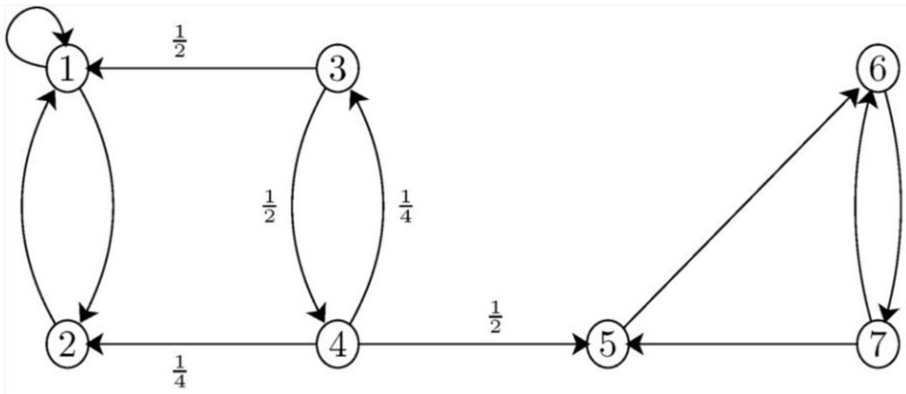
(b) If we know P(X1=1)=P(X1=2)=1/4, find P(X1=3, X2=2, X3=1).

$P(X_1 = 3) = 1 - P(X_1 = 1) - P(X_1 = 2)$
$= 1 - 1/4 - 1/4$
$= 1/2$

Property of DTMC:

$P(X_1 = 3, X_2 = 2, X_3 = 1)$
$= P(X_1 = 3) \cdot P(X_2 = 2 \mid X_1 = 3) \cdot P(X_3 = 1 \mid X_2 = 2)$
$= P(X_1 = 3) \cdot p_{32} \cdot p_{21}$
$= 1/2 \cdot 1/2 \cdot 1/3$
$= \boxed{1/12}$

6. Consider the Markov chain in the following figure. There are two recurrent classes, R1={1, 2} and R2={5,6,7}. Assuming initial state X0=3. Find the probability that the chain gets absorbed in R1.



$$\pi_j = \sum_i \pi_i P_{ij}$$

$$\pi_{R1} = 1, \quad \pi_{R2} = \emptyset$$

$$\pi_3 = 1/2(\pi_{R1}) + 1/2(\pi_4) = 1/2(1) + 1/2(\pi_4)$$

$$\pi_4 = 1/4(\pi_{R1}) + 1/2(\pi_{R2}) + 1/4(\pi_3)$$

$$= 1/4(1) + 1/2(\emptyset) + 1/4(\pi_3)$$

$$\pi_3 = 1/2 + 1/2\,\pi_4$$

$$\pi_4 = 1/4 + 1/4\,\pi_3$$

$$= 1/4 + 1/4(1/2 + 1/2\,\pi_4)$$

$$= 1/4 + 1/8 + 1/8\,\pi_4$$

$$\pi_4 = 3/7$$

$$\pi_3 = 1/2 + 1/2(3/7)$$

$$= \boxed{5/7}$$