Introduction: Fibonacci Numbers II

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Algorithmic Design and Techniques
Algorithms and Data Structures

Learning Objectives

- Produce a simple algorithm to compute Fibonacci numbers.
- Show that this algorithm is very slow.

Definition

$$F_n = \begin{cases} 0, & n = 0, \\ 1, & n = 1, \\ F_{n-1} + F_{n-2}, & n > 1. \end{cases}$$

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Grows rapidly.

Computing Fibonacci numbers

Compute F_n

Input: An integer $n \geq 0$.

Output: F_n.

Algorithm

FibRecurs(n)

if $n \leq 1$:
   return n

Algorithm

FibRecurs(n)

```
if n ≤ 1:
    return n
else:
    return FibRecurs(n − 1) + FibRecurs(n − 2)
```

Running time

Let $T(n)$ denote the number of lines of code executed by `FibRecurs(n)`.

If $n \leq 1$

FibRecurs(n)

```
if n ≤ 1:
    return n
else:
    return FibRecurs(n − 1) + FibRecurs(n − 2)
```

If $n \leq 1$

$T(n) = 2.$

If $n \geq 2$

FibRecurs(n)

```
if n ≤ 1:
    return n
else:
    return FibRecurs(n − 1) + FibRecurs(n − 2)
```

If $n \geq 2$

FibRecurs(n)

```
if n ≤ 1:
    return n
else:
    return FibRecurs(n - 1) + FibRecurs(n - 2)
```

$T(n) = 3$

If $n \geq 2$

FibRecurs(n)

```
if n ≤ 1:
    return n
else:
    return FibRecurs(n − 1) + FibRecurs(n − 2)
```

$$T(n) = 3 + T(n-1) + T(n-2).$$

Running Time

$$T(n) = \begin{cases} 2 & \text{if } n \leq 1 \\ T(n-1) + T(n-2) + 3 & \text{else.} \end{cases}$$

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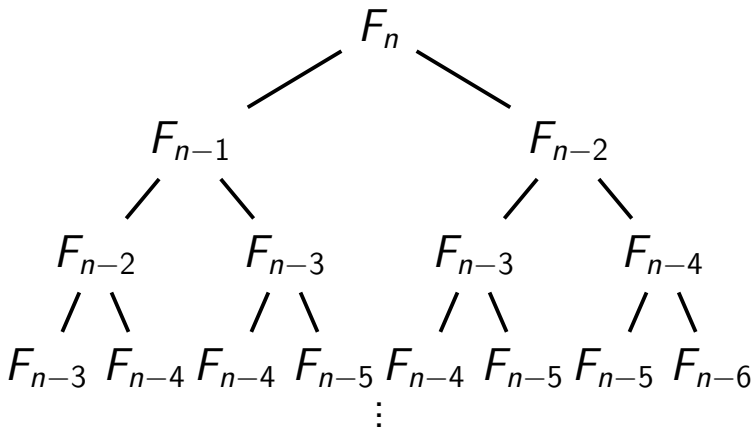
Therefore $T(n) \geq F_n$

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$T(100) \approx 1.77 \cdot 10^{21}$ \qquad (1.77 sextillion)

Running Time

$$T(n) = \begin{cases} 2 & \text{if } n \leq 1 \\ T(n-1) + T(n-2) + 3 & \text{else.} \end{cases}$$
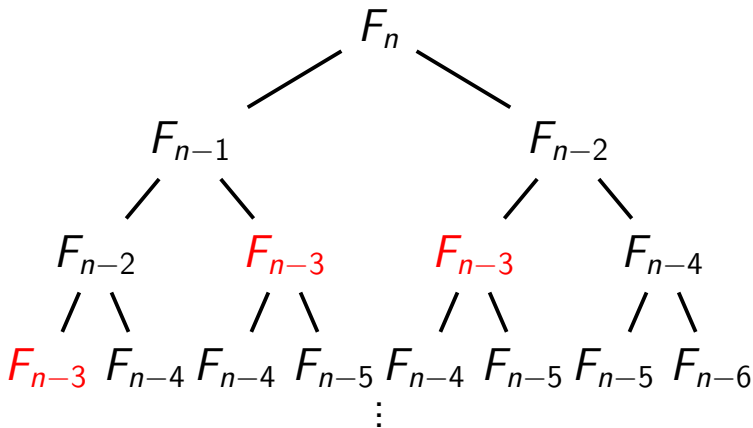
Therefore $T(n) \geq F_n$

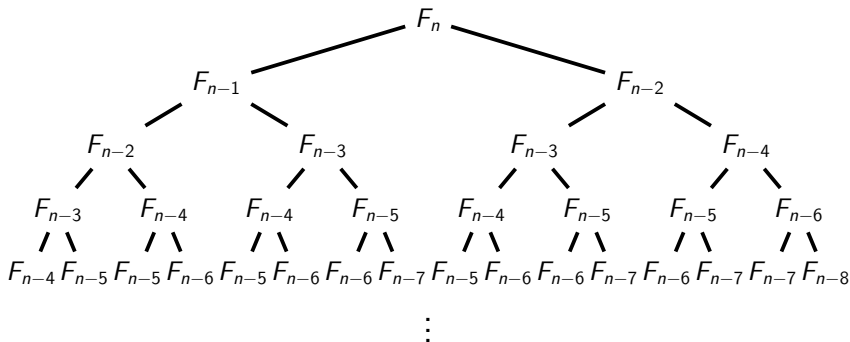$T(100) \approx 1.77 \cdot 10^{21}$ \qquad (1.77 sextillion)

Takes 56,000 years at 1GHz.

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