QESA Implementation and Uses

A Literature Review by Mridul Sarkar

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1 Abstract

The QESA algorithm from 'Tight bounds on quantum searching' presents an algorithm for finding one solution, multiple solutions, and an unknown number of solutions. The highlight of this paper is the generalized Grover's algorithm along with useful analysis of time complexity. We can use the analysis of the time complexity to derive the number of iterations needed on this generalized Grover. A few researches have modified this algorithm to achieve more desirable results, though the essential components of QESA are preserved in some shape or form.

2 Introduction

In this review there will be an analysis of the research that came from QESA and an implementation of QESA on a quantum comptuer simulation. It is important to understand how to use this algorithm in order for knowledge to grow. Papers have been derived from QESA improving on the original iteration bounds, time complexity bounds, the Grover's algorithm itself, and applying this algorithm to various scientific fields. One motivation for QESA is to find the minimum of an unsorted table in O($\sqrt{N}$) though use of QESA within QMSA. The QESA algorithm itself is a modified Grover's to find the optimal solution(s).

3 QESA Fundamentals

QESA utilizes a tight iteration bound to achieve a O($\sqrt{N}$).

When analyzing QESA's pseudo-code it can be interpreted that the code's run time is dependant on the amount of iterations on Grover. These iterations are determined by a m value set initially at 1 with and upper bound of $4N/3$. A random rational is chosen between 1 and 4/3 represented as z. We pick the number of iterations at random on the set of positive integers bounded by m. By setting m to 1 initially QESA only iterates Grover once. It then checks if the desired result is found. If the result is not found we multiply m and z together and take the floor to determine the next number of iterations.

If these steps are observed carefully it is easy to see QESA can only iterate once when called upon. The numbers m and z are a direct result of constraining an unfavorable probabilistic outcome with the number of iterations. The larger takeaway is that we only need to apply generalized Grover once to achieve a favorable probabilistic outcome. This is the essence of QESA.

An important aspect of QESA that has only been mentioned but not analyzed is the generalized Grover's Algorithm given under "Implementation Considerations" of 'Tight bounds on quantum searching'. It is a simple implementation of Grover's Algorithm. We will analyze this algorithm and implement it on a quantum computer simulation in Q sharp. One of the hardest parts of new technology is acclimating to the user interface and new features. This demonstration is given in hopes to make an algorithm bite sized and approachable. The analysis here will hopefully aid in analysis of future quantum algorithms when implementing them on quantum computers. INSERT CODE and Explanation

4 Applications of QESA

Recently QESA has been used in 'A Hybrid "Quantum and Classical" method for Outlier Detection'. Additionally QESA was used for function maximization and gloabl optimisation. The applications of QESA are bountiful and intriguing.

The application of QESA is most obviously seen in the paper 'A quantum algorithm for finding the minimum'. This paper proposes QMSA which uses QESA as its backbone, computationally, while setting a strict time complexity bound to find the minimum of table in O($\sqrt{N}$).

Quantum Minimum Searching Algorithm

1. Choose threshold index $0 \le y \le N-1$ uniformly at random.
2. Repeat the following and interrupt it when the total running time is more than $22.5\sqrt{N} + 1.4\lg^2 N$.[1] Then go to stage 2(2c).
    (a) Initialize the memory as $\sum_j \frac{1}{\sqrt{N}}|j\rangle|y\rangle$. Mark every item j for which $T[j] < T[y]$.
    (b) Apply the quantum exponential searching algorithm of [2].
    (c) Observe the first register: let y' be the outcome. If $T[y'] < T[y]$, then set threshold index y to y'.
3. Return y.

The iterations of generalized Grover (step 2(b): QESA) can be approximated by the run time (step 2) at $15\sqrt{N}$ iterations to achieve a probabilistic outcome with fifty percent favorability as proposed by Durr and Hoyer. Though it was later found that this approximation can be improved to $6.8\sqrt{N}$ iterations in 'A Quantum Algorithm for finding the Maximum'. Additionally we can allow for $13/6\sqrt{N}$ iterations to achieve an outcome with favorability higher than fifty percent. There is room for improvement on this upper bound. Research has been done for a definite lower bound time complexity for QMSA. Through this analysis it was proven that QMSA is the fastest quantum algorithm with the most favorable probabilistic outcome for the problem it solves.

Another area of interest within this algorithm is its use of generalized Grover. In 'An Optimized Quantum Maximum or Minimum Searching Algorithm and its Circuits" a modified Grover's Algorithm is used and its high accuracy is highlighted. The paper using Grover-Long proposes the most optimal algorithm for the case of multiple solutions and unknown number of solutions. As mentioned earlier, QMSA answers to finding the most optimal quantum algorithm for finding a unique solution.

5 Conclusion

The research derived from QESA is significant and broad. The use of QESA is exciting and undeniable. QMSA with its QESA bakcbone is equatable to Shor's, Grover's, and other algorithms that have become a fundamental part of quantum computing. Using QMSA will now be easy as loading a library and using the algorithm to find a unique solution. The applications are immense and with this knowledge there is hope for even more applications to develop. Pseudo code is understandable but to translate it to a seemingly novel and complex system like a quantum computer can be a challenge. The purpose of this review was to go over QESA and its applications while also allowing for ease of use of QMSA and more broadly the generalized Grover in QESA to allow for novel applications to develop.

QESA borrows from Grover's in its functionality. Here lies the core intention of this review. As new algorithms continue to be derived from other algorithms or simply constructed from intuition; it is important to take a look at what has come before. This reflection allows for avoidance of missteps, clear objectives, and innovative applications and algorithms. After a brief chat with Dr. Hoyer I realized that QMSA and QESA lead to some fundamental questions in quantum information. It should be noted that in 'An Optimized Quantum Maximum or Minimum Searching Algorithm and its Circuits' the initialization of qubits were different than QMSA. Can we improve QESA by simply encoding our information to qubits in a different way? Is there any level of bias we introduce into our system by the encoding of our quantum data?

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