EE302 - Feedback Systems **Spring 2019**

Lecture 5

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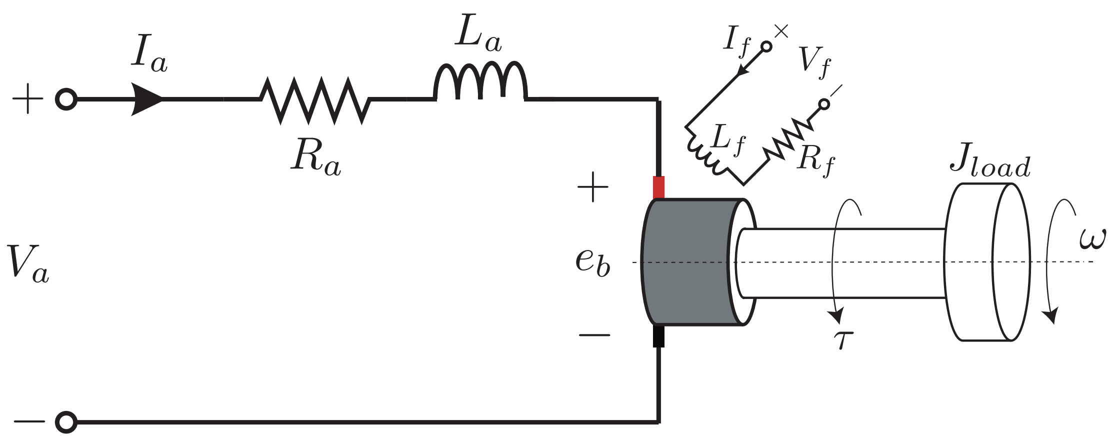
5.1 DC Motor Modeling

A generel "ideal" DC motor can be moddled as in the Figure below.



The dependent and "independent" variables associated with the idealized DC motor model and important relations/equations regarding the electro-mechanical interactions are given below.

V_a	Armature voltage
i_a	Armature current
V_f	"Field voltage"
i_f	"Field current"
V_b	Back emf
ω	Rotor angular velocity
τ	Generated torque
Φ	Air-gap magnetic flux

$$\Phi(t) = K_f I_f(t)$$
$$\tau(t) = K_m \Phi(t) I_a(t)$$
$$e_b(t) = K_b \omega(t)$$

Note that if both $i_f(t)$ and $i_a(t)$ are non-constant the electric-motor model won't be LTI. In order to have an LTI representation, there are two options

- Armature controlled DC motor: Φ is kept constant

$$\tau(t) = K_m \Phi I_a(t) = K_\tau^a I_a(t)$$

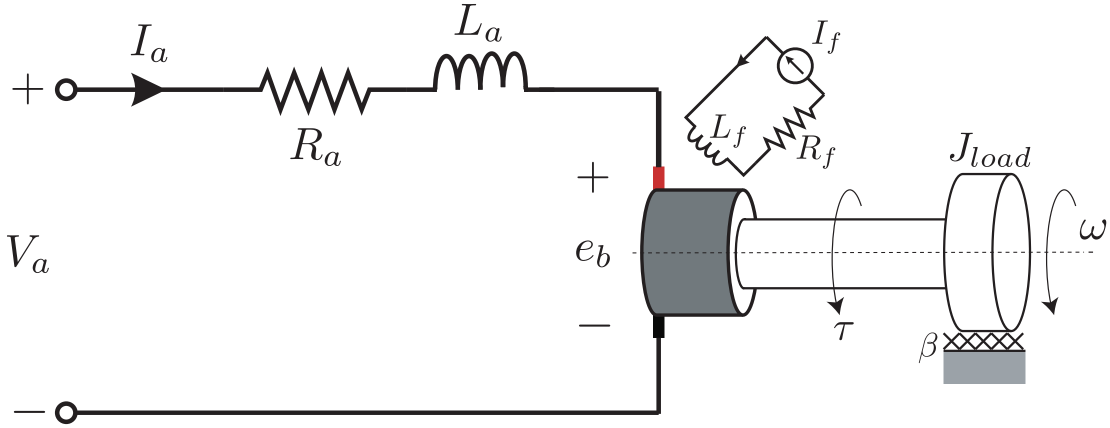
- Field controlled DC motor: i_a is kept constant

$$\tau(t) = K_m K_f I_a I_f(t) = K_\tau^f I_f(t)$$

5.1.1 Armature Controlled DC Motor

Majority of "DC" Motors are controlled (and indeed manufactured) with this approach. Either there is a permanent magnet which satisfies the constant Φ or a constant current is supplied through the coils that generates the magnetic field.

Let's model the following electro-mechanical system where the DC motor is armature controlled and given that $y(t) = \omega(t)$ and $u(t) = V_a(t)$.



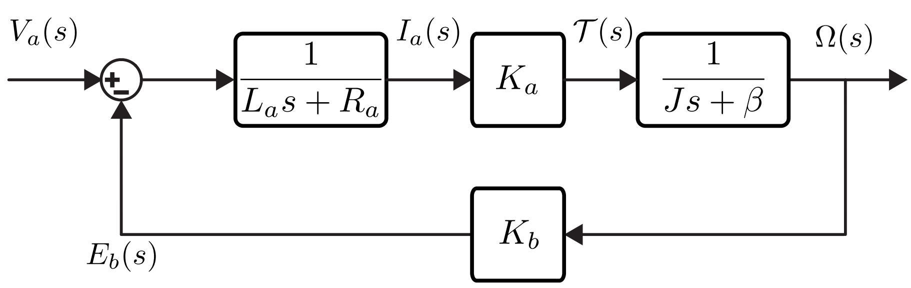
We already know the transfer function of the mechanical sub system:

$$\Omega(s) = \frac{1}{Js + \beta}\mathcal{T}(s)$$

Now let's write the remaining equations in Laplace domain

$$\begin{aligned}
I_a(s) &= \frac{1}{L_a s + R_a}\left(V_a(s) - E_b(s)\right) \\
\mathcal{T}(s) &= K_a I_a(s) \\
E_b(s) &= K_b \Omega(s)
\end{aligned}$$

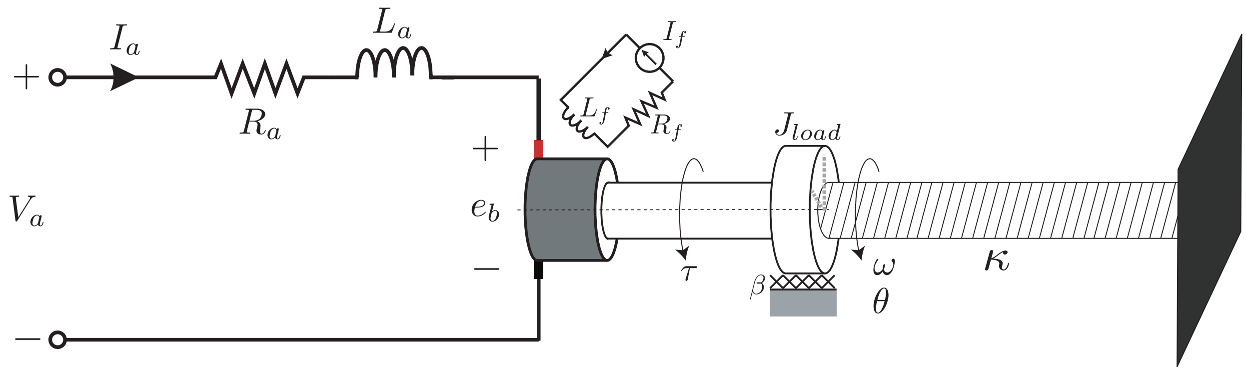
where $K_a \equiv K_\tau^a$. Now let's build a block-diagram topology



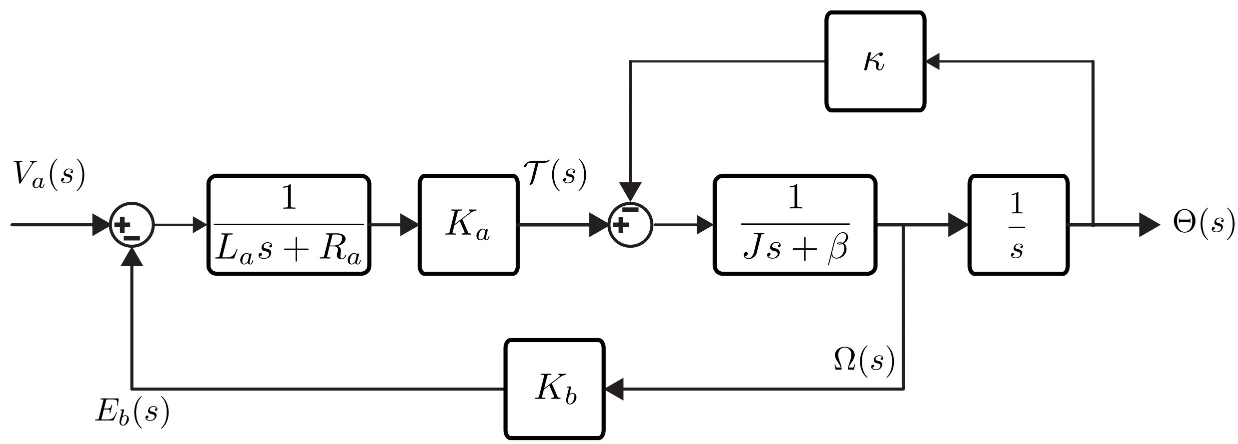
If we simplify the block diagram, we obtain the transfer function form

$$\begin{aligned}
G(s) = \frac{\Omega(s)}{V_a(s)} &= \frac{\frac{K_a}{(L_a s + R_a)(Js + \beta)}}{1 + \frac{K_a K_b}{(L_a s + R_a)(Js + \beta)}} \\
&= \frac{K_a}{L_a J s^2 + (L_a \beta + R_a J)s + R_a \beta + K_a K_b}
\end{aligned}$$

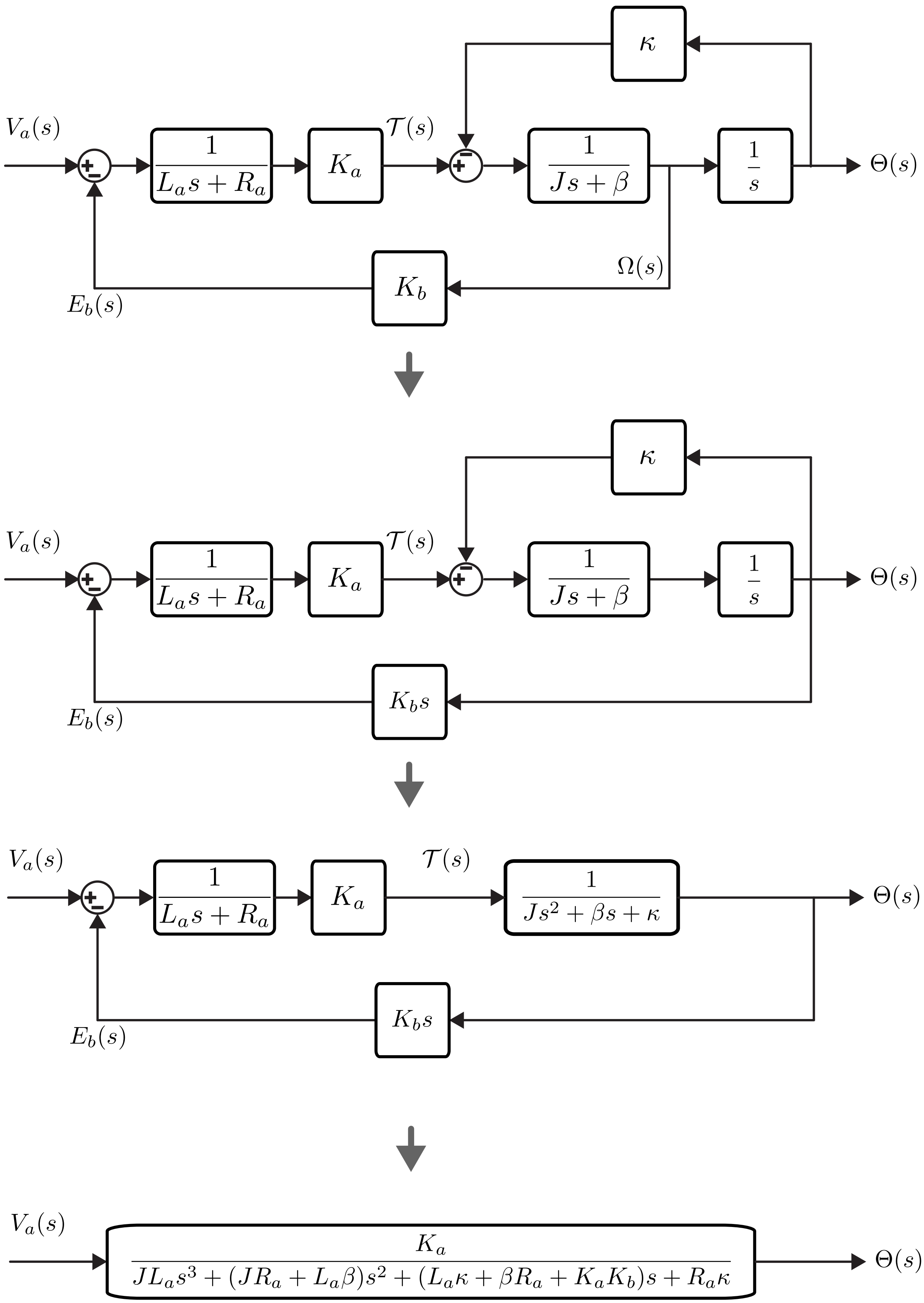
Example 1: Given that V_a is the input and θ is the output, construct a block-diagram for the following electro-mechanical system and then compute the transfer function.



Solution: A block diagram topology can be constructed by modifying the previous block diagram (armature controlled DC motor without torsional spring).



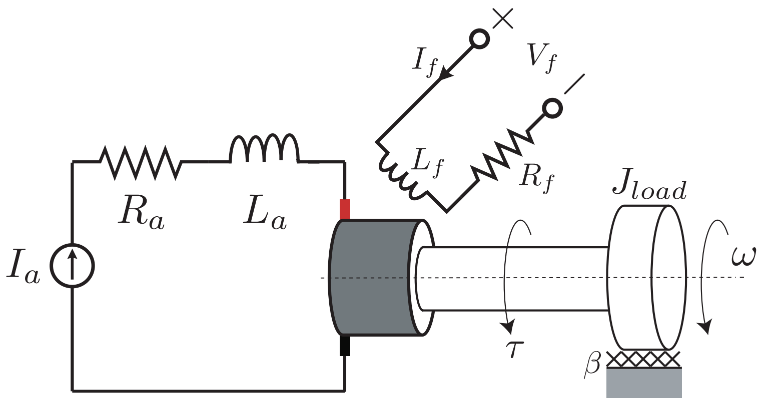
Then the transfer function can ne derived using block-diagram simplification methods as given in the next page


$V_a(s)$
$\frac{1}{L_a s + R_a}$
K_a
$\mathcal{T}(s)$
κ
$\frac{1}{Js+\beta}$
$\frac{1}{s}$
$\Theta(s)$
$\Omega(s)$
K_b
$E_b(s)$
$V_a(s)$
$\frac{1}{L_a s + R_a}$
K_a
$\mathcal{T}(s)$
κ
$\frac{1}{Js+\beta}$
$\frac{1}{s}$
$\Theta(s)$
$K_b s$
$E_b(s)$
$V_a(s)$
$\frac{1}{L_a s + R_a}$
K_a
$\mathcal{T}(s)$
$\frac{1}{Js^2+\beta s+\kappa}$
$\Theta(s)$
$K_b s$
$E_b(s)$
$V_a(s)$
$\frac{K_a}{JL_a s^3 + (JR_a + L_a\beta)s^2 + (L_a\kappa + \beta R_a + K_a K_b)s + R_a\kappa}$
$\Theta(s)$

5.1.2 Field Controlled DC Motor

In the field controlled DC motors, magnetic flux is actively controlled by adjusting electrical current/voltage. We assume that I_a is constant (LTI constraints). Since, there is no "feedback" in this field controlled DC motor model, the electrical circuit is isolated from the mechanical one.

Let's model the following electro-mechanical system where the DC motor is field controlled and given that $y(t) = \omega(t)$ and $u(t) = V_f(t)$.

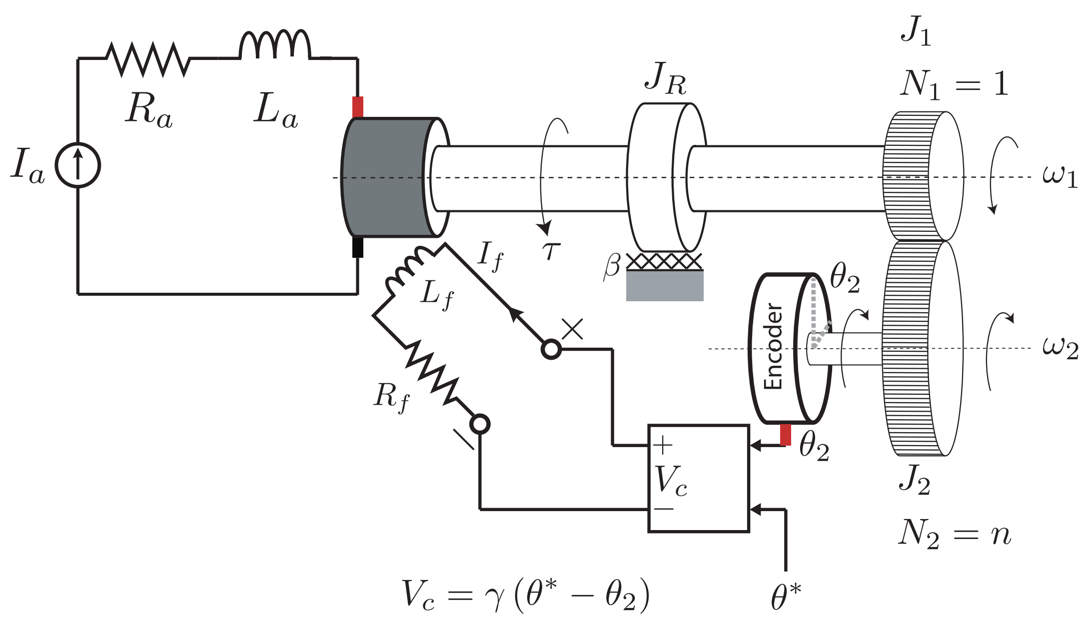


$$\frac{\Omega(s)}{\mathcal{T}(s)} = \frac{1}{Js + \beta}$$
$$\frac{I_f(s)}{V_f(s)} = \frac{1}{L_f s + R_f}$$
$$\mathcal{T}(s) = K_f I_f(s)$$

where $K_f \equiv K^f_{tau}$. Finally transfer function can be computed as

$$G(s) = \frac{K_f}{JL_f s^2 + (JR_f + \beta L_f)s + (\beta R_f + K_f)}$$

Example 2: Consider the following closed-loop field controlled electro-mechanical circuit. It is given that $\theta^*(t)$, i.e. reference angle signal, is the input and θ_2, angular displacement of the second gear, is the output. In the system, there is an encoder which reads the angular displacement and sends it to a controller box. The other input of this box is the reference signal. The box produces an output voltage, $V_c = \gamma\,(\theta^* - \theta_2)$, and feeds it to the input terminal of the V_f. Compute the transfer function.



Solution: A block diagram topology can be constructed by modifying the previous block diagram (armature controlled DC motor without torsional spring).

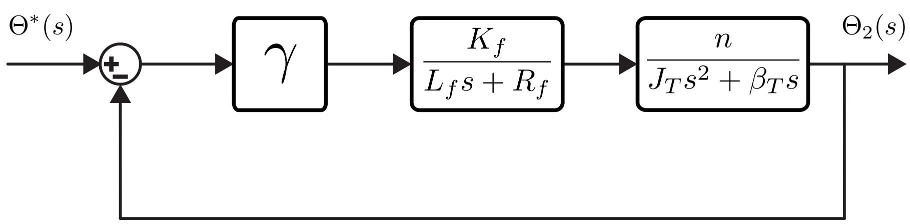
Let's first find a transfer function from τ to ω_2 and θ_2. The easiest way of computing this is using the concept of reflected inertia, damping, and torque.

$$\Omega_2(s) = \frac{\bar{\mathcal{T}}(s)}{J_T s + \beta_T} = \frac{n}{(n^2 J_R + n^2 J_1 + J2)\, s + n^2 \beta} \mathcal{T}(s)$$
$$\Theta_2(s) = \frac{n\mathcal{T}(s)}{J_T s^2 + \beta_T s}$$

We know that Laplace domain equations for remaining parts take the form

$$\frac{\mathcal{T}(s)}{V_f(s)} = \frac{K_f}{L_f s + R_f}$$
$$V_f(s) = \gamma\left(\Theta^*(s) - \Theta_2(s)\right)$$

Now let's construct a block diagram representation



Finally, transfer function can be computed as

$$G(s) = \frac{\gamma K_f n}{J_T L_f s^3 + (J_T R_f + \beta_T L_f)s^2 + \beta_T R_f s + \gamma K_f n}$$