EE302 - Feedback Systems **Spring 2019**

Lecture 20

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20.1 State-Space Analysis of LTI Systems

20.1.1 State-Space to TF

Let's first re-visit the conversion from a state-space representation to the transfer function representations for LTI systems.

Note that a SS representation of an n^{th} order LTI system has the from below.

$$\begin{aligned}
&\text{Let } x(t) \in \mathbb{R}^n \ , \ y(t) \in \mathbb{R} \ , \ u(t) \in \mathbb{R}, \\
&\quad \dot{x}(t) = Ax(t) + Bu(t), \\
&\quad y(t) = Cx(t) + Du(t), \\
&\text{where } A \in \mathbb{R}^{n \times n} \ , \ B \in \mathbb{R}^{n \times 1} \ , \ C \in \mathbb{R}^{1 \times n} \ , \ D \in \mathbb{R}
\end{aligned}$$

In order to convert state-space to transfer function, we start with taking the Laplace transform of the both sides of the state-equation

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t) \\
sX(s) &= AX(s) + BU(s) \\
sX(s) - AX(s) &= BU(s) \\
(sI - A)X(s)) &= BU(s) \\
X(s) &= (sI - A)^{-1} BU(s)
\end{aligned}$$

Now let's concentrate on the output equation

$$\begin{aligned}
y(t) &= Cx(t) + Du(t) \\
Y(s) &= \left[C\left(sI - A\right)^{-1} B + D\right] U(s) \\
G(s) &= C\left(sI - A\right)^{-1} B + D
\end{aligned}$$

Example: Let p be a pole of $G(s)$, then show that p is also an eigenvalue of A.

Solution: Let

$$G(s) = \frac{n(s)}{d(s)}$$

If p is a pole of $G(s)$, then $d(s)|_p = 0$. Now let's analyze the dependence of $G(s)$ to the state-space form.

$$\begin{aligned} G(s) &= \left[C\left(sI-A\right)^{-1}B+D\right] \\ \left(sI-G\right)^{-1} &= \frac{\mathrm{Adj}\left(sI-A\right)}{\det\left(sI-A\right)} \\ G(s) &= \frac{C\mathrm{Adj}\left(sI-G\right)B+D\det\left(sI-A\right)}{\det\left(sI-A\right)} \end{aligned}$$

If p is a pole of $G(s)$, then

$$\det\left(sI-A\right)|_{s=p} = 0$$

Obviously p is an eigenvalue of A.

20.1.2 Similarity Transformations

Now let's define a new "state-vector" $\hat{x}$ such that

$$\begin{aligned} Px(t) = \hat{x}(t) \quad &\text{where} \\ P \in \mathbb{R}^{n\times n} \quad &, \det(P) \neq 0 \end{aligned}$$

Then we can transform the state-space equations using P as

$$\begin{aligned} P^{-1}\dot{\hat{x}}(t) &= AP^{-1}\hat{x}(t) + Bu(t) \quad , \quad y(t) = CP^{-1}\hat{x}(t) + Du(t) \\ \dot{\hat{x}} &= PAP^{-1}\hat{x}(t) + PBu(t) \quad , \quad y(t) = CP^{-1}\hat{x}(t) + Du(t) \end{aligned}$$

The "new" state-space representation is obtained as

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{A}\hat{x}(t) + \hat{B}u(t) \\ y(t) &= \hat{C}x(t) + \hat{D}u(t) \\ \hat{A} &= PAP^{-1}\ ,\ \hat{B} = PB\ ,\ \hat{C} = CP^{-1}\ ,\ \hat{D} = D \end{aligned}$$

Since there exist infinitely many non-singular $n \times n$ matrices, for a given LTI system, there exist infinitely many different but equivalent state-space representations.

Example: Show that $A \in \mathbb{R}^{n\times n}$ and $P^{-1}AP$, where $P \in \mathbb{R}^{n\times n}$ and $\det(P) \neq 0$, have the same characteristic equation

Solution:

$$\begin{aligned} \det\left(\lambda I - P^{-1}AP\right) &= \det\left(\lambda P^{-1}IP - P^{-1}AP\right) \\ &= \det\left(P^{-1}\left(\lambda I - A\right)P\right) \\ &= \det\left(P^{-1}\right)\det\left(\lambda I - A\right)\det\left(P\right) \\ &= \det\left(P^{-1}\right)\det\left(P\right)\det\left(\lambda I - A\right) \\ \det\left(\lambda I - P^{-1}AP\right) &= \det\left(\lambda I - A\right) \end{aligned}$$

20.1.2.1 Invariance of Transfer Functions Under Similarity Transformation

Consider the two different state-space representations

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \qquad & \dot{\hat{x}}(t) &= \hat{A}\hat{x}(t) + \hat{B}u(t) \\ y(t) &= Cx(t) + Du(t) \qquad & y(t) &= \hat{C}\hat{x}(t) + \hat{D}u(t) \end{aligned}$$

where they are related with the following similarity transformation

$$Px(t) = \hat{x}(t)\ ,\ \hat{G} = PAP^{-1}\ ,\ \hat{B} = PB\ ,\ \hat{C} = CP^{-1}\ ,\ \hat{D} = D$$

Let's compute the transfer function for the second representation

$$\begin{aligned} \hat{G}(s) &= \left[\hat{C}\left(sI - \hat{A}\right)^{-1}\hat{B} + \hat{D}\right] \\ &= \left[CP^{-1}\left(sI - PAP^{-1}\right)^{-1}PB + D\right] \\ &= \left[CP^{-1}\left(P\left(sI - A\right)P^{-1}\right)^{-1}PB + D\right] \\ &= \left[CP^{-1}P\left(sI - A\right)^{-1}P^{-1}PB + D\right] \\ &= \left[C\left(sI - A\right)^{-1}B + D\right] \\ \hat{G}(s) &= G(s) \end{aligned}$$

20.1.3 Canonical State-Space Realizations

We know that for a given LTI system, there exist infinitely many different SS representations. We previously learned some methods to convert a TF/ODE into State-Space form. We will now re-visit them and talk about the canonical state-space forms.

For the sake of clarity, derivations are given for a general 3^{rd} order LTI system.

20.1.4 Controllable Canonical Form

In this method of realization, we use the fact the system is LTI. Let's consider the transfer function of the system and let's perform some LTI operations.

$$\begin{aligned} Y(s) &= \frac{b_3s^3 + b_2s^2 + b_1s + b_0}{s^3 + a_2s^2 + a_1s + a_0}U(s) \\ &= \left(b_3s^3 + b_2s^2 + b_1s + b_0\right)\frac{1}{s^3 + a_2s^2 + a_1s + a_0}U(s) \\ &= G_2(s)G_1(s)U(s) \text{ where} \\ G_1(s) &= \frac{H(s)}{U(s)} = \frac{1}{s^3 + a_2s^2 + a_1s + a_0} \\ G_2(s) &= \frac{Y(z)}{H(z)} = b_3s^3 + b_2s^2 + b_1s + b_0 \end{aligned}$$

As you can see we introduced an intermediate variable $h(t)$ or with a Laplace transform of $H(s)$. First transfer function has static input dynamics, operates on $u(t)$, and produces an output, i.e. $h(t)$. Second transfer function is a "non-causal" system and operates on $h(t)$ and produces output $y(t)$. If we write the ODEs of both systems we obtain

$$\begin{aligned}\dddot{h} &= -a_2\ddot{h} - a_1\dot{h} - a_0 h + u\\ y &= b_3\dddot{h} + b_2\ddot{h} + b_1\dot{h} + b_0 h\end{aligned}$$

Now let the state-variables be $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} h \\ \dot{h} \\ \ddot{h} \end{bmatrix}$. Then, individual state equations take the form

$$\begin{aligned}\dot{x_1} &= x_2\\ \dot{x_2} &= x_3\\ \dot{x_3} &= -a_2x_3 - a_1x_2 - a_0x_1 + u\end{aligned}$$

and the output equation take the form

$$\begin{aligned}y &= b_3\left(-a_2x_3 - a_1x_2 - a_0x_1 + u\right) + b_2x_3 + b_1x_2 + b_0x_1\\ &= (b_0 - b_3a_0)x_1 + (b_1 - b_3a_1)x_2 + (b_2 - b_3a_2)x_3 + b_3u\end{aligned}$$

If we re-write the equations in matrix form we obtain the state-space representation as

$$\begin{aligned}\dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u\\ y &= \begin{bmatrix} (b_0 - b_3a_0) & (b_1 - b_3a_1) & (b_2 - b_3a_2) \end{bmatrix} x + [b_3]\, u\end{aligned}$$

If we obtain a state-space model from this approach, the form will be in *controllable canonical form.*

For a general n^{th} order transfer function controllable canonical form has the following A , B , C , & D matrices

$$\begin{aligned}A &= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} \quad , \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}\\ C &= \begin{bmatrix} (b_0 - b_na_0) & (b_1 - b_na_1) & \cdots & (b_{n-1} - b_na_{n-1}) \end{bmatrix} \quad , \quad D = b_n\end{aligned}$$

20.1.5 Observable Canonical Form

In this method will obtain a different minimal state-space realization, the form is called observable canonical form. The process is different and state-space structure will have a different topology. Let's start with a 3^{rd} transfer function and perform some grouping based on the s elements.

$$\begin{aligned}&Y(s) = \frac{b_3s^3 + b_2s^2 + b_1s + b_0}{s^3 + a_2s^2 + a_1s + a_0}U(s)\\ &Y(s)\left(s^3 + a_2s^2 + a_1s + a_0\right) = \left(b_3s^3 + b_2s^2 + b_1s + b_0\right)U(s)\\ &s^3Y(s) = b_3s^3U(s) + s^2\left(-a_2Y(s) + b_2U(s)\right) + s\left(-a_1Y(s) + b_1U(s)\right) + \left(-a_0Y(s) + b_0U(s)\right)\end{aligned}$$

Let's multiply both sides with $\frac{1}{s^3}$ and perform further grouping

$$Y(s) = b_3U(s) + \frac{1}{s}\left(-a_2Y(s) + b_2U(s)\right) + \frac{1}{s^2}\left(-a_1Y(s) + b_1U(s)\right) + \frac{1}{s^3}\left(-a_0Y(s) + b_0U(s)\right)$$

$$Y(s) = b_3U(s) + \frac{1}{s}\left[\left(-a_2Y(s) + b_2U(s)\right) + \frac{1}{s}\left\{\left(-a_1Y(s) + b_1U(s)\right) + \frac{1}{s}\left(-a_0Y(s) + b_0U(s)\right)\right\}\right]$$

Let the Laplace domain representations of state variables $X(s) = \begin{bmatrix} X_1(s) \\ X_2(s) \\ X_3(s) \end{bmatrix}$ defined as

$$\begin{aligned}
X_1(s) &= \frac{1}{s}\left(-a_0Y(s) + b_0U(s)\right) \\
X_2(s) &= \frac{1}{s}\left\{\left(-a_1Y(s) + b_1U(s)\right) + \frac{1}{s}\left(-a_0Y(s) + b_0U(s)\right)\right\} \\
X_3(s) &= \frac{1}{s}\left[\left(-a_2Y(s) + b_2U(s)\right) + \frac{1}{s}\left\{\left(-a_1Y(s) + b_1U(s)\right) + \frac{1}{s}\left(-a_0Y(s) + b_0U(s)\right)\right\}\right]
\end{aligned}$$

In this context output equation in s and time domains simply takes the form

$$Y(s) = X_3(s) + b_3U(S) \quad \rightarrow \quad y(t) = x_3(t) + b_3u(t)$$

Dependently the state equations (in s and time domains) take the form

$$\begin{aligned}
sX_1(s) &= -a_0X_3(s) + (b_0 - a_0b_3)U(s) & \rightarrow \quad \dot{x}_1 &= -a_0x_3 + (b_0 - a_0b_3)u \\
sX_2(s) &= X_1(s) - a_1X_3(s) + (b_1 - a_1b_3)U(s) & \rightarrow \quad \dot{x}_2 &= x_1 - a_1x_3 + (b_1 - a_1b_3)u \\
sX_3(s) &= X_2(s) - a_2X_3(s) + (b_2 - a_2b_3)U(s) & \rightarrow \quad \dot{x}_3 &= x_2 - a_2x_3 + (b_2 - a_2b_3)u
\end{aligned}$$

If we re-write all equations in matrix form, we obtain the state-space representation as

$$\begin{aligned}
\dot{x} &= \begin{bmatrix} 0 & 0 & -a_0 \\ 1 & 0 & -a_1 \\ 0 & 1 & -a_2 \end{bmatrix} x + \begin{bmatrix} (b_0 - b_3a_0) \\ (b_1 - b_3a_1) \\ (b_2 - b_3a_2) \end{bmatrix} u \\
y &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x + [b_3]\, u
\end{aligned}$$

If we obtain a state-space model from this method, the form will be in *observable canonical form.* Thus we can call this representation also as *observable canonical realization.* This form and representation is the dual of the previous representation.

For a general n^{th} order system controllable canonical form has the following A , B , C , & D matrices

$$A = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & \cdots & 0 & -a_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -a_{n-2} \\ 0 & 0 & \cdots & 1 & -a_{n-1} \end{bmatrix} \quad , \quad B = \begin{bmatrix} (b_0 - b_na_0) \\ (b_1 - b_na_1) \\ \vdots \\ (b_{n-2} - b_na_{n-2}) \\ (b_{n-1} - b_na_{n-1}) \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \quad , \quad D = b_n$$

Diagonal Canonical Form

If the transfer function of the LTI system has distinct poles, we can expand it using partial fraction expansion

$$Y(s) = \left[b_3 + \frac{c_1}{s-p_1} + \frac{c_2}{s-p_2} + \frac{c_3}{s-p_3}\right] U(s)$$

Now let's concentrate on the candidate "state variables" and try to write state evaluation equations

$$\begin{aligned} X_1(s) &= \frac{1}{s-p_1} U(s) \quad \rightarrow \quad \dot{x}_1 = p_1 x_1 + u \\ X_2(s) &= \frac{1}{s-p_2} U(s) \quad \rightarrow \quad \dot{x}_2 = p_2 x_2 + u \\ X_3(s) &= \frac{1}{s-p_3} U(s) \quad \rightarrow \quad \dot{x}_3 = p_3 x_3 + u \end{aligned}$$

where as output equation can be derived as

$$y(t) = b_3 u(t) + c_1 x_1(t) + c_2 x_2(t) + c_3 x_3(t)$$

If we combine the state and output equations, we can obtain the state space form as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \begin{bmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \mathbf{x}(t) + b_3 u(t) \end{aligned}$$

where

$$\mathbf{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad , \quad A = \begin{bmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad , \quad C = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \quad , \quad D = b_3$$

The form obtained with this approach is called diagonal canonical form. Obviously, this form is not applicable for "some" systems that has repeated roots.

For a general n^{th} order system with distinct roots diagonal canonical form has the following A , B , C , & D matrices

$$\begin{aligned} A &= \begin{bmatrix} p_1 & 0 & \cdots & 0 & 0 \\ 0 & p_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & p_{n-1} & 0 \\ 0 & 0 & \cdots & 0 & p_n \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} \\ C &= \begin{bmatrix} c_1 & c_2 & \cdots & c_{n-1} & c_n \end{bmatrix} \quad , \quad D = b_n \end{aligned}$$

Example: Given that

$$G(s) = \frac{s^2 + 8s + 10}{s^2 + 3s + 2}$$

find a controllable, observable, and diagonal canonical state-space representation of the given TF.

Solution:

If we follow the derivation of controllable canonical form for a second order system we obtain the following structure

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} (b_0 - b_2 a_0) & (b_1 - b_2 a_1) \end{bmatrix} x + [b_2]\, u \end{aligned}$$

where

$$a_0 = 2\ ,\ a_1 = 3\ ,\ b_0 = 10\ ,\ b_1 = 8\ , \&\ b_2 = 1$$

Thus, the state-space representation takes the form

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 8 & 5 \end{bmatrix} x + [1]u \end{aligned}$$

Observable canonical form is the dual of the controllable canonical form thus for the given system, we know that

$$\begin{aligned} A_{OCF} &= A_{CCF}^T = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \\ B_{OCF} &= C_{CCF}^T = \begin{bmatrix} 8 \\ 5 \end{bmatrix} \\ C_{OCF} &= B_{CCF}^T = \begin{bmatrix} 0 & 1 \end{bmatrix} \\ D_{OCF} &= D_{CCF} = [1] \end{aligned}$$

In order to find the diagonal canonical form, we need to perform partial fraction expansion

$$G(s) = \frac{s^2 + 8s + 10}{s^2 + 3s + 2} = 1 + \frac{3}{s+1} + \frac{2}{s+2}$$

then SS matrices for the diagonal canonical form can be simply derived as

$$\begin{aligned} A_{DCF} &= \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \\ B_{DCF} &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ C_{DCF} &= \begin{bmatrix} 3 & 2 \end{bmatrix} \\ D_{DCF} &= [1] \end{aligned}$$

Example: Consider the following general state-space representation

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = Cx(t) + Du(t)$$

Now let's consider the following state-space representation

$$\dot{\bar{x}}(t) = A^T \bar{x}(t) + C^T u(t),$$
$$y(t) = B^T \bar{x}(t) + Du(t)$$

Show that these two state-space representations results in same transfer function form

Solution: For the second representation we have

$$\begin{aligned}\bar{G}(s) &= \bar{C}\left(sI - \bar{A}\right)^{-1}\bar{B} + D \\ &= B^T\left(sI - A^T\right)^{-1} C^T + D\end{aligned}$$

Since $\bar{G}(s)$ is a scalar quantity we can take its transpose

$$\begin{aligned}\bar{G}(s) &= [\bar{G}(s)]^T = [B^T\left(sI - A^T\right)^{-1} C^T + D]^T \\ &= (C^T)^T\left(\left(sI - A^T\right)^{-1}\right)^T (B^T)^T + D \\ &= C\left(\left(sI - A^T\right)^T\right)^{-1} B + D \\ &= C\left(sI - A\right)^{-1} B + D \\ \bar{G}(s) &= G(s)\end{aligned}$$

This result also shows that controllable and observable canonical representations are similar.

20.2 Stability & State-Space Representations

Let's consider the state-representation of an LTI system

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

Given LTI system is called *asymptotically stable* if, with $u(t) = 0$ and $\forall x(0) \in \mathbb{R}^n$, we have

$$\lim_{t\to\infty} ||x(t)|| = 0$$

Theorem: A state-space representation is asymptotically stable if and only if all of the eigenvalues of the system matrix, A, have negative real parts, i.e.

$$\forall x_0 \in \mathbb{R}^n, \lim_{t\to\infty} ||x(t)|| = 0 \iff \forall i \in \{1,\ \cdots,\ n\}, \mathrm{Re}\{\lambda_i\} < 0$$

Ex: Show that if a state-space representation is asymptotically stable then its transfer function representation is BIBO stable.

Solution: Previously we showed that if p is a pole of $G(s)$, then it is also an eigenvalue of A, since we can write $G(s)$ as

$$G(s) = \frac{C\mathrm{Adj}\,(sI - G)\,B + D\det(sI - A)}{\det(sI - A)}$$

If the state-space representation is asymptotically stable then we know that for each pole, p_i of $G(s)$ we have $\mathrm{Re}\{p_i\} < 0$ which makes the input–output dynamics BIBO stable. In conclusion,

Asymptotically stable $\Rightarrow$ BIBO stable

Example: Consider the following state-space form of a CT system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} x(t)$$

- Is this system asymptotically stable?
- Is this system BIBO stable?

Solution: Let's compute the eigenvalues of A

$$\det\left(\begin{bmatrix} \lambda & -1 \\ -1 & \lambda \end{bmatrix}\right) = \lambda^2 - 1$$
$$\lambda_{1,2} = \pm 1$$

Thus the system is NOT Asymptotically Stable. Now let's check BIBO stability condition. First, compute the $G(s)$

$$\begin{aligned}
G(s) &= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} s & -1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} s & 1 \\ 1 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{s^2-1} \\
&= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} s & 1 \\ 1 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{s^2-1} \\
&= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix} \frac{1}{s^2-1} \\
&= \frac{-(s-1)}{s^2-1} \\
&= \frac{-1}{s+1}
\end{aligned}$$

Indeed, the system is BIBO Stable.

In conclusion

- Asymptotically Stable $\Rightarrow$ BIBO Stable
- BIBO Stable $\not\Rightarrow$ Asymptotically Stable