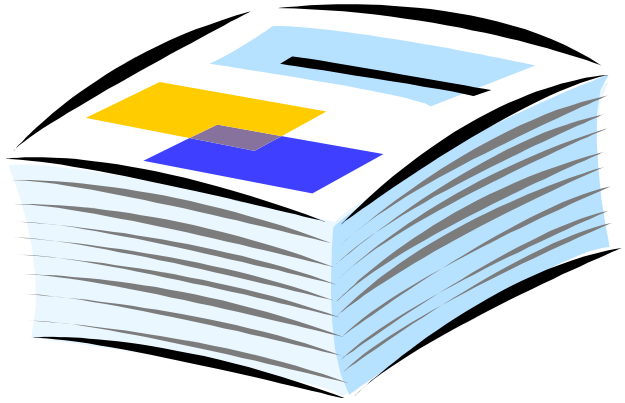
Simulation Examples

~ By Hand

~ Using Excel

Chapter 2



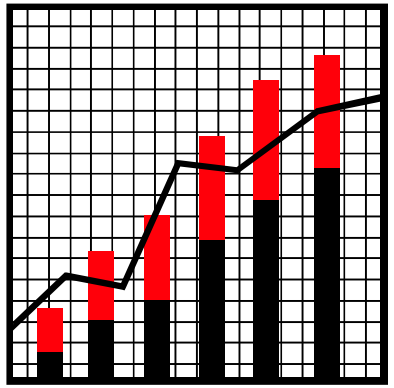
Why do examples by hand or spreadsheet??

- Insight to system
- Hands-on
- Helps with programming

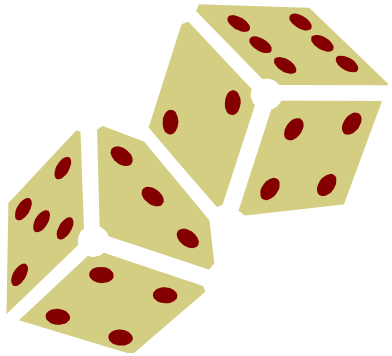
- Complex systems not amenable to spreadsheet simulation

Process

- Determine Characteristics of system
- Construct simulation table
- Generate & compute values



Key Components



- Random Numbers
  - Number: between 0 & 1
  - Variable: some quantity; perhaps from a known distribution
- Descriptive Statistics
  - Values used for describing a systems and making predictions about its behavior

Random Variable

- A quantity determined by some random experiment
- Examples
  - Number of heads obtained when flipping a coin 10 times
  - Number of customers arriving in an hour
  - Maximum length of a queue during the day
  - Shortest service time for a customer for the day

Randomness

- True Random vs. Pseudo-Random
- Random number sequence
  - Uniformly distributed
  - Each number statistically independent of previous numbers
- Where?
  - Random Number Generators (functions)
  - Random Number Tables

1 2 5 3 8 2 5 0 8 3 7 5 2 5 8 6 2 5 9

Excel – Random numbers

- =RAND( )
  - Generates real values: 0 <= val < 1
- =RANDBETWEEN (low, high)
  - Generates integers: low <= val <= high
- To use in Excel
  - IF (RAND ( ) < 0.5, 0, 1)
  - IF (A2 <= 0.33, 0, (IF A2 <= 0.66, 1, 2)
- Problem with Excel….

Other sources of random numbers

- Authors provide Visual Basic functions in the sample spreadsheets on web site.
  - We will not use these.
  - Discussed in 2.1.2 and 2.1.3
- Random Number Tables in text
  - Table A1 (p. 592) – uniform
  - Table A2 (p. 593) – normal
- Limitations: Excel & VB functions – don't use in professional work

Random Number Generator (RNG) Features

- **RNG** is a mathematical function
  - **Different strategies**
- **Period**: Number of values generated before sequence repeats
- **Seed:** Initialization value for a RNG

Example: Coin Tossing

- Monte Carlo Simulation
- Fair coin → Head/Tail equally likely
- IF (RAND ( ) < 0.5, "H", "T")

Example: Random Service Times

1. Integer value 1 to 10, inclusive
    1. =RANDBETWEEN (1, 10)
2. Integer value with given probability
    1. 3 @ 30%; 6 @ 45%, 10 @ 25%
    2. Develop cumulative probability
        1. 0 - .3 → 3
        2. .3 - .75 → 6
        3. .75 – 1 → 10
    3. IF (A2 <= 0.3, 3, (IF A2 <= 0.75, 6, 10)
4. Why not? IF (RAND() <= 0.3, 3, (IF RAND <= 0.75, 6, 10))

Arrival Times

- Arrival Time vs. Inter-Arrival Time
- Arrival time – Clock time of arrival
- Inter-Arrival Time: time between successive arrivals
- Example: Initialize: Clock = 0

Inter-Arrival Time	Arrival Time (Clock)
3	3
7	10
2	12

Queuing(Waiting Line) Systems

- Calling population
  - Infinite vs. Finite population
- Nature of arrivals
  - Arrival Rate vs. Effective Arrival Rate
- Service mechanism
  - Single vs. Multiple vs. Sequential
- Service time
- Queue discipline

Arrivals & Services

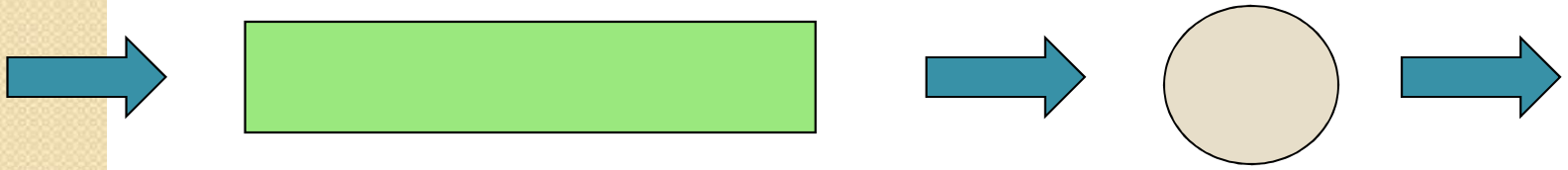
- Generally defined by a distribution (random)
- Arrivals
  - Time between arrivals – inter-arrival time
- Service
  - Service times

Arrival rate must be less than the service rate.
What if it is not? Unstable, explosive

Queue Basics

- System State
  - Number & status of entities (units)
- Event
  - Circumstance that causes a change in system state
- Clock
  - Relative time

Single Server & Queue



Arrive Queue Server Depart

What are the state variables?

What are the events?

Refer to flow diagrams – Pg. 42 +

Future Events List (FEL)

- Can Generate Events
  - up-front
    - Before simulation begins
    - OK for small/short simulations
  - on-the-fly
    - As needed
    - Used for professional/complex simulations
- Generate Inter-arrival times & Service times

Brief Example

Cust #	IAT	A-time *Clock	S-begin *Clock	S-time	S-end *Clock
1	0	0	0	2	2
2	2	2	2	1	3
3	4	6	6	3	9
4	3	7	9	2	11
5	2	9	11	1	12
6	6	15	15	4	19

Other simulation items

- What else can we keep track of during the simulation?
  - Wait time in queue
  - Time in system
  - Server idle time

- Calculate these for previous example.

Other simulation items

- What can we calculate at the end of simulation?
  - Average inter-arrival time
  - Average service time
  - Server utilization (% busy)
  - *Average queue length
- Calculate for previous example.

Common Stats to Calculate

- Customer
  - Time in queue, Time in system, Probability of waiting in queue, Inter-arrival time
  - Averages, max, min
- Server
  - Utilization, Service times (max, min, average)
- Queue
  - Length (current, average, max, min)

System State vs. Performance Measure * Current vs. After Simulation

1. Current queue length
2. Server status (busy, idle)
3. Customer wait time

1. Average, max, min queue length
2. Average, min, max service time; utilization
3. Average wait time, max, min

Simulation Statistics

- Numerous standard statistics of interest
- Some results calculated from parameters
  - Used to verify the simulation
- Most calculated by program

Statistics – Performance Measures

Average Wait time for a customer

$$= \frac{\text{total time customers wait in queue}}{\text{total number of customers}}$$

Average wait time of those who wait

$$= \frac{\text{total time of customers who wait in queue}}{\text{number of customers who wait}}$$

More Statistics

Proportion of server busy time

$$= \frac{\text{number of time units server busy}}{\text{total time units of simulation}}$$

Average service Time

$$= \frac{\text{total service time}}{\text{number of customers serviced}}$$

More Statistics

Average time customer spends in system

$$= \frac{\text{total time customers spend in system}}{\text{total number of customers}}$$

Probability a customer has to wait in queue

$$= \frac{\text{number of customers who wait}}{\text{total number of customers}}$$

Traffic Intensity

- A measure of the ability of the server to keep up with the number of the arrivals
- TI= (service mean)/(inter-arrival mean)
- If TI > 1 then system is unstable & queue grows without bound

Server Utilization

- % of time the server is busy serving customers
- If there is 1 server
  - SU = TI = (service mean)/(inter-arrival mean)
- If there are N servers
  - SU = 1/N * (service mean)/(inter-arrival mean)

Weighted Averages

- Necessary when unequal probability of values.
- Example: Service times: 20% take 5 minutes, 38% take 8 minutes, 42% take 11 minutes.
- What is the average service time?
  - Is it (5 + 8 + 11) / 3 = 8 ???

Correct Answer

20% take 5 minutes, 38% take 8 minutes,
42% take 11 minutes.

AST = .2 * 5 + .38 * 8 + .42 * 11

= 1 + 3.04 + 4.62

= 8.66

Spreadsheet Homework
DUE:

Page 78+ (Show all work & document)

2: Calculate expected number of customers per day & number of bagels needed: Based in these values, what is expected cost, income, & profit. Don't simulate.

4: Calculate expected # of calls 9 am – 5 pm. & avg. service time. What is utilization of taxi? What is utilization if 2 taxis? Complete an Excel Simulation for 9 to 5 day with 1 taxi. (Print off values & formulas version. Document well.)

#51: Calculate best case, worst case, and average case scenario for the student. What are the maximum & minimum loan amounts that he will need? Don't simulate.