

GTU Department of Computer Engineering
CSE 222/505 - Spring 2020
Homework 7-Question 1

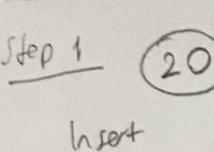
Mert Can BEŞİRLİ
1801042663

①

$$A = \{20, 30, 8, 47, 39, 18, 26, 63\}$$

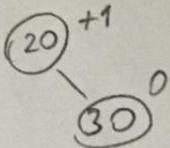
AVL Tree

Step 1



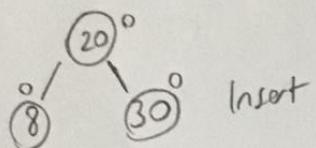
Insert

Step 2



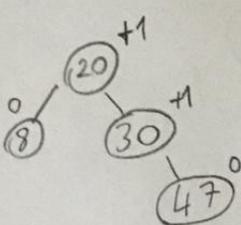
Insert

Step 3



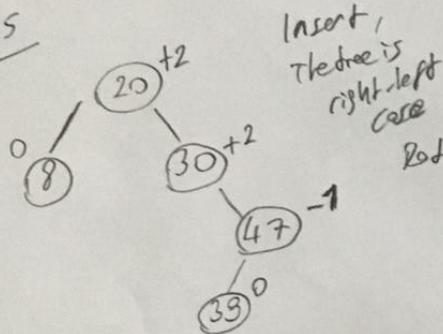
Insert

Step 4



Insert

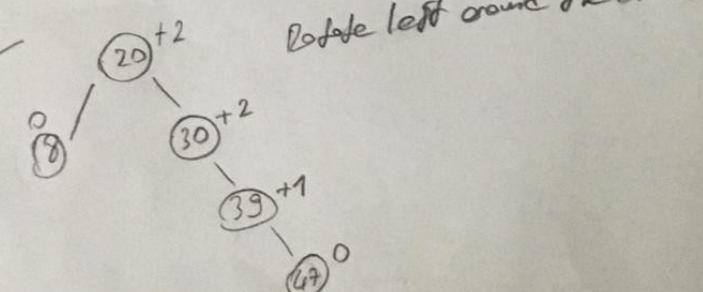
Step 5



Insert,
The tree is
right-left
case

Rotate right around the child

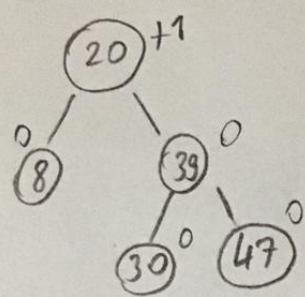
Step 6



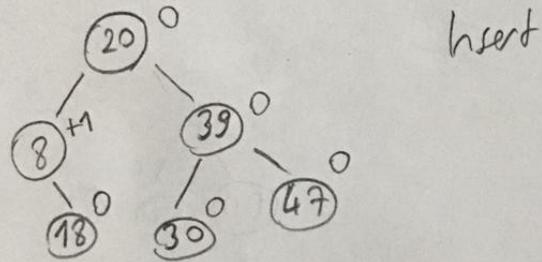
Rotate left around the child

(2)

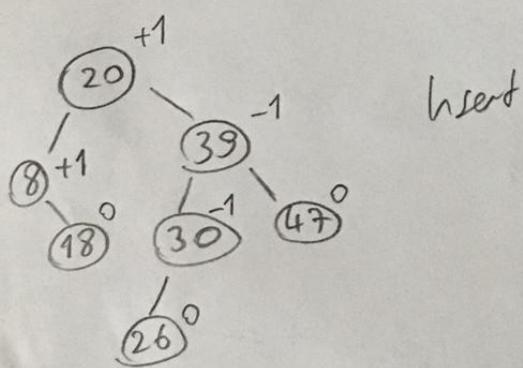
Step 7



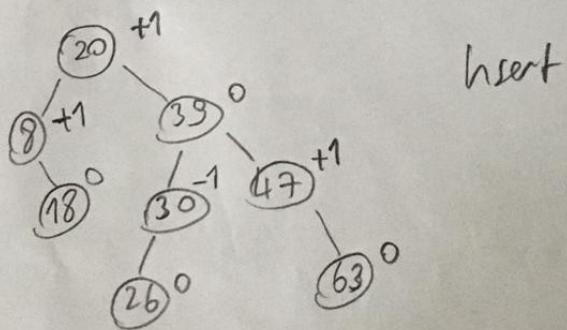
Step 8



Step 9

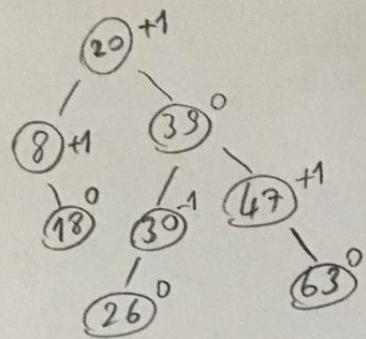


Step 10



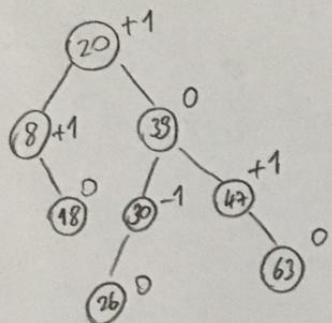
(3)

Step 11



Remove 20 node

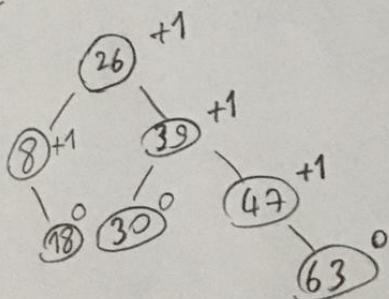
Step 12



Find minimum value in the
right subtree

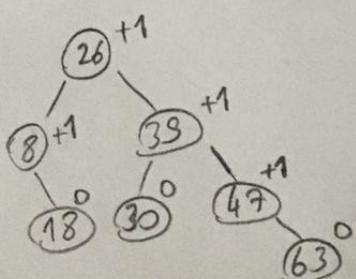
Find 26

Step 13



Copy the find of node
of the node into deletion
node, then remove node.

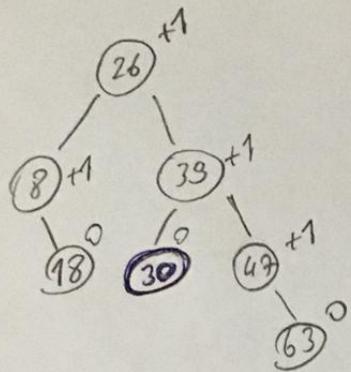
Step 14



Remove 30 node

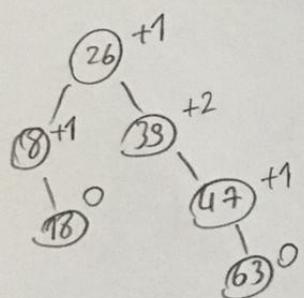
(4)

Step 15



Find the node with the value 30

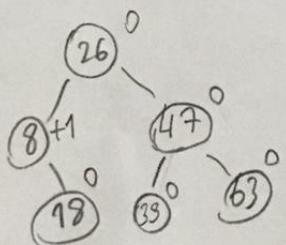
Step 16



It has no child so remove it.

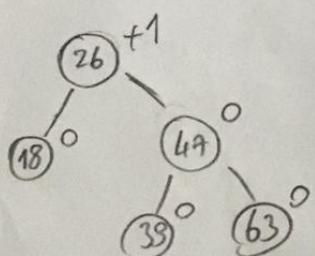
Rotate left (+2)

Step 17



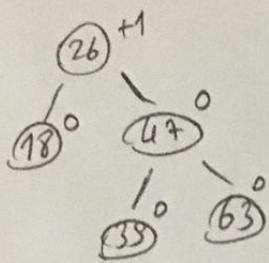
Remove 8 node

Step 18



Find 8 and remove it.

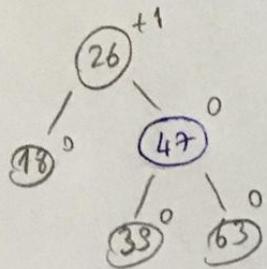
Step 18



(5)

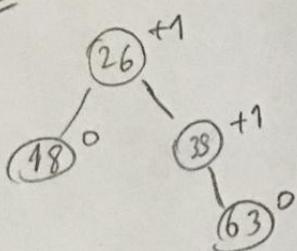
Remove 47, must find 47

Step 20



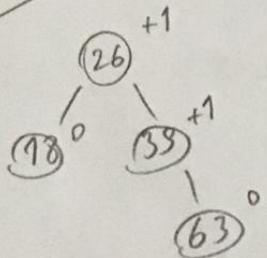
Find 47, it has two child,
find minimum value child

Step 21



Find minimum value child then copy
content deletion node, remove node

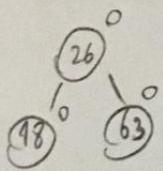
Step 22



Remove 39, must find 39

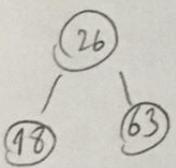
(6)

Step 23



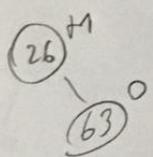
Find 39, it has no child, then remove it.

Step 24



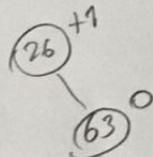
Remove 18, must find 18

Step 25



Find 18, remove it

Step 26



Remove 26, must find 26

Step 27



Find 26, remove it

Step 28



Remove 63

Step 29

Remove all elements (First in, First out)

$$A = \{20, 30, 8, 47, 39, 18, 26, 63\}$$

(7)

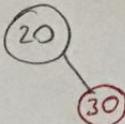
Red-Black Tree

Step 1



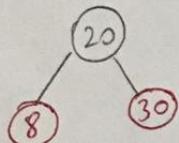
Tree is empty. So insert new node as root node with black color. (insert 20)

Step 2



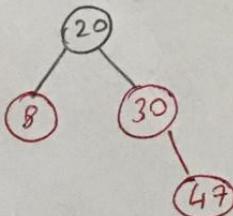
Tree is not empty. So insert new node with red color. (insert 30)

Step 3



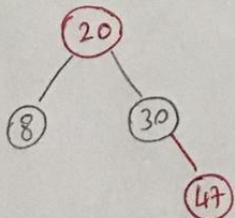
Tree is not empty. So insert new node with red color. (insert 8)

Step 4



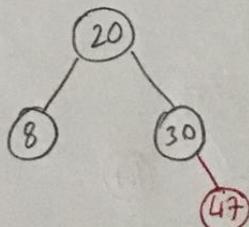
If parent is red, and its sibling is also red, they can both be changed to black, and the grandparent to red.

Step 5

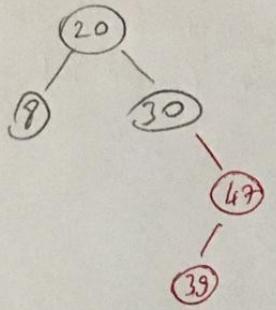


Root is always black,
recolor root

Step 6



Step 7



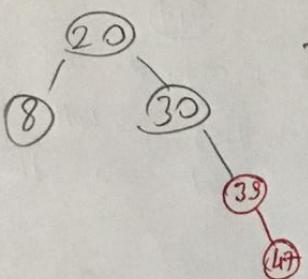
(8)

Insert 39 node, red color

Right-left case

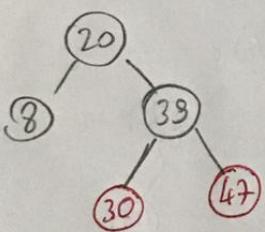
first right rotation

Step 8



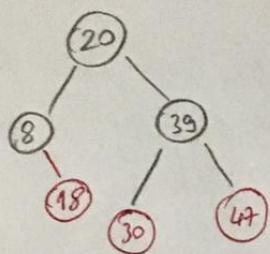
Then, left rotation and recolor

Step 9



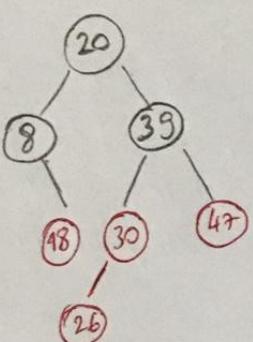
(9)

Step 10



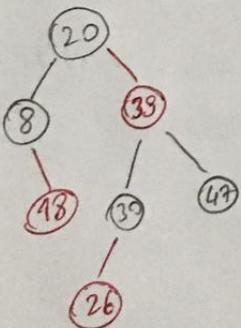
Insert 18, red color

Step 11

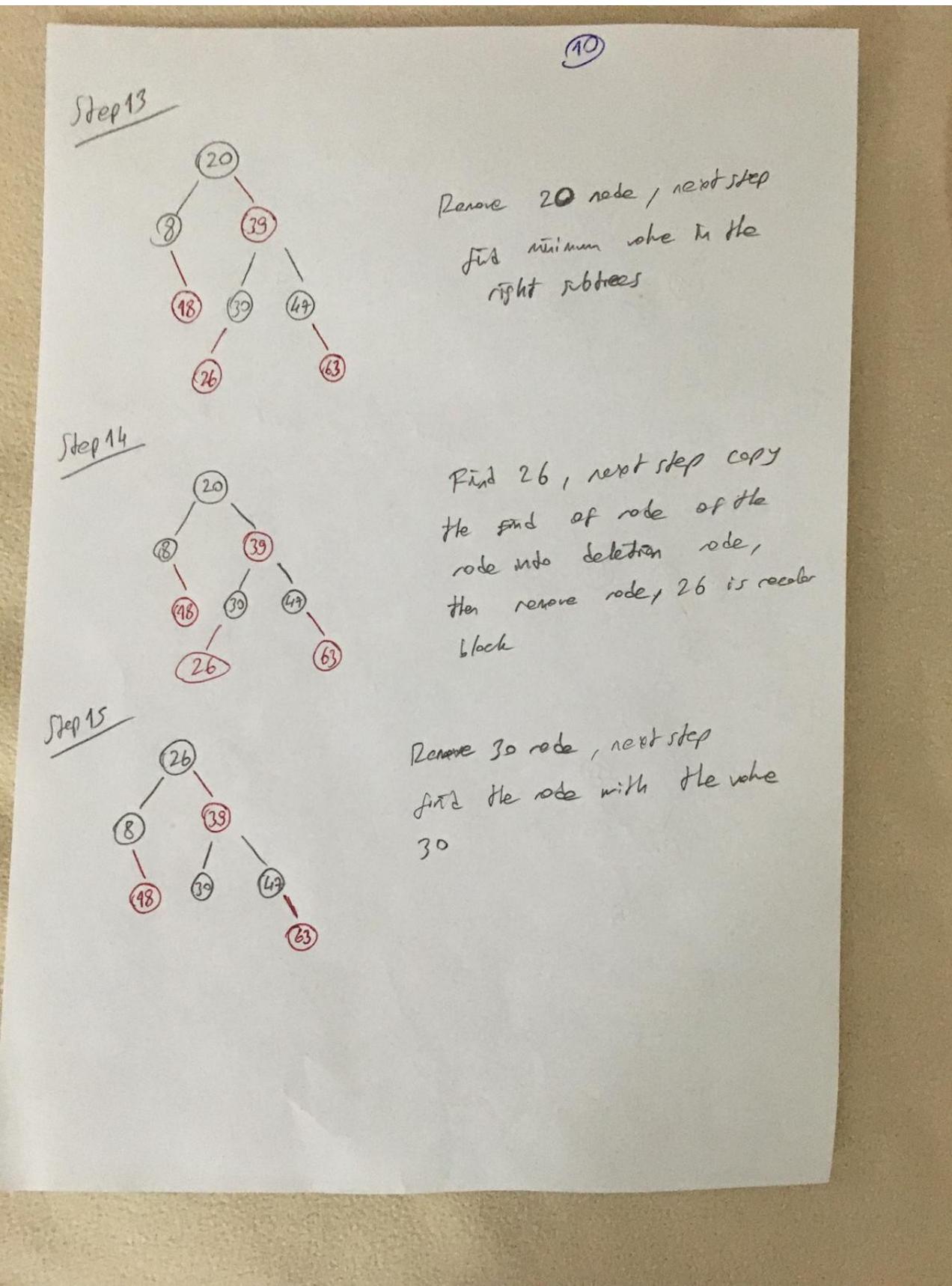


Insert 26, next step recolor

Step 12

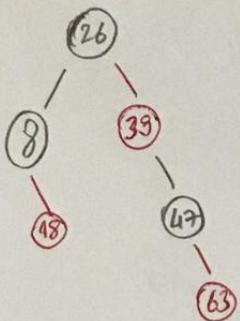


Recolor tree



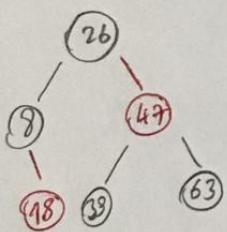
(11)

Step 16



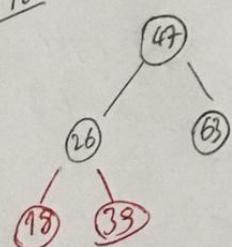
find 30, if it has no child so remove
it. Next step
rotate left & recolor

Step 17



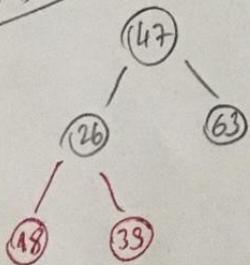
rotate and recolor, next step
remove 8, find it

Step 18



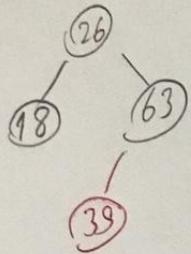
rotate and recolor

Step 19



Remove 47, find it

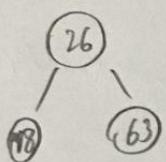
Step 20



(12)

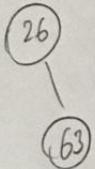
Rotate right and recolor
next step remove 39, find it

Step 21



Remove 39, it has no child

Step 22



Remove 18, it has no child

Step 23



Remove 26, copy content of right child, next step remove 63

Step 24

Remove all nodes, clear tree

$$A = \{20, 30, 8, 47, 39, 18, 26, 63\}$$

(13)

2-3 Tree

Step 1

(20)

Insert 20

Step 2

[20 30]

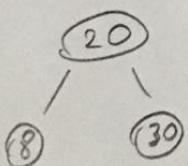
Insert 30

Step 3

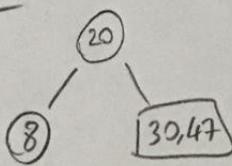
[8 20 30]

Insert 8, because a node can't store three values, the middle value propagates up to the 2-node parent and this leaf node splits into new 2 nodes

Step 4



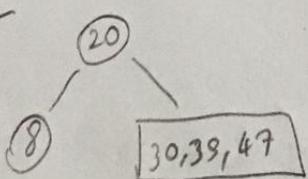
Step 5



Insert 47,

Because this node
is a 2-node,
we insert directly
into the node creating
a 3-node

Step 6

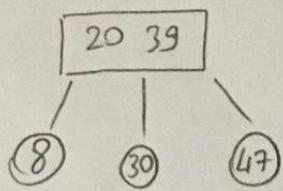


Insert 39,

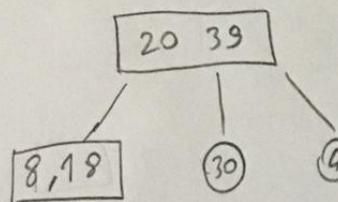
because a node can't store three
values, the middle value propagates
up to the 2-node parent and this leaf
node splits into new 2 nodes.

Step 7

(14)

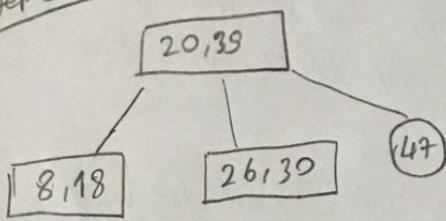


Step 8



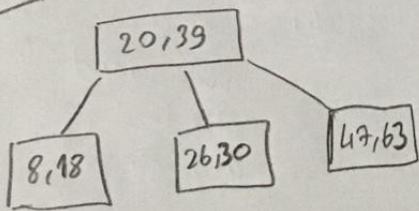
Insert 18

Step 9



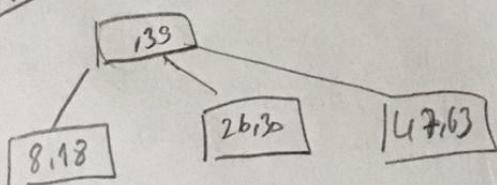
Insert 26

Step 10



Insert 63

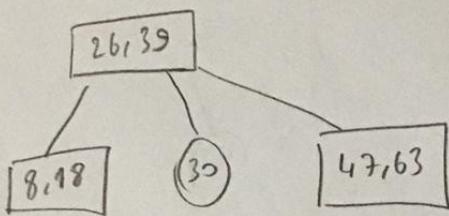
Step 11



Remove 20

The node becomes empty

Step 12

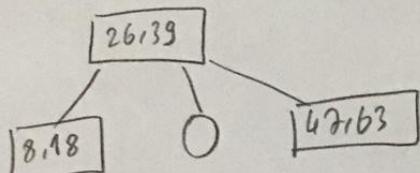


(15)

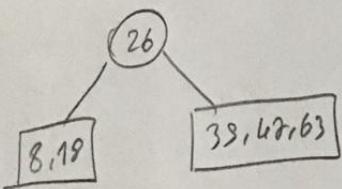
Split node and move 26
the up

Remove 30

Step 13



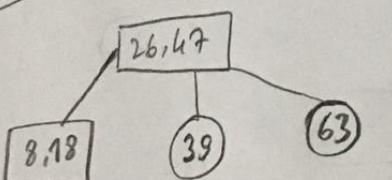
Step 14



Merge 39 with a missing
child

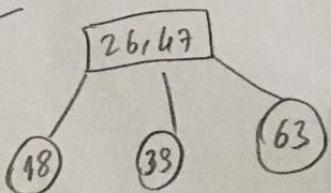
Split the node and move
the middle value (47) up

Step 15



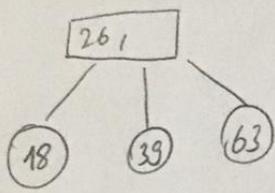
Remove 8

Step 16



Remove 47

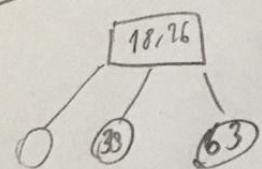
Step 17



(16)

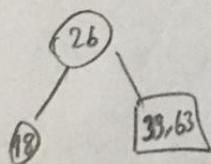
47 is not in a leaf, replace it with its leaf predecessor
(18)

Step 18



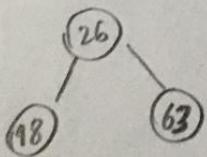
The left leaf is now empty.
Merge the child.

Step 19



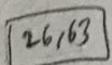
Remove 39

Step 20



Remove 18

Step 21



Remove 26

Step 22



Remove 63

Step 23

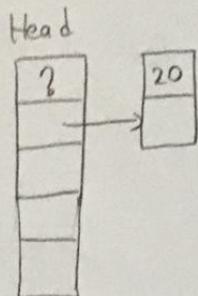
Remove all nodes in tree.

$$A = \{20, 30, 8, 47, 33, 18, 26, 63\}$$

(17)

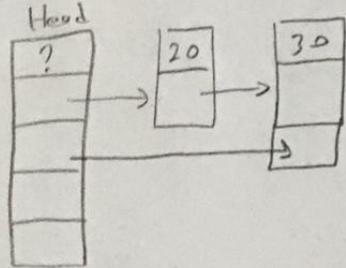
Skip List

Step 1



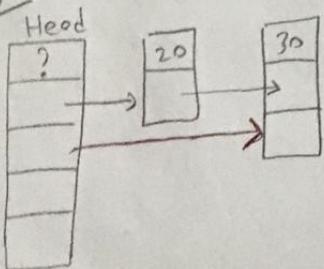
Insert 20 in level 1

Step 2



Insert 30 in level 2,
20 < 30

Step 3

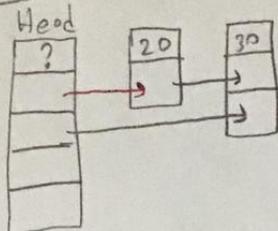


Insert 8 in level 1,
start with the highest level
in case, level 2, (8 finds where
insert)

The value of this node is 30,
since 8 < 30, we move back to
the predecessor node and search
the next lower level

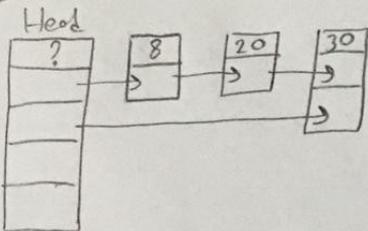
(18)

Step 4



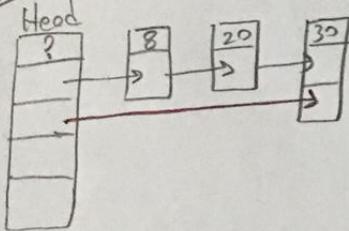
The value of the next node is 20.
8 < 20, so insert backward 20.

Step 5



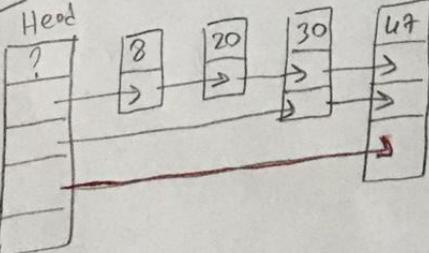
Insert 47 in level 3

Step 6



The value of node is 30,
since 47 > 30, insert after 30

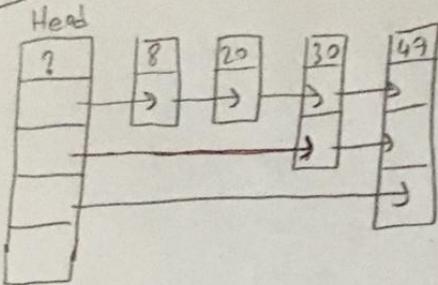
Step 7



Insert 38 in level 2,

start with highest list, in this
case level 3, the value of
this 47, so 38 < 47/
we move back to the
predecessor node and search
the next lower level

Step 8



(18)

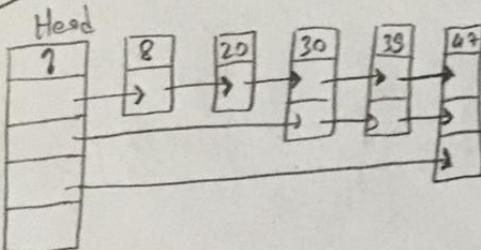
The value of the next node is 30, $39 > 30$

so we move to the next node on this level.

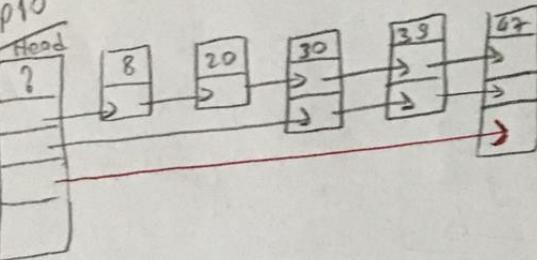
$39 < 47$ so we stop and insert it.

Insert 18 in level 1

Step 9

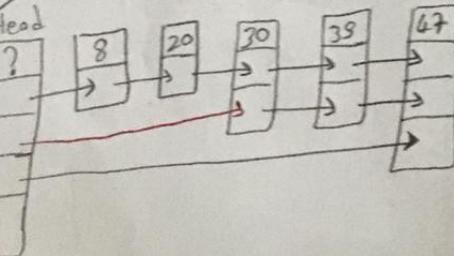


Step 10



Start with the highest list, in this case, level 3, the value of this node is 47, so $18 < 47$, we move back to the predecessor node and search the next lower level

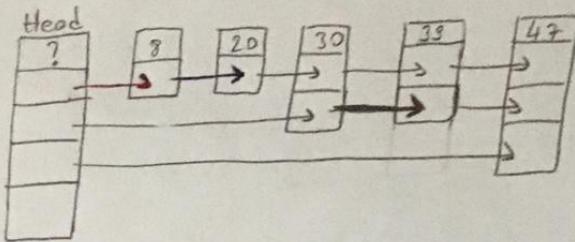
Step 11



The value of the next node is 30, $18 < 30$, so we move back to the predecessor node and search the next lower level.

(20)

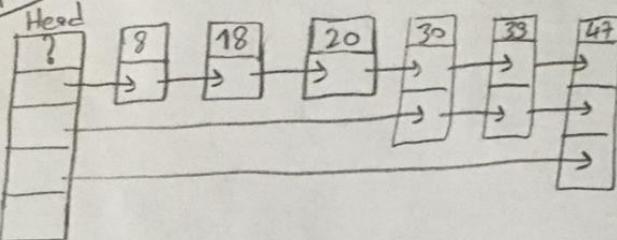
Step 12



The value of the next node is 8.
 $18 > 8$, so move next node on this level

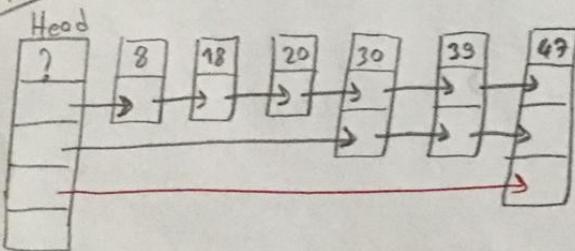
The value of node is 20
 $18 < 20$, insert here.

Step 13



Insert 26 in level 2

Step 14

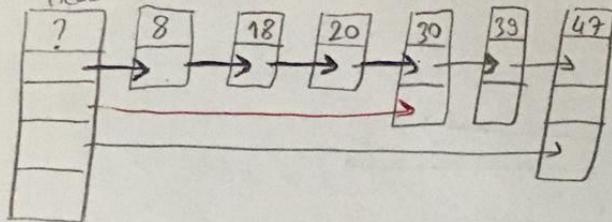


Start with the highest list, in this case, level 3.
The value of this node is 47, since

$26 < 47$ we move back to the predecessor node and search the next lower level.

Step 15

Head



(21)

The value of the next node is 30,
 $26 < 30$, we move
 the predecessor and
 search the next lower
 level

Step 16

$26 > 8$, move next node

Step 17

$26 > 18$, move next node

Step 18

$26 > 20$, move next node

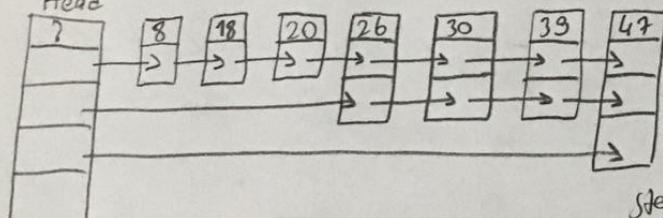
Step 19

$26 < 30$, we stop and

insert here.

Step 20

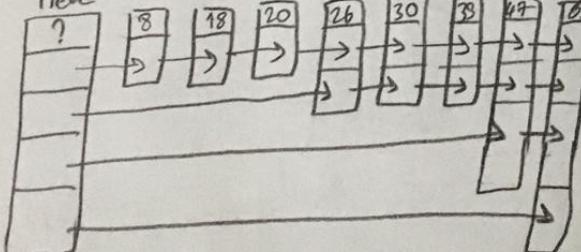
Head



Insert 63 in
 level 4,

Step 22

Head

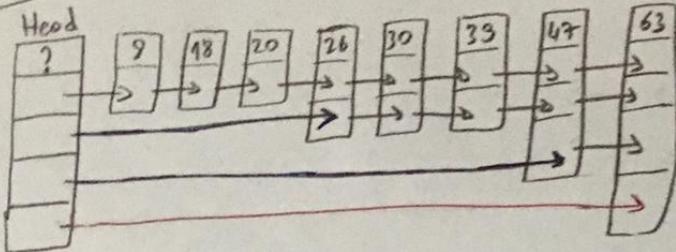


Step 21
 Start with the
 highest list, in this
 case, level 3. The value of node
 is 47, since $63 > 47$, so
 insert after 47.

(22)

Step 23

Head



Remove 20,
first search 20,
start with the
highest list. In
this case, level 4

The value of this node is 63, since $20 < 63$, we move
back to the predecessor node and search the next
lower level.

Step 24

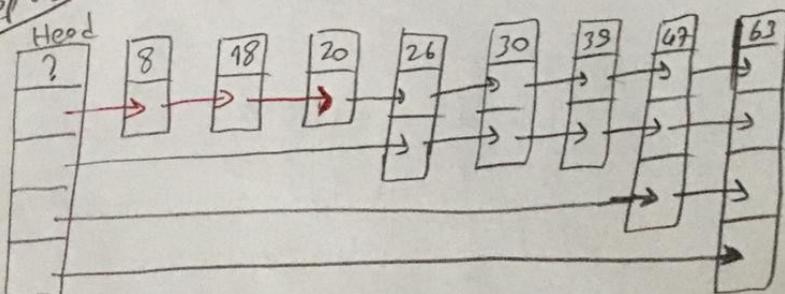
The value of the next node is 47, since $20 < 47$
so we move back to the predecessor node and
search the next lower level.

Step 25

The value of the next node is 26, $20 > 26$ /
so we move back to the predecessor node and
search the next lower level.

Step 26

Head



The value of the next node is 8, $20 > 8$, so no next
node on this level.

(23)

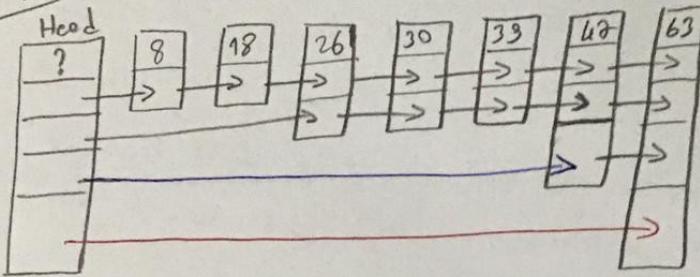
Step 27

The value of the next node is 18, 20 < 18,
so move next node on this level.

Step 28

20 is found, delete it.

Step 29

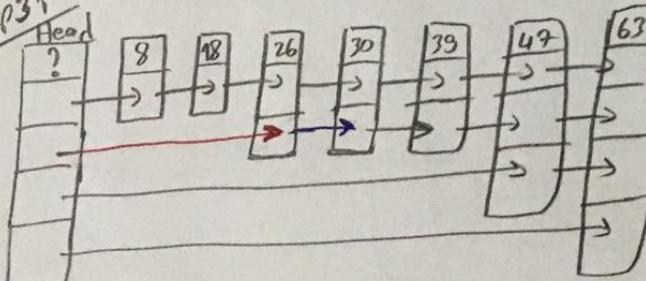


Remove 30, firstly
search 30,
start with the
highest list, in
this case, levels.

The value of this node is 63, since $30 < 63$, we move
back and search the next lower level.

Step 30 The value of next node is 47, since $30 < 47$ so we
move back to the predecessor node and search the next
lower level.

Step 31

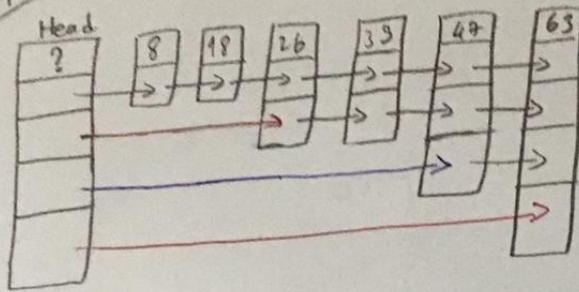


The value of node is 26,
since $30 > 26$, move
next node.

The value of node is 30.
30 is found, remove it.

(24)

Step 32



Remove 8, first search
8, start with highest
list, in this case, in
level 4.

The value of this node is

63, since $8 < 63$, we move
back to the predecessor node and search the next lower level.

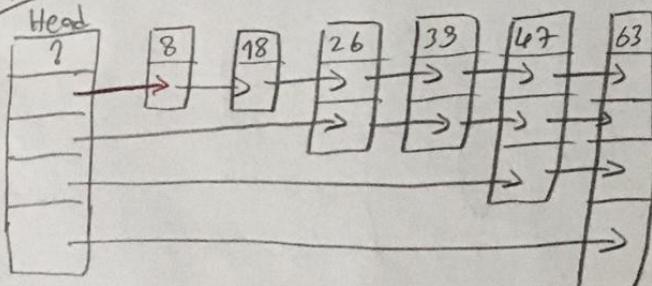
Step 33

The value of the next node is 47, since $8 < 47$
so we move back to predecessor node and search the next lower level.

Step 34

The value of the next node is 26, since $8 < 26$
so we move back to predecessor node and search the next lower level.

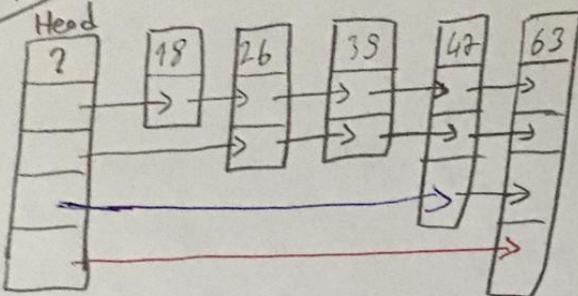
Step 35



The value is 8,
found it, delete 8

(25)

Step 36



Remove 47, first search 47.

Start with highest list, in this case, in level 4.

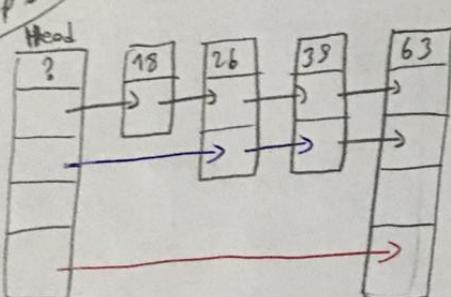
The value of this node is 63, since $47 < 63$, we

move back to the predecessor node and search the next lower level.

Step 37

The value of the next node is 47, found it. Delete 47.

Step 38



Remove 39, first search 39.

Start with highest list, in this case, in level 4.

The value of this node is 63, since $39 < 63$, we move

back to the predecessor node and search the next lower level.

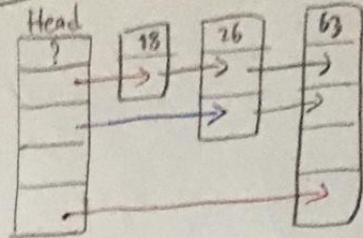
Step 39

The value of the next node is 26, since $39 > 26$, so move next node in this level.

Step 40

The value of the next node is 39, found it, remove 39.

Step 61



(26)

Remove 18, firstly search
18, start with highest list, in
this case, in level 4.

The value this node is 63,
since $18 < 63$, we move back
to the predecessor node and search the next lower
level.

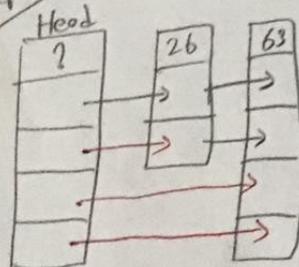
Step 62

The value of the next node is 26, since $18 < 26$,
so we move back to predecessor node and search the
next lower level.

Step 63

The value of the next node is 18, found it, delete 18.

Step 64



Remove 26, first search 26.

Start with highest list, in this case,
in level 4.

The value this node is 63, since
 $26 < 63$, we move back to the
next lower level.

predecessor node and search the next lower level.

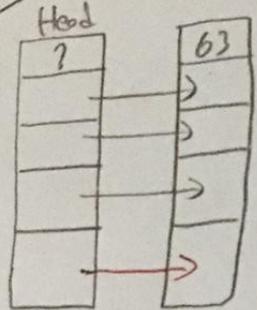
Step 65

The value this node is 63, since $26 < 63$, move next lower
level

Step 66

The value this node is 26, found it, remove 26.

Step 47



(27)

Remove 63, first search 63.

Start with highest list, in this case
in level 4.

The value is 63, found it, remove 63.

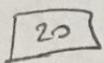
Step 48

Remove all elements.

(28)

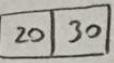
$$A = \{20, 30, 8, 47, 39, 18, 26, 63\}$$

Step 1



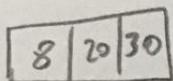
Insert 20

Step 2



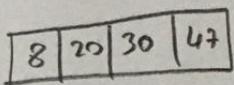
Insert 30, 20 < 30

Step 3



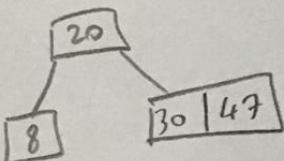
8 < 20, insert 8

Step 4

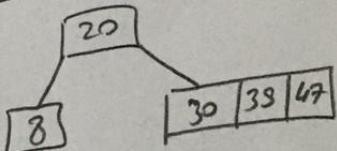


Insert 47, after 30

Step 5

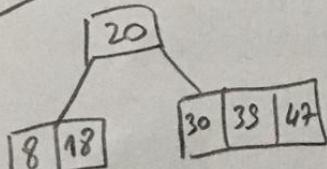


Step 6



Insert 39, comparable. Then insert
after 30.

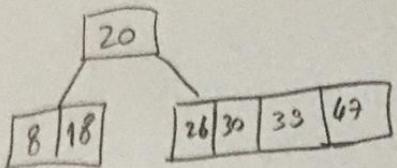
Step 7



18 < 20, 18 > 8, insert 18 to tree.

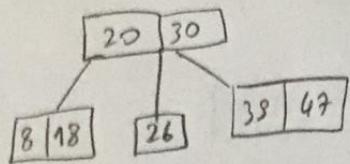
(2.8)

Step 8

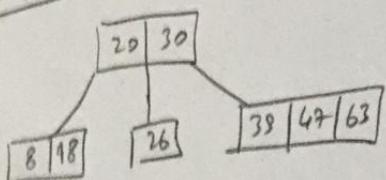


Insert 26, 20 < 26, 26 < 30

Step 9

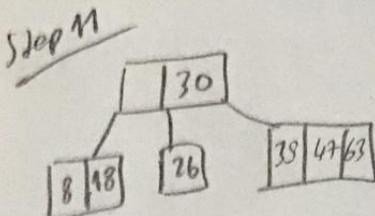


Step 10

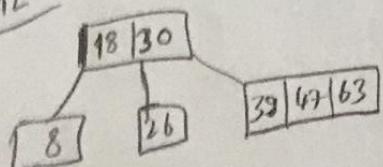


63 > 30, 63 > 38, 63 > 47, insert here.

Remove 20.

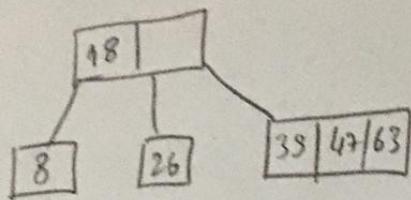


Step 12



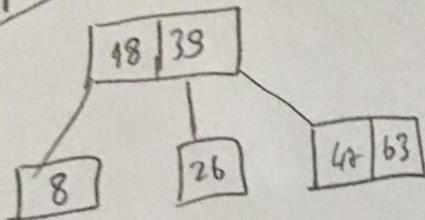
(30)

Step 13

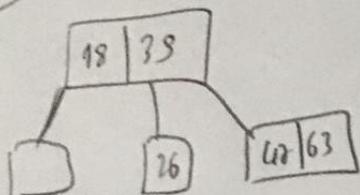


Remove 30.

Step 14

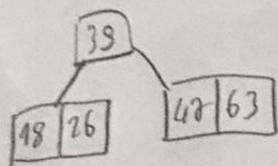


Step 15

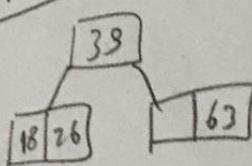


Remove 8

Step 16

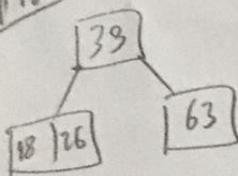


Step 17

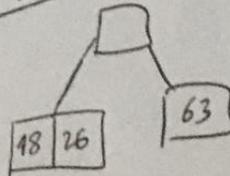


Remove 47

Step 18

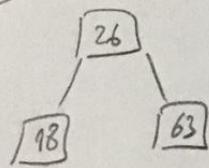


Step 19



Remove 39

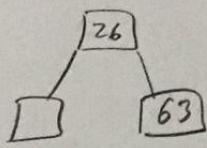
Step 20



Step 21

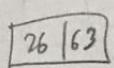
(31)

Remove 18



Step 22

Remove 26



Step 23



Remove 63

Step 24

Remove all nodes