CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 1

Bilgin, Mert Can
e265518@ceng.metu.edu.tr

LastName2, FirstName2
exxxxxxx@ceng.metu.edu.tr

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1. (a) If z = x + yj, then z* = x - yj
Replace them in 2z + 5 = j - z*, and we get the following equation:
2(x+yj) + 5 = j - (x - yj)
= 2x + 2yj + 5 = j - x + yj = 3x + 5 + j(y - 1) = 0
So, y = 1 and x = -5/3

$$|z|^2 = x^2 + y^2 = 25/9 + 1 = 34/9$$

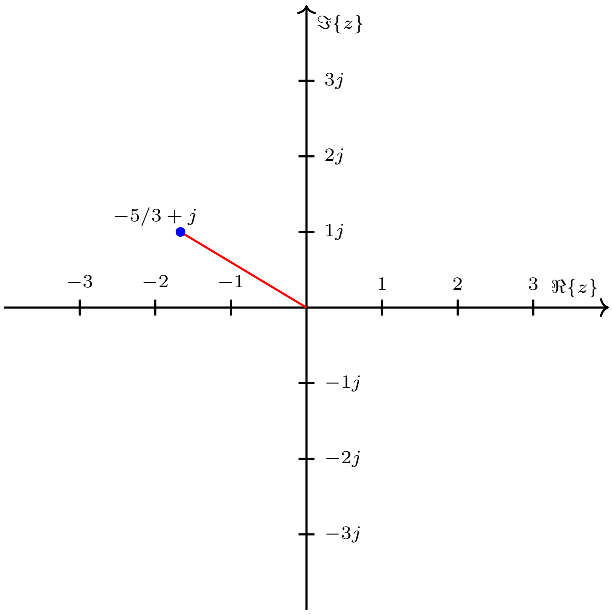


Figure 1: z = -5/3 + j

(b)

$$z^5 = 32j, z = (32j)^{1/5}$$

We know j:

$$j = e^{j\pi/2}$$

$$z = (32e^{j\pi/2})^{1/5} = 2e^{j\pi/10}$$

(c) After multiplying the denominator by the conjugate of it, we get

$$\frac{(1+j)(\frac{1}{2}+\frac{\sqrt{3}}{2}j)(j+1)}{-2} = \frac{(j^2+2j+1)(\frac{1}{2}+\frac{\sqrt{3}}{2}j)}{-2}$$

$$=> \frac{j-\sqrt{3}}{-2} = \frac{\sqrt{3}-j}{2} = \frac{\sqrt{3}}{2} - \frac{1}{2}j$$

$$=> r = \sqrt{a^2+b^2} = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{-1}{2})^2} = \sqrt{\frac{3}{4}+\frac{1}{4}} = 1$$

$$\theta = \arctan(\frac{-1}{2}.\frac{2}{\sqrt{3}}) = \arctan(-\frac{1}{\sqrt{3}}) = -\frac{\pi}{6}$$

(d) It is already in the polar form, but it can be written in rectangular form:

$$z = r\cos(\theta) + jr\sin(\theta) = j\cos(-\frac{\pi}{2}) + j^2\sin(-\frac{\pi}{2}) = 1$$

2.

$$y(t) = x(\frac{1}{2}t+1)$$

$$y(0) = x(1), y(2) = x(2), y(4) = x(3), y(-2) = x(0), y(-4) = x(-1), y(-6) = x(-2), y(8) = x(-3)$$

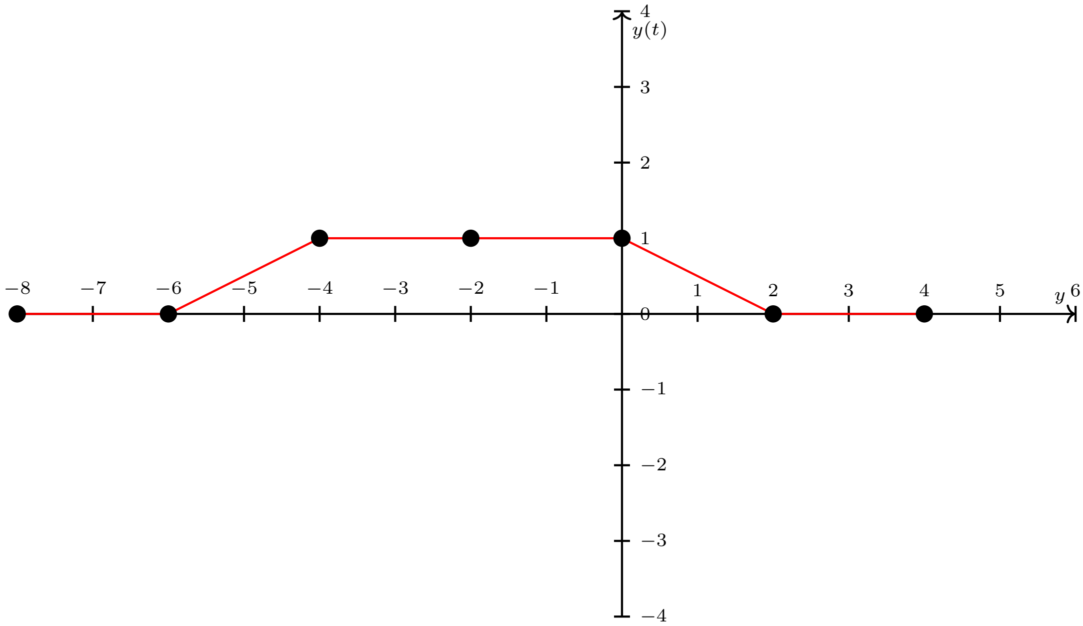


Figure 2: t vs. y(t)

3. (a) First, we need to plot x[-n] and x[2n - 1] separately.

$$x[-n] = 0 for n = -8, -6, -5, -3, 0, 1$$

$$x[-n] = -1 \ for \ n = -1, x[-n] = 2 \ for \ n = -2, x[-n] = -4 \ for \ n = -4, x[-n] = 3 \ for \ n = -7$$

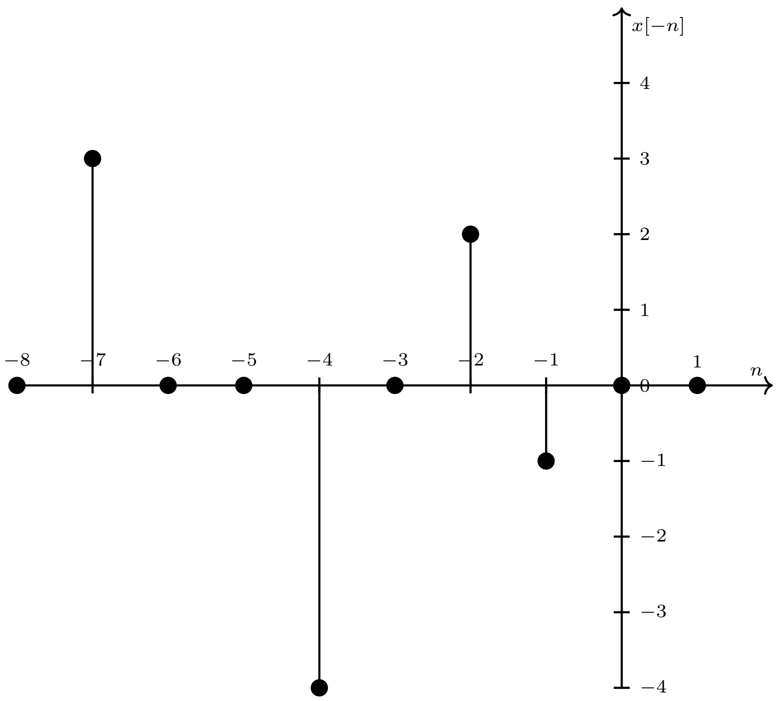


Figure 3: x[-n]

Because non-integer values are not defined, the signals at these points are squished.

$$x[2n-1] = 0 \ for \ n = 0, 2, 3 \ and \ x[2n-1] = -1 \ for \ n = 1 \ and \ x[2n-1] = 3 \ for \ n = 4$$

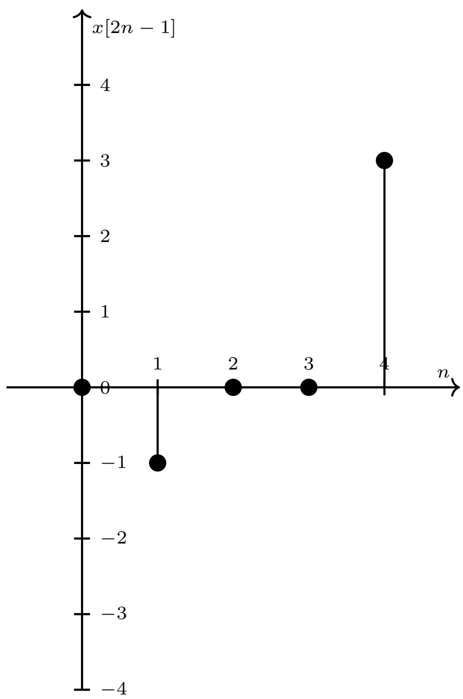


Figure 4: x[2n-1]

Finally, we can add these signals as in Figure 5.

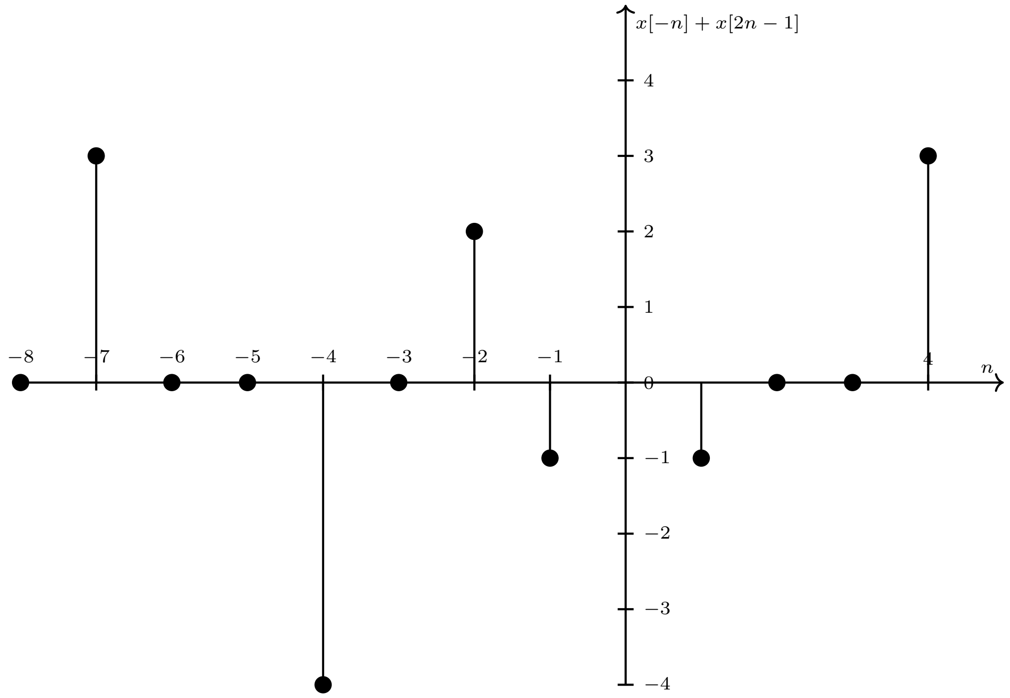


Figure 5: x[-n] + x[2n - 1]

(b)

$$x[-n] + x[2n-1] = 3\delta(t+7) - 4\delta(t+4) + 2\delta(t+2) - \delta(t+1) - \delta(t-1) + 3\delta(t-4)$$

4. (a)

$$x(t) = 5\sin(3t - \frac{\pi}{4})$$

Yes it is periodic because it is a sinusoidal function.
If we can take 3t from the function we can say:

$$3T = 2\pi k$$

$$T = \frac{2\pi}{3}k, T_0 = \frac{2\pi}{3}$$

(b) Firstly, we need to check if each signal is periodic. Cos function is periodic because n is a multiple of pi. Sin function is periodic too because its n also a multiple of pi. Secondly, we need to convert these periods to frequencies. Let's say frequency of cos function is f1, and frequency of sin function is f2.

$$f1 = \frac{w}{2\pi} = \frac{13\pi}{10}.\frac{1}{2\pi} = \frac{13}{20}$$

$$f2 = \frac{w}{2\pi} = \frac{7\pi}{10}.\frac{1}{2\pi} = \frac{7}{20}$$

We can see that we need 20 as a multiplier to make frequencies an integer, so

$$f = \frac{1}{20} => N_0 = 20$$

(c)

$$x[n] = x[n+N_0] = \frac{1}{2}\cos[7(n+N_0) - 5] = \frac{1}{2}cos[7n + 7N_0 - 5]$$

Hence,

$$7N_0 = 2\pi m$$

Because no values satisfy the above equation, the signal is not periodic.

5. (a)
$$x(t) = u(t-1) - 3u(t-3) + u(t-4)$$

(b)
$$\frac{dx(t)}{dt} = \delta(t-1) - 3\delta(t-3) + \delta(t-1)$$

Hence, it can be drawn as in the Figure 6.

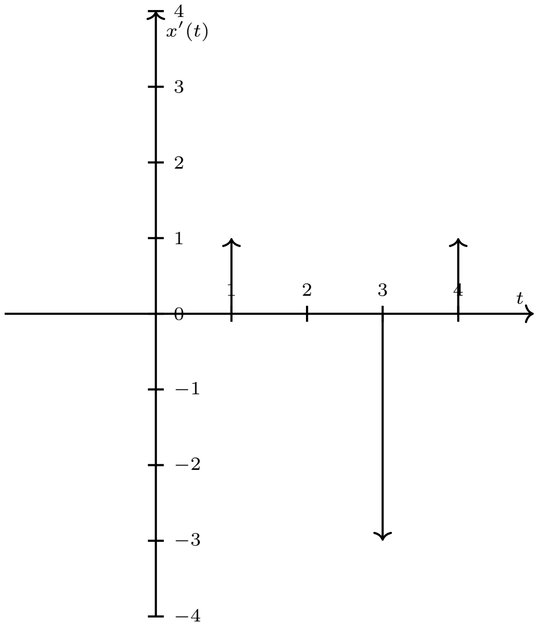


Figure 6: x'(t)

6. (a) Memory: It has memory because the output depends on the future.
Stability: It is unstable because the output grows unbounded with time even the input is bounded.
Causality: It is not causal because its output depends on future.
Linearity: Both superposition and scaling properties are satisfied, so it is linear.
Invertibility: Yes it is invertible because the system has a unique inverse mapping from y(t) to x(t).
Time-invariance: No it is not time-invariant because its behavior changes with respect to a shift in time.

(b) Memory: It has memory because the output depends on the past.
Stability: It is unstable because it is unbounded for k-¿infinity.
Causality: Yes it is causal because its output depends only on the input up to and including time n.
Linearity: Both superposition and scaling properties are satisfied, so it is linear.
Invertibility: Yes it is invertible because distinct inputs lead to distinct outputs.
Time-invariance: Yes it is time-invariant because shifting input in time causes an identical shift in the output.

7. (a) The code as the following:

```
import numpy as np
import matplotlib.pyplot as plt

# read CSV file and store values into numpy array
xn = np.genfromtxt('chirp_part_a.csv', delimiter=',')

# calculate x[-n] and store it into a new numpy array x_n
x_n = xn[::-1]

# calculate the even component of the signal and store it into a new numpy array evenPa
evenPart = 0.5 * (xn + x_n)
# calculate the odd component of the signal and store it into a new numpy array oddPart
oddPart = 0.5 * (xn - x_n)


# create a figure with two subplots
fig, axs = plt.subplots(2, 1)

# plot evenPart in the first subplot
axs[0].plot(evenPart)
axs[0].set_title('Even Part of the Signal')
axs[0].set_xlabel('Values')
axs[0].set_ylabel('Amplitude')

# plot oddPart in the second subplot
axs[1].plot(oddPart)
axs[1].set_title('Odd Part of the Signal')
axs[1].set_xlabel('Values')
axs[1].set_ylabel('Amplitude')

# adjust the layout of the subplots
fig.tight_layout()

# display the plot
plt.show()
```

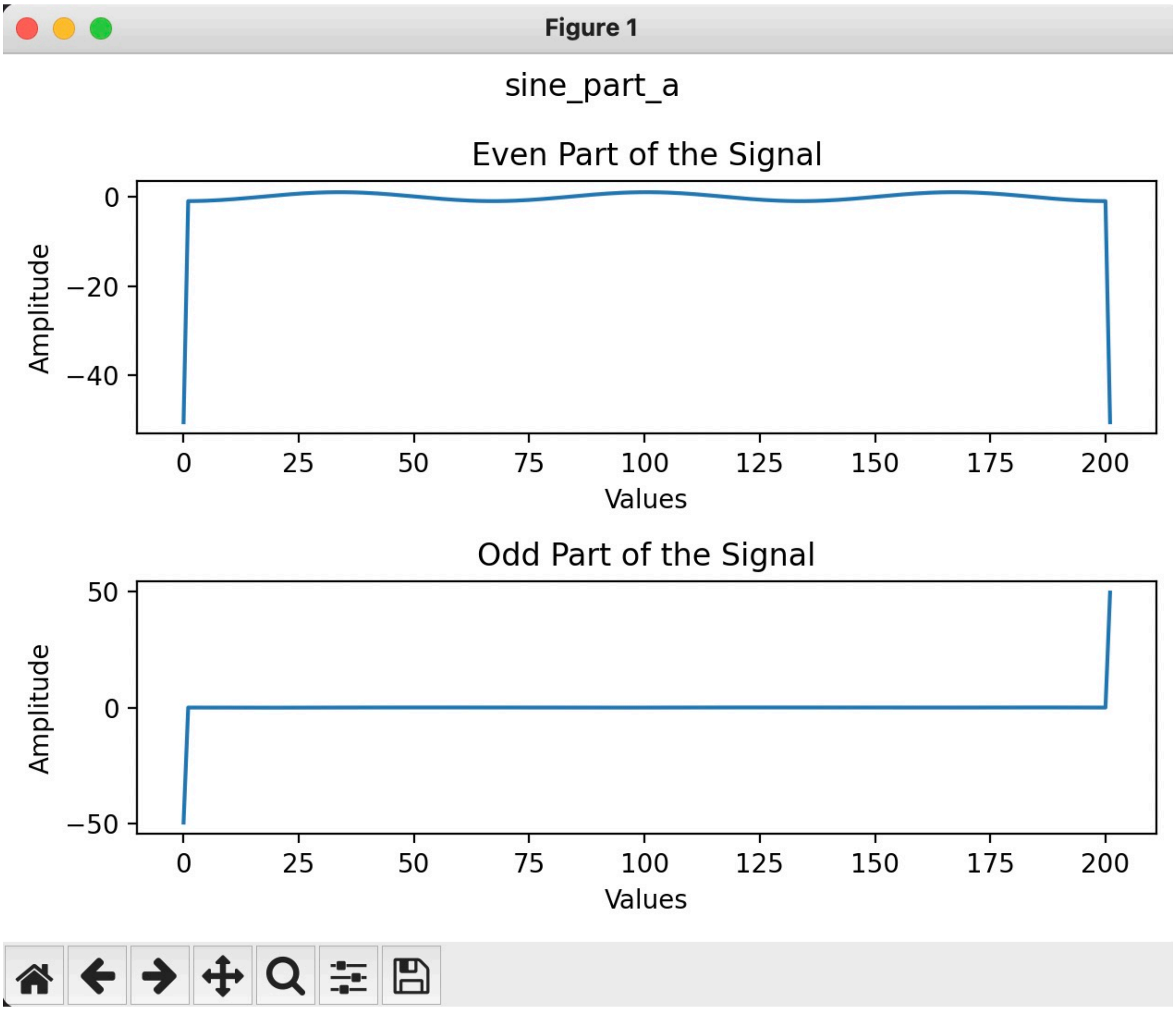


Figure 1
sine_part_a
Even Part of the Signal
Amplitude
0
−20
−40
0
25
50
75
100
125
150
175
200
Values
Odd Part of the Signal
Amplitude
50
0
−50
0
25
50
75
100
125
150
175
200
Values

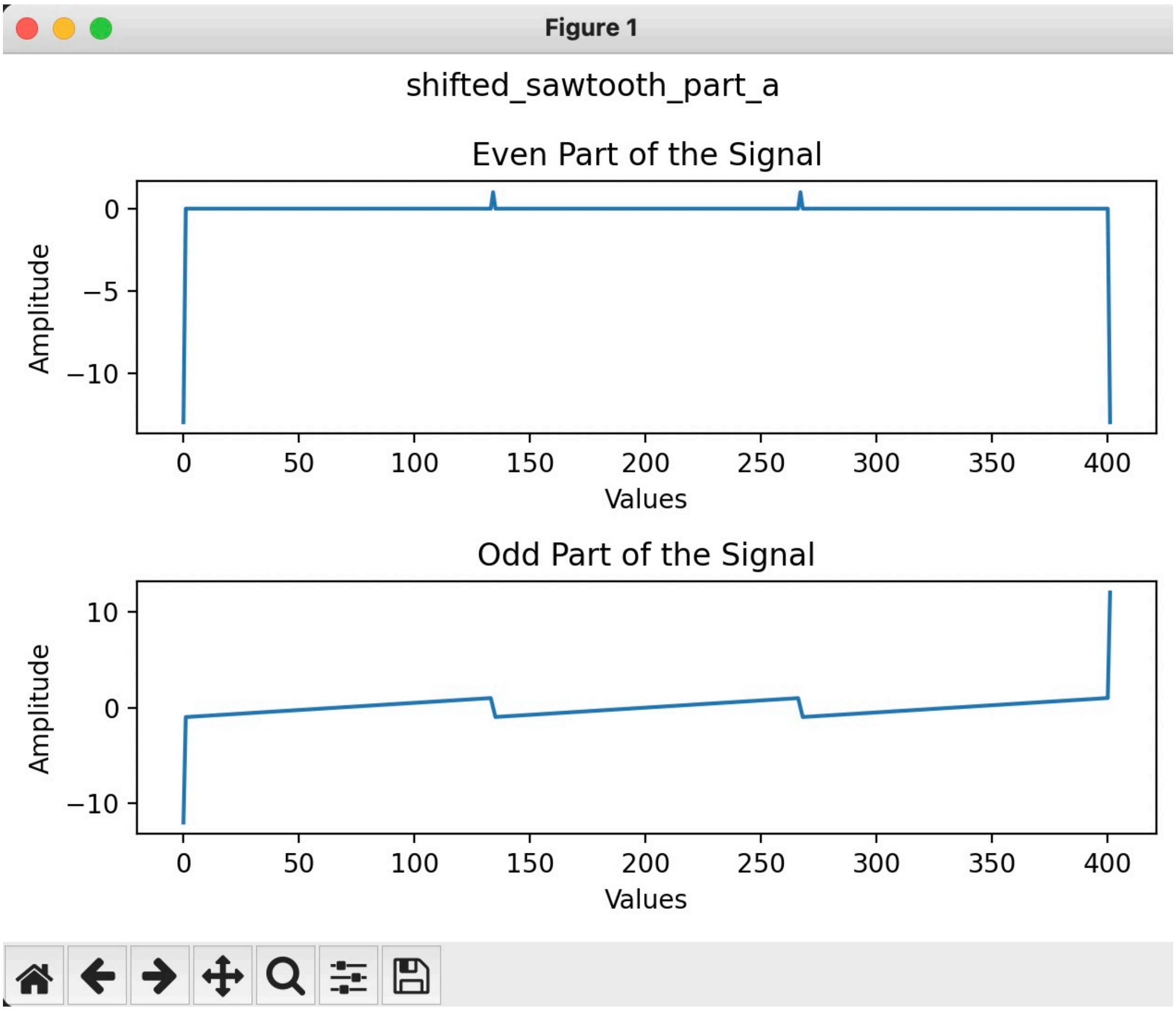


Figure 1
shifted_sawtooth_part_a
Even Part of the Signal
Amplitude
0
−5
−10
0
50
100
150
200
250
300
350
400
Values
Odd Part of the Signal
Amplitude
10
0
−10
0
50
100
150
200
250
300
350
400
Values

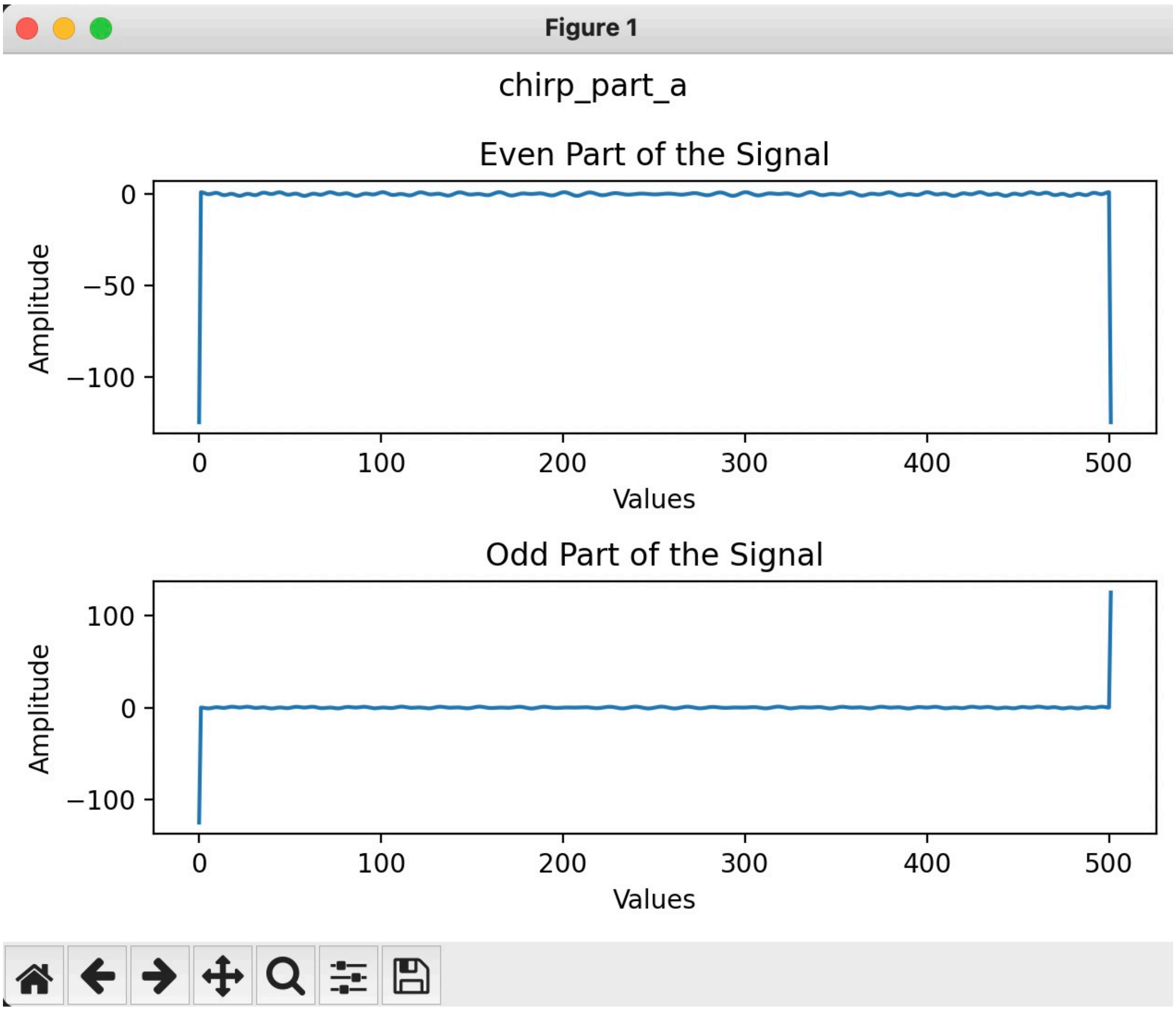
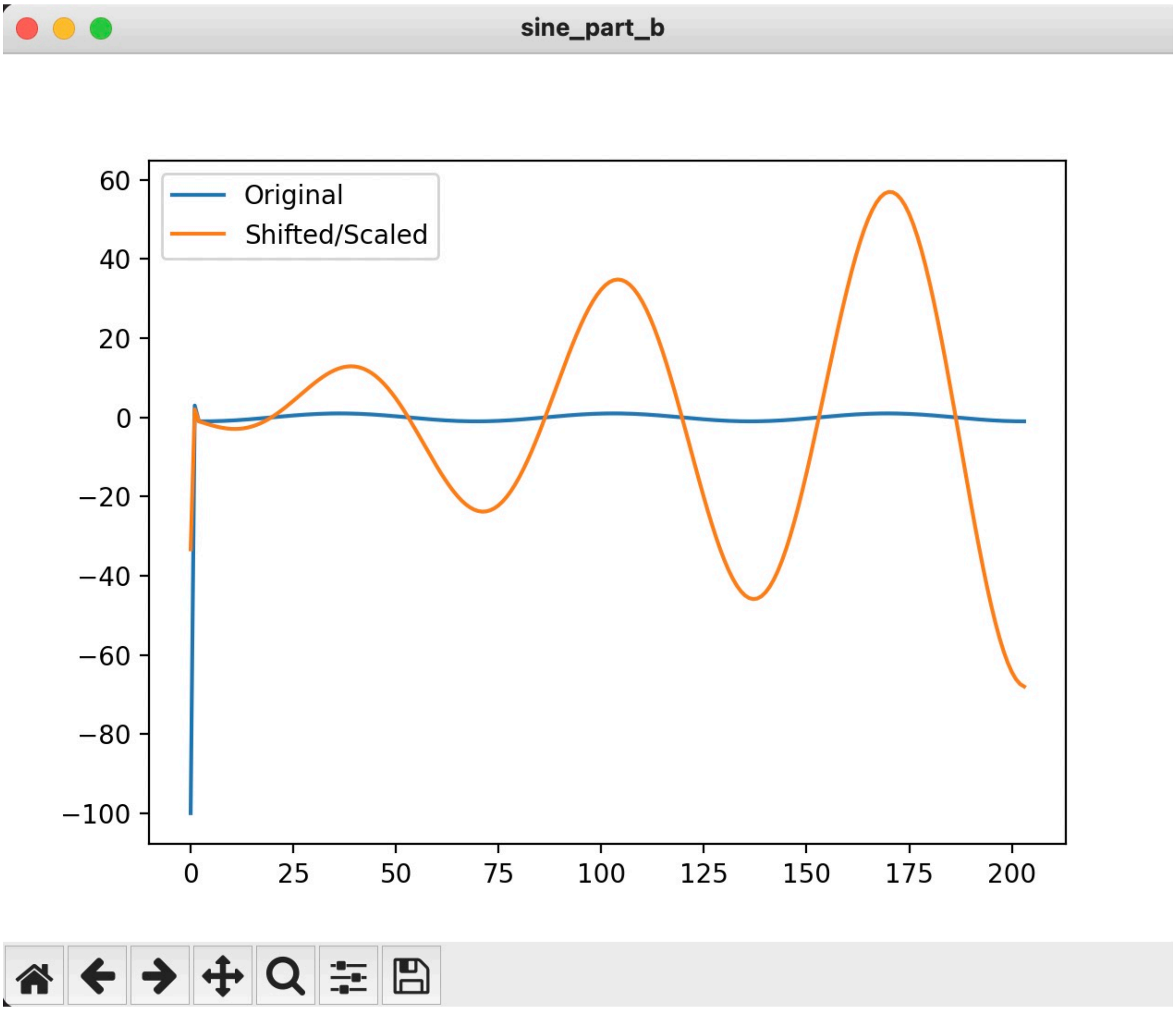
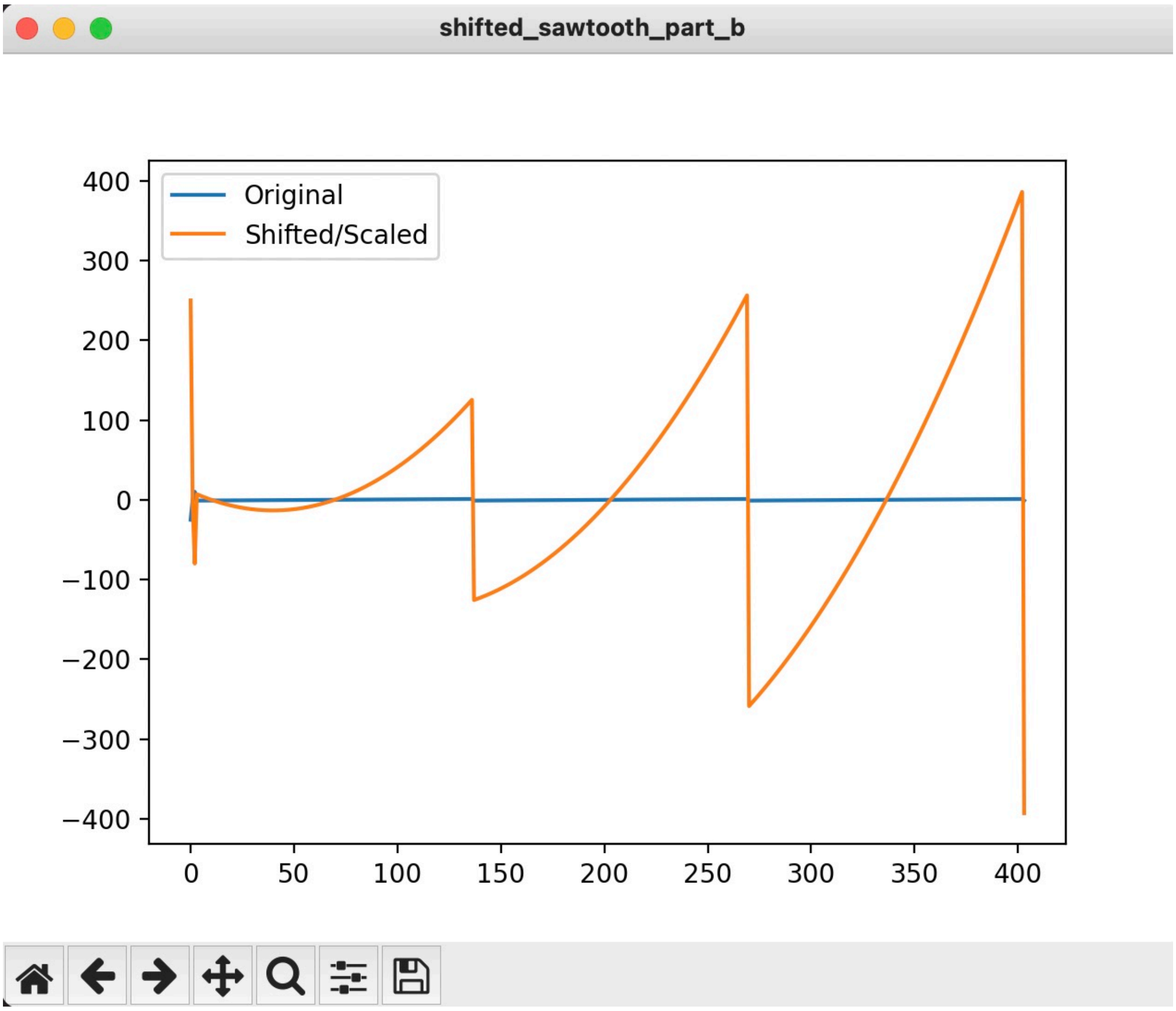


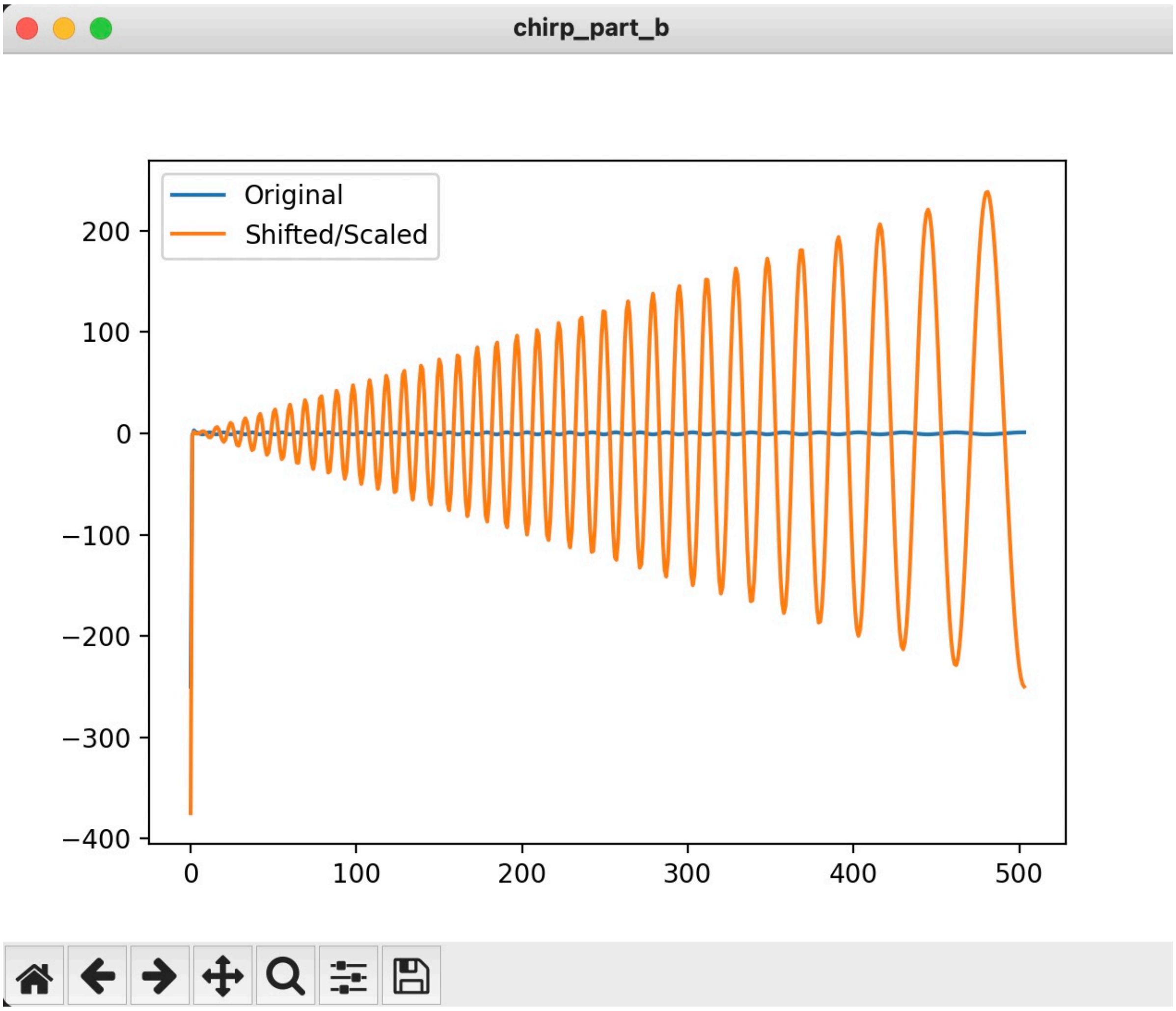
Figure 1
chirp_part_a
Even Part of the Signal
Amplitude
0
−50
−100
0
100
200
300
400
500
Values
Odd Part of the Signal
Amplitude
100
0
−100
0
100
200
300
400
500
Values

(b) The code is as the following:

```
import numpy as np
import matplotlib.pyplot as plt

# read CSV file and store values into numpy array
xn = np.genfromtxt('chirp_part_b.csv', delimiter=',')

# set the second and the third elements of xn as variables a and b
a = int(xn[1])
b = int(xn[2])

# calculate shifted and scaled xn
n = np.arange(len(xn))
shiftedScaledxn = xn * (n - b) / a
# plot original and shifted/scaled signals
fig = plt.figure(num='chirp_part_b')
plt.plot(n, xn, label='Original')
plt.plot(n, shiftedScaledxn, label='Shifted/Scaled')
plt.legend()
plt.show()
```


sine_part_b
Original
Shifted/Scaled
60
40
20
0
−20
−40
−60
−80
−100
0
25
50
75
100
125
150
175
200


shifted_sawtooth_part_b
Original
Shifted/Scaled
400
300
200
100
0
−100
−200
−300
−400
0
50
100
150
200
250
300
350
400


chirp_part_b
Original
Shifted/Scaled
200
100
0
−100
−200
−300
−400
0
100
200
300
400
500