CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 2

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April 13, 2023

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1. (a)

$$x(t) = y'(t) - 5y(t)$$

(b)

$$y(t) = y_h(t) = Ae^{st} => Ase^{st} - 5Ae^{st} = 0$$

$$A(s-5)e^{st} = 0 => s = 5 => y_h(t) = Ae^{5t}$$

$$y_p(t) = (K_1e^{-t} + K_2e^{-3t})u(t)$$

$$-K_1e^{-t} - 3K_2e^{-3t} - 5K_1e^{-t} - 5K_2e^{-t3} = (-6K_1e^{-t} - 8K_2e^{-3t})u(t) = (e^{-t} + e^{-3t})u(t)$$

$$K_1 = -\frac{1}{6}, K_2 = -\frac{1}{8} => y(t) = Ae^{5t} - \frac{1}{6}e^{-t} - \frac{1}{8}e^{-3t}$$

Because y(0) = 0:

$$A - \frac{1}{6} - \frac{1}{8} = 0 => A = \frac{7}{24}$$

$$y(t) = \frac{7}{24} - \frac{1}{6}e^{-t} - \frac{1}{8}e^{-3t}$$

2. (a)

$$n = -1 : x[-1] = 1, h[-1] = 2[$$

$$n = 0 : x[0] = 2, h[0] = 0$$

$$n = 1 : x[1] = 0, h[1] = 1$$

$$\sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$=> y[-2] = x[-1]h[1], y[-1] = x[0]h[-1] = 4, y[0] = x[-1]h[1] + x[0]h[0] = 1, y[1] = x[0]h[1] = 2$$

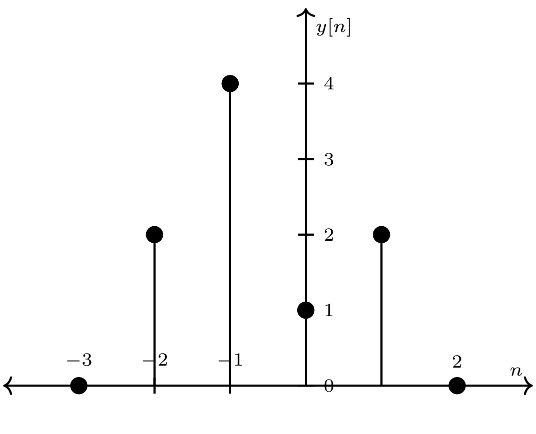

Figure 1: y[n]

(b)

$$y(t) = \int_{-\infty}^{\infty} x'(t)h(t-\tau)d\tau$$
$$x'(t) = \delta(t-1) + \delta(t+1)$$
$$\int_{-\infty}^{\infty} [\delta(t-1) + \delta(t+1)]h(t-\tau)d\tau$$
$$h(t-1) + h(t+1) = e^{-t-1}\sin(t-1)u(t-1) + e^{-t+1}\sin(t+1)u(t+1)$$

3. (a)

$$y(t) = x(t) * h(t) = \int_{\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$
$$\int_{-\infty}^{\infty} e^{-\tau}u(t)e^{-2(t-\tau)}u(t-\tau)d\tau = e^{-2t}\int_{0}^{t} e^{\tau}d\tau$$
$$e^{-2t}(e^{t}-1) = e^{t} - e^{-2t}u(t)$$

(b)

$$y(t) = \int_{-\infty}^{\infty} x(t)h(t-\tau)d\tau$$
$$\int_{-\infty}^{\infty} (u(t) - u(t-1))h(t-\tau)d\tau$$
$$\int_{-\infty}^{\infty} u(t)h(t-\tau)d\tau - \int_{-\infty}^{\infty} u(t-1)h(t-\tau)d\tau$$
$$\int_{0}^{t} e^{3t-3\tau}d\tau - \int_{1}^{t} e^{3t-3\tau}d\tau = \frac{-e^{3t}(1+e^{-3})}{3}$$

4. (a)

$$n=2: y[2] - y[1] - y[0] = 0 => y[2] = 2$$
$$n=3: y[3] - y[2] - y[1] = 0 => y[3] = 3$$
$$n=4: y[4] - y[3] - y[2] = 0 => y[4] = 5$$

It is Fibonacci Series, also we can solve it as:

$$r^2 - r - 1 = 0,\ r = \frac{1 \pm \sqrt{5}}{2}$$
$$y[1] = A\frac{1+\sqrt{5}}{2} + B\frac{1-\sqrt{5}}{2} = 1$$
$$A = \frac{1+\sqrt{5}}{2\sqrt{5}}, B = \frac{1-\sqrt{5}}{2\sqrt{5}}$$
$$y[n] = \frac{(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n}{\sqrt{5}}$$

(b)

$$y(t) = As^3e^{st} - 6As^2e^{st} + 13Ase^{st} - 10Ae^{st} = 0$$
$$A(s^3 - 6s^2 + 13s - 10)e^{st} = 0$$
$$s_1 = 2, s_2 = 2+i, s_3 = 2-i$$
$$y(t) = C_1e^{2t} + C_2e^{2t}cos(t) + C_3e^{2t}sin(t)$$

Because y(0) = 0, $C_1 + C_2 = 1$

$$y'(t) = 2C_1e^{2t} + C_2(2e^{2t}cos(t) - e^{2t}sin(t)) + C_3((2e^{2t}sin(t)) + e^{2t}cos(t))$$

Because y'(0) = 0, $2C_1 + 2C_2 + C_3 = \frac{3}{2})$

$$y''(t) = 4C_1e^{2t} + C_2(4e^{2t}cos(t) - 2e^{2t}sin(t) - (2e^{2t}sin(t) - e^{2t}cos(t))) + C_3(4e^{2t}sin(t) + 2e^{2t}cos(t) + (2e^{2t}cos(t) - e^{2t}sin(t)))$$

Because y"(0) = 0, $4C_1 + 3C_2 + 4C_3 = 3$
After solving the constants, we get: $C_1 = \frac{6}{7}, C_2 = \frac{1}{7}, C_3 = -\frac{1}{2}$

$$y(t) = \frac{6}{7}e^{2t} + \frac{1}{7}e^{2t}cos(t) - \frac{1}{2}e^{2t}sin(t)$$

5. (a)

$$y_p(t) = Acos(5t) + Bsin(5t)$$
$$y_p'(t) = -5Asin(5t) + 5Bcos(5t)$$
$$y_p''(t) = -25Acos(5t) - 25Bsin(5t)$$

If we substitute, we get:

$$-19Acos(5t) + 25Bcos(5t) - 19Bsin(5t) - 25Asin(5t)$$
$$-19A + 25B = 1$$
$$-19B - 25B = 0$$
$$A = -\frac{19}{986}, B = \frac{25}{986}$$
$$y_p(t) = -\frac{19}{986}cos(5t) + \frac{25}{986}sin(5t)$$

(b)

$$r^2 + 5r + 6 = 0$$
$$r_1 = -2, r_2 = -3$$
$$y_h(t) = C_1e^{-2t} + C_2e^{-3t}$$

(c)

$$y(t) = y_h(t) + y_p(t) = C_1e^{-2t} + C_2e^{-3t} - \frac{19}{986}cos(5t) + \frac{25}{986}sin(5t)$$
$$y(0) = 0 => C_1 + C_2 - \frac{19}{986} = 0 => C_1 + C_2 = \frac{19}{986}$$
$$y'(0) = 0 => -2C_1e^{-2t} - 3C_2e^{-3t} - \frac{95}{986}sin(5t) + \frac{125}{986}cos(5t)$$
$$C_1 = -\frac{2}{29}, C_2 = \frac{3}{34}$$
$$-\frac{2}{29}e^{-2t} + \frac{3}{34}e^{-3t} - \frac{19}{986}cos(5t) + \frac{25}{986}sin(5t)$$

6. (a)

$$x[0] = w[0] - \frac{1}{2}w[-1]$$
$$x[1] = w[1] - \frac{1}{2}w[0]$$
$$x[2] = w[2] - \frac{1}{2}w[1]$$

It forms a geometric sequence with common ratio 1/2, so

$$h[n] = 2^{-n}u[n]$$

(b)

(c)

7. (a)

(b)