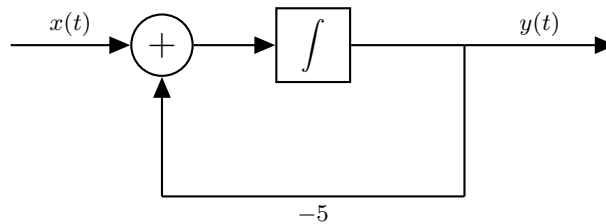




### Regulations:

- **Grouping:** You are strongly encouraged to work in pairs.
- **Drawing Plots:** Clearly label the coordinate axes and make sure that your plots are not open to different interpretations.
- **Submission:** You need to submit a pdf file named 'hw2.pdf' to the odtuclass page of the course. You need to use the given template 'hw2.tex' to generate your pdf files. Otherwise you will receive zero.
- **Deadline:** 23:55, 16 April, 2023 (Sunday).
- **Late Submission:** Not allowed.

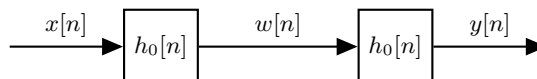
1. (15 pts) Consider an LTI system given by the following block diagram:



- (a) (5 pts) Find the differential equation which represents this system.
- (b) (10 pts) Find the output  $y(t)$ , when the input  $x(t) = (e^{-t} + e^{-3t})u(t)$ . Assume that the system is initially at rest.
2. (10 pts) Evaluate the following convolutions.
- (a) (5 pts) Given  $x[n] = 2\delta[n] + \delta[n+1]$  and  $h[n] = \delta[n-1] + 2\delta[n+1]$ , compute and draw  $y[n] = x[n] * h[n]$ .
- (b) (5 pts) Given  $x(t) = u(t-1) + u(t+1)$  and  $h(t) = e^{-t} \sin(t)u(t)$ , calculate  $y(t) = \frac{dx(t)}{dt} * h(t)$ .
3. (15 pts) Evaluate the following convolutions.
- (a) (7 pts) Given  $h(t) = e^{-2t}u(t)$  and  $x(t) = e^{-t}u(t)$ , find  $y(t) = x(t) * h(t)$ .
- (b) (8 pts) Given  $h(t) = e^{3t}u(t)$  and  $x(t) = u(t) - u(t-1)$ , find  $y(t) = x(t) * h(t)$ .
4. (10 pts) Solve the following homogeneous difference and differential equations with the specified initial conditions.
- (a) (5 pts)  $y[n] - y[n-1] - y[n-2] = 0$ ,  $y[0] = 1$  and  $y[1] = 1$ .
- (b) (5 pts)  $y^{(3)}(t) - 6y''(t) + 13y'(t) - 10y(t) = 0$ ,  $y''(0) = 3$ ,  $y'(0) = \frac{3}{2}$  and  $y(0) = 1$ .
5. (15 pts) Consider the differential equation given below,

$$y''(t) + 5y'(t) + 6y(t) = x(t).$$

- (a) (6 pts) Find the particular solution for  $x(t) = \cos(5t)$ .
- (b) (3 pts) Find the homogeneous solution.
- (c) (6 pts) Find the general solution assuming that the system is initially at rest.
6. (20 pts) Consider the following discrete time LTI system which is initially at rest:



where  $w[n] - \frac{1}{2}w[n-1] = x[n]$ .

- (a) (10 pts) Find  $h_0[n]$ .
- (b) (5 pts) Find the overall impulse response,  $h[n]$ , of this system.
- (c) (5 pts) Find the difference equation which represents the relationship between the input  $x[n]$  and the output  $y[n]$ .

7. (15 pts) Programming.

Write a computer program to take discrete convolution of 2 signals. (You are not allowed to use any `xx.convolve()` function from any library.) Your function takes 4 inputs: the first signal  $x[n]$ , the starting index of the first signal  $s_i^x$ , the second signal  $h[n]$  and the starting index of the second signal  $s_i^h$  (Starting indexes and signals are in the same format as the ones in HW1) and returns the output signal  $y[n]$  and the starting index of the output signal  $s_i^y$ .

- (a) (5 pts) Generate a shifted discrete impulse function  $\delta[n - 5]$  in the given signal form and plot the output function that is the result of your discrete convolution function when  $x[n]$ ="the signal in hw2\_signal.csv" and  $h[n]=\delta[n - 5]$ . What is the effect of convolution with  $\delta[n - 5]$ ? Comment on that.
- (b) (10 pts) The N-Point moving average filter is defined as follows:

$$h[n] = \begin{cases} \frac{1}{N} & \text{if } 0 \leq n \leq N - 1 \\ 0 & \text{otherwise} \end{cases}$$

Generate a N-point moving average filter  $m[n]$  in the given signal form and plot 4 output functions that is the result of your discrete convolution function when  $x[n]$ ="the signal in hw2\_signal.csv" and  $h[n]=m[n]$  by setting  $N=3,5,10,20$ . What is the effect of convolution with  $m[n]$ ? What are the differences between different  $N$  values?

You should write your code in **Python 3**. You are not allowed to use any library other than **matplotlib.pyplot** and **numpy**.