ELECTRICAL AND ELECTRONICS ENGINEERING
&
COMPUTER ENGINEERING

EEE 248 | CNG 232
Logic Design

21 | SPRING | 22

HW I
Number of Questions: 4

Due: APRIL 11, 2022
Good Luck

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Question	Achieved	Points
1		8
2		8
3		22
4		28
5		34
TOTAL		100

Question 1

a. $(38.45)_{10}$

$38 = 19 \times 2 + 0$
$19 = 9 \times 2 + 1$
$9 = 4 \times 2 + 1$
$4 = 2 \times 2 + 0$
$2 = 1 \times 2 + 0$
$1 = 0 \times 2 + 1$

$\Rightarrow 100110$

$0.45 \times 2 = 0.9 + 0$
$0.9 \times 2 = 0.8 + 1$
$0.8 \times 2 = 0.6 + 1$
$0.6 \times 2 = 0.2 + 1$
$0.2 \times 2 = 0.4 + 0$
$0.4 \times 2 = 0.8 + 0$
$0.8 \times 2 = 0.6 + 1$
$0.6 \times 2 = 0.2 + 1$

$\Rightarrow 0.0111\overline{0011}$

, so $(38.46)_{10} = (100110.0111\overline{0011})_2$

b. $(10110.010)_2$

$\Rightarrow (0 \times 2^0) + (1 \times 2^1) + (1 \times 2^2) + (0 \times 2^3) + (1 \times 2^4) = 22$

after dot: $(0 \times 2^{-1}) + (1 \times 2^{-2}) + (0 \times 2^{-3}) = 0.25$

; therefore, $(10110.010)_2 = (22.25)_{10}$

c. $(723.52)_8$

$\Rightarrow$ 7 2 3 . 5 2
↓ ↓ ↓ ↓ ↓
000 111 010 011 . 101 01000

$\Rightarrow$ take groups of four $\Rightarrow (1D3.A8)_{16}$

d. $(8EA3.4F)_{16}$

8 E A 3 . 4 F
↓ ↓ ↓ ↓ ↓ ↓
1000 1110 1010 0011 0100 1111

, so $(8EA3.4F)_{16} = (1000111010100011.01001111)_2$

Question 2:

a. 1001.01 × 11.11

```
=>    100101
   ×    1111
   ----------
        100101
       100101
      100101
 +   100101
 -------------
   1000101011
```

, there are 2 digits after the dot:

1001.(01)

, and there are 2 digits after the dot:

11.(11)

, 2+2=4, so we need to put the dot 4th place from the right.

; therefore, 1001.01 × 11.11 = 100010.1011

b. 0111.10 / 10.11

```
dividend ←           → divisor
=>  11110  | 1011
  - 1011   |------------------
  -------  | 10.1011101000
    0100000
     1011
  ---------
     010100
      1011
  ---------
      010010
       1011
  ---------
       01100
       1011
  ---------
       0011000
```

0111.10 / 10.11 = $10.\overline{1011101000}$

Question 3

a.

$29 = 14 \times 2 + 1$
$14 = 7 \times 2 + 0$
$7 = 3 \times 2 + 1$
$3 = 1 \times 2 + 1$
$1 = 0 \times 2 + 1$

$+29 = 0011101$

2's complement ↓

$-29 = 1100011$

,

$41 = 20 \times 2 + 1$
$20 = 10 \times 2 + 0$
$10 = 5 \times 2 + 0$
$5 = 2 \times 2 + 1$
$2 = 1 \times 2 + 0$
$1 = 0 \times 2 + 1$

$+41 = 0101001$

2's complement ↓

$-41 = 1010111$

i. $(+29) + (-41)$

$$\begin{array}{r} \overset{1}{0}\overset{1}{0}\overset{1}{1}\overset{1}{1}\overset{1}{1}\overset{1}{0}1 \\ + \quad 1\,0\,1\,0\,1\,1\,1 \\ \hline 1\,1\,1\,0\,1\,0\,0 \end{array} \Rightarrow$$

$1110100 \rightarrow$ **-0001100**

2's complement

ii. $(-29) + (+41)$

$$\begin{array}{r} \overset{1}{1}\quad\overset{1}{1}\overset{1}{0}\overset{1}{0}\overset{1}{0}1\overset{1}{1} \\ + \quad 0\,1\,0\,1\,0\,0\,1 \\ \hline 1\,0\,0\,0\,1\,1\,0\,0 \end{array} \Rightarrow$$

$10001100 \rightarrow$ the most significant digit can be discarded, so $\Rightarrow$ **0001100**

iii. $(-29) + (-41)$

$$\begin{array}{r} \quad\overset{1}{1}\overset{1}{0}\overset{1}{0}\overset{1}{0}1\overset{1}{1} \\ + \quad 1\,0\,1\,0\,1\,1\,1 \\ \hline 1\,0\,1\,1\,1\,0\,1\,0 \end{array} \Rightarrow$$

2's complement

$1011010 \rightarrow$ **-01000110**

Question 3

b. $+347 = 0101011011$

$+192 = 0011000000$

$-192 = 1101000000$

$+347 + (-192) = 0101011011 + 1101000000$

$$\begin{array}{r} 0101011011 \\ +\ 1101000000 \\ \hline 0010011011 \end{array}$$

$+347 = 0347$

$+192 = 0192 \longrightarrow$ 10's complement 9808

$$\begin{array}{r} 0347 \\ +\ 9808 \\ \hline 10155 \end{array} \Rightarrow 155$$

Question 4

i. $F = a' + ab + ac' + abc'$

$\Rightarrow (a'+a)(a'+b) + (ac')(1+b) \Rightarrow a'+b+ac' \Rightarrow (a'+a)(a'+c')+b$

$\Rightarrow \boxed{a'+c'+b}$

ii. $F = x' + xyz + x(y \oplus z) + xy'z'$

$\Rightarrow x' + xyz + x[(y+z)(y'+z')] + xy'z'$

$\Rightarrow (x'+x)(x'+y)(x'+z) + [(xy+xz)(xy'+xz')] + xy'z'$

$\Rightarrow (x'+x)(x'+y)(x'+z) + (xyxy' + xxyz' + xxy'z + xxzz') + xy'z'$

$\Rightarrow x'x' + x'z + x'y + yz + x(y+z+y'+z') + xy'z'$

$\Rightarrow x'(1+z+y+z) + x + xy'z'$

$\Rightarrow x' + x + xy'z' \Rightarrow x' + x(1+y'z') \Rightarrow x'+x = \boxed{1}$

iii. $F = ab'c + a'b'c + abc$

$\Rightarrow c(ab' + a'b' + ab) \Rightarrow c[b'(a+a') + ab] \Rightarrow c(b'+ab)$

$\Rightarrow c[(b'+a)(b'+b)] \Rightarrow \boxed{c(b'+a)}$

iv. $F = (a+bc'+cd)(b'+ef)$

$\Rightarrow$ It cannot be simplified more

$\Rightarrow$ This form has the minimum number of literals

Question 4

v. $F(a,b,c) = \Pi(1,3,6,7)$

a	b	c	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

$\Rightarrow F = (x+y+z')(x+y'+z')(x'+y'+z)(x'+y'+z')$

$\Rightarrow (x+y+z')(x'+y'+z')(xx'+xy'+xz+y'x'+y'y'+y'z+z'x'+z'y'+z'z)$

$\Rightarrow (x+y+z')(x'+y'+z')[x(y'+z)+y'(x'+1+z)+z'(x'+y')]$

$\Rightarrow (x+y+z')(x'+y'+z')[x(y'+z)+y'+z'(x'+y')]$

$\Rightarrow (x+y+z')(x'+y'+z')[y'(x+1+z')+xz+z'x']$

$\Rightarrow (x+y+z')(x'+y'+z')(y'+xz'+x'z')$

$\Rightarrow (xx'+xy'+xz'+x'y+yy'+yz'+x'z'+y'z'+z'z')(y'+xz+x'z')$

$\Rightarrow (xy'+xz'+x'y+yz'+x'z'+y'z'+z'z')(y'+xz+x'z')$

$\Rightarrow [xy'+x'y+z'(x+y+x'+y'+1)](y'+xz+x'z')$

$\Rightarrow (xy'+x'y+z')(xz+x'z'+y')$

$\Rightarrow xy'z + xx'y'z + xy' + x'xyz + x'yz' + x'yy' + xzz' + x'z' + y'z'$

$\Rightarrow xy'z + xy' + x'yz' + x'z' + y'z'$

$\Rightarrow y'(xz+x+z') + x'yz' + x'z' \Rightarrow y'[x(z+1)+z'] + x'z'(y+1)$

$\Rightarrow y'(x+z') + x'z' \Rightarrow xy' + y'z' + x'z' \Rightarrow \boxed{xy' + x'z'}$

b.

i.

x	y	z	F	G	H
0	0	0	0	0	0
0	0	1	1	0	1
0	1	0	1	1	0
0	1	1	1	0	1
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	0

ii] $F = \pi(0,4,5) = (x+y+z)(x'+y+z)(x'+y+z')$

$\Rightarrow (xx' + xy + xz + yx' + yy + yz + x'z + yz + zz)(x'+y+z')$

$\Rightarrow (xy + xz + yx' + yy + yz + zx' + zy + zz)(x'+y+z')$

$\Rightarrow (y(x+x'+1+z+z) + xz + zx')(x'+y+z')$

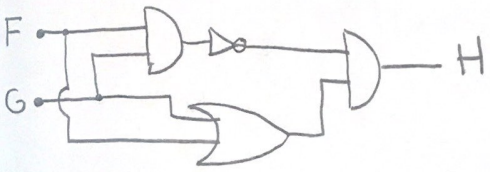
$\Rightarrow (y+z)(x'+y+z')$

$\Rightarrow x'y + y + yz' + x'z + yz + zz'$

$\Rightarrow y(x'+1+z'+z) + x'z$

$\Rightarrow y + x'z$ $\xrightarrow{\text{POS}}$ $\boxed{(y+x')(y+z)}$

iii]



Question 5

i.

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	1	0
11	1	0	0	1
10	1	0	0	1

ii. Prime implicants : $B'D$, $BC'D'$, $A'B'$, $C'D'$, $A'C'$

iii. Essential Prime Implicants : $A'D$, AD'

iv. Minimal SOP : $AD' + A'D + A'C' + A'B'$

iv. Minimal POS : De Morgan of SOP $= (A'+D)(A+D')(A+C)(A+B)$

Question 5.

vi.. Minimal SOP is: $AD' + A'D + A'C' + A'B'$

$\Rightarrow$ We know that $X \uparrow y = X' + y'$

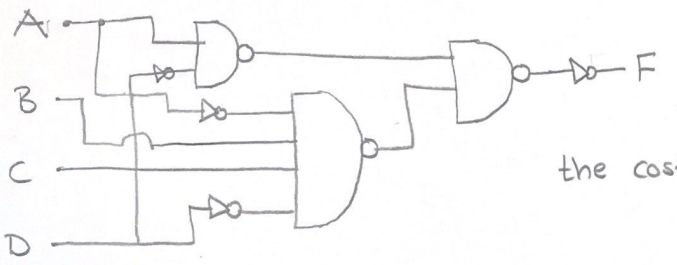
By using that:

$$(A'+D) \uparrow (A+D') \uparrow (A+C) \uparrow (A+B)$$

$$\Rightarrow (A \uparrow D') \uparrow (A' \uparrow D) \uparrow (A' \uparrow C') \uparrow (A' \uparrow B')$$

$$\Rightarrow (A \uparrow D') \uparrow A' \uparrow (D' \uparrow C \uparrow B)$$

$$\Rightarrow (A \uparrow D') \uparrow (A' \uparrow B \uparrow C \uparrow D')$$



, so there are 4 inverters,
2 2-input NAND gate, and
1 4-input NAND gate.

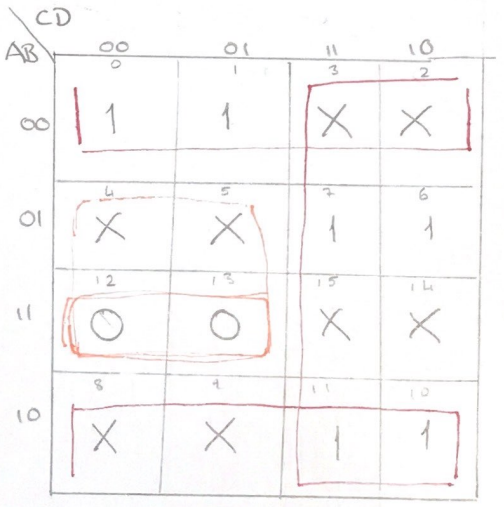
the cost is $4 \times 1 + 2 \times 2 + 1 \times 4 = \boxed{12}$

6.

i. Truth table:

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	X
0	0	1	1	X
0	1	0	0	X
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	X
1	0	0	1	X
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	X
1	1	1	1	X

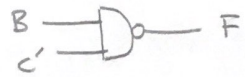
K-map $\Rightarrow$



Therefore,

SOP form = $\boxed{B' + C}$

Question 5

ii. $B' + C \Rightarrow (\bar{B}.C')' \Rightarrow$ 

iii. POS form = BC'

iv. $(\bar{A}BC') = \bar{A}$ NOR B' NOR C

$\Rightarrow$